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NUMERICAL MODEL UPDATING OF THE UCSC-CROSS (CHILE) USING EXPERIMENTALLY EXTRACTED DATA AND BY APPLYING REGIONAL SENSITIVITY METHODS

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Abstract

Calibration of numerical models has become indispensable to reduce error in the prediction of structural performance of complex systems. Several methods have been developed in the last two decades to calibrate a numerical model, but the implementation of these procedures in engineering common practice is still infrequent. Hence, the analysis-verification-calibration-design cycle is usually incomplete.

In this study, a 24-m high cross-shape truss structure was model using a finite elements approach. The finite element code was validated by comparing its results with other finite element models commercial package. The numerical model was calibrated using experimental data extracted from the actual structure located at the San Andrés Campus of the UCSC in Concepción, Chile.

A grid of 9 accelerometers was installed in the actual structure to record its response due to ambient excitation. The modal frequencies were extracted from these acceleration records using two operational modal analysis methods (SSI and FDD), identifying 4 predominant frequencies.

Model calibration was performed by applying a regional sensitivity method based on Monte-Carlo sampling. This method adjusts the modal response of the structural model to the real response measured on the actual structure. More than 2000 realizations were performed. The parameters chosen for calibrating the model were the Young´s modulus, truss-section thickness, material´s equivalent density and support flexibility. The simulation was carried out using the Matlab Toolbox MCAT.

A significant improvement of the model prediction was achieved by the calibration that allows a more precise estimation of structural performance under extreme conditions (wind gusts, earthquakes and others).

Keywords: Modal testing; Ambient excitation; Model calibration; Regional sensitivity methods; Monte-Carlo sampling.

1. Introduction

In the last 25 years, numerical modeling of structural systems has become a routinary task in engineering. In the present days, most of the engineering firms work with packages of software (usually based on finite element approaches) to assist the design of complex structures. This is certainly an advantage for the new generation of structural designers, but at the same time it becomes a risk, because rarely these models are validated by contrasting their predictions with experimental data. Hence, the analysis-verification-calibration-design cycle is usually incomplete.

Models are required that properly predict the performance of existing structures under operational conditions and extreme loadings, such as strong winds, earthquakes and other imposed loads. There is an increasing concern regarding the development of techniques to determine the degree of coherence among model predictions and the actual behavior of structures.

In the last two decades, a number of techniques have been developed to quantitatively determine the level of coherence between the actual structural response recorded in an existing building and the response predicted by numerical models [1-4]. One of the most attractive methods consists of measuring the dynamic response of a structure when it is excited by low intensity actions. These actions induce an elastic vibration response of the structure from which the modal properties (frequencies, mode shapes and damping) can be estimated by applying system identification techniques. This kind of analysis is known in the structural dynamics community as Modal Testing [5, 6]. The modal response recorded in the actual structure is compared to the modal response computed by the numerical model. Then, the differences identified between both sets of modal properties are minimized by modifying physical properties (geometry, density, mechanical properties, boundary conditions, etc.) within a range of plausible values. This is a process known as Model Updating [7, 8].

Modal testing of large scale structures can be undertaken by performing ambient vibration tests. In an ambient vibration test, the excitation is not controlled but the signal can usually be considered as a stationary random process. Consequently, only the response data can be used to estimate the dynamic parameters. A number of system identification methods have been proposed to obtain the modal parameters from vibration measurements. These methods can be classified as time-domain techniques and frequency-domain techniques, and usually more than one method and both approaches are used in a project.

Model updating is essentially a process of adjusting certain parameters of the finite element model. There are numerous sources of modeling error, some of them resulting from idealization assumptions or simplifications and others related to intrinsic discretization errors introduced by the numerical methods employed which cannot be corrected. However, there is another group of errors associated with incorrect assumptions about model parameters which can be minimized by adjusting the model [9]. The final goal of model updating is to produce corrected and verified models that can be used for a better prediction of structural response under extreme actions (earthquakes, wind-gusts, impacts or any other imposed excitation). It also will contribute to a more precise diagnosis of structural health and improve the design of the needed retrofit interventions.

