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HIGHWAY BRIDGE DAMPING EVALUATION: FORCED AND AMBIENT VIBRATION TESTING

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Abstract

The identification of the main dynamic properties – frequencies, mode shapes and damping – is an important step in the analysis of existing structures and in the design of rehabilitation solutions. Dynamic tests are now frequently carried out on bridges to provide this type of data for numerical model calibration. Vibration frequencies and mode shapes are important parameters, but damping, which plays a crucial role in seismic analyses, is rarely the focus of reported experimental investigations. The accurate and repeatable evaluation of damping remains a challenge in dynamic testing. Ambient vibration test results have shown their limitations in this regard, and forced vibration tests are often the preferred method, when possible, to extract this key parameter.

This paper presents full-scale dynamic tests carried out on three highway bridges in eastern Canada, located in moderate seismicity zone. The tested structures all have two spans that vary between 27 and 35m and have little to no skew. They were selected to correspond to a generic regular bridge type. Each structure is supported by a central reinforced concrete bent, and superstructure types include cast-in-place prestressed concrete, steel and prefabricated AASTHO type V beams, respectively.

The focus of this experimental program is to compare ambient and forced vibration techniques as a means to evaluate equivalent viscous damping. This important parameter is then used to calibrate three dimensional finite element models of the bridges. A series of forced-balanced triaxial accelerometers and velocity transducers were used to extract frequencies, mode shapes and modal damping. The ambient vibration tests were carried out under normal operational conditions without interrupting traffic. The bridges were then closed for traffic to carry out forced vibration tests using an electrodynamic shaker mounted on the decks at specific locations selected to excite the first few modes in both the vertical and transverse directions. Complete frequency response curves were obtained by varying the operating frequency of the shaker by small increments, up to 20 Hz. The paper describes the experimental procedures, extracted structural properties and finite element model calibration. Results from both testing techniques are compared, and their combination is shown to produce reliable results and therefore lead to better calibration of FE models.

Keywords: Highway Bridge; Ambient vibration tests; Forced vibration tests, Dynamic properties; Damping.

1. Introduction

Vibration tests are widely used to identify dynamic properties which are essential to understand dynamic structural behavior and calibrate numerical models. Generally, the main objectives of such tests are the evaluation of vibration frequencies and mode shapes. These tests can also provide information on damping, which plays a major role in the seismic response of bridges. In this paper, we will focus on experimental damping measurements, obtained from ambient and forced vibration tests on three highway bridges. It is generally accepted that forced vibration tests lead to better estimates of damping, because the frequency and amplitude of the input force are controlled [1]. However, damping values obtained by ambient and forced vibration tests have rarely been compared, particularly for bridges [2].

This paper presents a case study of damping measurements obtained on three highway bridges in Eastern Canada, using both ambient and forced vibration testing methods. Experimental procedures are explained and extracted structural properties are exposed to discuss differences between the methods. The paper describes the experimental procedures, extracted structural properties and finite element model calibration. Results from both testing techniques are compared and show that frequency responses generated by forced vibrations lead to more reliable damping measurements and therefore to a better calibration of FE models.

2. Selected bridges

Three highway bridges located in Eastern Canada were selected for this study. The tested structures all have two spans that vary between 27 and 35m and have little to no skew. They were selected to correspond to a generic regular bridge type. Each structure is supported by a central reinforced concrete bent, and superstructure types include cast-in-place prestressed concrete, steel and prefabricated AASTHO type V beams, respectively. Two of the structures were recently built while the other dates from the 70s.

2.1 Bridge 1

The first bridge, built in 2011, is a reinforced concrete overpass, 54.4m long and 17.4m wide. The superstructure includes four rectangular cast-in-place continuous prestressed beams (1.50 m wide by 0.85m high) and a 250mm slab. A 4.25 m wide concrete divider strip separates the two traffic lanes. The bridge is seated on confined elastomeric bearings, which are all fixed in the transverse direction. Bearings at the abutments are free in the longitudinal direction. The bridge has a 16 degrees skew. Fig. 1 and Fig. 2 show the bridge and its dimensions.

Fig. 1 – Bridge 1 (over future Highway), built in 2011

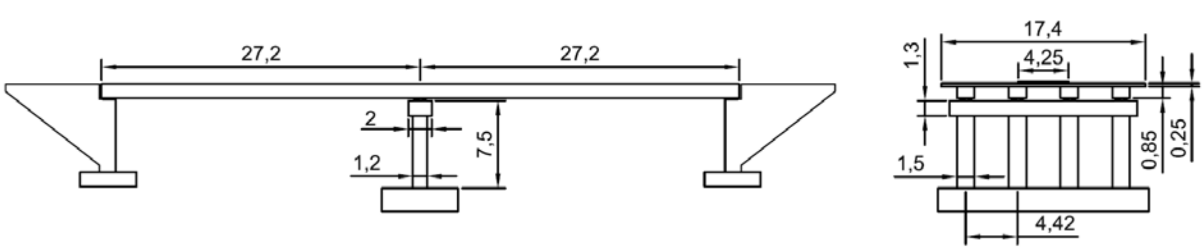


Fig. 2 – General dimensions of Bridge 1 (dimensions in meters)

2.2 Bridge 2

The second bridge is a 2010 steel beam overpass, 59.4m long and 11.4m wide. The four continuous steel beams are 1.2m high and the superstructure has a 6 degrees skew and a 200mm slab. It was built for two lanes of traffic and a bike path. The beams are supported over the bent by confined elastomeric bearings, and by multilayered elastomeric bearings at the abutments. They are all fixed in the transverse direction, but fixed in the longitudinal direction only at the west abutment. Fig. 3 and 4 show the bridge and its dimensions.

