16th World Conference on Earthquake, 16WCEE 2017
Santiago Chile, January 9th to 13th 2017
Paper N° 1039 (Abstract ID)
Registration Code: S-C1461960518

PROBABILISTIC ANALYSIS OF STEEL FRAME SUBJECTED TO SINGLE AND REPEATED EARTHQUAKES

G.P. Balomenos[(1)], M.D. Pandey[(2)]

[(1)] *Ph.D. Candidate, Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, ON, Canada, gbalomen@uwaterloo.ca*
[(2)] *Professor, Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, ON, Canada, mdpandey@uwaterloo.ca*

Abstract

A 3-story 4-bay steel moment resisting frame (MRF) is analyzed using the finite element analysis (FEA) software OpenSees. The frame is subjected to dynamic analysis, based on selected ground motions of past earthquakes, considering input uncertainties. Based on the fact that time history analysis is time consuming, probabilistic FEA may not be practical for dynamic analysis problems. Therefore, the probabilistic FEA is implemented using the multiplicative dimensional reduction method (M-DRM), which is automated in OpenSees using the Tcl programing language, in order to calculate the statistical moments of the response, e.g., mean and variance. This study shows that M-DRM can be considered as an efficient alternative method for the probabilistic analysis of these types of problems. In addition, this study examines how the same input uncertainties affect the structural response, e.g., inter-story drift, when the structure is subjected to different ground motions. First, the frame is subjected to single ground motion records under uncertainties in material properties. The results in terms of coefficient of variation (COV) indicate that the frame's structural response is not primarily affected from the material variations. However, considering as a random variable the applied mass in each node, the results indicate that the structural response uncertainty is slightly increased and it is observed that the COV of the inter-story drift can be highly increased for a specific earthquake. Taking into account that repeated earthquakes have occurred at brief intervals of time, this study also examines the structural response of an already deformed frame subjected to a subsequent ground motion under input uncertainties. Therefore, the frame is subjected to two repeated earthquakes, i.e., combining the previous single ground motions. The results indicate that the material uncertainty does not seem to affect the inter-story drift variance, while the mass uncertainty does. However, more earthquake scenarios are to be examined, since the proposed framework can be easily applied for probabilistic studies of structures subjected to single and/or repeated earthquakes under input uncertainties.

Keywords: Dynamic analysis, Probabilistic analysis, Multiplicative dimensional reduction method, Steel frame, OpenSees

1. Introduction

Finite element analysis (FEA) is widely used for evaluating the structural performance. However, structural performance may depends on the variability of the input parameters, i.e., material properties, geometry, applied forces, etc., since these values might never be the same even under seemingly identical conditions [1]. Thus, uncertainties can play a major role when it comes to assess the performance of a structure, which can be classified in two main categories known as aleatory and epistemic [2]. For dynamic analysis problems, aleatory uncertainties can be referred to the earthquake loading randomness and epistemic uncertainties can be referred to randomness due to not complete knowledge, e.g., in material properties, model parameters, analysis methods, etc. [3]. Therefore, the structural behavior under seismic loads can be influenced due to variabilities in both ground motion records and scientific assumptions/omissions [4]. Although, the earthquake loading is considered as the most dominant source of uncertainty, research has only partial addressed several epistemic uncertainties [3], while considering epistemic uncertainties for repeated earthquakes scenarios has not been of interest yet.

Probabilistic analysis can be used for incorporating these uncertainties with the FEA, because it allows characterizing the deterministic quantities of interest as random variables [5]. Monte Carlo simulation (MCS) is considered as an easy to implement probabilistic analysis technique, where the FEA code is called repeatedly. However, this approach can be highly computational demanding due to the required repeated analyses [6]. Considering that the time variant nature of the dynamic analysis also requires an enormous computational time [7], the MCS computational cost might be prohibited for analyzing structures subjected to single and/or repeated ground motion records under uncertainties. The multiplicative dimensional reduction method (M-DRM) provides fairly accurate results within a feasible computational time for the probabilistic FEA of structures subjected to time history analysis [8]. Thus, M-DRM is adopted in this study for the nonlinear FEA of a steel moment resisting frame (MRF) subjected to both single and repeated earthquakes under input uncertainties.