The simplest version of model updating is reducing model discrepancies by applying the trial-and-error method, but this procedure can be extremely laborious and highly ineffective for complicated systems. In recent years many efforts have been undertaken to automate this process and one of the most traditional approaches is based on the construction of sensitivity matrices [10].

There is another group of techniques that have been effectively implemented in other fields of civil engineering, but they have not yet been explored in structural engineering. That is the case of regional sensitivity methods which are widely employed to calibrate hydrological and environmental models [11, 12]. The basic idea of regional sensitivity analysis is to partition the samples of an input factor with a potential influence on model behavior into two sub-samples (i.e., behavioral and non-behavioral) according to the given criterion (e.g. quantiles of the output distribution, or thresholds derived from observed behavior). If the distributions of the input factor in the two sub-samples are dissimilar, then that factor is considered influential in the model. The comparison of the two distributions is done by a Smirnov test, in which the maximum distance between the two empirical cumulative distributions is taken as the sensitivity indicator [13].

In this study, a cross-shape truss structure was instrumented with a set of 9 accelerometers to record its response due to ambient excitations. The modal frequencies were extracted from these acceleration records using two operational modal analysis methods: Stochastic Subspace Identification [14] and Frequency Domain Decomposition [15]. A numerical model of the structure was generated by applying a finite elements approach The model was calibrated using experimental data extracted from the actual structure by applying a regional sensitivity method based on Monte-Carlo sampling.

2. Structure Description

The UCSC-Cross is located at the San Andres Campus of the Universidad Católica de la Santísima Concepción in Concepción, Chile. It was erected in 1987 for the visit of the Pope John Paul II to the City of Concepción, and it became an iconic image of the university since its foundation in 1991. It is a 24 m high truss structure built using two types of steel profiles with hollow circular section. Each cross arm is 4.5 m long. The external diameters of the sections are 64 mm and 89 mm. The thicknesses of the sections are unknown. The bar joints correspond to welded connections. The cross is connected by bolted base plates to individual foundation blocks. A general view of the structure is presented in Fig. 1 and the most significant dimensions are presented in Fig. 2.

Fig. 1 – General view of UCSC-Cross

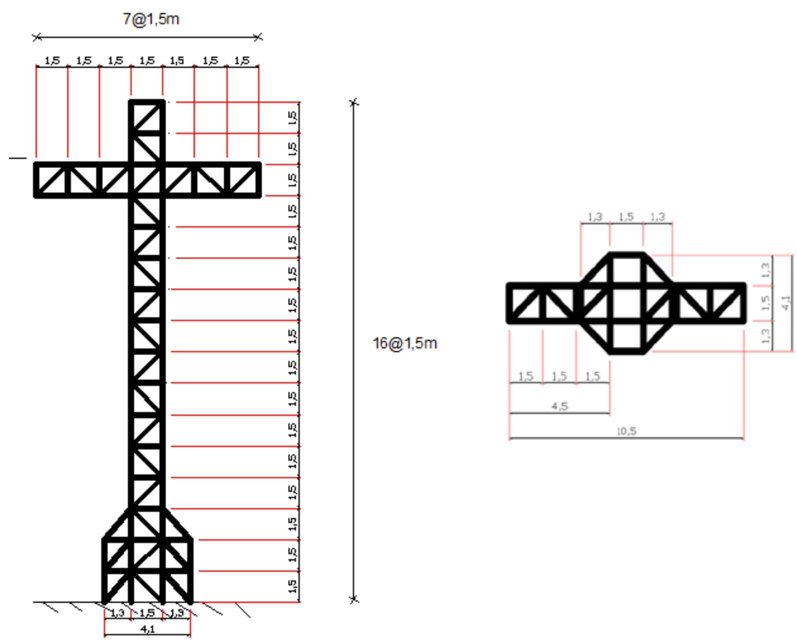


Fig. 2 – Main dimensions of UCSC Cross.
(a) Front view, and (b) Plane view

3. Numerical Model

A numerical model was generated in Matlab following a classical finite element approach for tridimensional truss elements. The model was formed by 336 frame elements and 116 nodes, according to the detail presented in Table 1. The support of the structure was modeled as hinges connected to the base plate. Also, fully restricted supports were considered in others analyses, not showing significant differences in the results. The structural parameters considered for the initial model are shown in Table 2.

