16th World Conference on Earthquake Engineering, 16WCEE 2017
Santiago Chile, January 9th to 13th 2017
Paper N° 756
Registration Code: S-W1463221525

A DIRECT PERFORMANCE BASED SEISMIC DESIGN METHOD FOR IRREGULAR STRUCTURES

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Abstract

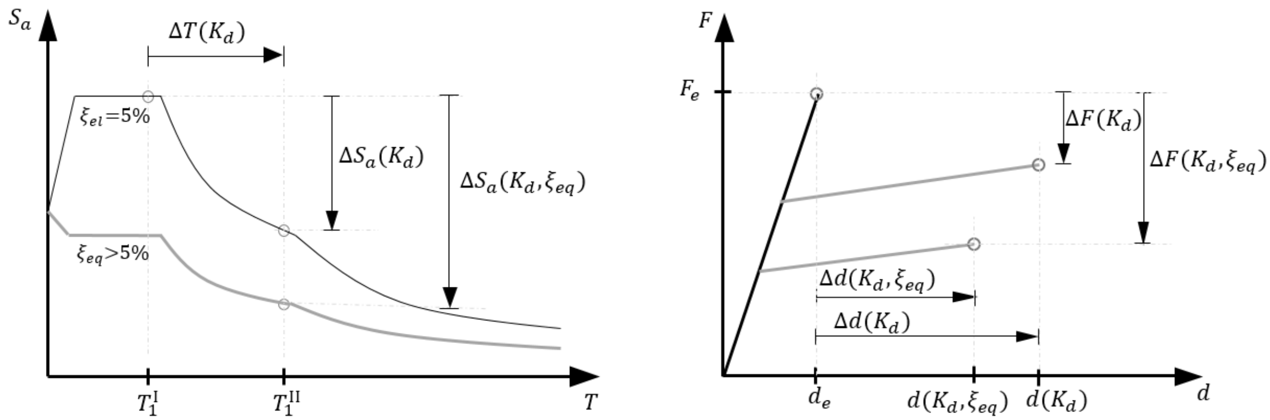
The problem of designing structures to achieve a specified performance limit state has gain interest in seismic design practice. Currently, some methodologies have been proposed in order to take into account for inelastic behavior of the structures in design phases; in that sense, a performance limit state can be provided. However, most of these approaches are iterative processes and depend, in some cases, on the experience of the designer. Otherwise, they are based on the concept of equivalent structures, only adequate for regular structures. In this paper, a direct performance based seismic design methodology for irregular structures with damage control is proposed. This method is based on the superposition of two elastic analyses. The strength of the method is the selection of local damage regions (hinges) intentionally chosen by the designer. This distribution of hinges defines the zones where damage is allowed and the desired failure mechanism in the design. A combination of dynamic modal-spectral analyses using a damage parameter (α) to control the damage intensity in the plastic hinges and the non-structural damage through allowable displacement or drifts. The methodology will be presented for planar frame structures presenting vertical irregularities. Furthermore, the effects of higher modes of vibrations is highlighted.

Keywords: Performance-Based Seismic Design, Damage Control, Non-linear behavior, Irregular Structures, Higher Vibration Modes.

1. Introduction

In the last years, strong earthquakes (Canterbury 2011, Maule 2010, Ecuador 2016 among other) have showed that extensive level of damage that we can expect even in structures designed with modern codes; confirming the need to improve seismic design methods to control residual deformation or damage. Some performance-based seismic design (PBSD) methods have been proposed in order to achieve adequate inelastic behavior through practical design steps. However, the majority of these methods require iterative schemes involving non-linear analyses; hence, their use has been limited. Currently, one of the most extended methods is the Direct Displacement Based Seismic Design Method (DDBSD) proposed by Priestley & Kowalsky [1], which does not require iterations or non-linear analysis. Nevertheless, it is intended for regular structures, which can be reasonably approximated by a single degree of freedom (SDOF) system. In general, current direct methods still need improvement in order to catch the effects of some phenomena that occur in the process of damage, as the influence of higher modes of vibration, the change of load pattern due to damage progresses, among others. In this paper, a design methodology is proposed in order to handle most of these aspects in a simple manner.

The proposed approach is an extension from a previous nonlinear design method (NLSD) proposed by Bairán, et al. [2] developed for static loads, it is based on two elastic analyses that are superposed in order to estimate the nonlinear behavior in design phase without any iteration. Two elastic structural models are analyzed, one for the reference undamaged structure and another damaged (named auxiliary). A distribution of perfect hinges is selected by the designer in the auxiliary structure. This distribution of hinges defines the damageable zones and desired failure mechanism.



a. Structure modal – spectral response variation on a design spectrum.

b. Structure force – displacement response variation for a cyclic strong motion.

Fig. 1 – Response variation depending on seismic damage process.

The main goal in extending this approach to dynamic loads is to predict the change of load that is depending on the degrading of the structure stiffness. Moreover, if there is inelastic cyclic loading, energy dissipation take place during the process of yielding; therefore, seismic load changes as well.