Fig. 3– Bridge 2, over four lane highway, built in 2010

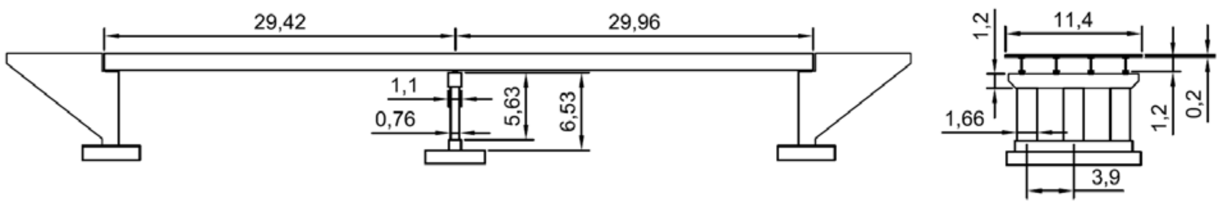


Fig. 4 – General dimensions of Bridge 2 (dimensions in meters)

2.3 Bridge 3

The third bridge was built in 1977 and is a 70.1m long and 11.98m wide bridge made of 5 prestressed AASTHO Type V beams. The bridge has two traffic lanes and no skew. The beams are supported by multilayered elastomeric bearings. The supports are all fixed in the transverse direction and those at the abutments are free in the longitudinal direction. Fig. 5 and 6 show the bridge and its general dimensions.

Fig. 5– Bridge 3, over four lane highway, built in 1977

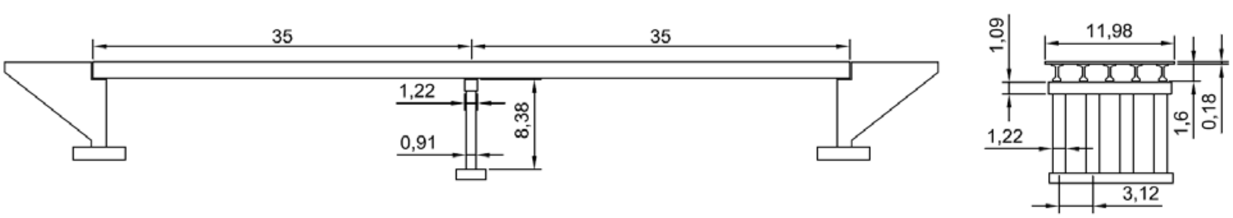


Fig. 6 – General dimensions of Bridge 3 (dimensions in meters)

3. Tests setup

All bridges were subjected to both ambient and forced vibration tests. The ambient vibration tests were first carried out during the normal operation of the bridge, without shutting down traffic. Forced vibration tests using a small electrodynamic shaker were then completed at night time while the bridges were closed to traffic.

3.1 Ambient vibration tests

Ambient accelerations and velocities due to traffic and wind were recorded on the bridge decks at several locations. For bridge 1, six roving SYSCOM uniaxial velocity transducers were used. Seven different sensor setups were necessary to cover the entire bridge and obtain data in the transverse and the vertical directions. Two reference sensors were kept at fixed positions in each setup. Fig. 7(a) shows the measurement locations.

For bridges 2 and 3, four Kinemetrics triaxial accelerometers were used on the superstructure, with one reference sensor. With these triaxial sensors, six setups were used as shown on Fig. 7(b). This led to a more accurate representation of mode shapes, including the longitudinal direction that was not recorded on the first bridge. For all bridges and every setup, data were recorded during 20 minutes at a frequency of 400 Hz. The Enhanced Frequency Domain Decomposition method (EFDD) method [3] was used to analyze data.

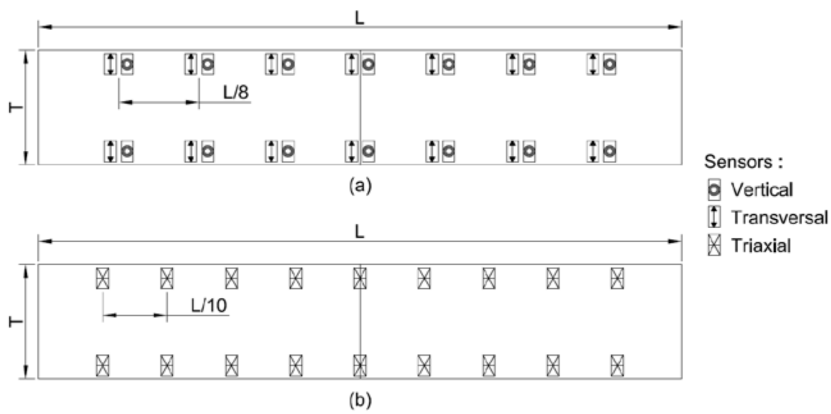


Fig. 7 – Configuration of ambient vibration tests for (a) Bridge 1; and (b) Bridges 2 and 3

3.2 Forced vibration tests

While traffic was shut down on bridges, an ELECTRO-SEIS APS-400 shaker, with a maximum force output of 445N, was mounted on the bridge decks first to produce either a horizontal (transverse direction) load, or a vertical load. The shaker locations shown in Fig. 8 are slightly off-centered to excite torsional modes. For Bridge 1 the shaker acted in the transverse direction only, for Bridge 2, both directions (transverse and vertical) where used, and finally only the vertical direction was used for Bridge 3.