The quality of the modeling approach was verified by comparing the results of a static and modal analysis performed in our Matlab code with the results of a model built in the commercial package of finite elements (SAP2000), showing an excellent correlation (differences smaller than 1% in stresses and deformations).

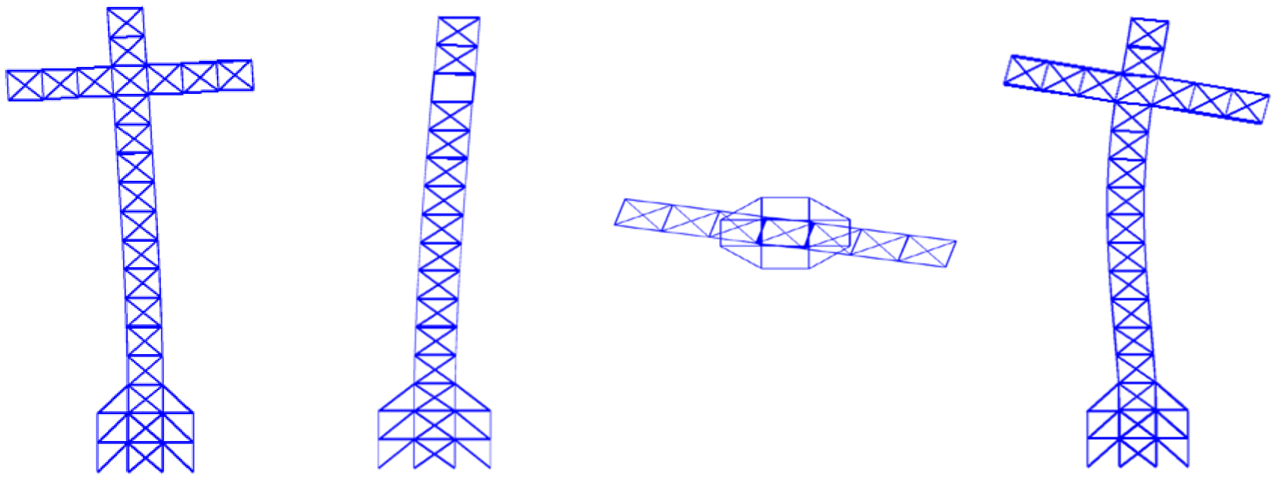
Table 1 – Elements of the numerical model

	Elements diameter [mm]	Number of elements
Longitudinal elements	89	96
Transversal elements	64	120
Diagonal elements	64	112
	89	8

Table 2 – Structural parameter considered for the initial numerical model

Parameter	**Value**
Diameter 1 [mm]	89
Diameter 2 [mm]	64
Poisson´s ratio	0.3
Young´s Modulus [MPa]	$2.10*10^5$
Thickness 1 [mm]	1.5
Thickness 2 [mm]	2.0
Equivalent density [kg/m3]	7800

Some results of the modal analysis is presented in Fig. 3. It can be observed that the first mode corresponded to a translational mode that activated lateral movement in the cross. The resonant frequency of mode 1 was 1.976 Hz. Mode 2 had a frequency equal to 1.992 Hz and mobilized the cross in the frontal direction. Mode 3 exhibited a torsional behavior and had a frequency of 3.456 Hz. Mode 4 corresponded to a lateral mode with a significant vertical component in the arms of the cross. The fourth modal frequency was 10.509 Hz.



