



P-DELTA EFFECTS IN THE TORSIONAL RESPONSE OF STRUCTURES

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Abstract

Nonlinear dynamic analysis is an accepted procedure to assess the performance of building structures during earthquakes. Several documents have emerged to provide guidance in terms of mathematical modeling, ground motion selection and scaling, and acceptability of results. Due to computational advances, one of the newer requirements provided by these standards is to perform a three dimensional analysis and to include P-Delta effects. Unfortunately, the same provisions do not provide details on methods for incorporating P-Delta effects into the mathematical model, and as a result important response characteristics, including the potential for global torsional collapse may be overlooked. The issue at hand is the potential for not including or improperly modeling the P-Theta effect, which is an amplification of rotations about the vertical axis due to gravity loads. In this paper, the P-Theta effect is investigated for a torsionally irregular nine-story buckling restrained braced frame system. Three methods for incorporating the P-Delta and P-Theta effects are illustrated. The first method, which uses a single leaning column at the building's center of mass, properly includes P-Delta effects but does not capture P-Theta effects. The second method uses four leaning columns, each located at the centroid of a quadrant of the buildings. This method captures P-Delta effects and P-Theta effects, although the influence of P-Theta effects is underestimated. Finally, each column of the structure, including gravity columns, is explicitly modeled, and geometric stiffness is assigned to the column based on its tributary gravity load. This method is deemed the most accurate, and captures detrimental behavior, including collapses, that the other methods miss.

Keywords: *Nonlinear Analysis, Accidental Torsion, Geometric Nonlinearities, Codes and Standards*

1. Introduction

Nonlinear dynamic response history (NLRH) analysis is becoming an accepted procedure to assess the performance of building structures during earthquakes. In support of this trend, several documents [1-5] have emerged to provide guidance in terms of mathematical modeling, ground motion selection and scaling, and specification/evaluation of acceptance criteria. Additionally, several standards (or prestandards) [6-11] provide specific requirements for performing such analysis. A review of these documents has indicated various areas of agreement and disagreement. One of the most striking areas of disagreement is related to methodologies required to capture accurate three-dimensional response, and more specifically, whether or not accidental torsion is required. However, ASCE 7-16 [12] explicitly requires the three-dimensional modeling, and for torsionally irregular systems, incorporation of accidental torsion when NLRH is performed.

The inclusion of accidental torsion for torsionally irregular systems in ASCE 7-16 was in part based on research performed by DeBock et al. [13] where using the FEMA P-695 Methodology [14], it was found that accidental torsion is not warranted in the design of torsionally regular structures in low to moderate seismic hazard areas, but is needed for torsionally irregular structures in high seismic hazard areas. Another study that had an important influence on the decision of including accidental torsion in NLRH analysis was presented by Flores et al. [15]. This investigation showed that accidental torsion significantly increased global displacements at the edges of the building, and can strongly influence dynamic instability, especially when the structure is torsionally irregular. An important factor in both studies was the treatment of P-Delta effects, and particularly the P-Theta effect which is the amplification of global torsional rotation about the vertical axis due to destabilizing gravity load effects.

In the new research presented in this paper, the influence of three-dimensional P-Delta effects in NLRH analysis is investigated through the evaluation of the response of a 9-story steel building with Buckling Restrained Braces (BRB) used to resist lateral loads. The building was designed considering accidental torsion requirements given by ASCE7-10 [7]. The structure is regular in plan and height and has an extreme torsional irregularity. Three mathematical models of the building were developed, wherein the only difference is the method used to incorporate P-Delta effects. The influence of P-Delta modeling on the building's torsional response is illustrated by performing nonlinear static pushover and nonlinear dynamic analyses. Accidental torsion was induced in all analysis, either by applying the lateral loads at an eccentricity (pushover) or by shifting the location of the center of mass (dynamic).

2. Second Order Effects in 3-D Structural Analysis

Most analysts recognize the importance of including second-order effects. These effects cause amplification of lateral displacements (the P-Delta effect) as well as amplification of system torsional rotation (referred to herein as the *P-Theta effect*, where Theta is the global rotation about the vertical axis). Wilson and Habibullah [16] provide a thorough discussion of such effects, and present approximate methods for incorporating them in analysis of building systems with idealized rigid diaphragms.

If both the gravity system and the lateral system are physically modeled in 3-D analysis, both the P-Delta and the P-Theta effects are automatically captured when element stiffness formulations include geometric stiffness. Such models with spatially distributed columns may be used for both rigid and semi-rigid diaphragm idealizations. If the gravity columns are not included in the model, the destabilizing gravity loads tributary to these columns must be accounted for, and this is often done using a "leaner column" which, in essence, lumps some or all of the P-Delta effects into one element with zero elastic stiffness and negative geometric stiffness. It is very important to note, however, that use of a single leaner column at the center of mass will not capture P-Theta effects at all, and the use of a few leaner columns at inappropriate locations may not capture P-Theta effects entirely. In both cases the result is that important torsional response will be underestimated. Where the system is initially torsionally irregular the misuse of leaner columns may fail to recognize cases where torsion induces structural collapse.