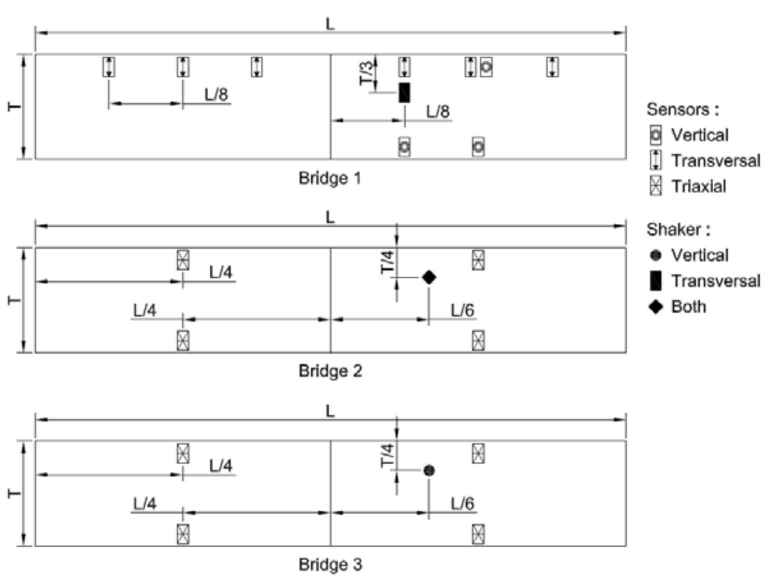


Fig. 8 – Configuration of forced vibration tests for (a) Bridge 1; (b) Bridge 2 and (c) Bridge 3

A sinusoidal load was applied using incremental frequency steps (0.01 to 0.05 Hz) to the bridge decks. Tests started at 2 Hz and were carried out up to 17 Hz for Bridge 1; 9 Hz and 8 Hz for Bridge 2 and 3, respectively, in order to record the first few modes. The same velocity and acceleration sensors were used to record data during these series of tests. A high sampling frequency rate of 1000 Hz was used for eight seconds periods for each frequency step. An accelerometer mounted on the shaker allowed for the exact measurement of the input force.

A smaller number of sensor locations was used in this case because the objective here was to (i) confirm the resonances observed during the ambient tests (ii) to obtain the better frequency response curves in order to identify modal damping. Also, forced vibration tests require more time and this was a factor as the bridge was shut down during the whole tests.

4. Results

Ambient test data were analysed with the ARTeMIS software [4], using the EFDD method to identify the key dynamic properties [3], including the frequencies, mode shapes and damping, which is evaluated based on the logarithmic decrement for each mode. Results are presented below for each bridge, with frequencies identified on normalized singular value plots and some of the corresponding mode shapes are illustrated below. The modes shapes are labelled with the letter "B" for bending modes, the letter "T" for torsional modes and the letter "H" for transverse (Horizontal) modes. Horizontal modes are of particular interest here, since the finite element models that are calibrated with the experimental results are then used to carry out linear and non-linear seismic analysis in that direction.

Forced vibration tests data are sinusoidal responses (acceleration or velocities) recorded at specific frequencies. As the shaker was successively operated throughout the required range, at 0.01 to 0.05 steps, a large number of sinusoidal responses were obtained for each recording position. The amplitude and the phase were then extracted from each of those records, for each frequency step. The amplitudes were normalized by the shaker force, as this force increases during the tests with the square of the operation frequency. Complete frequency response curves were thus constructed for each measurement position. The frequencies were then identified directly on these curves, and modal damping was evaluated by the half-power bandwidth method. Results are presented in the form of normalized frequency responses with the maximum value scale to unity, as the measured vertical response was approximately ten times stronger than the horizontal response.

4.1 Bridge 1

Figure 9 shows the vibration frequencies identified by peak picking on the normalized singular value plots resulting from the ambient recordings for the first bridge. The first four vertical modes are shown, as well as the first two horizontal ones, and the corresponding shapes are shown in Fig. 10. The evaluated modal damping ratios vary between 1.0 and 2.5 %. Table 1 summarizes the extracted values for Bridge 1.

Frequency response curves obtained from the forced vibration tests are shown in Fig. 11. The shaker was acting in the horizontal plane (transverse) direction. The first curve shows results obtained with the uniaxial velocity sensors mounted in the vertical direction. Four vibration modes (one in bending and three in torsion) were identified at frequencies that were quite close to those obtained from the ambient tests (the first mode being identical), with differences varying from 0 to 0.13 %. Damping values were generally lower than those obtained from the logarithmic decrement of the individual modal responses. The second curve shows results obtained with the uniaxial sensors mounted in the horizontal (transverse) direction. Two additional transverse modes are visible in Fig 11, with frequencies that are in agreement with those observed with ambient data. As stated above, the bridge shutdown time was a factor, and a smaller number of measurement locations were used during these tests (Fig. 8a). It was still possible to extrapolate mode shapes by looking at the response phases. In the transverse direction, the phases of the sensors located on the first span were synchronous with respect to the second span for the first mode (H1) and asynchronous for the second mode (H2), which indicates the same mode shapes found during the ambient vibrations tests. An indication of the correlation between two sets of mode shapes that is widely used is the Modal Assurance Criterion [5], usually calculated between a measured and a computed set of modal vectors, which varies from 0 to 1 (1 indicating a perfect correlation). MAC values were calculated here to show the correlation between both sets of experimental data, in order to compare ambient and forced vibration results. These values were only obtained in the horizontal plane for the first bridge, because of the limited measurement points used for the forced vibration measurements in the vertical direction. They are close to 1, and therefore show good agreement between both sets of results (Table 1).