The steel MRF provides primarily resistance to the lateral load due to the rigid beam-to-column connection, which does not allow the frame to displace laterally without the beams and columns having bend [9]. Thus, the MRFs are popular in high seismicity areas for several reasons, such as high ductility and architectural versatility. However, the 1994 Northridge earthquake resulted to more than 100 failures of steel beam-column connections [10], while the 1995 Kobe earthquake highlighted the severity of the problem [11]. Apart the principal earthquake effect on the structure, steel MRFs can experience further damage when subjected to one or more aftershocks, within a short period of time following the mainshock [12]. Thus, the current study will also investigate the effect of repeated earthquakes on the structural response due to input uncertainties.

2. Multiplicative dimensional reduction method

In probabilistic FEA the structural response, e.g., inter-story drift, is evaluated as a function of several input variables as $Y = h(\mathbf{x})$, where Y is the FEA response and $\mathbf{x}$ is the vector of input random variables, i.e., $\mathbf{x} = \mathrm{x}_1, \mathrm{x}_2, \dots, \mathrm{x}_\mathrm{n}$. Using M-DRM the response function is approximated as:

$$Y = h(\mathbf{x}) \approx h_0^{(1-n)} \times \prod_{i=1}^{n} h_i(x_i) \tag{1}$$

where, h_0 is the response when all random variables are fixed to their mean value, n is the number of random variables and $h_i(x_i)$ is an i^{th} one-dimensional cut function $(i = 1,2,\dots,n)$. Then, a k^{th} moment of the response function can be approximated using the previous M-DRM approximation as:

$$E[Y^k] \approx E\left[\left(h_0^{(1-n)} \times \prod_{i=1}^{n} h_i(x_i)\right)^k\right] \tag{2}$$

where, $E[.]$ is the mathematical expectation operation, e.g., $E[Y]$ is the mean value for $k = 1$. For independent input random variables Eq. (2) can be written as:

$$E[Y^k] \approx h_0^{k(1-n)} \prod_{i=1}^{n} E\left[\left(h_i(x_i)\right)^k\right] \tag{3}$$

Then, considering the mean and the mean square of an i^{th} cut function as $\rho_i = E[h_i(x_i)]$ and $\theta_i = E\left[\left(h_i(x_i)\right)^2\right]$, respectively, and with the use of Eq. (3), the mean (μ_Y) and mean square (μ_{2Y}) of the response are approximated as:

$$\mu_Y = E[Y] \approx h_0^{(1-n)} \times \prod_{i=1}^{n} \rho_i$$
$$\mu_{2Y} = E[Y^2] \approx h_0^{(2-2n)} \times \prod_{i=1}^{n} \theta_i \tag{4}$$

The evaluation of the mean $(k = 1)$ or any other k^{th} moment of the response requires the calculation of a k^{th} moment of all the cut functions through one dimensional integration. This integration is optimized using the Gauss quadrature formulas as:

$$E\left[\left(h_i(x_i)\right)^k\right] = \int_{X_i} [h(x_i)]^k f_i(x_i) dx_i \approx \sum_{j=1}^{L} w_j \left[h_i(x_j)\right]^k \tag{5}$$

where, L is the number of the Gauss quadrature points, x_j and w_j are the coordinates and weights, respectively, of the Gauss quadrature points $(j = 1,2,\dots,L)$ and h_i $(i = 1,2,\dots,n)$ is the response when an i^{th} cut function, i.e., input random variable, is set at a j^{th} Gauss quadrature point. Then, the variance of the response is calculated as [2]:

$$V_Y = \mu_{2Y} - (\mu_Y)^2 \approx (\mu_Y)^2 \times \left[\left(\prod_{i=1}^{n} \frac{\theta_i}{\rho_i^2}\right) - 1\right] \tag{6}$$

The standard deviation of the response (σ_Y) is then calculated as the square root of the variance and the coefficient of variation (COV) of the response is finally calculated as the ratio of the standard deviation to the mean. The coefficient of variation is dimensionless and is considered as positive even though the mean may be a negative value. Small value of the response's COV indicates a small amount of uncertainty in the response due to the input random variables [13].