This paper specifically investigates the role that P-Theta effects play in assessing the torsional response of buildings. This is done by performing analysis with and without P-Theta effects, and comparing the computed

response. Additionally, both the lateral and torsional stability coefficients are computed and presented for each system. The lateral stability coefficients, Q_Δ , are determined by performing a static linear-material analysis for the structure under gravity and lateral loads, without accidental torsion, with and without P-Delta effects, and computing the following quantity at each story level:

$$Q_\Delta = 1 - \frac{\Delta_o}{\Delta_f} \quad (1)$$

where Δ_o is the story drift computed without P-Delta and Δ_f is the story drift including P-Delta. The torsional stability ratios, Q_θ , are determined by running a static linear-material analysis for the structure loaded with gravity load and accidental story torques, with and without P-Theta effects, and computing the following quantity at each story level:

$$Q_\theta = 1 - \frac{\theta_o}{\theta_f} \quad (2)$$

where θ_o is the difference in torsional rotations at the top and bottom of the story without P-Theta and θ_f is the same quantity computed for analysis that includes P-Theta. Lateral deflection and torsional rotation amplifiers, λ_Δ and λ_θ , respectively, are determined as follows:

$$\lambda_\Delta = \frac{1}{1 - Q_\Delta} \quad (3)$$

$$\lambda_\theta = \frac{1}{1 - Q_\theta} \quad (4)$$

3. Building Description

The system analyzed, illustrated in Figure 1, is nine stories tall, with a rectangular plan consisting of two 30-ft bays on one direction, and eight 30-ft bays in the other direction. The system was adapted from a similar square-plan building described in the ATC 76 project [17], and analyzed for the purpose of assessing torsional performance in Flores et al. [15]. For the study reported herein the design was revised because in the ATC 76 project the displacement amplification factor (C_d) was taken equal to the response modification factor ($R=8$). In the current design, C_d was taken equal to the value given by ASCE7-10 [7] which is 5. This difference had an important influence in the design of the buckling restrained braced (BRB) frames because it changed from a structure design controlled by drift to one controlled by strength.

The lateral load resisting system consists of four bays BRB frames, with two bays in each direction (Figure 1). The yielding core in the BRBs use ASTM A992 steel with nominal yield strength of 50 ksi. Three variations of this system are investigated, wherein the *only difference* between the systems is the placement of the leaning columns to capture P-Delta effects. All systems have BRBs on gridlines A and C to resist loads in the E-W direction and for N-S loads the BRBs are positioned only along gridlines 4 and 6 as shown in Figure 1. Based on ASCE 7-10 definitions, the system has a Type-1b (extreme) torsional irregularity. The building was designed considering the effects of accidental torsion during the design stage. Following what is specified by ASCE7-10, loads were applied with accidental eccentricity equal to 5% the building's width and interstory drifts were checked at the corners. This system from now will be called Model A-2 (this is one of several models investigated in a broader study that will be discussed in a future paper). The member sections of the building are shown in Table 1. The difference of the member sections between the BRBs in the N-S and E-W direction is due to the influence of accidental torsion in the N-S direction.

The gravity system consists of a metal deck and concrete floor slab supported by an assembly of steel beams and columns. The total seismic weight of the system, W , is 14490 kips, which represents a weight density of 8.6 pounds per cubic foot. The design floor load was 100 psf dead and 50 psf live. Seismic design was based on ASCE 7-10 and the 2005 AISC Seismic Specification [18]. Design level spectral accelerations S_{DS} and S_{DI} are

1.0g and 0.6g, respectively. These are the Seismic Design Category D_{max} spectral accelerations in FEMA P-695 [14].

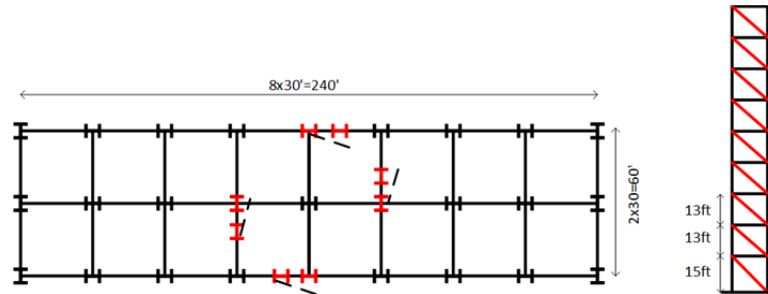


Figure 1 Structural System Analyzed (Model A-2)

Table 1 Model A-2 Member Sections

Model A-2						
Level	North-South			East-West		
	Column	Beam	BRB (in ²)	Column	Beam	BRB (in ²)
Roof	W14x370	W21x62	9	W14x120	W21x62	7
Level 8	W14x370	W24x76	9	W14x120	W24x76	7
Level 7	W14x370	W24x76	10	W14x120	W24x76	8
Level 6	W14x398	W24x76	10	W14x193	W24x76	8
Level 5	W14x398	W27x94	11	W14x193	W24x84	9
Level 4	W14x455	W27x94	11	W14x311	W24x84	9
Level 3	W14x455	W27x94	11	W14x311	W24x84	9
Level 2	W14x500	W27x94	12	W14x342	W24x84	10
Level 1	W14x500	W27x94	12	W14x342	W24x84	10

