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Finite Elements vs. Equivalent-Frame Models for URM walls' in-plane behaviour

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Abstract

As a consequence of the large number of masonry buildings and their important historical value, there is an ongoing effort to investigate their structural behaviour. This has led to a continuous development of new methodologies and techniques applied to masonry structures, including an increasing application of non-linear analyses.

The considerable computational effort required by detailed models, e.g. Finite Elements (FE) models, highlighted the need to develop simplified modelling procedures. In this context, the well-known Equivalent Frame Method (EFM) has become very successful. This simplified approach is based on the assumption that the behaviour of the masonry walls is modelled with a system of mono-dimensional beam elements (pier and spandrel panels). The great simplicity of implementation and interpretation of the results contributed to the widespread application of the method in both the scientific field and the professional practise.

Despite the huge effort devoted to improve the effectiveness of the EFM, its application can be considered really effective only for new or existing regular buildings. For these structures, in fact, the presence of regular configurations and good connection between walls and floors guarantee a "box-like" behaviour under horizontal (e.g. seismic) loads. The "box like" system is a necessary requisite to obtain the in-plane behaviour of the masonry walls well described by EFM.

This method is more difficult to apply to existing irregular buildings and, in particular, in the case of old structures in historical centres. In these cases, irregular geometry, the presence of deformable diaphragms and the interaction with other structures in aggregate configurations are critical issues for the reliability of Equivalent-Frame (EF) procedures.

A comparison between EF and FE models is deemed useful for investigating some of these uncertainties and for exploring the reliability of EF procedures in the case of existing masonry buildings. In [1], a parametric investigation was performed for several types of masonry regular buildings with different geometrical configurations and for walls affected by geometrical irregularities. The effects of the presence of irregularities on EFM accuracy are presented in this paper.

Keywords: Masonry, URM structures; Equivalent Frame method, Irregularity.

1. Introduction

The structural modelling of masonry constructions is a relatively recent topic, if compared with the ancient and consolidated use of masonry as building material. Over time, masonry buildings have been built only based on experience. In the majority of the cases, therefore, no structural design or numerical simulation have been employed to study the actual performance of such structures.

The recent progresses in the computational capacity of modern computers together with the high historical value connected to masonry buildings have largely contributed to develop numerical methods useful for studying masonry behaviour. A large number of structural modelling methods, characterized by different levels of detail and accuracy, have been consequently developed for masonry constructions over the last decades [2]. However, the difficulties associated with the material properties characterization and with the complexity of the structural configurations, make the numerical modelling of masonry structures a challenging open research topic.

With the aim of reaching a good compromise between the modelling accuracy and the required computational effort, a simplified modelling approach, known as Equivalent-Frame Method (EFM), has become widely used recently. In this approach, masonry walls are studied as plane frames, which allows a remarkable reduction of the number of degrees of freedom and hence of the computational effort compared to more detailed and accurate methods. The EFM is simple to implement and the results are easy to interpret, and this has contributed to its widespread use.

Despite the huge effort devoted to improving the effectiveness of the EFM, it is worth underlining that unreinforced masonry (URM) constructions often do not follow the structural and mechanical assumptions on which this method is based. The absence of rigid floor diaphragms and the consequent absence of a box-like behaviour, the heterogeneity of the materials and construction techniques, the presence of complex structural elements such as vaults or domes, and many other issues, pose serious doubts on the effectiveness of EFM to simulate the actual behaviour of existing structures.

Up to now, the accuracy of EFM in predicting the actual behaviour of masonry structures has been demonstrated, by means of experimental and numerical applications [3], only for *regular* buildings. These buildings, characterized by walls with a regular distribution of openings and well connected to each other, seem to follow well the assumptions of EFM. In case of *non-regular* buildings, namely masonry constructions characterized by high geometrical and mechanical irregularities, greater uncertainties affect the accuracy of the equivalent static schemes.

In this work, the presence of geometrical irregularities in the wall configuration is investigated to evaluate the effects on the EFM modelling accuracy. For some typical configurations, the precision of the EFM is evaluated comparing the results obtained with more detailed Finite Element Method (FEM). The study presented here is part of a wider research aimed at studying the accuracy of EFM for both regular and irregular URM walls for different geometrical irregularities [1]. The first results of this research were already presented for sample cases of regular walls [4], while here the attention is focused on the implications of geometrical irregularities on the wall in-plane behaviour and consequently on the accuracy of EFM in simulating it.

2. Equivalent-Frame Modelling of URM walls

In the wide scenario of modelling approaches currently available for masonry constructions, the Equivalent-Frame method (EFM) represents a simplified procedure used to study the seismic behaviour of masonry structures [5, 6]. The significant simplicity of the implementation of this method has led to consider EFM as a reliable alternative to more detailed modelling procedures such as the Finite Element Method (FEM).