The benefit of M-DRM is the low computational cost since M-DRM combined with the Gaussian quadrature reduces remarkably the total number of evaluations of the response function. M-DRM requires $(nL + 1)$ function evaluations to calculate all the moments, where 1 corresponds to the function evaluation for which all the input random variables are set equal to their mean values. For instance, a problem with 10 random variables and a 5-point Gauss quadrature scheme will require 51 function evaluations.

3. Steel moment resisting frame

A 3-story 4-bay structure is selected from literature [14], which represents a hypothetical office building located in Vancouver (BC, Canada), has a symmetric structural layout and consists of four steel MRFs located at its perimeter (Fig. 1). All 3-stories and 4-bays are each 3.96 m high and 9.14 m wide (center-to-center), respectively. Only the East-West direction of the MRF is considered in this study, resulting to the 2D analysis of a 3-story 4-bay steel MRF (Fig. 2). The columns of the steel MRF are fixed to the ground level, while it is assumed to have rigid beam-to-column connections. The steel MRF is connected with a fictitious leaning column through rigid links at each story level, in order to take into account the effect of the interior gravity frames for the FEA. The seismic weight distribution is shown in Fig. 2, from which the seismic weight is equal to 4567 kN for the floor 2 and 3 and equal to 4850 kN for the roof.

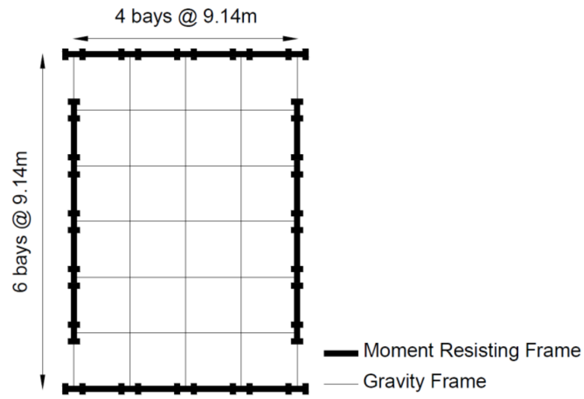

Fig. 1 – Plane view of the 3-story 4-bay building showing the moment resisting frames and the gravity frames

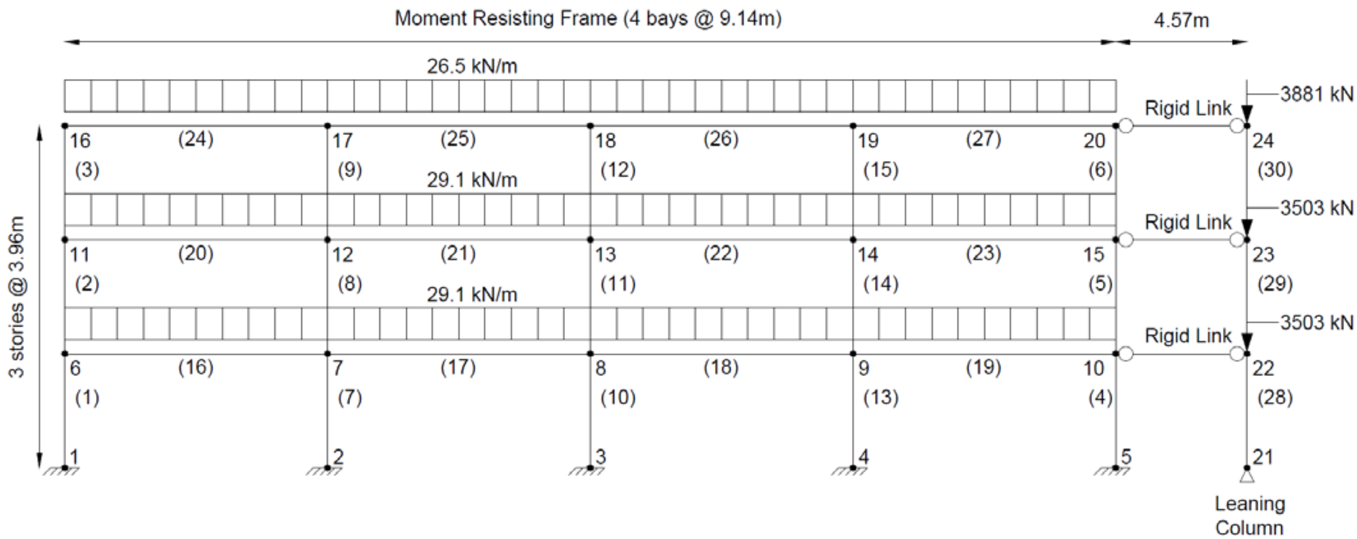

Fig. 2 – Side view of East-West direction of the steel moment resisting frame showing geometry, seismic weight distribution, node numbers and element numbers (in parenthesis)