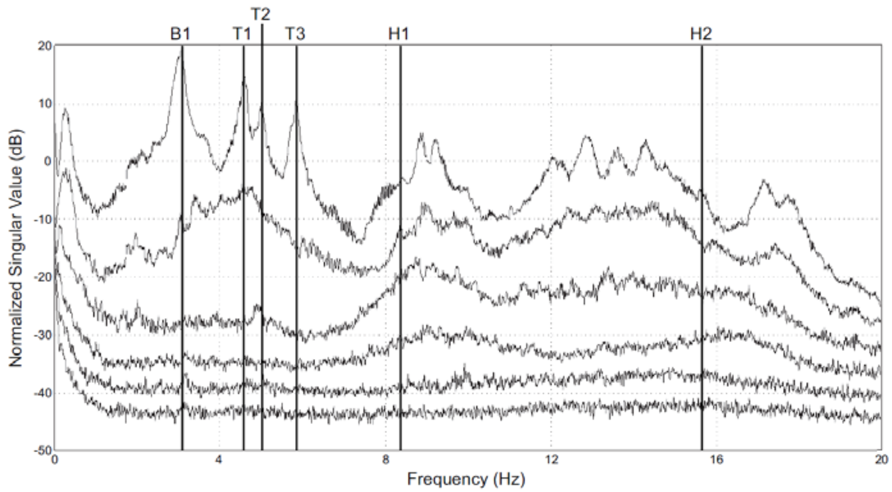


Fig. 9 – FDD results – Bridge 1

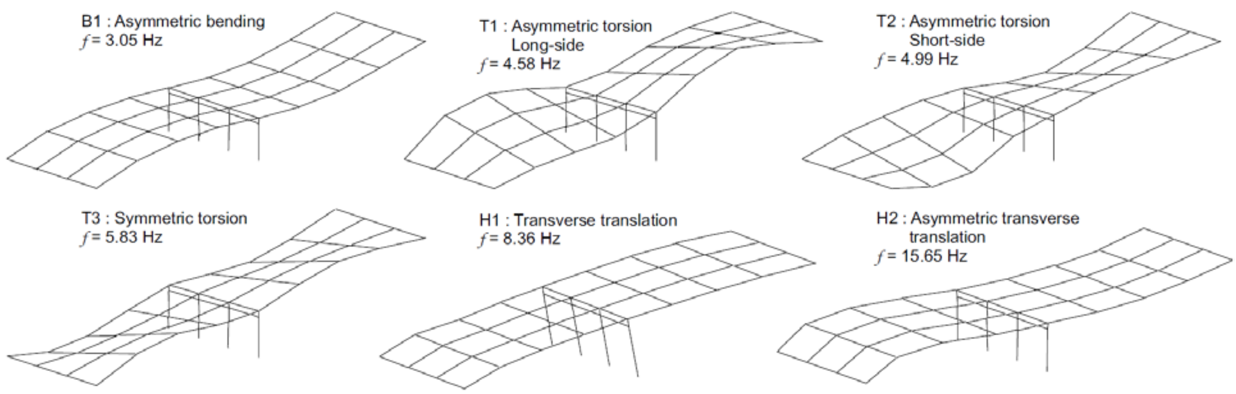


Fig. 10 – Mode shapes obtained with ambient vibration tests – Bridge 1

Table 1 – Comparison of ambient and forced vibration test results: Bridge 1

Vibration mode	**Ambient vibration tests**		**Forced vibration tests**		**MAC**
	Frequency (Hz)	Damping (%)	Frequency (Hz)	Damping (%)	
B1	3.05	2.43	3.05	1.64	N/A
T1	4.58	1.31	4.50	0.94	N/A
T2	4.99	1.06	4.90	0.74	N/A
T3	5.83	1.18	5.70	0.83	N/A
H1	8.36	1.80	8.32	1.04	0.99
H2	15.65	1.50	16.50	1.65	0.96