EFM predicts the in-plane behaviour of masonry walls by identifying them with a plane frame. Each resisting masonry wall is discretized with mono-dimensional elements which represent the behaviour of finite portions of the structures. The masonry panels are usually characterized by axial, flexural and shear flexibility and connected by rigid nodes, hence creating an ideal plane frame configuration.

The identification between a masonry wall and a plane frame configuration, on which the Equivalent-Frame (EF) approach is based, leads to the distinction of three types of macro-elements. A preliminary geometrical identification of such elements can be made by extending the opening contours. In this way, the wall is divided in vertical and horizontal structural panels, namely piers and spandrels, working together to carry out and transfer gravitational and seismic actions. The connections between piers and spandrels is finally modelled with fully rigid nodes.

It is clear that the geometrical definition of the macro-elements affects the total strength and stiffness of the EF model used for the walls simulation. Particular attention is required in the definition of the piers' effective height. The piers represent the main resisting components of the EF models, and their stiffness can strongly influence the global stiffness and the seismic performance of the wall.

The problem of the piers' effective height was addressed for the first time by Dolce [7], who derived a geometrical criterion based on principles of equivalent stiffness. Starting from the FEM simulation of a large number of pier-spandrel modules, Dolce proposes a geometrical rule for the definition of piers' effective height that is aimed to reproduce the equivalent stiffness of the corresponding masonry panels, also taking into account the mutual interaction with the surrounding spandrels. The effective height of the piers is defined as function of a reference dimension (*h'* in Fig. 1a) corresponding to the distance between the midpoints of the line connecting the vertices of two consecutive openings. The lines defining the parameter *h'* have a limit inclination equal to 30° [7]. The parameter *h'* is then modified, as a function of the global geometry of the wall's level under consideration, to derive the piers' effective height.

As an alternative proposal for the piers' effective height, Augenti [8] proposed a criterion based on the assumption of "strong spandrel and weak pier" provided by FEMA 356 [9]. The criterion, revised and updated by Augenti and co-workers (references in [8]), is based on the direct observation of the damage patterns observed in ordinary masonry buildings subjected to past earthquakes. The piers' effective height is considered as a function of the direction of application of seismic actions, so each pier is modelled with the same height of the consecutive opening from the side of the earthquake loading. In contrast with Dolce's criterion, the rule provided by Augenti leads to the definition of two different models in case of asymmetric walls (Fig. 1b).