As mentioned above, when a strong motion is acting on a structure there is an evolution of modal properties and internal forces when damage starts to occur. This is illustrated in Fig. 1 showing the period shift (ΔT) of the first vibration mode of a "un-damage structure" $\left(T_1^{\mathrm{I}}\right)$ to its final position $\left(T_1^{\mathrm{II}}\right)$. The latter notation represents the "damage structure" with its corresponding damage stiffness (K_d). The spectral acceleration load shift is represented by(ΔS_a). However, another reduction for the spectral acceleration in the design spectrum correspond to the final damping of the system represented as an equivalent ratio $\left(\xi_{eq}\right)$. Fig. 1.b illustrates a force – displacement evolution. It presents the reduction of base shear (ΔF) and the displacement shift (Δd) provoked by the energy dissipation and the reduction of stiffness.

These two models are superposed using a damage parameter (α) that controls the damage intensity in the plastic hinges (as plastic rotation) and the non-structural damage through allowable displacement or drifts. In this paper, the methodology will be presented for planar frame structures showing vertical irregularities. The validation of the methodology will be shown by a design of an irregular concrete frame structure, and it further assessment through non-linear time-history. Discussions about the easiness of use and design robustness will also be presented.

2. Current PBSD methods

The aim of PBSD is to consider the inelastic behavior of material, to control the local and global damage, but also, to make the process simple in practice. However, there exist some design methods that meet, in some way, most of the aspects mentioned before. One that is considered very accurate is the "deformation-based seismic design method for irregular structures" proposed by Kappos & Stefanidou [3]. This method involves the use of non-linear dynamic analyses of the structure. However it has as drawbacks that, to apply it, the designer must have enough knowledge and experience in the execution and interpretation of these non-linear analyses, this increases the probability of human error.

In the work of Liao [4], it is proposed the method of "performance – based plastic design", in which it must pre-select a target drift and yield mechanism as performance criteria. The design base shear for selected hazard level is determined by equating the work needed to push the structure monotonically up to the target drift to the corresponding energy demand of an equivalent SDOF oscillator. However, for irregular structures, the concept of equivalent structure is not appropriate. Moreover, it can be questionable only to account for the fundamental vibration mode in inelastic behavior.

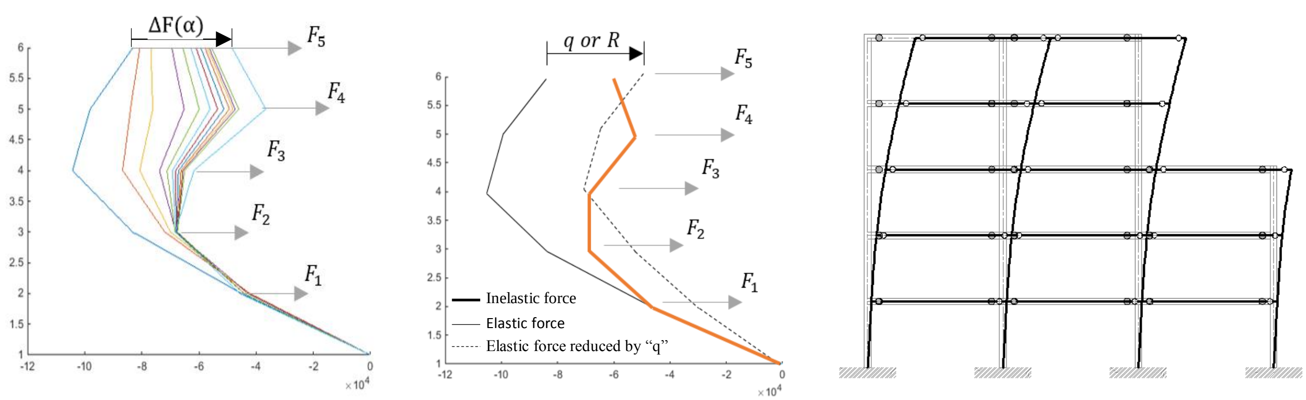
A method proposed by Ayala, et al. [5] is a the “displacement based seismic design method with damage control for RC building design”. This method is able to analyze irregular structures, as the first method mentioned. It allows to manage the local damage at will, it is formulated from basic approximations to concepts of structural dynamics used in design practice. Nevertheless, the method is sensible to the importance of proposing a realistic damage distribution and more rigorous relationship between stiffness of structural elements in order to have a good approximation. On other hand, a comparison of spectral displacement and design displacement should be made in the process as a condition. That means that an iteration is involved in the process.

An attractive method due to its apparent simplicity is the “direct displacement based seismic design method”, proposed by Priestley & Kowalsky [1]. However, its application has some limitations in its formulation and process. The method is based on the characterizing the behavior of non-linear MDOF structures by the means of an equivalent linear structure as a SDOF system; which is not always appropriated, especially for irregular structures layout. Moreover, the effect of higher vibration modes in inelastic behavior can provoke a poor approximation of the actual response.

At present, direct methods only accounts for the first mode of vibration of the elastic structure, thus limiting its applicability in more general situations. Methods for irregular structures are iterative, need to converge and involve several time-history analyses. However, it is known that when a damage occurs, e.g. cracking or local yielding, a variation of the stiffness is produced, hence, modal properties are affecting the seismic demand and distribution of inertial forces as it is shown in Fig. 2. Inelastic modes shape implies modification of the natural periods and the mass participation factor, making the structure more sensible to higher mode contributions, as the structure becomes more flexible, i.e. damage increases.