(a) Mode 1
1.976 Hz

(b) Mode 2
1.992 Hz

(c) Mode 3
3.456 Hz

(d) Mode 4
10.509 Hz

Fig. 3 – Modal response obtained by the finite element model

4. Experimental Analysis

As explained above, the cross is located in the San Andres Campus of UCSC, close by a main highway, and exposed to wind action. Considering that, an operational modal analysis approach was followed to identify the modal properties of the actual structure. A set of 9 accelerometers was attached to the structure to capture the vibration response due to ambient excitations. The accelerometers were triaxial capacitive transducers (MEMSIC CXL04GP3) connected by cables to a National Instrument data acquisition system (NI9205) controlled by a LabView code developed by the authors. Fig 4 presents the accelerometer installing process assisted by a team of Concepción Fire Brigade. Fig 5 indicates the location of the sensors in the actual structure.

Fig. 4 – Installation of accelerometers in the Cross

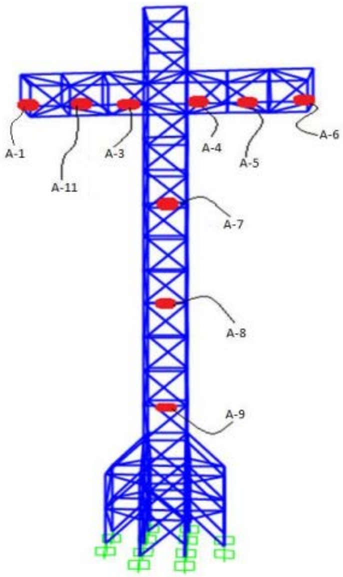


Fig. 5 – Accelerometers distribution in the structure

Structural response was recorded during 60 minutes with a sample rate of 400 Hz. Two measurements were performed per day during 8 days. These measuring campaigns were conducted at different hours in the day to assess the influence of intensity of the excitation source (vehicle traffic) in the structural response. Then, the records were then cleaned using a 5^{th} order Butterworth low-pass filter with a 100 Hz cut off frequency.

Two system identification methods were employed to determine the modal properties of the structure: Stochastic Subspace Identification (SSI) [14] and Frequency Domain Decomposition (FDD) [15]. The SSI method is a data-driven time-domain technique that employs QR-factorization and singular value decomposition to identify the matrices of the dynamic state-space model. Once the state space model of the structure is found, the modal parameters (natural frequencies, damping ratios and mode shapes) can be determined by eigenvalues decomposition. In general, it is not possible to determine the system order beforehand. Therefore, it is necessary to repeat the analysis with different system orders and verify the repeatability of the results. This procedure is performed by constructing stabilization charts (Fig 6). In this graph, the dots represent the fundamental frequencies of the poles (modes) identified considering models with different system orders (SO). The red dots are associated with those frequencies that are similar to another frequency detected in the precedent model, while the blue circles around the dots represents those poles that have a similar mode shape to a pole detected in the precedent model. Those poles that reveal stability in terms of similar frequencies and mode shapes (usually aligned in a vertical column in the graph) are very likely to represent vibration modes.

The FDD method is an extension of the classical peak-picking method. The FDD algorithm assumes that the excitation applied on the structure has a random nature and can be described as a white-noise. Thus, the excitation power spectral density function (PSD) becomes a constant (S) and, consequently, the FRF peaks can be directly identified from the peaks of the response PSD function. These peaks on the PSD function are assumed as resonant frequencies and mode shapes can be determined by applying Single Value Decomposition procedures. The PSD curves obtained for this experiment are presented in Fig 7.