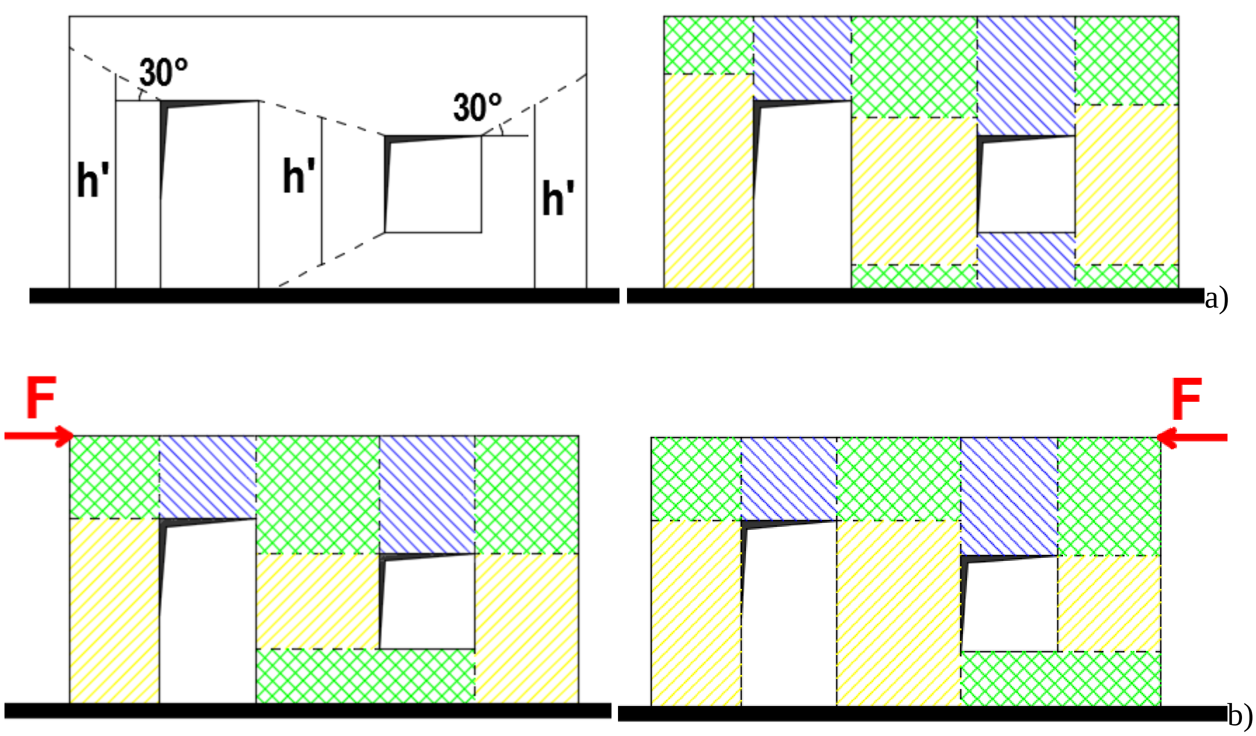

Fig. 1 – Criteria for the definition of piers' effective height proposed by (a) Dolce [3] and (b) Augenti [4].

The work presented here is aimed at investigating the uncertainties introduced in the EF modelling approach by the presence of irregularity in URM walls. Given the assumptions on which the two criteria for the definition of piers' effective height are based, strong differences can be produced in the EF models by their alternative application in case of irregular walls. This can result in a remarkable variation of the walls' stiffness and consequently of the walls' structural performance leading to a significant variability of the modelling results.

2.1 Non-linear mechanical characterization of EF models

From the first appearance of EFM, several non-linear constitutive laws and damage models have been formulated to guarantee a reliable simulation of masonry mechanical behaviour and of the main failure modes observed in masonry structures subjected to seismic actions. Despite a great effort, strong limits and uncertainties still affect the accuracy of these simplified modelling procedures. Their effectiveness is particularly dubious in the case of existing masonry constructions. Existing buildings are usually very far from matching the main assumptions that guarantee a reliable application of the simplified method, such as rigid diaphragms, box-like behaviour and homogeneous material.

Without entering in details in the description of the numerous mechanical models provided for implementing EFM, a recent formulation for the mechanical characterization of masonry panels is adopted here [10]. In this paper, each mono-dimensional element works as a system of three springs in series calibrated to simulate the non-linear response of masonry. The association of these springs allows the combination of a distributed plasticity model, based on the fiber discretization of the resisting sections, with a lumped non-linear shear spring.

The fiber approach, originally introduced to describe reinforced concrete structures [11], proposes the discretization of the panels' cross-section into longitudinal fibers. The mechanical characterization of each fiber is carried out by considering only an axial deformability and by calibrating *ad hoc* non-linear constitutive σ–ε laws. The combination of the fibers' non-linear behaviour allows the simulation of masonry flexural mechanisms. In the specific case of the models presented here, the uniaxial constitutive law provided by Kent and Park [12] is used for the mechanical characterization of the fibers. This law is essentially an elasto-plastic law with a softening branch that reproduces the main properties of masonry, such as the initial linear elastic branch, the non-linear phase before the peak stress and the softening branch of the post-peak phase.

The model is completed with a non-linear lumped spring calibrated to simulate shear mechanisms, i.e. diagonal cracking and shear sliding. A multi-linear V-γ law is used to implement the shear mechanisms. This law is calibrated by evaluating the global shear capacity of each masonry panel and the angular deformation corresponding to the peak shear strength.

2.2 Irregularity in masonry walls: Classification and Modelling issues

Existing URM buildings are often characterized by complex geometrical configurations, both in the case of monumental constructions and in ordinary residential buildings. The lack of regularity in the arrangement of openings, the absence of rigid floor diaphragms and the presence of complex structural details are recurrent features for such buildings. At the same time, these elements introduce many uncertainties in the definition of the static schemes, compromising the effectiveness of EFM for their simulation.

Among the numerous features affecting the modelling accuracy, we focused our attention on the presence of irregularities in the geometrical configuration of the wall. The concept of regularity to which the present work is referred regards the geometrical configuration of the single load-bearing walls. With reference to URM walls with openings, the distinction between regular and irregular configuration is essentially based on the openings arrangement. Therefore, a generic URM wall is regular if a perfect alignment in both vertical and horizontal directions characterizes its openings (Fig. 2a). Conversely, in case of at least one misalignment between the openings, the structural configuration of such wall has to be considered as irregular (Fig. 2b).