In the methodology here proposed, means to account for the above mentioned phenomena are presented. The proposal provides an approach to evaluate them during the design process and control the collapse mechanism according to the chosen designer strategies. In the following section, the method is described.

This methodology also includes the variation of the distribution of forces along the height of the structure, that changes due to the influence of higher vibration modes during the process, as well as the proposed level damage and distribution of damage.



a. Evolution of seismic load pattern with α

b. Comparison of inelastic seismic force for DLA and normal reduction q factor

c. Building with all beams hinged (undeformed & deformed shape)

Fig. 2 – Evolution of seismic load pattern in a design process

3. Direct performance based seismic design for irregular structures

The methodology proposed is named "Double Linear Analysis" (DLA). As it is based on the superposition of two linear analyses, one a reference elastic structure and the other an auxiliary structure (damaged structure). The main difference with respect to traditional seismic design with respect to static loading is that the superposition is based on the final results of internal forces and deformation of two normal linear modal spectral dynamic analysis (MSDA), as shown in Fig. 3.

The first structure is subjected to both elastic gravitational and seismic forces, from it, the elastic internal forces and deformation can be obtained. The auxiliary structure includes a series of internal perfect hinges, which are distributed according to the designer decision based on the first analysis. Those perfect hinges are the points where structural damage will be allowed in the design.

The general steps of the DLA are quite direct and relatively simple, without iterations loops as shown below:

1. Perform a linear MSDA on the reference elastic structure and obtain the elastic responses.

2. Decide a strategy of plastic hinge location based on the need of reduction of internal forces and where internal forces change is to be located. This defines the model of the auxiliary structure.

3. Perform a linear MSDA on the auxiliary structure that is topologically similar to the references structure, but includes perfect hinges in the region defined.

4. Plot superposition curves as a function of parameter (α) and select the factor to satisfy local damage, displacement and drift.

5. Design reinforcement in the plastic range for M_d and θ_p and in the elastic region according to capacity design.

In Fig. 3 steps are sketched. Fig. 3 b.1, b.2 and b.3 represent the first step, it consists in to perform a conventional MSDA. In this step, the elastic moments (M_e) and elastic displacements (δ_e) are obtained from the elastic seismic force (Fs_e) and gravitational load (q_g). After deciding a configuration of plastic hinges (step 2), the MSDA in the auxiliary structure (step 3) is carry out, this step is shown in Fig. 3.c.1, 2, and 3. The seismic demand in step 3 is different, and generally lower than the one on the elastic reference structure due to its higher flexibility and energy dissipation.

The response of the auxiliary structure represents the maximum possible deformation for the given distribution of perfect hinges in terms of displacement (δ_u) and ductility demand (θ_p) at each hinge. With the combination result as it is shown in Fig. 3 d.1, d.2 and d.3, structural design based on a given strength and ductility demand can be performed. In elements without hinges, a redistribution of internal forces will be observed after combination of responses Fig. 3.c and d.

In step 2, the designer should take a decision about the quantity and the location of damaging points. In Fig. 4.a, it is represented, from de points (p) to (a), the response of a linear analysis corresponding to an elastic stiffness matrix $[K_e]$. The decision of the distribution of hinges could be based on convenient levels of internal forces redistribution, reduction of base shear or a level of damage to be controlled as a function of plastic rotation in hinges, or a combination of both. In the same Fig. 4.a, from point (p) to (b), the linear response behavior of this flexible structure with a lower stiffness matrix $[K_u]$ is evident and observed. The Fig. 4.b shows a moment – rotation diagram evolution in a hinge. In this figure, point (a) is the reduced bending moment as a result of the yielding of plastic hinges and the redistribution of forces produced. However, after considering the hysteretic energy dissipation, the strength demand is represented by point (b). The maximum reduction of bending moment would be reached if the maximum possible rotation in a hinge is achieved. If this is the case, the strength demand in that hinge should be enough to resist gravity loads or static bending moment M_{st}, as shown in point (c). In that case, internal forces will be redistributed in other elements that should resist all seismic forces.