The steel MRF consists of 27 wide-flange section members, with 345 MPa grade steel and 248 MPa grade steel for the columns and beams, respectively, where this difference in the grade steel is based on ductility consideration [14]. In the original study [14], a capacity design optimization was performed resulting to an optimum design of W310x158 for exterior columns, W360x179 for interior columns, W610x82 for floor 2 beams, W530x66 for floor 3 beams and W460x82 for roof beams. The basic dimensions of each selected cross section are presented in Table 1, while more information can be found in the Canadian Handbook of Steel Construction [15]. The gravity columns were considered as hollow structural sections HSS254x254x13 [15], while the leaning column has a cross sectional area equal to 89,000 mm^2 and a moment of inertia equal to 845×10^6 mm^4 [14].

The frame is modeled and analyzed using the OpenSees FEA software [16]. The force-based beam-columns elements are employed (with five Gauss-Labbato numerical integration points) and the steel behavior is modeled using the bilinear elastoplastic stress-strain relationship with 5% hardening [17]. The summation of the mass of all the members connected to a joint result to the lumped mass at a joint of the frame. For the time history analysis are selected the same ground motions with the original study [14]. These ground motions were taken from the PEER Strong Motion Database (http://peer.berkeley.edu/smcat/) and were scaled based on the design response spectrum for Vancouver, as specified by the National Building Code of Canada [18], such that their response spectra should be equal or bigger than the design response spectrum throughout the period of interest. The adopted ground motions together with the scale factors are shown in Table 2.

Table 1 – Beams and columns cross sections

Variable	Steel section	Depth (mm)	Flange Width (mm)	Flange Thickness (mm)	Web Thickness (mm)
Exterior columns	W310x158	327	310	25.1	15.5
Interior columns	W360x179	368	373	23.9	15.0
Floor 2 beams	W610x82	599	178	12.8	10.0
Floor 3 beams	W530x66	525	165	11.4	8.9
Roof beams	W460x82	460	191	16.0	8.9

Table 2 – Selected earthquakes records for the steel MRF

Earthquake	Station	Magnitude	PGA (g)	Record duration (sec)	Scale Factor
1979 Imperial Valley	El Centro Array #12	6.5	0.143	39.01	2.1
1989 Loma Prieta	Belmont Envirotech	6.9	0.108	39.99	4.5
1989 Loma Prieta	Presidio	6.9	0.2	39.985	1.5

4. Probabilistic analysis

4.1 Single earthquakes under material uncertainty

First, are considered as uncertain the material properties only, i.e., the modulus of elasticity E, the yield strength f_y and the hardening ratio b (ratio between post-yield tangent and initial elastic tangent). Each member of the steel MRF is assigned one random variable for each material property. These random variables are independent and identically distributed across the steel MRF members (Table 3). Thus, in total there are 81 (27×3) independent input random variables. The fifth-order Gauss Hermite integration scheme is adopted, resulting to the M-DRM method with 81 × 5+1 = 406 FEA trials. The frame is subjected to each of the three previous earthquakes, considering the same input material uncertainties. In this study, the response of the steel frame in terms of inter-story drift is recorded. The M-DRM approximation is used for the calculation of the inter-story drift statistics (Table 4). The results indicate that the material uncertainty does not play a significant role to the response variance, since it has been estimated a coefficient of variation (COV) less than 1.5% for each time history analysis.