4. Mathematical Model

Each system was analyzed in three dimensions using OpenSees [19]. Floor diaphragms were assumed rigid in-plane and flexible out of plane. The gravity system was included in the analysis, but did not contribute to the lateral strength and stiffness. P-Delta effects were included using three different approaches, each incorporating the P-Delta transformation within OpenSees. The first, as illustrated in Figure 2a, used a "leaning column" at the center of the building, wherein the P load on the column represented the entire gravity load of the system. This method includes only the translational P-Delta effect, and in the remainder of this paper it is referred to as P- Δ . The second approach was the same as the one taken by DeBock et al. [13] in their study about accidental torsion. The method incorporates, as displayed in Figure 2b, four "leaning columns" placed at the centroid of each building quadrant, wherein the P load on the column represented 25% of the gravity load of the system. This method includes translational P-Delta effects and, as demonstrated later, partially includes the rotational P-Theta effects. Thus from now on in this study, this approach is referred as P- $\Delta\theta_p$ where the symbol θ_p in the second case indicates that torsional P-Delta effects are included partially in addition to P- Δ . The third approach distributes the story P loads using the tributary area to each of the individual columns. In this method the torsional and P-Theta and translational P-Delta are incorporated entirely, so for the remainder of this study this approach is referred as P- $\Delta\theta$. The influence of the different P-Delta modeling approaches on the computed response is the main topic of investigation of this paper.

Material nonlinearities were included in the beams, columns, and braces of the BRB systems. Beams and columns were modeled using displacement control fiber elements and the BRBs were modeled using a phenomenological model. The analytical approach used to model the BRB system is discussed in detail in the study by Atlayan [20]. In the dynamic analyses the effect of accidental torsion was introduced by modifying the diaphragm mass distribution such that the desired mass eccentricity was achieved. Inherent damping was

modeled as Rayleigh damping by setting the critical damping ratio to 2% at the fundamental and fifth modes of the structure.

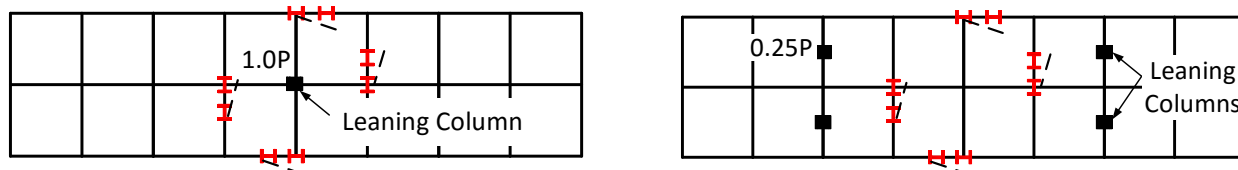


Figure 2 Modeling P-Delta effects a) Model with P- Δ effects b) Model with P- $\Delta\theta_p$ effects

5. Torsional Properties of Systems

The lateral and rotational stability coefficients were computed for each story of the building (Eqs. 1 and 2), and the maximum values along the height computed for Model A-2 are reported in Table 2. Also provided are the corresponding amplification factors (Eqs. 3 and 4).

Table 2 Stability Coefficients and Amplification Factors

System	Q_Δ	λ_Δ	$Q_{\theta p}$	$\lambda_{\theta p}$	Q_θ	λ_θ
A-2	0.032	1.033	0.152	1.179	0.258	1.347

The lateral stability coefficient for system A-2 is 0.032. Note that ASCE 7-05 and newer versions of this standard require that P-Delta analysis be included only where the maximum lateral stability ratio is greater than 0.10. Thus, in this case the building would not require P-Delta to be included. Moreover, ASCE 7 does not require P-Theta evaluations of any kind. However, as noted from Table 2 the torsional stability coefficient is greater than 0.10 for both cases: when rotational P-Theta is included partially and when it is included completely. The torsional stability for the latter case is very significant at 0.258 indicating an increase in static plan-wise torsional rotations by a factor of 1.347 due to P-Theta effects alone. With a lateral stability coefficient this high (larger than 0.25), ASCE7 would require the design to be modified. Note also that there is a 14% increment when the two methods used to incorporate P-Theta effects are compared.

Another indicator of the likelihood of torsional response is the torsional irregularity factor (TIF) which is equal to the maximum ratio, over all stories, of the building edge inter-story drift to the building center inter-story drift when an equivalent lateral load is applied at a 5% eccentricity. In ASCE 7 this factor is used to determine if a torsional irregularity occurs. If the TIF is greater than 1.2 and less than 1.4 the building is torsionally irregular, and if the ratio is greater than or equal to 1.4 the system is extremely irregular. The TIFs computed for Model A-2 with different approaches for including geometric nonlinearity are shown in Table 3. It is interesting to note that the P- Δ model reports slightly lower TIFs than those obtained when no geometric nonlinearity is used. However the TIFs increase progressively when P- $\Delta\theta_p$ and P- $\Delta\theta$ are included respectively.