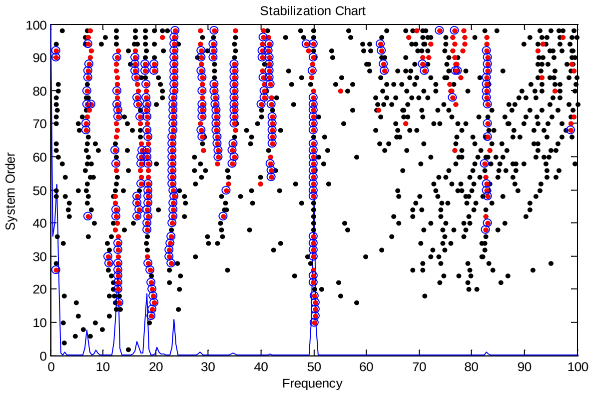


Fig. 6 – Stabilization charts of SSI metdod

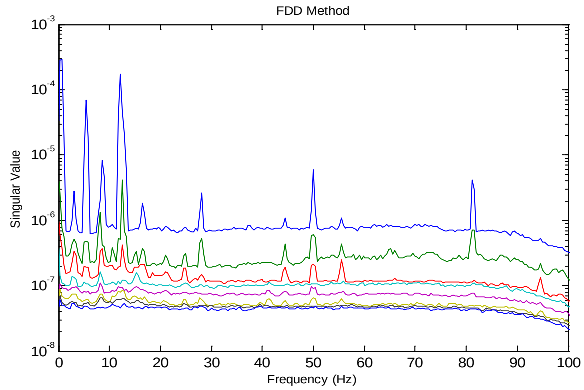


Fig. 7 – Power spectral density curves of FDD method

The frequencies were identified from those systematically repeated poles in the different tests and detected by both system identification techniques. The first three modal frequencies detected by the structure are presented in Table 3. These frequencies were paired with the frequencies predicted by the numerical model paying attention to the predominant displacements activated for each mode and the mass participation calculated by the numerical model. It can be noted that the first experimentally detected frequency was paired with two frequencies of the numerical model. This analyst's decision was made considering on the similitude between these two numerical modes, and assuming that our system identification method was unable to decouple these modes. This judgement was based on the main analyst experience and it was demonstrated to be correct, because during the calibration other mode pairing choices were studied with deficient results.

Table 3 – Experimentally detected and numerically calculated modal frequencies

	Mode 1	**Mode 2**	**Mode 3**	**Mode 4**
Numerical model (Hz)	1.976	1.992	3.456	10.509
Experimental FDD (Hz)	1.172		2.637	5.469
Experimental SSI (Hz)	1.104		2.687	5.567

5. Model Calibration

The model calibration was performed using a regional sensitivity method based on a Monte-Carlo sampling procedure for the structural parameters. This method randomly selected combinations of physical parameters that may affect the structural response from a range of plausible values. A number of simulations were performed using these inputs parameters and the simulated responses were compared to the experimental response. The similitude was assessed using a frequency error function known as Mean Absolute Relative Error (MARE) that is described by Eq. (1):

$$MARE = \frac{1}{n}\sum_{i=1}^{n}\frac{|f_{cal} - f_{exp}|}{f_{exp}} \quad (1)$$

where f_{cal} is the modal frequency calculated by the numerical model, f_{exp} is the modal frequency experimentally identified by the modal tests and n is the number of modal frequencies considered for the comparison. A smaller value of MARE implies a better level of similitude between the calculated and the experimentally observed response.

The above described procedure was performed by the Matlab Toolbox Monte-Carlo Analysis Toolbox (MCAT) [16]. This toolbox allowed the analyst to perform a huge number of simulations given a set of input parameters and a range of values for each of them. After each simulations the MARE was calculated and stored by the software. The results of these simulations in terms of MARE values and the sensibility of MARE to the parameter value were graphically displayed as it can be seen in Fig 8. With this information on sight the parameter ranges were progressively narrowed until the model became insensitive to the parameter in the range of analysis. That optimal solution was then considered as a possible solution of the problem. However, this solution might be not unique and its physical meaning needed to be validated.