Table 3 – Statistical properties for material random variables

Parameter	Distribution	Mean	COV	Reference
E of steel columns and beams (27 RVs)	Lognormal	200,000 N/mm^2	5.0%	[19]
f_y of steel columns (15 RVs)	Lognormal	345 N/mm^2	10%	[19]
f_y of steel beams (12 RVs)	Lognormal	248 N/mm^2	10%	[19]
b of steel columns and beams (27 RVs)	Lognormal	0.05	10%	[19]

Notes: RVs = random variables; COV = coefficient of variation

Table 4 – Inter-story drift statistics: Steel MRF subjected to single earthquakes under material uncertainty

	El Centro		Belmont		Presidio	
Location	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**
Story 1	1.40	1.25	1.08	1.00	1.26	0.89
Story 2	2.39	0.51	1.67	1.16	2.02	0.26
Story 3	2.08	0.95	2.43	0.74	2.01	0.57

4.2 Single earthquakes under mass uncertainty

The mass at each node of the steel MRF is considered as uncertain, only. Each node takes the half mass of each element, which is framing to that node, and this value is considered as the mass mean value for each node. The dead loads, which form the mass of a structure, can be considered as random variables with a COV equal to 0.1 [20]. Thus, the mass at each node is assumed to have a lognormal distribution with a 10% COV, resulting to 18 (18×1) independent input random variables in total, since they are not correlated. The M-DRM method requires 18 × 5+1 = 91 FEA trials, since the fifth-order Gauss Hermite integration scheme is adopted. The inter-story drift statistics (Table 5) show an increased COV compared to the material uncertainty results. Especially, for the earthquake 1989 Loma Prieta (Belmont Envirotech) the COV of the story 2 inter-story drift is 13.70%. Thus, the mass uncertainty may play an important role to the response uncertainty compared to the material uncertainty. In addition, the results indicate the importance of the ground motion selection in these types of analyses, since there can be a significant variation to the COV of the response. For example, for the 1989 Loma Prieta earthquake the inter-story drift of story 2 has a 2.05% COV using the Presidio station ground motion, while this COV is highly increased to 13.70% using the Belmont Envirotech station ground motion.

Table 5 – Inter-story drift statistics: Steel MRF subjected to single earthquakes under node mass uncertainty

	El Centro		Belmont		Presidio	
Location	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**
Story 1	1.41	5.92	1.11	4.60	1.26	1.80
Story 2	2.37	5.12	1.68	13.70	2.03	2.05
Story 3	2.05	7.96	2.41	7.85	2.01	2.75

4.3 Repeated earthquakes under material uncertainty

It has been observed that structures can be subjected to repeated earthquakes, which may occur at brief time intervals [21]. Thus, the steel MRF is subjected to two repeated earthquakes, i.e., combining the aforementioned scaled ground motion records (Fig. 3). A time gap, i.e., zero ground acceleration, is applied between the two hypothetical seismic events, in order to cease the moving of the structure due to damping [22]. This time gap is assumed to be equal to 40 seconds [21]. For instance, the hypothetical sequence for applying the 1979 Imperial Valley (El Centro Array #12) earthquake followed by the 1989 Loma Prieta (Belmont Envirotech) is shown in Fig. 3, where t denotes the duration of each ground motion record (Table 2).