Table 3 Torsional Irregular Factors

Model	No P- Δ or P- θ	P- Δ	P- $\Delta\theta_p$	P- $\Delta\theta$	Irregularity?
A-2	1.93	1.90	2.02	2.12	Extreme

As a final check of the torsional sensitivity of the buildings, the first three periods of vibration were computed with and without an accidental eccentricity, and using all the P- Δ the P- $\Delta\theta_p$ and the P- $\Delta\theta$ methods. The accidental eccentricity was induced by moving the mass a certain percentage with respect to the centroid of the building. The results of the analysis are presented in Table 4.

Table 4 Periods of Vibration Model A-2 (seconds)

Model	Mode	Model Analyzed with No Accidental Torsion				Mode Type	Model Analyzed with 5% Accidental Torsion				Mode Type
		No P- Δ or P- Θ	P- Δ	P- $\Delta\Theta_p$	P- $\Delta\Theta$		No P- Δ or P- Θ	P- Δ	P- $\Delta\Theta_p$	P- $\Delta\Theta$	
Model A-2	1	4.538	4.559	4.869	5.177	Torsion	4.623	4.586	4.623	5.267	Torsion
	2	2.928	3.068	3.068	3.068	Lateral	2.928	3.068	2.928	3.068	Lateral
	3	2.501	2.586	2.586	2.586	Lateral	2.456	2.537	2.456	2.543	Lateral

From Table 4, when no accidental torsion is included, it can be seen that the first mode is torsional, with a period of 4.538 s when geometric nonlinearity is not included. The first mode period barely increases when the P- Δ model is used (4.559 s), but increases to 4.869 s and 5.177 s when the P- $\Delta\Theta_p$ and P- $\Delta\Theta$ models are used, respectively. It is clear from the results that the P- Δ model influences the lateral modes, but not the torsional modes. It is noted also that the torsion mode for System A-2 is significantly larger than the lateral modes, an indicator of extreme torsional flexibility. By comparing the periods of vibration between the models that includes and does not include a mass eccentricity of 5% it may be seen there is a slight increase in the torsional period for each of the geometric nonlinearity assumptions when accidental torsion is incorporated.

6. Nonlinear Static Pushover Analyses

Prior to performing NLRH analysis on the systems, a series of nonlinear static pushover analyses were performed to evaluate the influence of modeling P-Delta effects. All pushover analyses were analyzed using displacement control with a first-mode lateral load distribution. The displacements were measured at the roof at the centroid of the building. The pushover is an analysis that takes to building to failure so it would not matter if it is measured at the centroid or the building's corner. However for the dynamic analysis was measured at the corner. Gravity load consisting of 1.05 times the dead load plus 0.25 times the live load and was applied prior to lateral loading. In these analyses two basic parameters were varied: a) the magnitude of accidental eccentricity as a percentage of the length perpendicular to the direction of load, and b) whether or not lateral loads were applied simultaneously in the orthogonal directions considering one of the loads (E-W direction) to be a percentage of the other load (N-S direction). The results of the pushover analyses for each of the cases are shown in the following subsections.

6.1 Including P-Delta Effects (P- Δ)

The model analyzed in this section is the one that includes only translational P- Δ effects. In order to evaluate these effects on the torsional response in the first series of analyses the lateral load was applied in the N-S direction only, at some eccentricity to the right from the center of mass. The results of these analyses are shown in Figure 3a. This figure also includes the pushover curve when the P- Δ is not considered in the analyses to demonstrate its importance. From the pushover curves it can be seen that induced torsion in the building causes early yielding but the post yield stiffness remains the same for all the values of accidental torsion. Figure 3b illustrates the influence of bidirectional loading on the extremely irregular System A-2, when the N-S loading eccentricity is 3% of the building width, and orthogonal (E-W) loading is applied at the center of mass at some percentage of the full load. The orthogonal load in the E-W direction is applied as a percentage of the full load applied in the N-S direction. This fraction of the load is going to be called from now on Orthogonal Load Factor (OLF) and it is considered to give an idea of how vulnerable the building is when is subjected to both ground motion components. The results shown in Figure 3b display the effect of having loads applied in both orthogonal directions and it is clear that for this model, which includes only translational P- Δ effects, there is no change in the pushover curve. Even for an OLF equal to 60%, the results are the same.