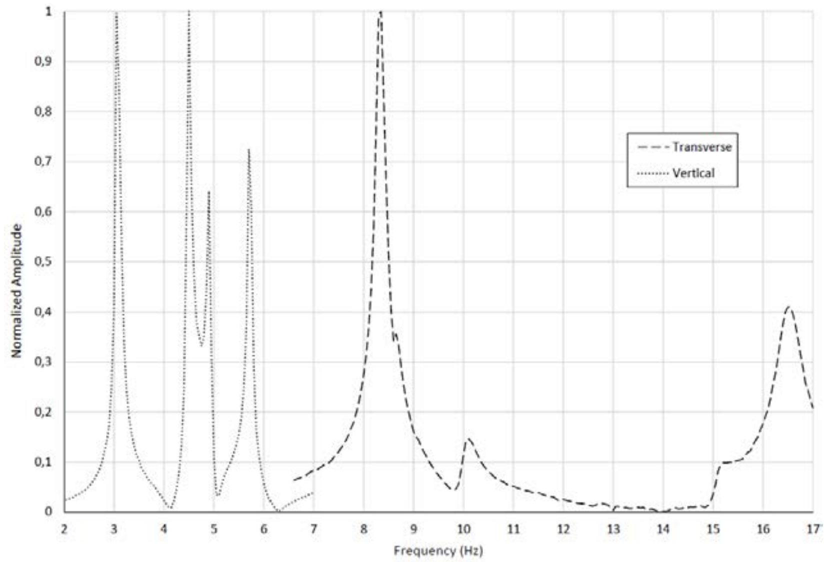


Fig. 11 – Frequency responses obtained from forced vibration tests – Bridge 1
(shaker acting in the horizontal, transverse direction only)

4.2 Bridge 2

Fig. 12 shows the EFDD results obtained for the second bridge. In this case, the first six peaks correspond to four bending modes and two torsional modes. The resulting mode shapes are shown in Fig. 13. There were no horizontal modes obtained with the recorded ambient data, although horizontal motion was detected in mode shapes #2, 4, 5, and 6 (confirmed by the forced-vibration tests). The extracted modal damping values vary between 0.89 and 1.22 %, as can be seen in Table 2. These damping values are somewhat lower than Bridge 1, which is expected since the superstructure is made of steel.

Forced vibration test results for this bridge are shown in Figs. 14 a) (shaker acting vertically) and 14 b) (shaker in the horizontal, transverse direction). The three curves in each graph indicate the responses of the triaxial accelerometers in the three directions, respectively. The use of this type of sensor (instead of uniaxial velocity sensors for Bridge 1), lead to a larger set of results and well-defined frequency response curves. The dominant peaks in each response curve were identified and are listed in Table 2. Again, lower values of damping were generally obtained for Bridge 2 during these tests.