In this study, the parameters selected as input values to be variated in the simulations were the Young´s Modulus of the steel bars, the equivalent density of elements considering the mass of steel and the paint covering, and the thicknesses of the tubular elements. These parameters were selected because there were high levels of uncertainty in their value and the model demonstrated to be sensitive to them. Other parameters were discarded as calibration parameters because their values were well known or measurable (bar element diameters), or they did not affect significantly the structural response (Poisson´s ratio). Therefore, they were selected as constant. Two thousand simulations were performed and the values chosen for the fixed parameters and the range of values considered for the variable parameters are presented in Table 4.

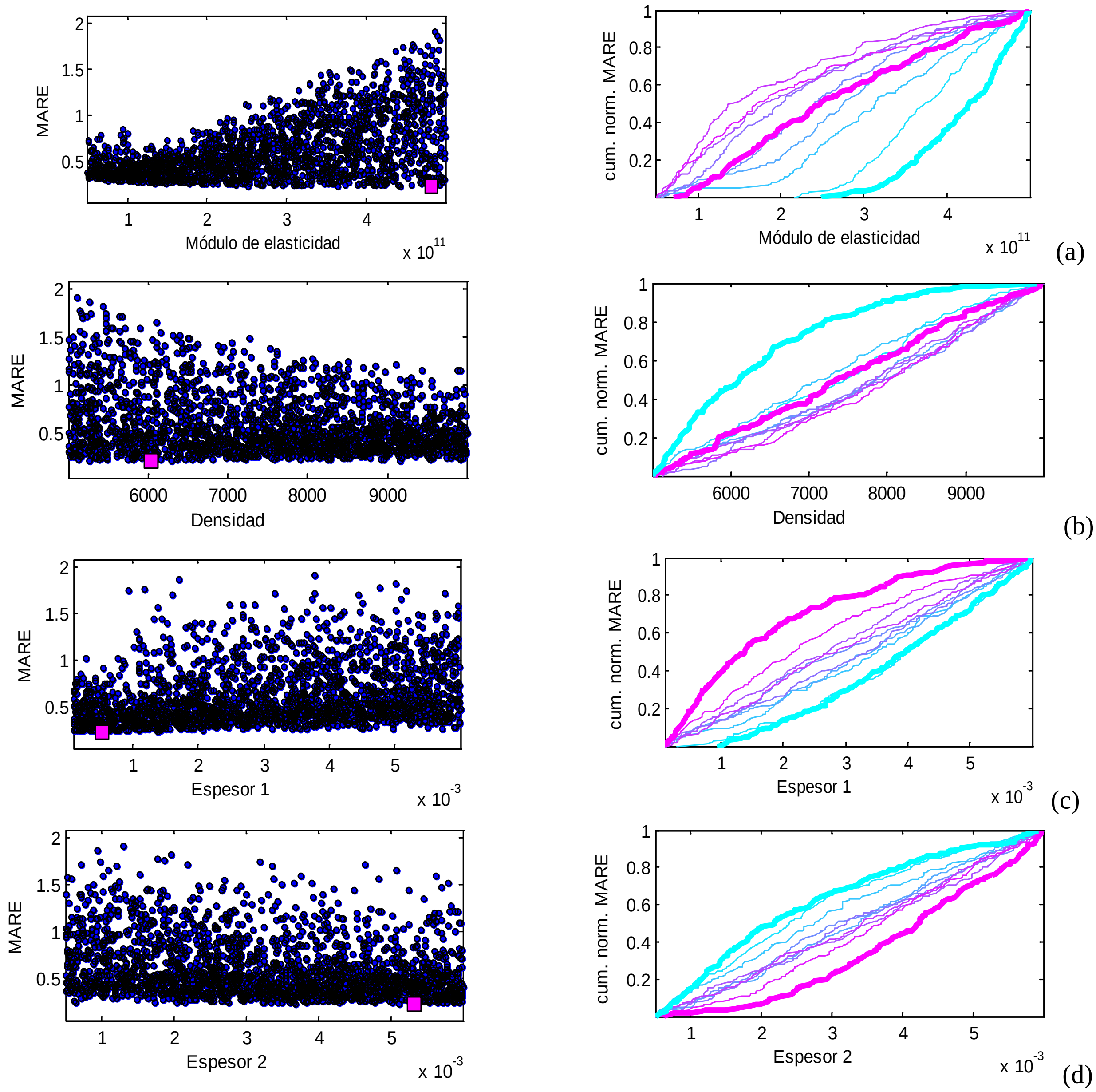


Fig. 8 – MARE value and Cummulative probability charts for impusts parametes. (a) Young’s modulus, (b) equivalent density, (c) thikness of 89 mm diameter bars and (d) thikness of 64 mm diameter bars.