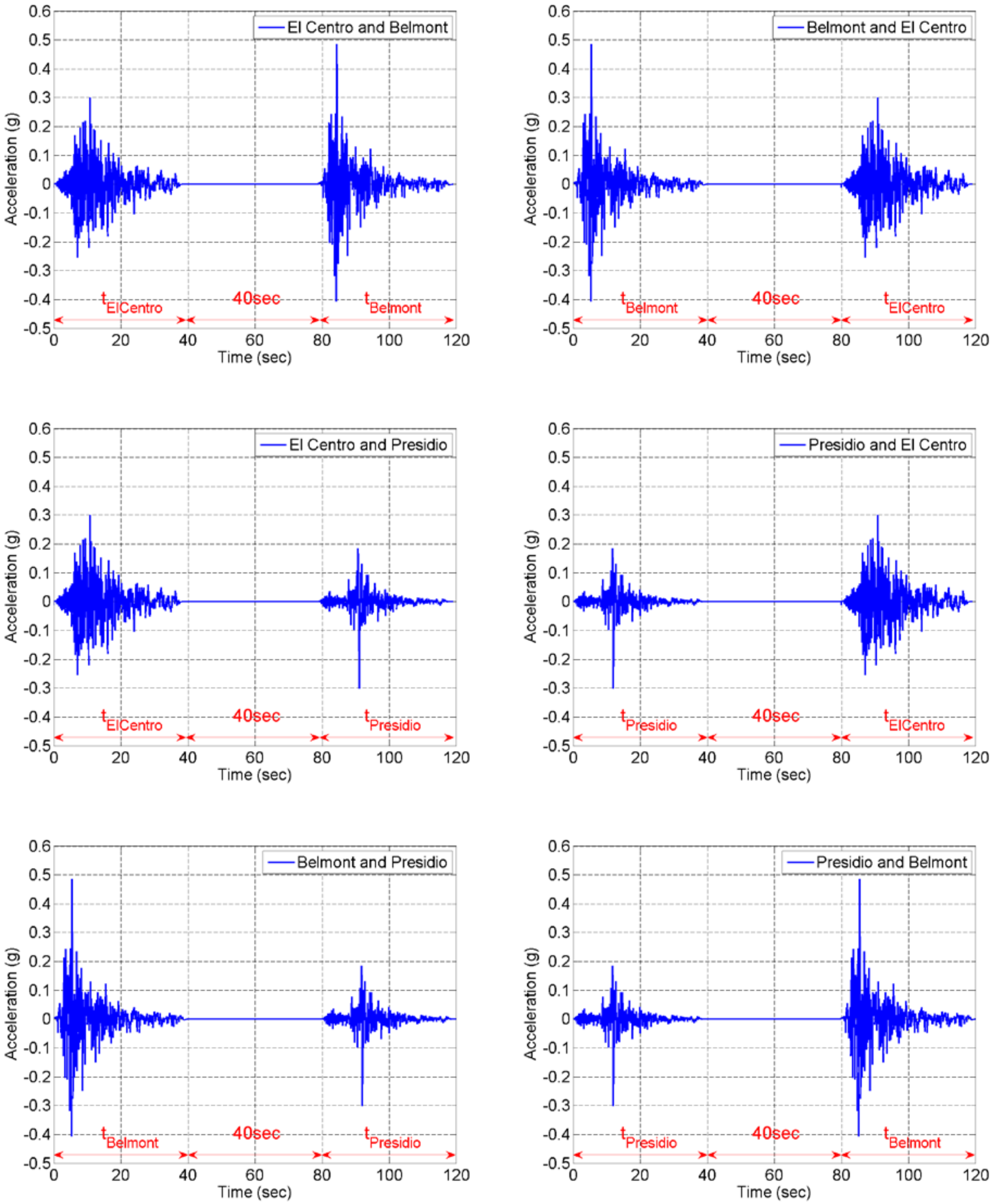

Fig. 3 – Seismic sequence of scaled ground motion records

The steel MRF is subjected to these repeated earthquakes considering as random variables the same material properties with the previous analyzed single event (Table 3). M-DRM is implemented with 406 FEA trials for each seismic sequence. It is observed that there is almost no increase in the COV of the inter-story drift (Table 6), compared to the single event results under material uncertainty (Table 4). Thus, the total response uncertainty is not primarily affected by the material uncertainty, for the repeated earthquakes scenarios.

Table 6 – Inter-story drift statistics: Steel MRF subjected to repeated earthquakes under material uncertainty

	El Centro and Belmont		**Belmont and El Centro**		**El Centro and Presidio**		**Presidio and El Centro**		**Belmont and Presidio**		**Presidio and Belmont**	
Location	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**
Story 1	1.40	1.25	1.31	1.12	1.40	1.43	1.50	1.57	1.16	0.92	1.26	0.89
Story 2	2.39	0.51	2.23	0.76	2.39	0.51	2.55	0.51	1.87	0.55	2.02	0.26
Story 3	2.28	0.83	2.43	0.74	2.17	0.89	2.25	0.84	2.43	0.74	2.29	0.82

4.4 Repeated earthquakes under mass uncertainty

The steel MRF is subjected to the previous hypothetical scenarios of repeated earthquakes, considering only the node masses as random variables, similar to the previous analyzed single event (Section 4.3). The inter-story drift statistics (Table 7) have an increased COV compared to the material uncertainty results under repeated earthquakes. Applying the 1989 Loma Prieta (Belmont Envirotech) earthquake followed by the 1979 Imperial Valley (El Centro Array #12) predicts the biggest COV for the inter-story drift, indicating the importance of the selected earthquakes and the major role of the mass uncertainty to the outcome response. In addition, the selected order of the applied ground motion record, i.e., which earthquake will be applied first and which second, does not seem to highly affect the uncertainty of the output response.