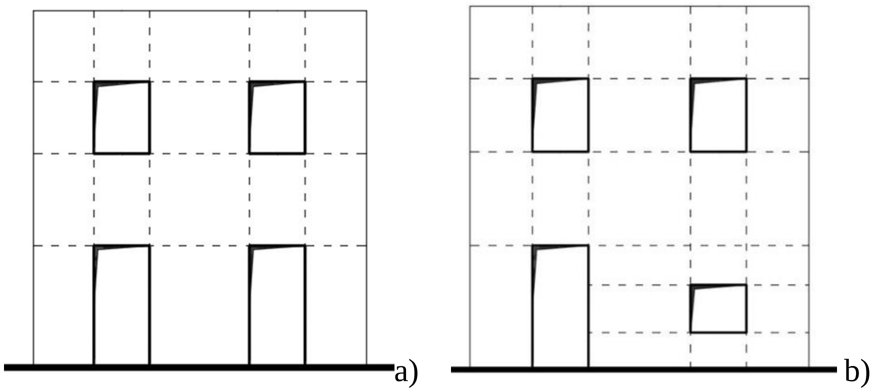

Fig. 2 – Geometrical configuration for regular (a) and irregular (b) masonry walls.

The definition of regularity and irregularity of masonry walls configurations provided here is directly derived from Parisi and Augenti [13]. In their work, Parisi and Augenti [13] provide the first systematic classification of irregularities trying to include the typologies recurrent in existing buildings. Four types of irregularities are identified, distinguishing between categories related to the misalignments of openings and to different number of openings between consecutive levels. Finally, non-dimensional indexes are defined for each category to provide a measure of the amount of irregularity affecting the wall.

Irregular walls can be considered critical elements in structural modelling of masonry buildings. The presence of geometrical irregularities can in fact induce stresses concentrations and increase the incidence of local damage mechanisms, resulting in the increase of the wall seismic vulnerability. The opening misalignment can in fact interfere with the effective height of masonry piers, resulting in a strong alteration of the effects produced by seismic actions [14]. These considerations suggest the great sensitivity of structural modelling for masonry walls to the presence of irregularities.

In [13], the studies carried out on this topic limited their attention to the capability of irregularities to reduce the seismic vulnerability of URM walls. A reliable applicability of the EF approach was accepted also in the case of irregular configurations. In Siano [1], the attention was moved for the first time on the effects produced by the irregularities on the EFM modelling accuracy.

3. Numerical Modelling of Irregular walls by Equivalent-Frame Method

A comparative study between simplified and rigorous modelling approaches, EFM and FEM respectively, was carried out on single URM walls affected by geometrical irregularities [1]. The percentage differences between the predictions provided by the two modelling approaches were computed and assumed as a measure to evaluate the accuracy of EFM for an increasing amount of irregularities. In such a way the capability of the irregularities to affect the in-plane behaviour of URM walls, and hence to influence the modelling results, were investigated for a wide range of geometrical irregularities. In this paper, two sample cases are discussed.

These sample walls were initially tested in linear field by comparing the results provided by EF and FE models calibrated according to the same static and mechanical assumptions. In this first phase, all models were submitted to the same gravitational loads and a system of horizontal actions proportional to the first mode of vibration. The equivalent static forces determined for each model were distributed over all the nodes of the wall in order to prevent stress concentration or other anomalies in the modelling assumptions. The linear models were simulated using the software Midas GEN © [15] both in the case of EF and FE approach.

In addition, the models were investigated using non-linear static analyses in order to take into account the strong non-linearity affecting masonry constructions. The use of non-linear models allowed to verify the implications of the presence of irregularities on the modelling accuracy in the non-linear field. An analogous comparative study between EF and FE non-linear models was therefore carried out by assuming as fixed the mechanical, boundary and loading conditions for the sample cases already tested in linear field.

The non-linear EF models were implemented by using a fiber discretization for the panels' cross-section for simulating masonry flexural behaviour. As already described in section 2.1, the non-linear mechanical model was completed by adding a lumped non-linear spring for the simulation of the shear mechanisms, i.e. diagonal cracking and sliding shear [10]. The non-linear EF models were also defined with the software Midas GEN © [15], which allowed the combination of the fiber discretization of beam elements with non-linear springs simulating shear constitutive laws.

For the simulation of the non-linear FE models, two-dimensional systems composed by plane-stress rectangular elements were defined using the software Midas FEA © [15]. The constitutive laws were defined in accordance with the ones implemented in the EF models. More specifically, an exponential law was adopted for the tension branch of the $\sigma-\varepsilon$ law, while a parabolic law was adopted for the compression branch.