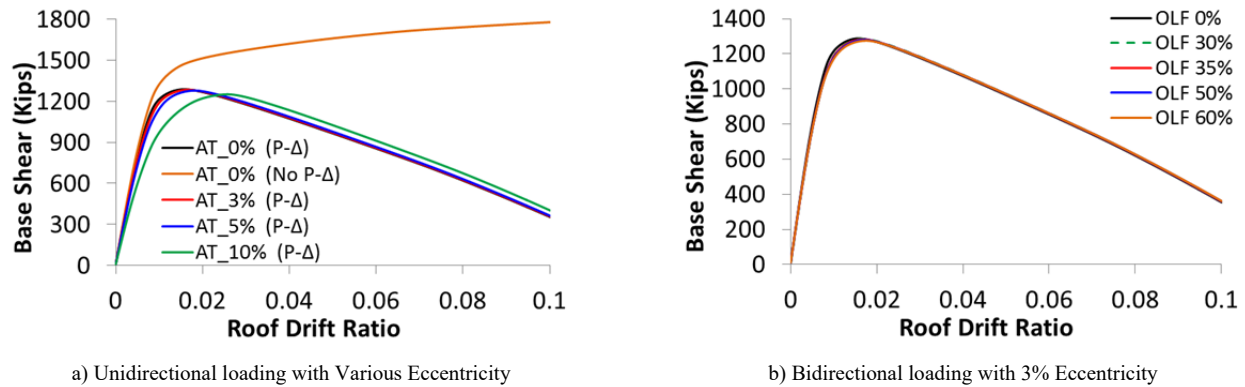


Figure 3 Pushover Curves Including P- Δ effects

6.2 Including P-Delta effects and partial P-Theta effects

The second model to be analyzed using a nonlinear static approach is the one that includes translational and partially includes the torsional P-Delta effects (P- $\Delta\theta_p$). The same methodology described in the previous sections was followed to load the building and induce a torsional response. The results of the loads applied in one and both directions are shown in Figure 4 a) and b). Figure 4a presents the results of applying the load unidirectionally in N-S direction with different accidental torsions. The pushover curves obtained up to an accidental torsion equal to 7% are the same as the ones shown in the previous case. However, after this point, larger amounts of accidental torsion produce a sudden change (bifurcation) where the strength of the system degrades rapidly. The results for the second analysis are shown in Figure 4b where it can be seen that unlike the results obtained for the previous case, the bifurcation of the pushover curves occurs at an OLF equal to 45%. An explanation for the sudden strength degradation is provided in Section 7 of this paper.

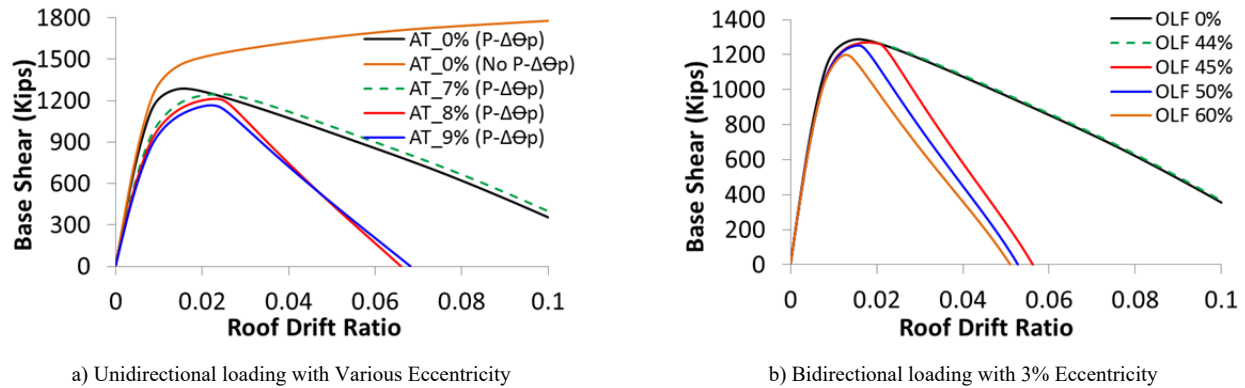


Figure 4 Pushover Curves Including P- $\Delta\theta_p$ effects

6.3 Including P-Delta and P-Theta effects

The third case to be analyzed is the one that includes translational and rotational P-Delta effects (P- $\Delta\theta$). From the previous results it is clear now that the methodology used to model P-Delta effects have a significant influence on the torsional response of a building. The influence of including all the gravity columns to incorporate P-Delta effects is seen in Figure 5. For the analysis in one direction, the bifurcation of the pushover curves occurs only at 4% of accidental torsion (Figure 5a). On the other hand, the OLF required to cause the sudden strength reduction when both orthogonal loads are applied simultaneously is equal to 23%. An OLF this low could mean that the building's dynamic response is susceptible when both ground motions are applied simultaneously.

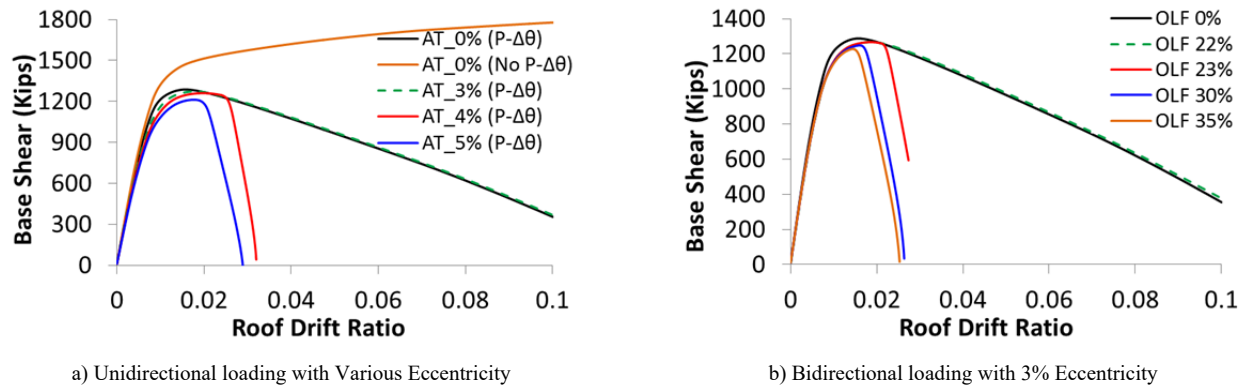


Figure 5 Pushover Curves Including P- $\Delta\theta$ effects

The results presented for all three cases demonstrated the importance of modeling P-Delta effects on the torsional response of a structure. Modeling with one leaning column is clearly erroneous and modeling with just four leaning columns fails to capture significant structural deficiencies.

