



NUMERICAL MODEL UPDATING OF THE DUOC-UC FOOTBRIDGE (CONCEPCIÓN, CHILE) USING EXPERIMENTALLY EXTRACTED DATA

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Abstract

Calibration of numerical models has become indispensable to reduce error in the prediction of structural performance of complex systems. Several methods have been developed in the last two decades to calibrate a numerical model, but the implementation of these procedures in engineering common practice is still infrequent. Hence, the analysis-verification-calibration-design cycle is usually incomplete.

In this study, a 52-m long footbridge located in Concepcion (Chile) was model using finite elements software. The footbridge consists of two 6.5 m high arches built of laminated timber. The main arch elements (curved beams) have a rectangular section of 1100 x 200 mm. A steel structure is attached to the curved beams, with columns spaced every 4 m. This steel structure supports a 100 mm thick timber slab with an 80 mm thick top layer of asphalt.

An operational modal analysis was conducted to dynamically characterize the footbridge. Accelerometers were attached at different points on the footbridge and one-hour ambient tests were performed on different days and hour times. Stochastic Subspace Identification (SSI) and Frequency Domain Decomposition (FDD) were employed to determine the dynamic properties of the structure (modal frequencies, damping and mode shapes). The experimental results were compared with the modal response calculated by the numerical model. Results are discussed in terms of the similitude between experimental and theoretical response. Finally, a model updating procedure was proposed to improve numerical model predictions.

Keywords: Modal analysis, model updating, system identification.

1. Introduction

Footbridges are generally slender structures that may experience large displacements or excessive vibrations due to ambient excitations (pedestrian flow, vehicles traffic, wind, earthquakes, etc.). These structures are usually designed using computational models that verify the strength of each of the constituent elements and, generally, strength conditions are easily satisfied. However, it is not exceptional finding structures that experienced vibrations which amplitude or frequency generates discomfort in users. This undesired structural behavior is usually not predicted by the numerical models because they do not necessarily represent the actual response of the structure in an exact manner [1]. Moreover, numerical models are generally not validated by any mean. Hence, it is impossible that these models can trustfully predict the actual structural performance if they are not calibrated using experimental measurements on the real structure.

Tools used to calibrate numerical models are modal testing and model updating. These methods consider measuring the low intensity vibrations that experienced structures due to ambient excitations [2-3]. The modal parameters that dynamically characterize the structures (frequencies, mode shapes and damping) are extracted from these operational vibration records. This information is then compared to the response predicted by numerical models and the models are adjusted to match the response measured on the real structure.

Our study aims to identify the modal properties (modal frequencies and mode shapes) of a pedestrian arch bridge built in laminated timber and steel. These experimental data will be then employed to obtain an improved numerical model of this.

2. Description of the structure

DUOC-UC footbridge is located over Avenida Paicaví, in front of the UCSC Medical Center in Concepción, Chile. The footbridge consists of two 52 m long and 6.5 m height arches. The main elements of the arch are four curved beams made of laminated timber of radiata pine hinged at both ends (supports and middle length of the arches). These beams have a rectangular section of 1100 x 200 mm. A steel structure is attached to the main beams, with columns spaced every 4 m. This steel structure supports a 100 mm thick radiata pine slab with an 80 mm thick top layer of asphalt (Fig. 1).

This footbridge is highly transited, because it connects two educational institutions (UCSC and DUOC-UC) with a main road. The peak of pedestrian flow occurs between 12:00 and 14:00 hours and between 17:00 and 19:00 during week days, especially from March to December. The peak of vehicular flow under the bridge occurs at the same time frame.



Fig. 1 – DUOC-UC footbridge

3. Experimental campaign

3.1 Instrumentation

The structure was instrumented using accelerometers (Memsic CXL04GP3) attached to the asphalt top-layer of the footbridge's traffic slab, avoiding to locate them on the potential antinodes of the slab pre-identified by an initial numerical model described in Section 4 of this paper (Fig. 2). These transducers are able to measure in the three main directions, with a sensitivity of $500 \pm \text{mV/g}$, in a range of $\pm 4\text{g}$ up to 100 Hz. The data acquisition system is constituted by a NI9205 voltage module mounted on a cDAQ-9174 chassis, manufactured by National Instruments. The accelerometers were connected to the data acquisition system by multipolar cables which length range from 10 to 50 m. These cables were shielded with aluminium paper, in order to avoid a signal contamination from high a voltage network located in the vicinity of the footbridge. Data recording was controlled by a routine programmed on a LabView platform. The measuring campaign was conducted in three different days as it is presented in Table 1.