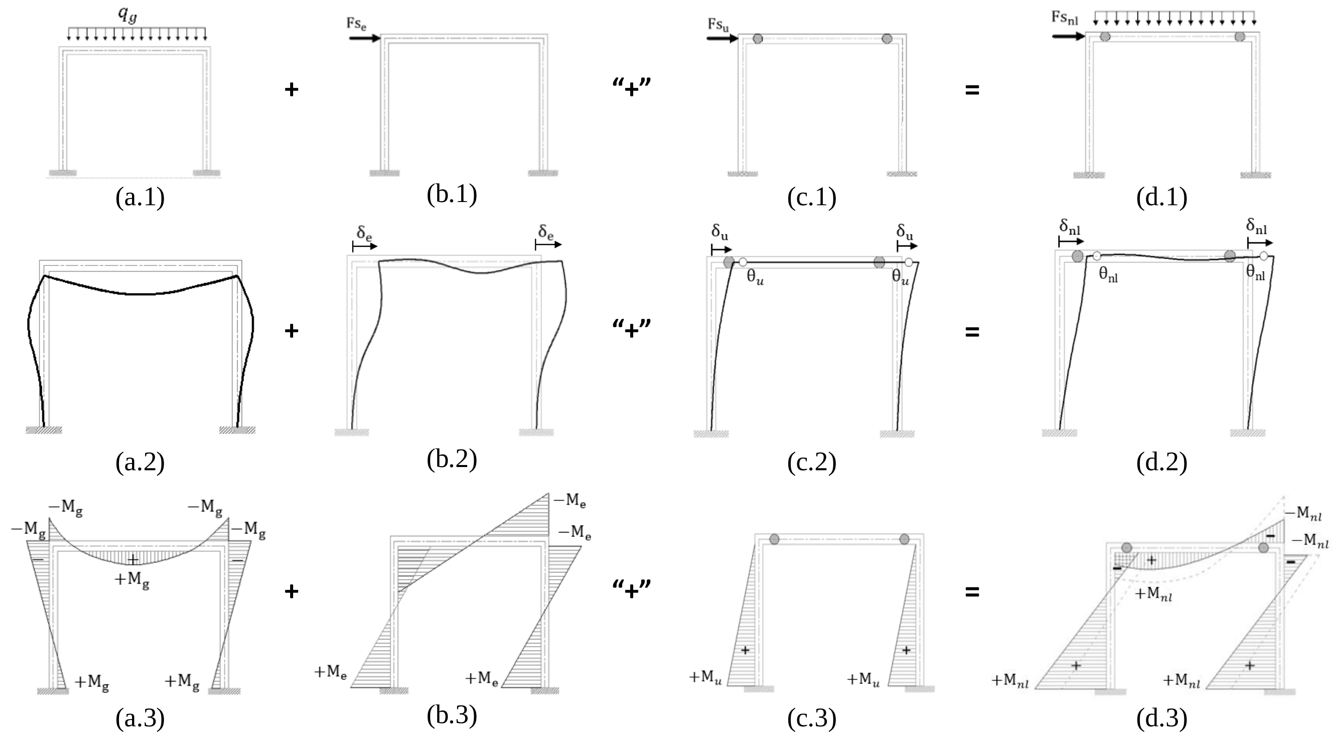
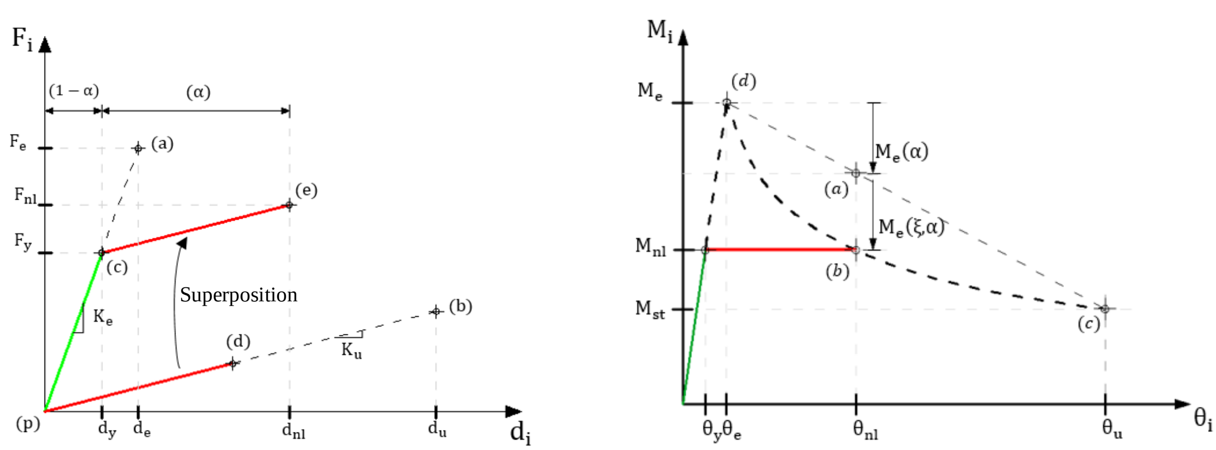


Fig. 3 – Final state estimation of design steps for the DLA. The symbol "+" stands for superposition combination



a. Superposition of elastic and auxiliary structure in a force – displacement diagram in terms of damage parameter alpha (α)

b. Moment – Rotation curve evolution in hinge (i)

Fig. 4 – Evolution of forces and internal force depending on the damage parameter alpha (α)

3.1 Combination of structures and damage

From the previous steps the superposition of deformation and internal forces of both structure may be performed so as to obtain a combined structure. For this goal a combination factor (α) is proposed in order to combine forces and displacements shown in Eq. (1) and (2), respectively. As will be shown latter, this factor later controls the damage taken place in the structure; hence, providing a way to select its value objectively. Therefore α will be referred as a plastic parameter; α will range from 0 to 1. In a fully structure with plasticized damaged regions, α will be equal to 1. A fully undamaged structure will be obtained with $\alpha = 0$. The combination of internal forces and deformation is thus,

$$F_{nl}^{i} = F_{e}^{i} \cdot (1-\alpha) \cdot \eta + F_{u}^{i} \cdot \alpha \cdot \eta \tag{1}$$

$$d_{nl}^{i} = d_{e}^{i} \cdot (1-\alpha) \cdot \eta + d_{u}^{i} \cdot \alpha \cdot \eta \tag{2}$$

In this notation, F_{nl}^{i} and d_{nl}^{i} are the final results of the combined forces and displacements at the node i. Hence, F_{e}^{i} and d_{e}^{i} are the forces and displacements in the elastic structure, respectively (step 1), and F_{u}^{i} and d_{u}^{i}, the forces and displacement in the auxiliary structure (step 3).