Table 4 – Input parameter considered for the regional sensitivity analysis

Parameter	**Value / Range of values**
Diameter 1 [mm]	89.0
Diameter 2 [mm]	64
Poisson´s ratio	0.3
Young´s Modulus [MPa]	$1.89*10^5 \sim 2.31*10^5$
Thickness 1 [mm]	3 ~ 5
Thickness 2 [mm]	3 ~ 5
Equivalent density [kg/m3]	7700 ~ 9400

In Figure 8, the plots on the left represents the value of MARE given different values of the parameters. Each dot represents the results of one simulation and the dot in magenta color corresponds to the regional minimum of MARE. Each line in the plots on the right show the cumulative probability associated to each input parameter grouped in deciles. The magenta curve represents the decil of the best simulations. The more stepped (or less linear) is this curve, the more significant is the effect of a parameter variation in the simulated response.

In the chart presented in Fig. 9, the grey zone corresponds to the 95% confidence zone for the frequencies calculated by the model with the initial range of values given in Table 3. Fig 10 shows the same chart, but this time considering the final range of values considered for adjusting the model. In both charts, the blue dots represents the experimentally determined frequencies. As it can be seen, the final solution was able to find satisfactory solutions only for the first three frequencies, but it was unable to match an adequate response for the fourth frequency. Also, a fifth experimentally detected frequency was matched by the calibrated model.

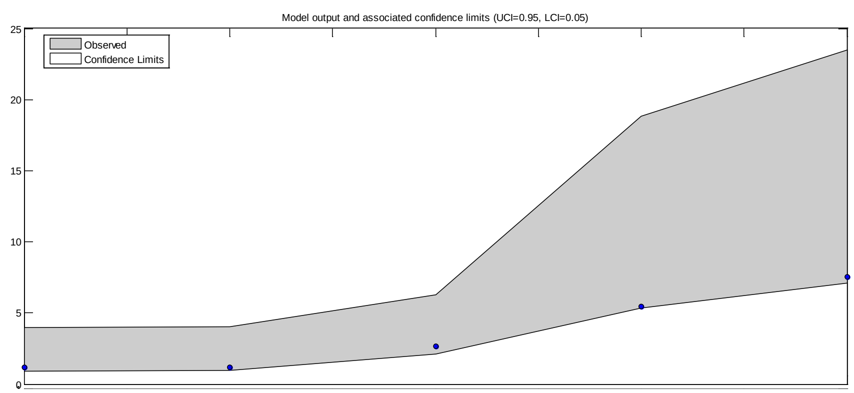


Fig. 9 – 95% confidence response zone for the parameters considering initial range of values

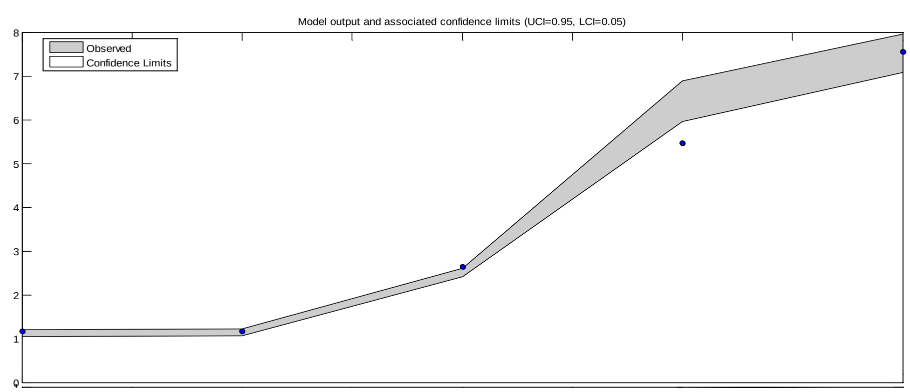


Fig. 10 – 95% confidence response zone for the parameters considering optimal range of values