Table 1 – Information of data collection

	Test 1	Test 2	Test 3
Date	19/03/2015	06/04/2015	07/04/2015
Starting time	16:30	16:30	15:30
Finishing time	17:30	17:30	16:30

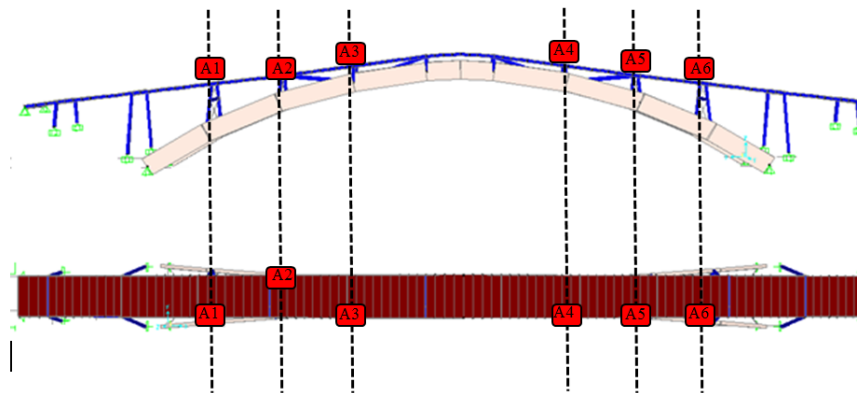


Fig. 2 – Position of the accelerometers. (a) Lateral view, and (b) Plane view.

3.2 Modal parameters identification

Two system identification methods were employed to determine the modal properties of the structure: Stochastic Subspace Identification (SSI) and Frequency Domain Decomposition (FDD). The SSI method [4] is a data-driven time-domain technique that employs QR-factorization and singular value decomposition to identify the matrices of the dynamic state-space model. Once the state space model of the structure is found, the modal parameters (natural frequencies, damping ratios and mode shapes) can be determined by eigenvalues decomposition. In general, it is not possible to determine the system order beforehand. Therefore, it is necessary to repeat the analysis with different system orders and verify the repeatability of the results. This procedure is performed by constructing stabilization charts (Fig. 3). In this graph, the dots represent the fundamental frequencies of the poles (modes) identified considering models with different system orders (SO). The red dots are associated with those frequencies that are similar to another frequency detected in the precedent model, while the blue circles around the dots represents those poles that have a similar mode shape to a pole detected in the precedent model. Those poles that reveal stability in terms of similar frequencies and mode shapes (usually aligned in a vertical column in the graph) are very likely to represent vibration modes.

The FDD method [5] is an extension of the classical peak-picking method. The FDD algorithm assumes that the excitation applied on the structure has a random nature and can be described as a white-noise. Thus, the excitation power spectral density function (PSD) becomes a constant (S) and, consequently, the FRF peaks can

be directly identified from the peaks of the response PSD function. These peaks on the PSD function are assumed as resonant frequencies and mode shapes can be determined by applying Single Value Decomposition procedures. The PSD curves obtained for this experiment are presented in Fig. 4.

The SSI and FDD methods were able to identify 9 modes. In general, the modal frequencies obtained by both methods coincide as can be observed in Table 2.

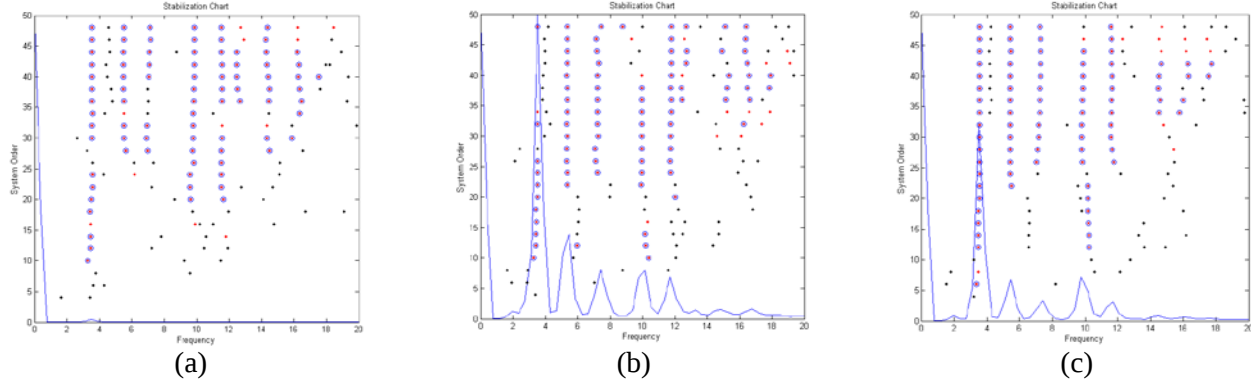


Fig. 3 – Stabilization charts of (a) Test 1, (b) Test 2 and (c) Test 3.