7. P-Theta Effects on Nonlinear Static Pushover Analyses

The previous sections showed a sudden change in the strength of the building when a certain amount of eccentricity was used to induce a torsional response to the building. In order to explain the reason for this behavior the sequence of yielding when the pushover analysis is performed is analyzed. Figure 6 displays the Model A-2 sequence of yielding for a unidirectional pushover analysis with an eccentricity equal to 3% and 4%. These accidental eccentricities are the limit point where the bifurcation of the pushover curve occurs. The colored lines drawn next to the BRB frames are correlated directly to the points drawn in the pushover curves and they represent the different states of the structure. The number (N) placed next to the colored lines represents the number of BRBs that are yielding at that specific point in the pushover analysis.

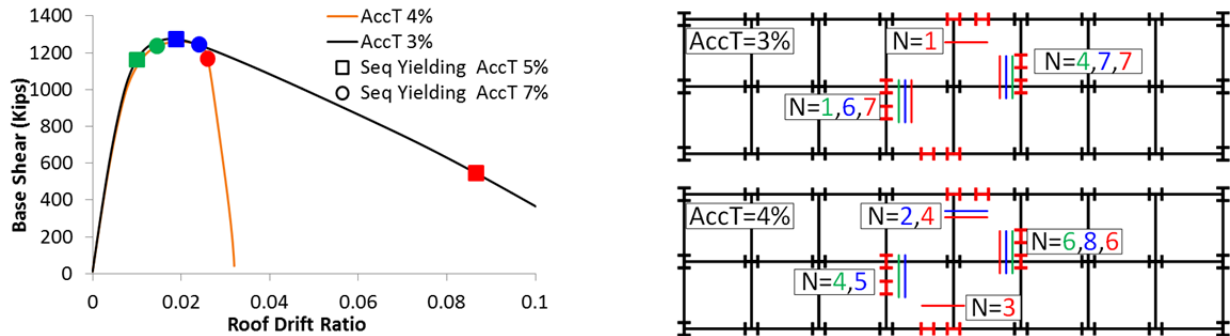


Figure 6 Sequence of Yielding Model A-2 including P- $\Delta\theta$ effects

By comparing the state of the system at the red dot and square in Figure 6, the bifurcation occurrence is explained. It can be seen that for a 3% eccentricity just one of the orthogonal BRB frames (E-W direction) is yielding while for 4% eccentricity, both orthogonal BRB frames are yielding. Therefore, the sudden degradation of the system capacity is due to yielding of the frames in the orthogonal direction. This outcome was also seen by De la Llera and Chopra [21] in their investigation.

8. Nonlinear Dynamic Analyses

While nonlinear static pushover analysis clearly illustrated the importance of adequately model P-Delta effects and P-Theta, NLRH provide additional insight into the performance of the system. This section shows the results of the NLRH analysis where each model was subjected to 11 ground acceleration recordings, representing 11 actual earthquake events selected from the P-695 Far-field record set listed in Table 5. The number of ground motions to be used in this investigation is the minimum required by ASCE7-16 when NLRH is to be performed.

Table 5 Input Ground Motions

Earthquake		PGA (N-S)	PGA (E-W)	Earthquake		PGA (N-S)	PGA (E-W)
1	Cape Mendocino-Rio Dell	0.45g	0.32g	7	Manjil-Abbar	0.41g	0.39g
2	Duzce-Bolu	0.52g	0.46g	8	Northridge-BH	0.34g	0.27g
3	Hector-Hector	0.37g	0.29g	9	Northridge-CC	0.40g	0.34g
4	Kobe-Nishi Akashi	0.52g	0.52g	10	San Fernando-LA	0.44g	0.37g
5	Kocaeli-Duzce	0.25g	0.22g	11	Superstition Hills-Poe	0.52g	0.35g
6	Landers-Yermo	0.24g	0.15g				

The horizontal component with the largest peak ground acceleration was selected for use for N-S direction shaking, and each component was amplitude scaled for consistency with MCE_R level shaking at the lateral period of vibration in the N-S and E-W direction. For each analysis the system was first subjected to gravity load, followed by ground shaking and the drifts were the largest measured at the corners of the building. As with the pushover analysis the parameter varied for the analyses was the amount of accidental eccentricity. The results shown in the following subsection are for the structures subjected to both components of ground motions simultaneously. Due to page limit restrictions the results of the structures subjected to only one ground motion component are not shown in this paper. However, the influence of subjecting the structure to both components is significant, as demonstrated in Flores et al. [15].