The prescribed loads were applied in steps in both modelling procedures considered. A first step was defined to apply the gravitational loads corresponding to the panels' self-weight and the loads transferred by the slabs. A unit weight equal to γ = 18.0 kN/m^3 was used. The slab loads are applied as uniform distributed loads equal to 15.9 kN/m for both the levels. Once the deformed shape produced by the gravitational loads was determined, a second loading step was defined for the application of the horizontal actions. These actions were defined with a loading profile proportional to the seismic masses and were applied with a monotonic incremental time function. The non-linear static analyses were carried out in displacement control by means of an iterative solution procedure based on the "initial stiffness" method.

3.1 Models' Selection

The sample schemes selected for the comparative study discussed here were defined by following the irregularity classification provided by Parisi and Augenti [13] and represent typical URM walls. Starting from a two-story URM wall with regular configuration, two sample cases were derived by including an irregularity at the first level, namely a misalignment in horizontal direction between the openings of the first level. The two schemes were defined by increasing the misalignment between the openings, which allowed to evaluate the effect of increasing irregularities on the accuracy of EF models.

The reference geometry was defined with global dimensions 8.60 m x7.20 m and with a constant thickness equal to 0.40 m for both levels (Fig. 3). In order to simplify as much as possible the model calibration, the reference geometry was symmetric, so equal interstorey height for both the levels and equal height for all the openings were assumed. These geometrical assumptions simplified the definition of piers' effective height and guaranteed an easier introduction of the irregularities in the reference scheme.

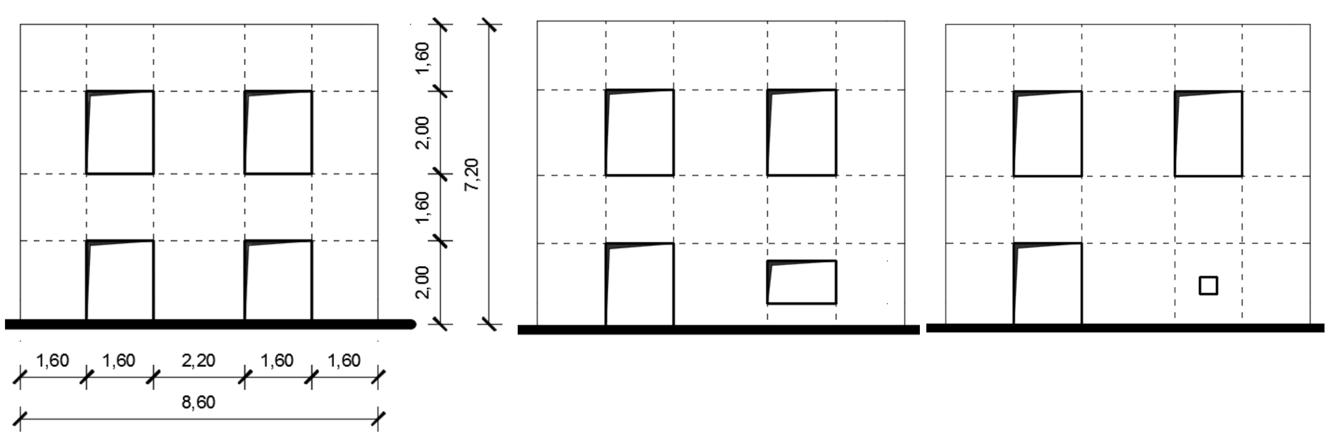

Fig. 3 – Geometry of the tested walls affected by increasing irregularities.

For each sample case, models were defined using both Dolce's criterion and the criterion of "strong spandrel and weak piers" as adapted to masonry walls by Augenti and co-workers [8]. This allowed studying the effect of the irregularities for both the approaches to define the effective height of piers. This is particularly relevant in case of presence of horizontal misalignment between the openings for the uncertainties that this kind of irregularity introduces in the definition of piers effective height. In both the criteria, in fact, the value of the piers effective height is a function of the openings height. For Augenti's criterion this has required to study two different models for the same geometrical configuration, depending on the direction of the seismic actions. On the contrary Dolce's method considers one model to predict the behaviour.

3.2 Comparative analysis of modelling performances EFM *vs* FEM

Starting from the comparison of the linear models, the distribution of internal actions and the absolute horizontal displacements were taken in consideration for measuring the modelling performance. Fig. 4 reports the percentage differences between the results provided by EF and FE models for the sample schemes selected. The results are expressed in terms of base shear distribution among the piers and horizontal displacements at the top of each pier. The results obtained from Dolce and Augenti's models are compared to the more refined FE model.