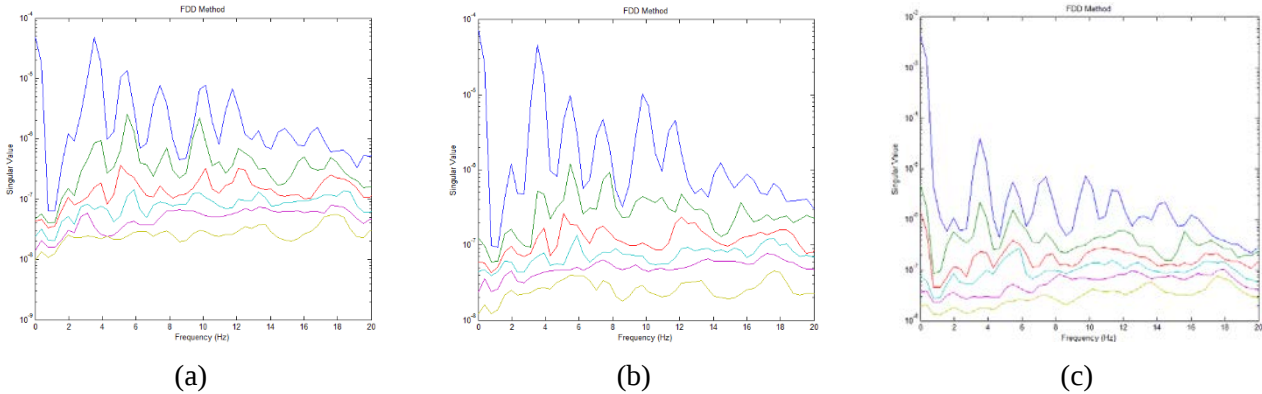


Fig. 4 – Power spectral density curves of (a) Test 1, (b) Test 2 and (c) Test 3

Table 2 – Experimental modal frequencies

Mode	SSI					FDD					Difference (%)
	Test 1 (Hz)	Test 2 (Hz)	Test 3 (Hz)	Average (Hz)	CoV (%)	Test 1 (Hz)	Test 2 (Hz)	Test 3 (Hz)	Average (Hz)	CoV (%)	
1	3.56	3.52	3.54	3.54	0.61	3.52	3.52	3.52	3.52	0.00	0.6
2	5.51	5.34	5.43	5.43	1.59	5.47	5.47	5.47	5.47	0.00	0.7
3	7.09	7.23	7.27	7.19	1.34	7.42	7.42	7.42	7.42	0.00	3.1
4	9.84	9.97	9.85	9.89	0.75	9.77	10.16	9.77	9.90	2.27	0.1
5	11.57	11.77	11.60	11.64	0.92	11.33	11.72	11.72	11.59	1.94	0.4
6	12.50	12.68	-	12.59	0.99	13.28	13.28	-	13.28	0.00	5.3
7	14.46	15.25	14.50	14.73	3.02	14.45	14.84	14.45	14.58	1.54	1.0
8	16.30	16.57	16.31	16.39	0.91	16.02	16.80	16.02	16.28	2.77	0.7
9	-	17.94	17.75	17.85	0.75	-	18.36	17.58	17.97	3.07	0.7

Modal assurance criterion (MAC) is used to compare experimental modal shapes obtained in the different test and those determined by different methods. MAC is an indicator that represents the degree of similitude between two mode shapes by determining the minimum square deviation (Eq. 1). Values close to unit indicate a high similitude between those two modes, and values close to zero do indicate no similitude between modes.

$$MAC(\phi_i, \phi_j) = \frac{|\phi_i^T \phi_j|^2}{(\phi_i^T \phi_i)(\phi_j^T \phi_j)} \quad (1)$$

where ϕ_i = modal vector identified for testing i , and ϕ_j = modal vector identified for testing j .

The results of this comparison are presented in Tables 3. It can be observed that the modes identified in Test 1 are similar to those identified in Test 2, especially in the first three modes, while the results of the comparison Test 1 vs Test 3 and Test 2 vs Test 3 are deficient. Consequently, Test 3 was discarded because this differences might be evidence of a faulty modal identification in this test.