8.1 Including P-Delta Effects

NLRH analysis was performed on each of the models with different approaches of including P-Delta effects. In order to evaluate the effects of P-Delta on the torsional response, an accidental torsion was introduced to the model by moving the center of mass a certain eccentricity while maintaining a constant total mass. The eccentricities at which the structures were subjected to were 3%, 5% and 7%. The roof drift time history for Model A-2 with translational P- Δ effects subjected to Duzce-Bolu and Landers-Yermo are shown in Figure 7 a) and b) respectively. It can be seen that in both cases the influence of the torsional response is minimum. Thus if only P- Δ effects are included it would not matter if accidental torsion is taken into consideration.

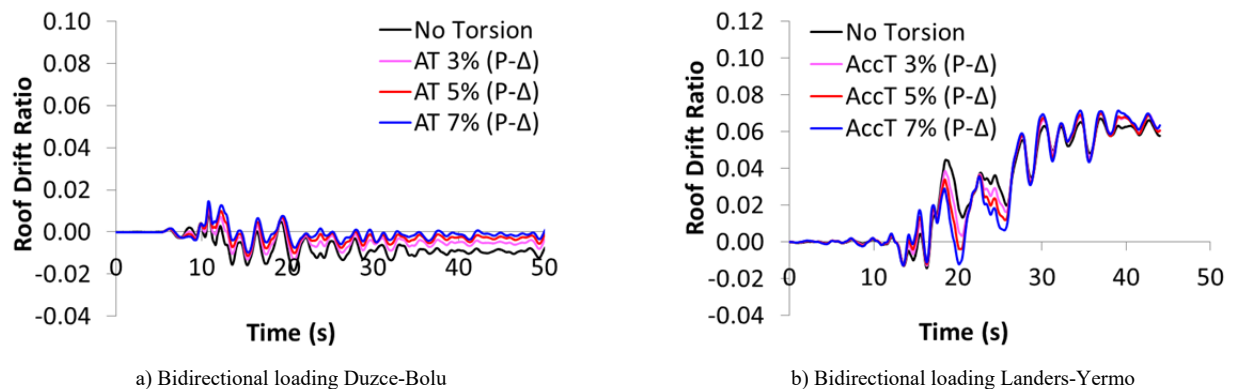


Figure 7 Total Drift Time History Response Model A-2 including P- Δ effects

8.2 Including P-Delta effects and partially P-Theta effects

The building analyzed to incorporate P- $\Delta\theta$ effects was subjected to the same ground motions as the first case. The results of these analyses are shown in Figure 8 a) and b). Figure 8 a) illustrates the effects of accidental torsion when the structure is subjected to the Duzce-Bolu earthquake. It can be seen that in this case the torsional effects worsen the response because P- θ effects are included. A similar response occurred when the building was subjected to the Landers-Yermo ground motion. For the latter case, an accidental torsion of 7% is causing according to FEMA 350 [22] collapse because it has a drift of 10%.

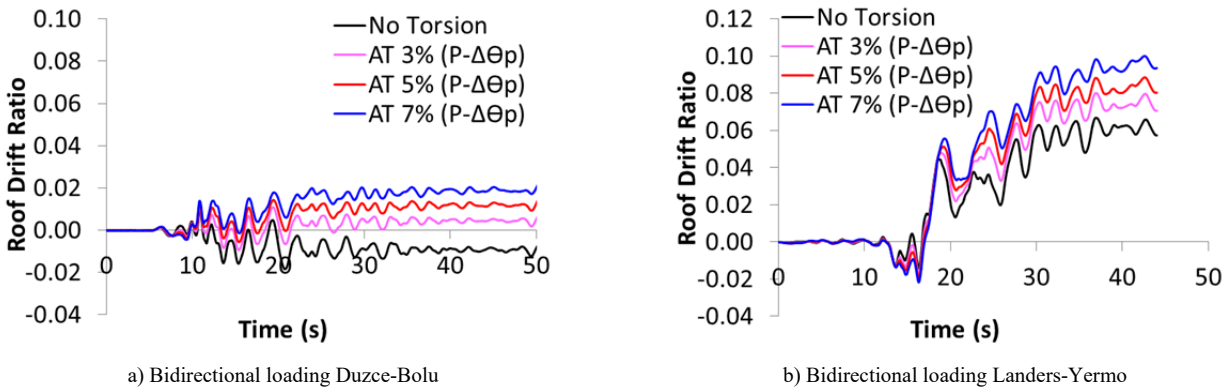


Figure 8 Total Drift Time History Response Model A-2 including $P-\Delta\theta_p$ effects

8.3 Including P-Delta and P-Theta effects

From the previous two cases it was seen that torsional effects worsen the building's response when $P-\theta$ effects are being considered even if it is just partially. Where $P-\Delta$ and $P-\theta$ effects are fully represented by modeling the geometric stiffness of all columns, the building was subjected to the same ground motions and the results for Duzce-Bolu and Landers-Yermo are shown in Figure 9 a) and b). Unlike the other two approaches, for this case the Duzce-Bolu ground motion is causing dynamic instability when accidental torsion is included. The same is occurring for Landers-Yermo where for all accidental eccentricities the building is collapsing.