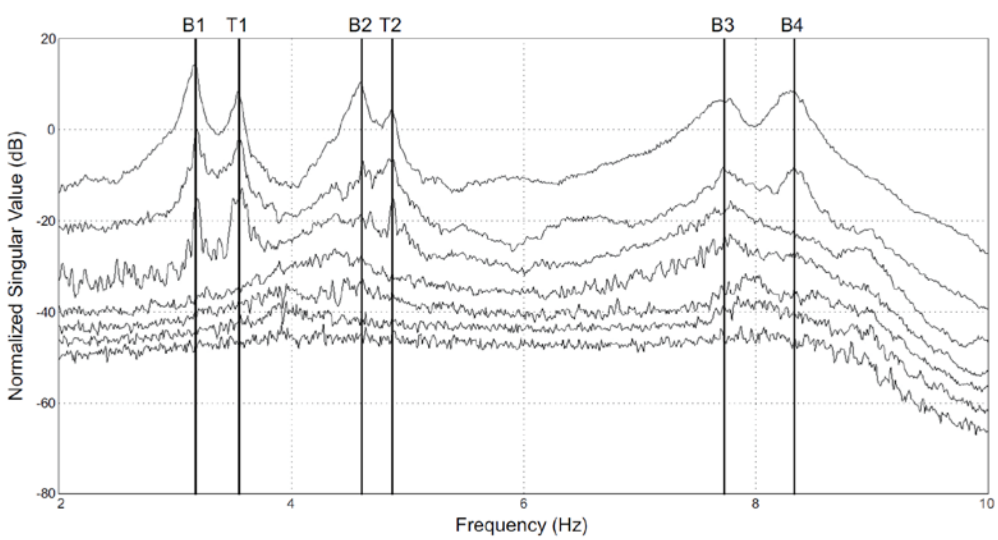


Fig. 12 – FDD results – Bridge 2

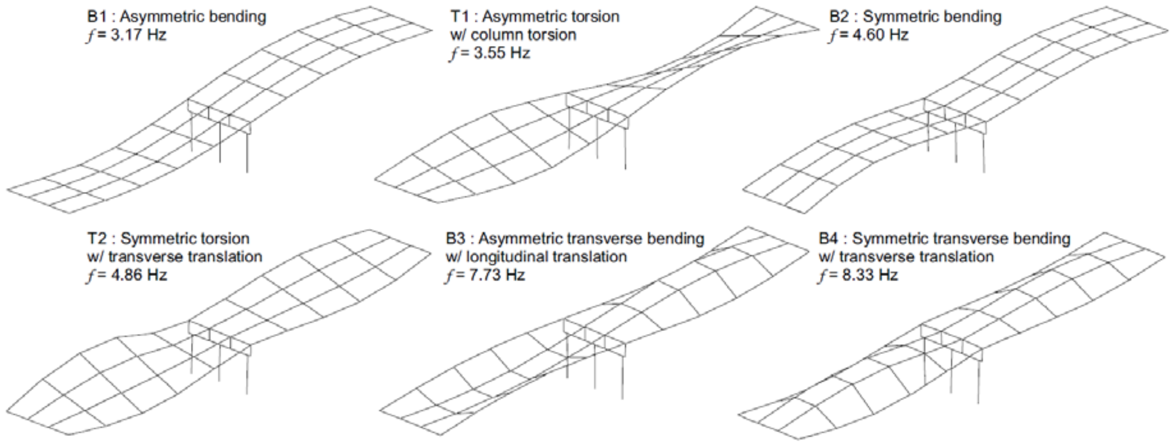


Fig. 13 – Mode shapes obtained with ambient vibration tests – Bridge 2

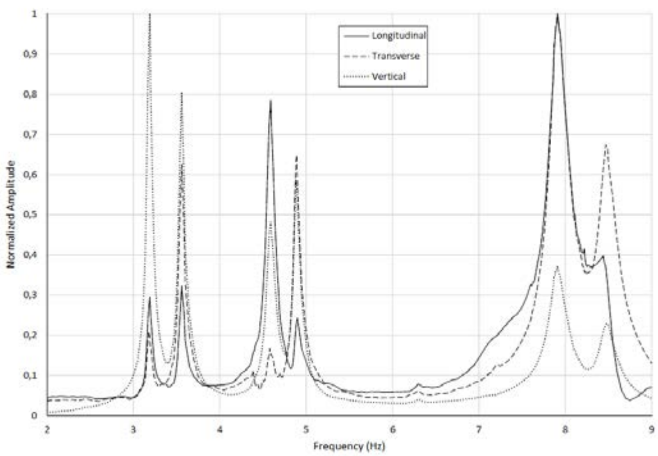
Table 2 – Comparison of ambient and forced vibration test results: Bridge 2

Vibration mode	**Ambient vibration tests**		**Forced vibration tests**		**MAC**
	Frequency (Hz)	Damping (%)	Frequency (Hz)	Damping (%)	
B1	3.17	1.01	3.19	0.80	0.97
T1	3.55	0.89	3.56	0.79	0.99
H1	N/A	N/A	4.25	1.80*	N/A
B2	4.60	1.22	4.59	0.88	0.99
T2	4.86	0.96	4.89	0.61	0.98
B3	7.73	1.18	7.91	1.14	0.97
H2	N/A	N/A	7.91	1.24	N/A
B4	8.33	1.02	8.47	1.09	0.97
H3	N/A	N/A	8.72	2.05*	N/A

*Upper estimate

Comparing the responses curves on Figs 14 a) and b), with the shaker acting in the vertical and horizontal directions, respectively, a peak appears on Fig 14b) at 4.25 Hz, which is the first transverse (horizontal plane) mode (labelled H1 in Table 2). The same figure shows two unaligned peak at 8.47 and 8.72 Hz, the latter being the second transverse mode (labelled H3 in Table 2). Since those peaks were not previously identified with the ambient tests, the frequency increment of the shaker was not "refined" close the peaks and kept at 0.05 Hz. Damping values are therefore difficult to evaluate here using the half-power bandwidth method, and given in Table 2 as upper estimates (with an asterisk). The actual values are most likely lower for these two transverse modes (H1 and H3).

The peak at 7.91 Hz found on both Fig 14 requires consideration as well. The horizontal response is very strong while the shaker is positioned vertically (Fig. 14 a), but it strongly reduces in amplitude with the shaker acting in the horizontal plane (Fig. 14 b). In both figures, however, the vertical response amplitude is approximately the same for this peak. This can only be the case if two closely-spaced modes are present, and only one is driven when the shaker is in the horizontal direction. Looking at the ambient vibration tests results, the mode identified at 7.73 Hz shows longitudinal motion along with bending, so this "hidden" mode is likely the first longitudinal mode (labelled H2 in Table 2). Placing the shaker in horizontal plane, but in the longitudinal direction (instead of transverse) could have corroborated this assumption. The damping ratio of this mode was evaluated with the longitudinal peak on Fig. 14 a). MAC values were calculated for the vertical modes and show a strong correlation between results from both testing methods.



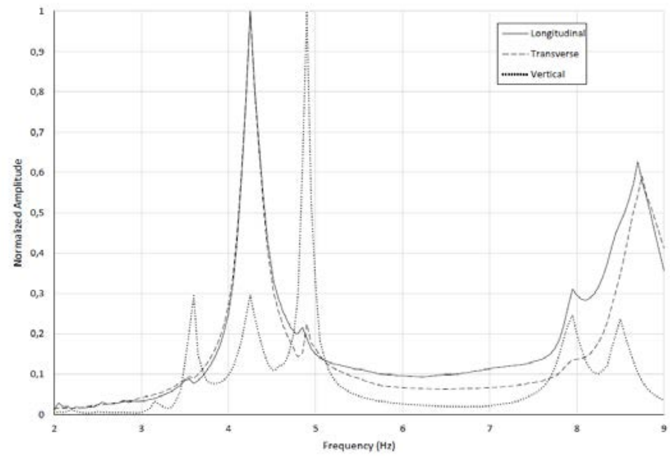


Fig.14 – Frequency responses obtained from forced vibration tests– Bridge 2:
a) vertical excitation and b) horizontal, transverse excitation)

4.3 Bridge 3

Fig. 15 shows the ambient test results, where vibration frequencies are identified, along with the corresponding mode shapes plotted in Fig. 16. Seven modes were found under 10 Hz and are identified in Table 3. In this case, a longitudinal mode was clearly identified (the sixth mode, labelled H2 in Fig 16). Damping values were found to vary from 0.5 to 1.7 %, most of them below 1 %. It is interesting to note that this RC bridge is older, and a study on a large number of bridges reported a reduction of elastic viscous damping over the life of a bridge [6].