In Fig. 4.a, the force and displacement variation from point (p) to (d) represents the plastic behavior of the combined structure and if it is overlap to the elastic structure will be the curve from point (c) to (e). The elastic part of the structure's behavior corresponds to the force and displacement variation from point (p) to (c). Thereby the combination and superposition of both structures represent the final or combined structure represented by an elastic and perfectly plastic behavior branch. The notation η is a reduction factor accounting for hysteretic dissipation. Its computation will be discussed in section 3.2.

3.2 Approach to local and global damage control

In the last step of this methodology, the final design of structure must be carried out by designing the steel reinforcement to satisfy the required strength and ductility demand, for the selected cross section size. To this end, the moment resistance demand is obtained as in Eq. (3) and the ductility demand is obtained from Eq. (4). The yield rotation $\theta_{y}^{(i)}$ is the first term of the Eq. (4)

$$M_{nl}^{i} = M_{e}^{i} \cdot (1-\alpha) \cdot \eta + M_{u}^{i} \cdot \alpha \cdot \eta \tag{3}$$

$$\theta_{nl}^{i} = \theta_{e}^{i} \cdot (1-\alpha) \cdot \eta + \theta_{u}^{i} \cdot \alpha \cdot \eta \tag{4}$$

The energy dissipation correction factor (η) in Eq. (1) to (4), is defined in terms of the ductility demand selected in each local damaging point (hinge). Hence, it also dependents on local type of hysteresis loop of the component, the kinematic of the irregular system in the dynamic action.

The ductility demand can be computed as in Eq. (5), as a function of the α coefficient.

$$\mu^{(i)} = \frac{\theta_{nl}^{(i)}}{\theta_{y}^{(i)}} = \left[1 + \mu_{max}^{(i)} \cdot \frac{\alpha}{(1-\alpha)}\right] \tag{5}$$

According to Dwairi, et al. [6] the hysteretic component of response in the form:

$$\xi_{hyst}^{(i)} = \mathrm{C} \cdot \left(\frac{\mu^{(i)} - 1}{\mu^{(i)} \cdot \pi}\right) \tag{6}$$

Where the coefficient C depends on the shape of the hysteretic loop, the type of record and shift of period of the structure. Different values of C can be found in the work of Dwairi, et al. [6]. The total damping of the system is the sum of elastic and hysteretic damping:

$$\xi_{sys} = \xi_{el} + \xi_{eq} \tag{7}$$

As it is commonly assumed, the elastic damping value is 5% acceptable in most of cases. The global damping may be found by the weighted average based on the energy dissipated by the different damaged regions, as:

$$\xi_{eq} = \frac{\sum_{i=1}^{n} \mathrm{M}_{nl}^{(i)} \cdot \theta_{nl}^{(i)} \cdot \xi_{hyst}^{(i)}}{\sum_{i=1}^{n} \mathrm{M}_{nl}^{(i)} \cdot \theta_{nl}^{(i)}} \tag{8}$$

The equivalent damping reduce the seismic demand through the correction factor η. Different relationship area available to relate η and ξ_{eq}; however, in this paper, the Eq. (9) in Eurocode-8 [7] will be used.

$$\eta = \sqrt{\frac{0.10}{0.05 + \xi_{sys}}} \tag{9}$$

Finally, having the internal forces at each element, it is necessary a structural design for the completed strength and ductility demands in yielding and non-yielding ($\theta_p = 0$) region. So that the structure could be able to resist with the same cross section by proposing steel reinforcement for the redistribution moment as it is shown in Fig. 3 (c.3). A direct design process for flexure for given ductility and strength demand can be found in [8].

4. Case study

4.1 Description

To illustrate an application and advantages of the DLA proposed in this paper, a 7 story building with vertical irregularities is designed. The structural layout and height of the example are chosen in order to evaluate the importance of higher vibration modes and the design method accuracy for irregular reinforced concrete structures.

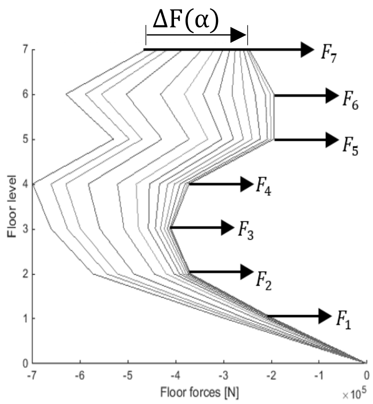
The preliminary design of the frame is done according to EC-02. The mechanical properties of materials used are $f_c' = 30$ MPa, Modulus of elasticity $E_c = 30$ GPa, and for steel reinforcement properties, the yield stress $f_y = 500$ MPa and modulus of elasticity $E_s = 200$ GPa. The table below shows the cross section of the building. A gravity load of 61.25 KN/m is applied and the nodal mass is taken as the self-weight and 80% of gravity load. the lengths of the spans are all 7 meters, and the heights of the floors are 3.5 meters.