Table 7 – Inter-story drift statistics: Steel MRF subjected to repeated earthquakes under mass uncertainty

	El Centro and Belmont		**Belmont and El Centro**		**El Centro and Presidio**		**Presidio and El Centro**		**Belmont and Presidio**		**Presidio and Belmont**	
Location	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**	**Mean (%)**	**COV (%)**
Story 1	1.41	5.92	1.32	8.09	1.42	5.33	1.50	4.67	1.17	3.09	1.26	1.64
Story 2	2.37	5.12	2.22	7.97	2.37	5.04	2.53	3.95	1.89	3.32	2.03	2.05
Story 3	2.32	5.81	2.41	7.50	2.17	4.76	2.21	6.17	2.41	7.82	2.27	7.29

4.5 Computational time

The single time history analysis of structures usually requires an enormous computational cost. Thus, the probabilistic dynamic analysis of repeated earthquakes can be a highly demanding computational task. M-DRM seems to overcome this challenge, since the required trials can be performed within a feasible computational time for both single (Table 8) and repeated earthquakes (Table 9), using a personal computer with Intel i7-3770 3rd Generation Processor and 16GB of RAM. For instance, M-DRM with 406 FEA trials and 91 FEA trials requires less than 12.5 hours and 3 hours, respectively, for each hypothetical scenario of repeated ground motion records. Therefore, M-DRM can be considered as an efficient tool for the probabilistic FEA of structures subjected to single and/or repeated earthquakes.

Table 8 – Computational time using M-DRM: Single earthquakes

	Computational time	
Single Earthquakes	**Material uncertainty (406 FEA trials)**	**Mass uncertainty (91 FEA trials)**
El Centro	104 min	25 min
Belmont	150 min	37 min
Presidio	98 min	35 min

Table 9 – Computational time using M-DRM: Repeated earthquakes

	Computational time	
Repeated Earthquakes	**Material uncertainty (406 FEA trials)**	**Mass uncertainty (91 FEA trials)**
El Centro and Belmont	717 min	122 min
Belmont and El Centro	737 min	122 min
El Centro and Presidio	667 min	134 min
Presidio and El Centro	660 min	131 min
Belmont and Presidio	660 min	172 min
Presidio and Belmont	645 min	168 min

5. Conclusions

This study presents the multiplicative form of dimensional reduction method (M-DRM), for the probabilistic analysis of structures subjected to both single and repeated earthquakes. Based on the Gauss quadrature scheme, M-DRM reduces significantly the required FEA trials. Thus, the statistical moments (mean and variance) of the structural response are obtained within a feasible computational time, making M-DRM an efficient alternative for the probabilistic analysis of high computational demanding problems. In this study, the M-DRM trials have been automated in OpenSees using Tcl programming, where the Gauss quadrature scheme requires the change of only one random variable per FEA trial. Therefore, for a small number of input random variables, M-DRM trials can be executed without using a programming language to automate the procedure within the deterministic FEA software. Taking advantage of the M-DRM computational efficiency, several dynamic analyses of a steel moment resisting frame (MRF) are performed considering as random variables the material properties and the node mass. The results indicate that the response variance, in terms of coefficient of variation (COV), is not primarily affected by the material uncertainty of the steel MRF subjected to single or repeated earthquakes. However, mass uncertainty may affect the variance of the structural response for a specific single earthquake scenario, i.e., for the 1989 Loma Prieta (Belmont Envirotech) the COV of the story 2 inter-story drift was highly increased to 13.70%. The mass uncertainty also may affect the variance of the structural response for the repeated ground motion scenarios. For example, applying the Belmont Envirotech station ground motion record followed by the Presidio, the COV of the story 3 inter-story drift was calculated equal to 7.82%. In general, mass uncertainty mostly contributes to the variance of the inter-story drift, compared to the material uncertainty. However, more single and repeated earthquakes scenarios under input uncertainties are to be examined.