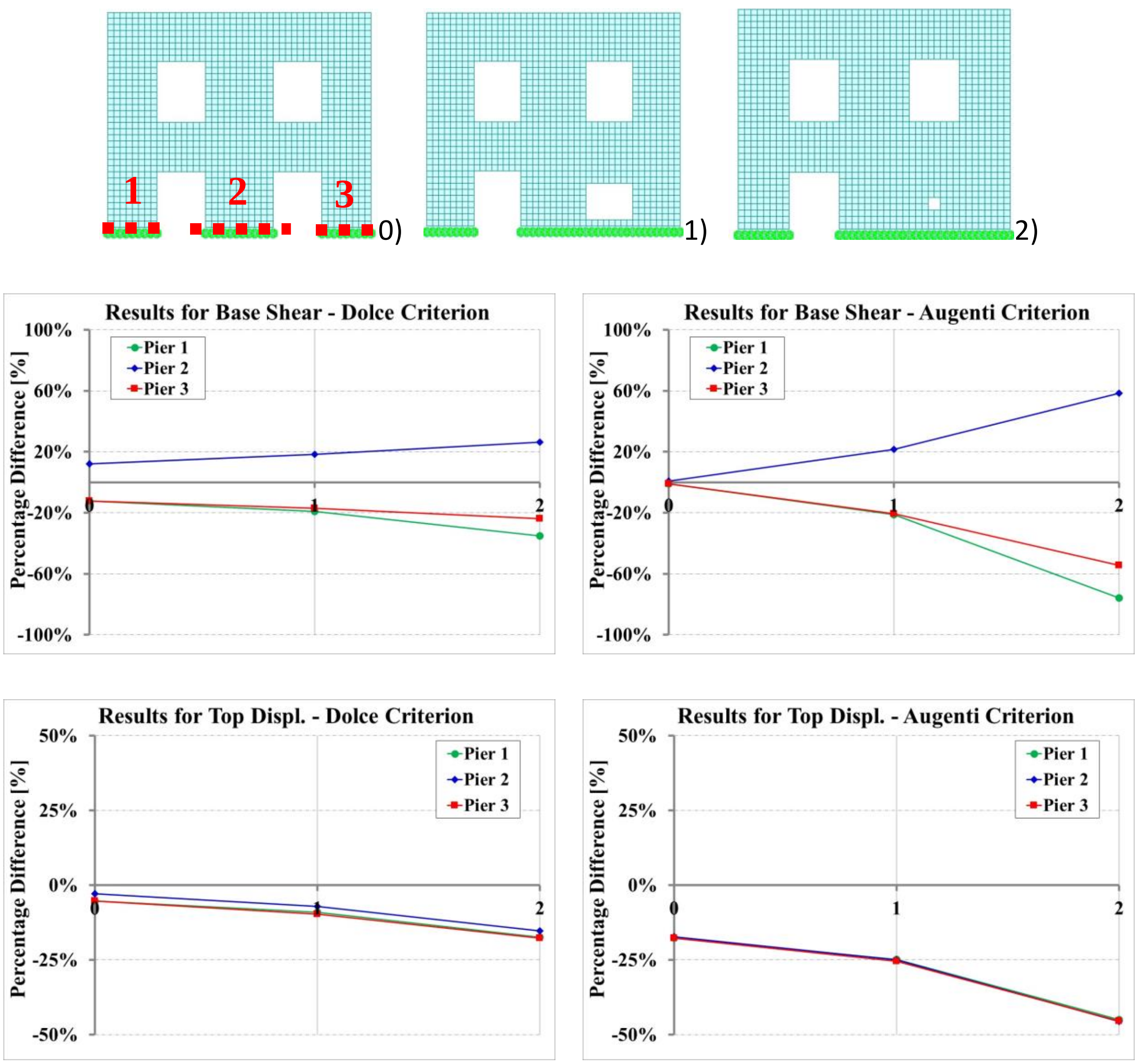

Fig. 4 – Percentage of difference between EF and FE linear predictions for base Shear and top Displacement for the models tested.

The comparison of the results obtained for the irregular and regular schemes highlights the effects produced by the irregularities on the modelling accuracy. For both irregular schemes, an increase of the percentage difference between the EFM and FEM results can be observed compared to the results provided for the regular scheme. This trend can be observed, with varying intensity, for both the predictions of shear distribution and horizontal displacements, confirming the strong limiting effects produced by the irregularity on the EFM modelling accuracy.

A different performance can be observed between the two criteria for the definition of the piers effective height. Both criteria result in an increase of the percentage difference between EFM and FEM results with the increase of the irregularity amount, but the results obtained with Augenti's criterion turn out larger differences in the prediction of both shear actions and displacements. These results reach high values of differences with respect to the FEM results for the sample case characterized by the highest amount of irregularity. In the linear field, Augenti's criterion seems consequently more sensitive to the presence of irregularity compared to Dolce's criterion.