Forced vibration test results for this bridge is shown on Fig. 17. Unlike the previous bridge, some of the modes do not produce significant vertical motion. For example, the three horizontal modes at 3.81, 5.54 and 7.39 Hz (H1, H2 and H3, respectively), do not show vertical peaks, which means that the modes shapes are predominantly horizontal. Mode shapes were also extracted from the forced vibration results and MAC values were calculated and again, with horizontal data being used for modes H1, H2 and H3. The higher damping value here for longitudinal mode H2 could be attributed to a higher amplitude of motion at the elastomeric bearings present at abutments.

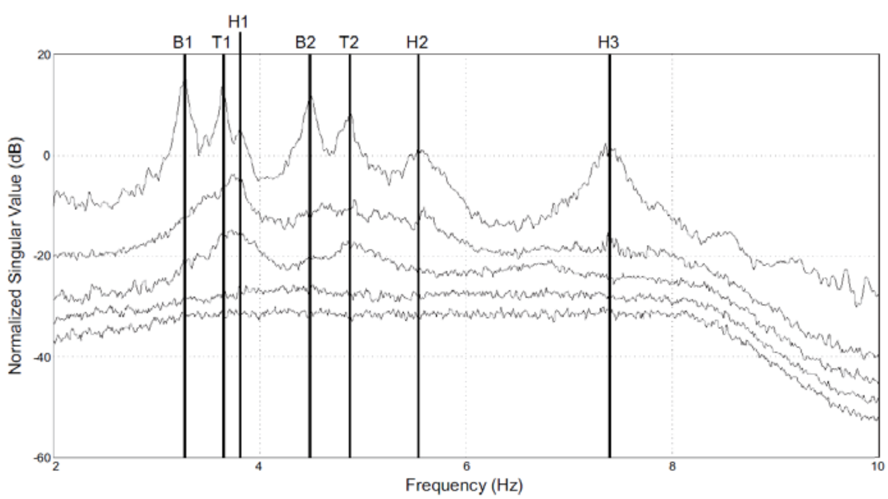


Fig. 15 – FDD results – Bridge 3

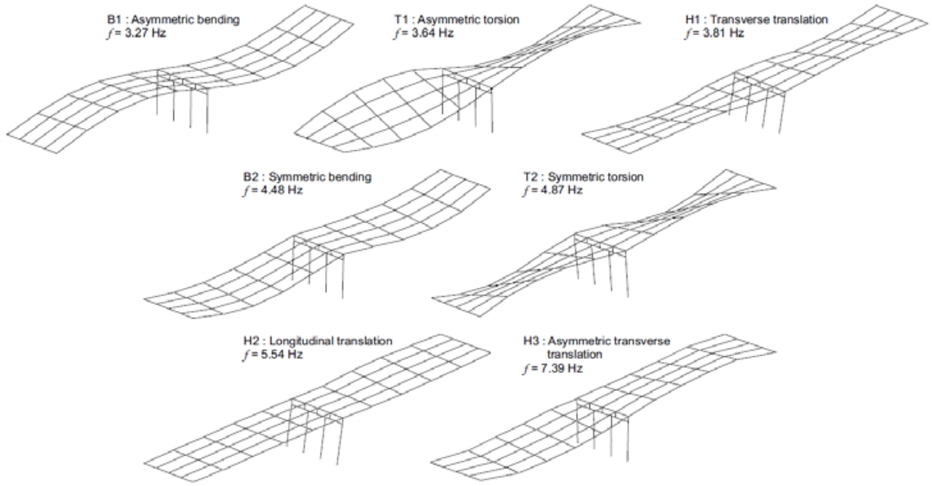


Fig. 16 – Mode shapes obtained with ambient vibration tests – Bridge 3

Table 3 – Comparison of ambient and forced vibration test results: Bridge 3

Vibration mode	Ambient vibration tests		Forced vibration tests		MAC
	Frequency (Hz)	Damping (%)	Frequency (Hz)	Damping (%)	
B1	3.27	0.80	3.26	0.82	1.00
T1	3.64	0.73	3.64	0.71	1.00
H1	3.81	0.54	3.82	0.96	0.98*
B2	4.48	0.88	4.48	1.01	1.00
T2	4.87	0.87	4.85	0.89	0.99
H2	5.54	1.69	5.63	1.71	0.98*
H3	7.39	1.13	7.48	1.38	1.00*

*Horizontal motions were used to determine the Modal Assurance Criterion

4.4 Results Comparison

Tables 1, 2 and 3, show that both testing techniques yield similar results when comparing vibration frequencies and mode shapes. It could be argued that ambient vibration tests should result in lower natural frequencies, because of the added mass of the vehicles. However, the differences are not significant (below 5 % for all nineteen modes found on the three bridges). When it was possible to extract mode shapes from the forced vibrations tests, MAC values showed an excellent correlation between both sets of data for all three bridges.