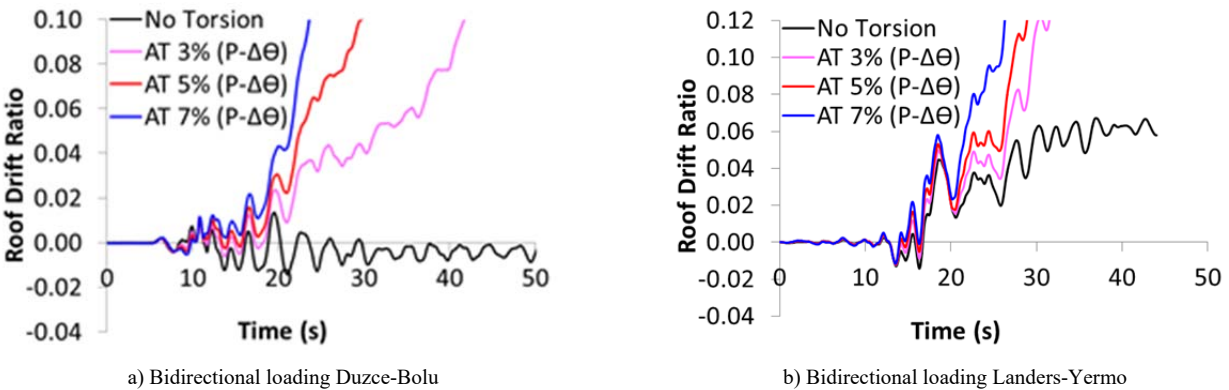


Figure 9 Total Drift Time History Response Model A-2 including $P-\Delta\theta$ effects

The results obtained from the NLRH demonstrated once more the fundamental importance of modeling $P-\theta$ effects adequately when the torsional response is to be evaluated. The effects of accidental torsion went from no effect whatsoever when only $P-\Delta$ effects are included to collapse of the structure when $P-\Delta\theta$ effects were incorporated adequately. The results were improved when $P-\Delta\theta_p$ was included in the model, but important response characteristic were underestimated. These results followed the same trend for all the 11 ground motions that were analyzed and this is shown in Table 6 with a summary of the collapses occurred for all the cases. Table 6 presents all the collapses that occurred for each of the approaches used to incorporate $P-\Delta$ and $P-\theta$ effects. The lack of the model to predict any collapses is a serious shortcoming of using a single leaning column at the center of the building. Using four leaning columns at the middle of the four building quadrants is an improvement, but $P-\theta$ effects are incorporated just partially. For this case only 1 collapse occurred for all the accidental eccentricities. Using four leaning columns might be sufficient but more research is required to define the exact location and gravity load for the columns. Finally it can be seen that if $P-\theta$ effects are incorporated adequately the number of collapses is equal to 6 for an accidental torsion equal to 5%. It is important to point out that 5% is the accidental torsion usually given by codes and that the number of collapses allowed by ASCE7-16 is one among the 11 ground motions analyzed. Therefore failing to include correctly these effects significantly overestimates the building response.

Table 6 Summary of Collapses under Dynamic Loading

	Model A-2			
Accidental Torsion	0%	3%	5%	7%
Bidirectional (P-Δ)	0	0	0	0
Bidirectional (P-Δθp)	0	1	1	1
Bidirectional (P-Δθ)	0	5	6	7

9. Conclusions

This study focused on the consequences of including P-Delta and P-Theta effects in analysis of the torsional response of structures. Three approaches for including such effects were considered. The first approach used only one leaning column at the centroid of the building, including only translational P-Δ effects. The second approach was the same as used in the study by DeBock et al., where only four leaning columns were placed at the centroid of each of the building's quadrant. This approach incorporates completely translational P-Δ effects but just partially the rotational P-θ effects. The third and last approach used all the gravity columns as leaning columns to incorporate completely translational P-Δ and rotational P-θ effects. The results obtained from using the three approaches when nonlinear static and dynamic analyses were performed are significantly different. In the case of the pushover analyses, a bifurcation point appeared when P-θ effects are incorporated, even when only partially included. This bifurcation point, where a sudden strength degradation occurred, was a result of the orthogonal frames yielding. This behavior was not seen at all for the first approach, it occurred for an accidental torsion equal to 8% for the second approach and an accidental torsion equal to 4% for the last method. The bifurcation seen on the pushover curves had a detrimental effect on the NLRH analysis. The incorporation of P-Δθ effects into the analyses caused the collapse of the structure for several cases. This was not seen at all for the first approach and it occurred only once when the P-θ effects were included partially.

After performing all the analyses and observing and analyzing all the results, it can be concluded that inclusion of P-θ effects are essential when a model is analyzed in three dimensions, especially if torsional response is to be evaluated. The approach of using four leaning columns might be viable by placing them in specific locations that capture rotational P-Delta effects adequately. However, it would be difficult to implement this approach for structures that have a complex geometry. Thus, the only rational approach is to use the "Full System Modeling" method wherein each gravity column is explicitly modeled in the correct location with the correct vertical load. Where the column is not part of the lateral load resisting system the stiffness and strength need not be included, although there might be an advantage to include this as shown by the studies by Flores et. al.[23, 24].

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