The optimal parameters obtained from the above described calibration process is presented in Table 5. The numerical model results obtained by applying these parameter values are shown in Table 6. It can be seen that the MARE in the initial numerical model was 68% and the prediction was significantly improved giving an updated model which MARE was 37%.

Table 5 – Parameter obtained after the calibration

Parameter	**Value / Range of values**
Diameter 1 [mm]	89
Diameter 2 [mm]	64
Poisson´s ratio	0.3
Young´s Modulus [MPa]	$1.89*10^5$
Thickness 1 [mm]	3
Thickness 2 [mm]	5
Equivalent density [kg/m3]	9400

Table 6 – Experimentally detected and numerically calculated modal frequencies at the beginning and end of the analysis

	Mode 1	**Mode 2**	**Mode 3**	**Mode 4**	**MARE**
Experimental (Hz)	1.133	1.133	2.662	5.518	
Initial numerical model (Hz)	1.976	1.992	3.456	10.509	
Initial numerical model error	74%	76%	30%	90%	68%
Updated numerical model (Hz)	1.530	1.554	3.221	8.532	
Updated numerical model error	35%	37%	21%	55%	37%

To complete the calibration process a further adjustment was incorporated to the model. The support conditions at the base of the cross were modeled as uniaxial springs that acted in the vertical direction. A stiffness equal to $4.75*10^6$ [N/m] was assigned to these springs which was considered as reasonable for the kind of soil identified at the base of the structure and the size of the foundation blocks. This second stage calibration was performed manually, considering that the other parameters where already optimized. The final outcome of this calibration process is presented in Table 7. It can be seen that the MARE was improved from 68% at the initial model to 11% in the final calibrated model.

Table 7 – Experimentally detected and final numerically calculated modal frequencies

	Mode 1	**Mode 2**	**Mode 3**	**Mode 4**	**MARE**
Experimental (Hz)	1.133	1.133	2.662	5.518	
Final numerical model (Hz)	1.159	1.172	3.082	6.753	
Final numerical model error	2%	3%	16%	22%	11%

6 Conclusions

The modal frequencies of the structure were satisfactorily identified by two system identification methods: SSI and FDD. Both techniques were able to identify the same modes with a maximum relative error smaller than 6%.

The numerical model of the structure constructed using a Matlab code was compared to a model elaborated using a commercial package of software (SAP 2000), obtaining the same results when subjected to static load. Also, the modal response predictions were coincident.

Even though the model was carefully generated using the best available information, the model predictions of the modal response did not coincide with the response measured on the actual structure. The average difference between the numerical and experimental frequencies was 68%, considering just the first four modes.

The regional sensitivity method implemented for calibrating the model demonstrated to be efficient in adjusting the numerical predictions. The mean absolute relative error for the calibrated model was reduced to 37%. This calibration process imposed a 10% reduction in the Young´s Modulus. The 89 mm diameter bars increased their thickness in 200% and the 64 mm diameter bars increased their thickness in 250%. The equivalent density that considers the mass of steel and the paint cover was increased in 20%.

A further calibration was performed by adding springs in the structure supports, replicating the soil stiffness. As results of this process the model error was reduced to 11%.

The result of the calibration process was considered as satisfactory, because the model error was reduced by modifying the structural parameters within a range of values that were physically reasonable. This new calibrated mode can be now used for more precisely predicting the response of the structure under extreme action.

The study presented here demonstrated that the regional sensitivity method based on a Monte-Carlo sampling was an effective tool for calibrating numerical models of structural systems.

4. Acknowledgements

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