Table 1 - Geometry of elements sections and modal properties of elastic (E) and auxiliary structure (A)

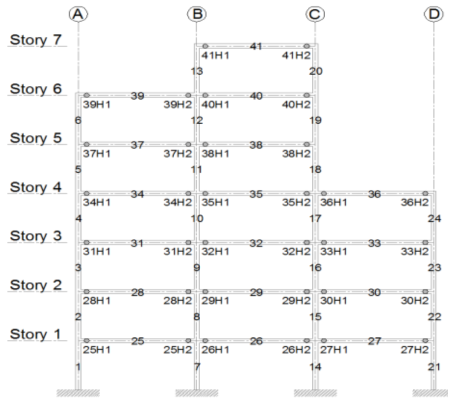
Element		Edge	Story	Width [mm]	Height [mm]	Mode	Periods [sec.]		Mass part. factor [%]	
							(E)	(A)	(E)	(A)
Column	C1	A-B-C-D	5-7	600	600	1	1.11	3.15	71.94	59.73
Column	C2	A-B-C	1-4	800	800	2	0.39	0.72	12.83	20.03
Column	C3	D	1-4	500	500	3	0.19	0.27	6.38	9.72
Beam	V1	-	1-7	350	500	4	0.13	0.15	3.37	4.22

4.2 Design

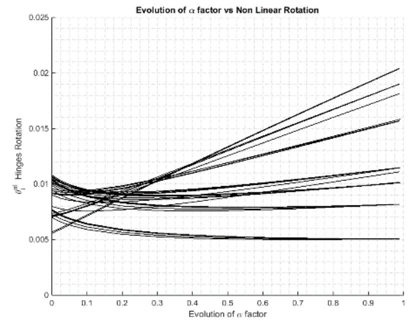
The structure layout designed with the DLA is shown in Fig. 5.b. The mechanism proposed is for all beams damaged for an approximation of the concept of “strong columns - weak beam”. Columns are designed to remain elastic. In Fig. 5.c to Fig. 5.f there is some of the parameter to take in to account for the selection of the damage factor (α). In this case alpha is taken as ($\alpha = 0.5$). The selection of this value in this case was taken with the evolution of displacement, where, the final displacement should be similar to the elastic displacement. However, other consideration could be as the maximum rotation of hinges as a parameter of sectional damage.



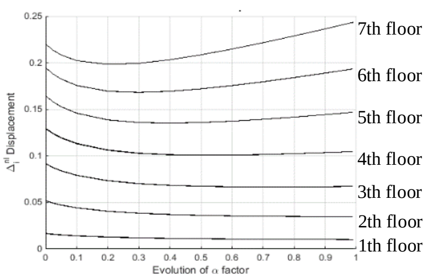
a. Maximum absolute floor – forces evolution with the mechanism proposed



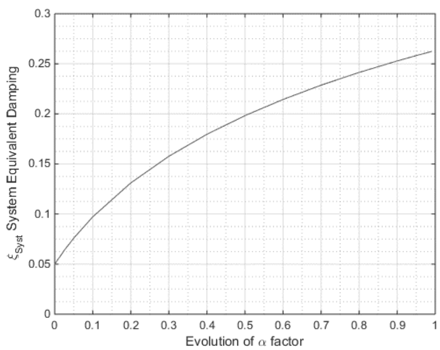
b. Mechanism proposed, all beams hinged



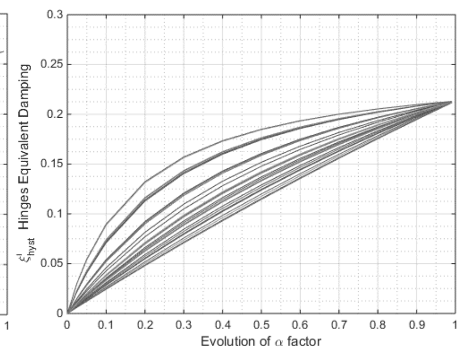
c. All hinges plastic rotation evolution



d. Maximum absolute Floor – displacement evolution



e. Structure Equivalent damping evolution



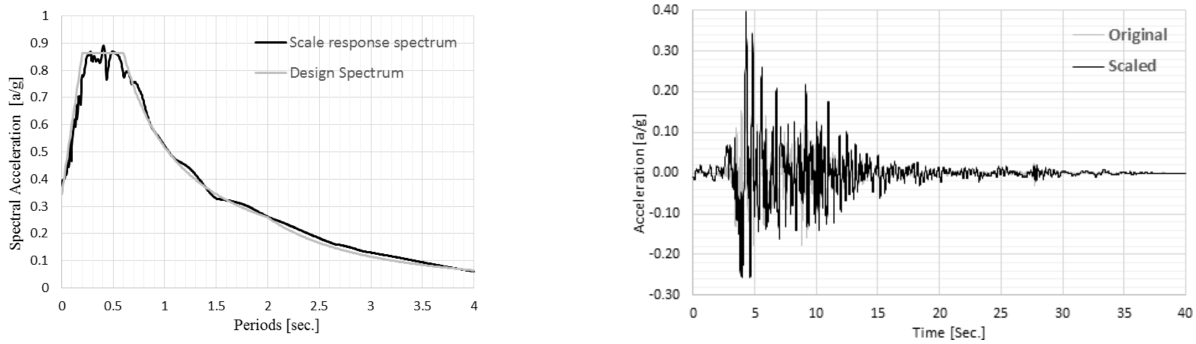
f. All hinges hysteretic damping evolution

Fig. 5 – Analysis for PBSD with the DLA.
Combination and superposition for a wide range of alpha factor (step 3)

4.3 Verification using non-linear time history analysis

For the purpose of validating the accuracy of the method, the seismic demand considered in the case of study, was a design spectrum from the EC8, with a PGA = 0.30g, a spectrum type 1 and a soil type C. The seismic input action is a record from L'Aquila earthquake scaled (Fig. 6.b) to fit to the design spectrum (Fig. 6.a). To validate the result obtained with the DLA; displacements and rotations of the frame were computed using non-linear step by step analysis under the same seismic demand for which it was designed. The non-linear time history analysis is carry out with the software SAP2000 considering a hysteretic rule as elastic-perfectly plastic behavior. P-delta effect is not considered in neither of the two analyses.