Continuing with the non-linear models, Fig. 5 compares the results provided by the EF and the FE non-linear models only for the two sample schemes affected by irregularity. The results are here expressed as global capacity curves for each model. The two EF models, obtained by applying the two aforementioned criteria for piers effective height, were taken into account also for the comparative study in non-linear field.

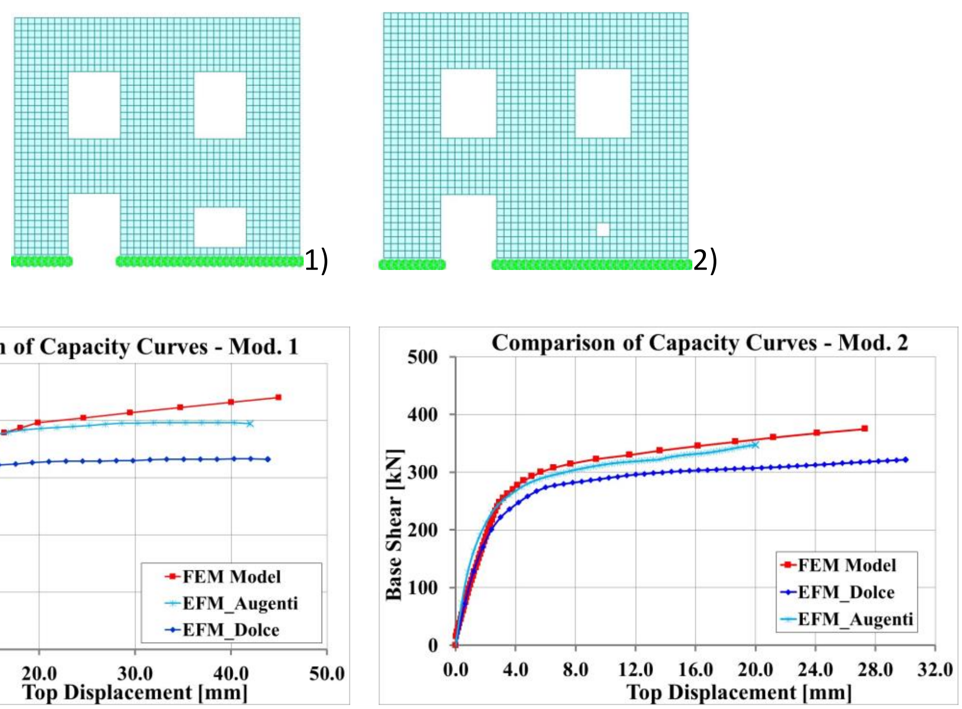

Fig. 5 – Comparison of the capacity curves obtained with FE and EF models for the two schemes affected by horizontal irregularity.

The comparison of the capacity curves obtained for the two irregular schemes confirmed the presence of appreciable differences between the results provided by the EF and the FE models. However, in both the sample cases the non-linear EF models demonstrated to be able to recover partially the strong ineffectiveness of the corresponding linear models. The differences between the capacity curves provided by FEM and those obtained with the two EF models were in fact lower than the difference between results obtained with the linear models in all the cases. The better performance of the non-linear models demonstrated the capability of the simplified approach to properly reproduce the internal force redistribution among the resisting elements induced by the incremental damaging conditions.

A slightly greater precision was observed in the results provided by the EF models using Augenti's criterion. The models defined according to Augenti's criterion provided lower differences in the prediction of the maximum shear capacity with respect to FEM results. On the contrary, Dolce's criterion reproduced better the shape of the capacity curve obtained with FEM, providing a better match also in the prediction of the initial elastic stiffness and the global deformability of the wall. These results confirm the observations already made for regular walls [1] about the modelling performance of the two criteria. Dolce's criterion predicts displacement with greater accuracy whereas Augenti's criterion is more precise at predicting stresses.

The non-linear models did not confirm the increasing trend of the differences between EFM and FEM results, observed for the linear models for the increasing irregularity. Further observations should be made about the effect of small openings on the modelling results. The sample case characterized by the highest amount of irregularity (Model 2 in Fig. 5) represents a critical example for the reduced dimensions of one of its openings. In case of very small openings, the definition of the structural elements surrounding them requires a great attention. Two strategies are possible in these cases, which are to model the piers surrounding the small opening as separated beam elements or to neglect the opening and introduce a unique resisting element with equivalent strength and stiffness.