6. Acknowledgements

The financial support for this study was provided by the Natural Science and Engineering Council of Canada (NSERC) and the University Network of Excellence in Nuclear Engineering (UNENE), which is gratefully acknowledged by the authors.

7. References

[1] Benjamin JR, Cornell AC (1970): *Probability, Statistics and Decision for Civil Engineers*. McGraw-Hill, 1st edition.

[2] Ang A H-S, Tang WH (2007): *Probability Concepts in Engineering: Emphasis on Applications in Civil & Environmental Engineering*. Wiley, 2nd Edition.

[3] Kazantzi AK, Vamvatsikos D, Lignos DG (2014): Seismic performance of a steel moment-resisting frame subject to strength and ductility uncertainty. *Engineering Structures*, **78**, 69-77.

[4] Vamvatsikos D (2014): Seismic performance uncertainty estimation via IDA with progressive accelerogram-wise Latin hypercube sampling. *Journal of Structural Engineering*, **140** (8), A4014015.

[5] Ditlevsen O, Madsen H (1996): *Structural reliability methods*. Wiley, 1st Edition.

[6] Papadrakakis M, Kotsopoulos A (1999): Parallel solution methods for stochastic finite element analysis using Monte Carlo simulation. *Computer Methods in Applied Mechanics and Engineering*, **168** (1-4), 305-320.

[7] Clough RW, Penzien J (1992): *Dynamics of structures*. McGraw-Hill, 2nd edition.

[8] Balomenos GP, Pandey MD (2016): Finite element reliability and sensitivity analysis of structures using the multiplicative dimensional reduction method. *Structure and Infrastructure Engineering*, **12** (12), 1553-1565.

[9] Bruneau M, Uang C-M, Whittaker A (1998): *Ductile design of steel structures*. McGraw-Hill, 1st edition.

[10] Christopoulos C, Filiatrault A, Uang C-M, Folz B (2002): Posttensioned energy dissipating connections for moment-resisting steel frames. *Journal of Structural Engineering*, **128** (9), 1111-1120.

[11] Gupta A, Krawinkler H (1999): Seismic demands for performance evaluation of steel moment resisting frames. *Technical Report No 132*, John A. Blume Earthquake Engineering Center, Stanford University, CA, USA.

[12] Li Q, Ellingwood BR (2007): Performance evaluation and damage assessment of steel frame buildings under main shock–aftershock earthquake sequences. *Earthquake Engineering & Structural Dynamics*, **36** (3), 405-427.

[13] Haldar A, Mahadevan S (2000): *Probability, Reliability and Statistical Methods in Engineering Design*. Wiley, 1st Edition.

[14] Xue Y (2012): Capacity design optimization of steel building frameworks using nonlinear time-history analysis. *PhD Dissertation*, Department of Civil and Environmental Engineering, University of Waterloo, Waterloo, ON, Canada.

[15] CISC (2012): *Handbook of Steel Construction*. Canadian Institute of Steel Construction, Willowdale, ON, Canada.

[16] McKenna F, Fenves G L, Scott MH (2000). *Open System for Earthquake Engineering Simulation*. University of California–Berkeley, Berkeley, CA, USA.

[17] Mazzoni S, McKenna F, Scott MH, Fenves GL, et al. (2007). *OpenSees command language manual–version 2*. University of California–Berkeley, Berkeley, CA, USA.

[18] NRCC (2010). *National Building Code of Canada*. National Research Council of Canada, Ottawa, ON, Canada.

[19] Haukaas T, Scott MH (2006). Shape sensitivities in the reliability analysis of nonlinear frame structures. *Computers & Structures*, **84** (15-16), 964-977.

[20] Ellingwood BR, Galambos TV, MacGregor JG, Cornell CA (1980). *Development of probabilities based load criterion for American national standard A58*. National Bureau of Standards, Washington, DC, USA.

[21] Amadio C, Fragiacomo M, Rajgelj S (2003). The effects of repeated earthquake ground motions on the non-linear response of SDOF systems. *Earthquake Engineering and Structural Dynamics*, **32** (2), 291-308.

[22] Hatzigeorgiou GD, Beskos DE (2009). Inelastic displacement ratios for SDOF structures subjected to repeated earthquakes. *Engineering Structures*, **31** (11), 2744-2755.