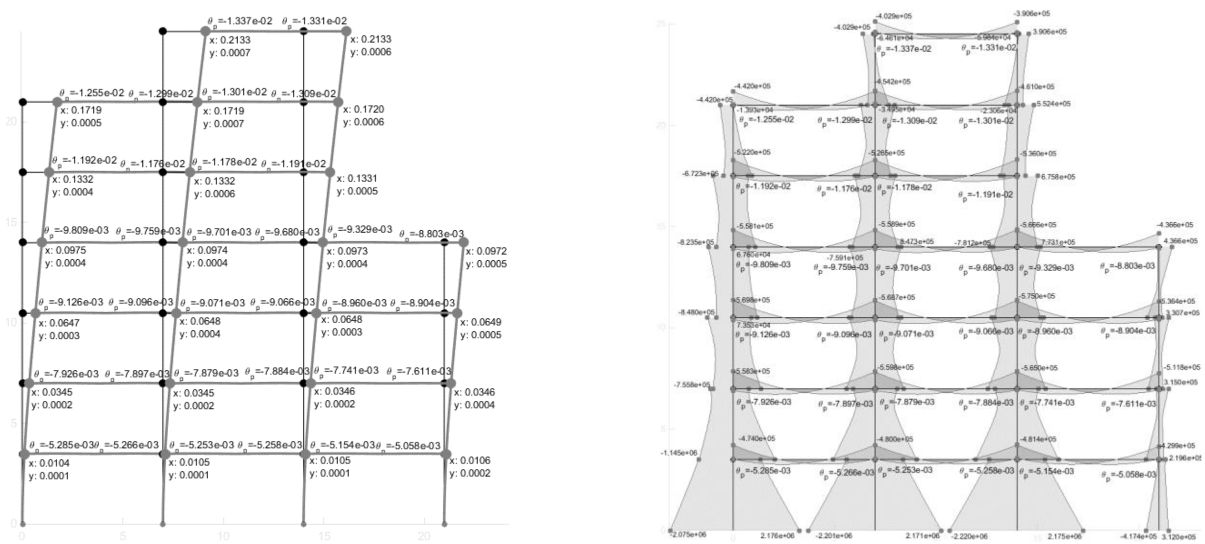
Fig. 7.a shows the structure deformation, as displacement and rotation demand for which the damaged sections should be designed. Maximum bending moment for the selected damage factor, are graphic Fig. 7.b. For this example, the accuracy of the method is shown in Fig. 8, Table 2, Table 3 and Table 4, there, displacement, rotation and bending moment computed by the DLA are compared with the result of non-linear time history analysis. All columns remain elastic as it is shown in Table 4, its bending moment are lower than the computed by DLA (Fig. 7.b. and Table 4). The based shear obtained in the DLA was 1234.9 KN, a difference of 14% lower respect to time-history analysis. A preliminary reason, is that the spectral acceleration (Sa), i.e., of the 1st mode of vibration, for the damaged structure in the design spectrum, has a 11% of difference in comparison with the (Sa) in the scale response spectrum, higher modes also are not 100% accurate with design spectrum. Finally, in this example the concept of weak-beams strong-columns is achieved accounting a performances based design for a given hazard.



a. Response spectrum compared with design spectrum EC8

b. Original accelerogram compared with the scaled signal

Fig. 6 – Seismic demand considered to design and seismic record to validate



a. Displacement deformed shape

b. Bending moment law

Fig. 7 – Deformation and internal forces result for the DLA with a damage factor ($\alpha = 0.5$)

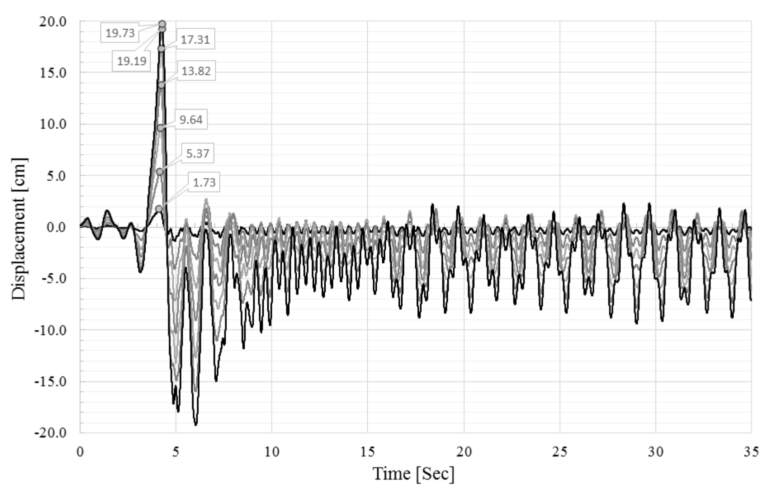


Fig. 8 – Non – linear time – history displacement of all the storey [cm]