The apparent contradiction between the results provided by the linear and non-linear models for the aforementioned case was overcome by looking at the distribution of shear actions among the single piers. As shown in Fig. 6, great anomalies were in fact observed in the capability of the EF non-linear models to predict the shear capacity of the single piers in the case of the model affected by the highest irregularity. High errors were obtained with the non-linear models in the prediction of both the shear capacity and the stiffness for the single pier, as already observed for the linear models (Fig. 4). These criticalities confirmed the importance of an in-depth evaluation of all the aspects of the numerical simulation to check the modelling accuracy of the EFM approach.

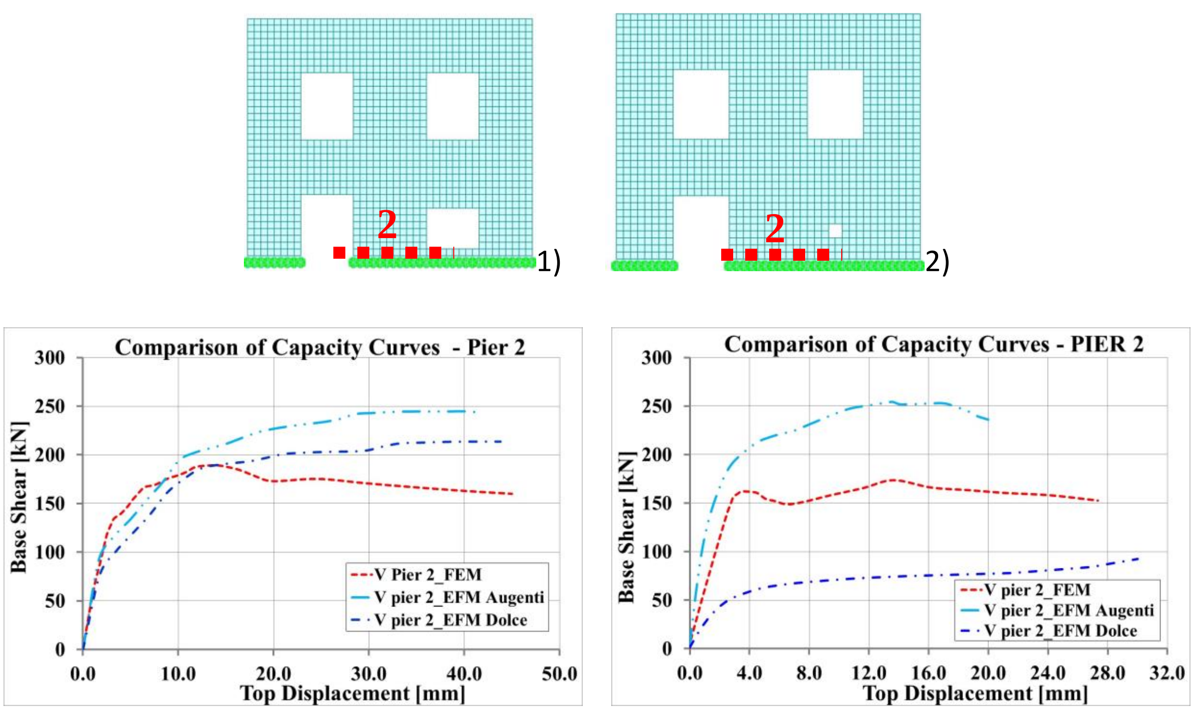

Fig. 6 – Comparison of the capacity curves obtained for a single pier of the two irregular schemes.

4. Conclusions

The study of the EFM modelling accuracy is presented with reference to irregular URM walls having as main purpose the evaluation of the irregularities effects on the performance of this simplified approach. Some geometrical configurations were defined by introducing a horizontal irregularity [13] on the same regular configuration and by increasing the amount of this irregularity.

The sample schemes obtained were simulated using both simplified and detailed modelling approaches, EFM and FEM respectively, and submitted to linear and non-linear static analyses. The results provided with the more rigorous FE models represented the reference to measure the EFM modelling accuracy. More specifically, the difference between the results provided by the two modelling approaches was assumed to measure the effectiveness of the simplified one. The results taken into consideration for the comparative study were both forces and displacements predictions.

The accuracy of the EF models decreased when the amount of irregularity increased. The non-linear models confirmed this trend in particular for the simulations of the shear capacity and the deformability of the single piers. The non-linear models showed a slightly better performance in simulating the global shear capacity of the irregular walls, although they showed a not negligible margin of error compared to FEM results.

The results presented here complete the study already presented about the EFM modelling accuracy for regular URM walls [4]. This work is part of wider research aimed at investigating the implications of the walls geometrical configurations on the EFM effectiveness in simulating their in-plane behaviour for both regular and irregular configurations [1].

5. Acknowledgements

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