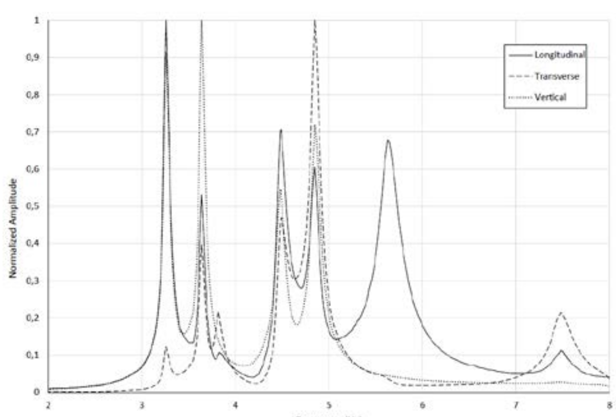


Fig. 17 – Frequency responses obtained from forced vibration tests – Bridge 3
(shaker acting in the vertical direction only)

Damping values show more variation when comparing ambient and forced vibration test results. Modal damping values are sometimes lower when measured by the forced vibration tests, but it is not always the case, especially for Bridge 3. Some studies showed that the measured viscous damping increases with the excitation amplitude [7]. However, it is possible for the differences to be attributed to the different methods used with each sets of data: logarithmic decrement on isolated modal peaks for ambient data, and half-power bandwidth for forced vibration data. With the latter method, the resulting frequency response curves exhibit much less noise (because of the nature of the single frequency input load) and their definition depends on the increment used for the tests. Such well-defined curves lead to better estimates of damping, but require longer and costlier testing methods.

With both testing methods, the measured damping values vary between 0.54 % and 2.43 %, with the majority of values around 1 %. This is much higher than the usual 5 % damping used in most seismic analyses. Although it can be argued that higher values of 5% are justifiable in a non-linear analysis for an extreme seismic event, such high values are perhaps not appropriate for linear investigations (e.g. Performance Based Design). Such low values of experimental damping values are in agreement with other investigations [8, 3, 2, 9, 10].

5. Model calibration

The main objective of this paper was to compare both testing techniques and the quality of their resulting dynamic properties. These properties are often used to calibrate finite element models in order to carry out seismic analysis and parametric studies (on damping effects, for example, which is the subject of an upcoming paper). Following the tests, three-dimensional finite element models were created for the three bridges using the OpenSees software [11] and calibrated with the experimental vibration frequencies and mode shapes.

The bridge piers were modelled with Force-Based Beam-Column elements and using the Cusson-Paultre confinement model [12]. Zero-length elements were used for the various bearings. All other elements (for the beams and decks, diaphragms, abutments, beam cap, foundations) have linear behavior. Calibration was carried out manually with the (linear) initial stiffness, by varying certain parameters (foundation thickness, soil stiffness, bearing stiffness, deck diaphragm stiffness). Table 4 compares the experimental and numerical frequencies, as well as the correlation (MAC values) between experimental and numerical mode shapes. Ambient testing results were used for this comparison, as mode sensors were deployed on the structures for these tests, with the exception of some horizontal modes for Bridge 2 (that were detected only using forced vibration tests). The correlation is quite high for the three models and most modes. Although the calibration process could be enhanced with optimization tools (automatic calibration), a higher level of precision was not required for this project, which was focused on the experimental evaluation of damping.

Table 4 – Comparison of experimental and numerical results

Bridge 1				**Bridge 2**				**Bridge 3**			
Vibration mode	**Frequency (Hz)**		**MAC**	**Vibration mode**	**Frequency (Hz)**		**MAC**	**Vibration mode**	**Frequency (Hz)**		**MAC**
	Tests	FEM			Tests	FEM			Tests	FEM	
B1	3.05	2.99	0.99	**B1**	3.19	3.05	0.94	**B1**	3.26	3.26	0.98
T1	4.58	4.51	0.98	**T1**	3.56	3.56	0.97	**T1**	3.64	3.67	0.99
T2	4.99	4.73	0.96	**H1**	4.25	4.23	0.95*	**H1**	3.82	3.82	0.98**
T3	5.83	5.28	0.97	**B2**	4.59	4.80	0.94	**B2**	4.48	4.69	0.94
H1	8.36	7.95	0.99	**T2**	4.89	5.09	0.98	**T2**	4.85	4.78	0.93
H2	15.65	15.02	0.93	**B3**	7.73	7.74	0.95	**H2**	5.63	5.63	0.99**
				H2	7.91	7.84	0.98*	**H3**	7.48	7.74	0.91**
				B4	8.47	8.43	0.94				
				H3	8.72	8.63	0.92*				

* Forced-vibration tests were used **Horizontal values compared

6. Conclusion

The paper compared two vibration testing techniques applied to three highway bridges. Ambient tests are widely used and can be completed without shutting down the bridge, using normal traffic or wind loads. Forced vibration techniques rely on an input force (shaker) and are costlier, more time consuming and involve shutting down traffic. However, since the input load is controlled in both amplitude and frequency, forced-vibration tests generally produce better frequency responses, and lead to more reliable estimates of certain properties like damping. Also, as was shown for one of the tested structures, some closely-spaced modes that are difficult to find with ambient tests can be identified by using a different excitation direction and different sensor configurations and orientations.

It is the combination of both tests methods that yields the best results. Prior knowledge of vibration frequencies (from ambient tests) indicated ranges where the frequency increments could be reduced in order to obtain clear and well-defined responses curves, therefore leading to better evaluations of modal damping values. Damping values from forced vibration tests were somewhat lower than those calculated from ambient tests, and both methods yielded values that were much lower than the commonly used 5% values. Values below 2% were measured in this study (lower for the steel bridge than the RC bridges), also obtained by other researchers, and this should be considered when carrying out linear analyses of highway bridges.

7. Acknowledgements

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