Table 2 – Ductility demand (θ_d) [rad] from non-linear time–history analysis (NLTH) compared to the DLA design

Hinge ID	26H1	26H2	29H1	29H2	32H1	32H2	35H1	35H2
NLTH	0.00541	0.00718	0.00815	0.00873	0.00799	0.00985	0.00788	0.00977
DLA	0.00525	0.00526	0.00788	0.00788	0.00907	0.00907	0.00970	0.00968
Hinge ID	38H1	38H1	40H1	40H2	41H1	41H2		
NLTH	0.00856	0.00803	0.01039	0.00832	0.01195	0.00951		
DLA	0.01178	0.01191	0.01301	0.01309	0.01337	0.01331		

Table 3 – Maximum displacement [cm] for non-linear time – history analysis (NLTH) compared with the prediction using the DLA design

Story	7	6	5	4	3	2	1
NLTH	19.73	19.19	17.31	13.84	9.63	5.36	1.73
DLA	21.33	17.20	13.31	9.72	6.49	3.46	1.06

Table 4 – Maximum bending moment [KN/m] in plastic hinge for non-linear time – history analysis (NLTH) compared with the prediction using the DLA design

Hinge ID	26H1	26H2	29H1	29H2	32H1	32H2	35H1	35H2	38H1	38H1
NLTH	473.83	486.45	542.28	555.60	542.35	560.81	525.93	552.14	501.91	518.18
DLA	471.44	481.37	549.43	565.04	557.09	574.97	539.01	566.56	513.54	536.00
Hinge ID	40H1	40H2	41H1	41H2	1H1	7H1	14H1	21H1		
NLTH	468.42	455.69	412.50	397.14	1846.9	1971.6	1996.9	375.7		
DLA	470.80	460.97	402.85	390.64	2175.8	2200.7	2220.3	417.37		

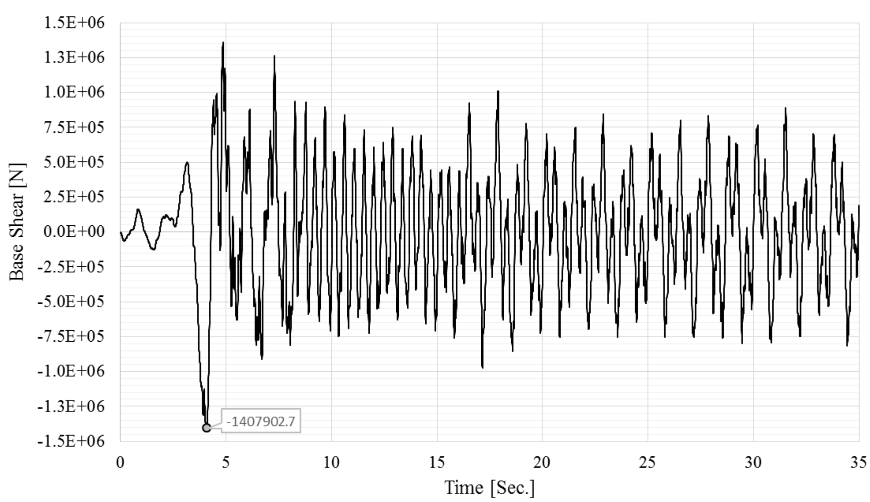


Fig. 9 – Non-linear time – history of the base shear [cm]

5. Conclusions

A performance-based seismic design method for irregular structure was presented in this paper. Through a simplified approach, the method is explicitly capable of considering the non-linear behavior in a direct design process. This simplification is based on two linear analyses, on elastic and auxiliary models and the adequate superposition of both by means of a damage factor (α). The method can be applied for complex structures accounting for different structural element types and different hysteretic dissipation rules. It was shown that the damage factor controls the inelastic behavior; however, it depends on the distribution of perfect hinges selected by the designer.

In the irregular case-study considered, the method succeeded in capturing the variations on the participation of vibration modes as damage progresses. In particular, the first mode's participation factor generally lowers while second and third gain importance; hence, affecting the lateral force distribution pattern. In the proposal, this pattern varies naturally as a function of the damage factor, the selected hinge distribution and the structural irregularities; hence, it provides the designer an objective way to take this effect into account. Furthermore, the approach adequately predicted the maximum floor displacements, local ductility demand in hinges and final internal forces as bending moment in hinges compared against a nonlinear time-history analysis of an irregular multi-story frame. Finally, the approach also predict an evolution of the distribution pattern of maximum internal forces as function of the damage factor; this resulted partially relevant in avoiding yielding in columns and formation of soft stories.

6. Acknowledgment

The present study was supported by the projects PROPPOSE – "Performance based design of partially prestressed concrete structures. Proposed new methodology, experimental verification and design criteria" (BIA2012-36848) and HORVITAL – "Contribution to the extension of the lifetime of existing structures through an adequate evaluation and strengthening with advanced materials" (BIA2015-64672-C4-1-R), funded by the Ministry of Economy and Competitiveness of Spain and the European Regional Development Funds.

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