The shapes of the three first modes obtained from the data extracted from Test 2 by SSI and FDD methods are compared in Fig. 5.

Table 3 – MAC values

Mode	SSI			FDD		
	Test 1 vs Test 2	Test 1 vs Test 3	Test 2 vs Test 3	Test 1 vs Test 2	Test 1 vs Test 3	Test 2 vs Test 3
1	0.811	0.807	0.955	0.817	0.805	0.967
2	0.794	0.279	0.439	0.892	0.357	0.642
3	0.966	0.613	0.599	0.878	0.795	0.897
4	0.220	0.016	0.471	0.390	0.083	0.406
5	0.620	0.290	0.508	0.611	0.192	0.669
6	0.862	--	--	0.906	--	--
7	0.002	0.740	0.044	0.000	0.016	0.561
8	0.226	0.044	0.171	0.340	0.05	0.024
9	--	--	0.269	--	--	0.385

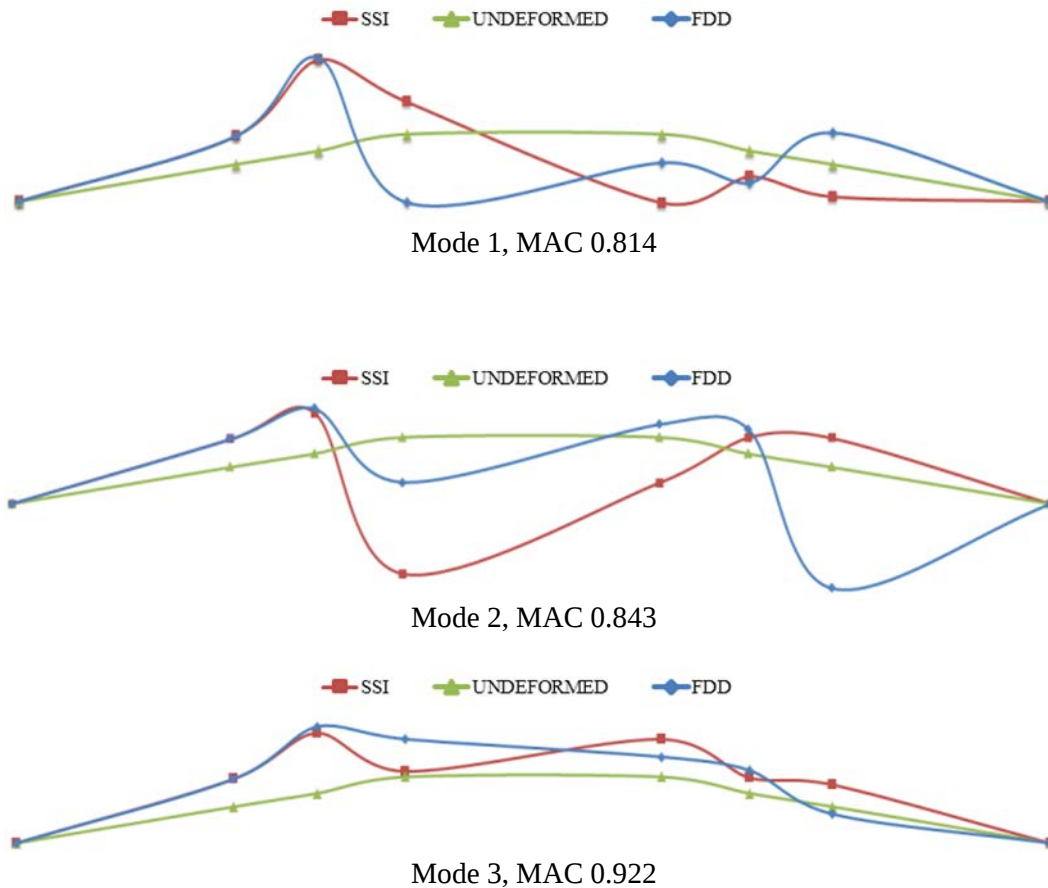


Fig. 5 – Experimental modal shapes

4. Numerical model

4.1 Model description

A model was generated using the finite elements software SAP2000 [6]. The numerical model has 256 nodes and consists of 330 frame elements and 102 shell type elements (Fig. 6). The traffic slab, constituted of timber slab covered by an asphaltic layer, was modelled with shell element (100 mm thick) which equivalent to weight was 1520 kg/m^3 . Timber mechanical properties (Young's modulus and Poisson's ratio) were assigned to the traffic slab as a first modelling approach. The properties of steel members, laminated timber beams and traffic slab are presented in Table 4.

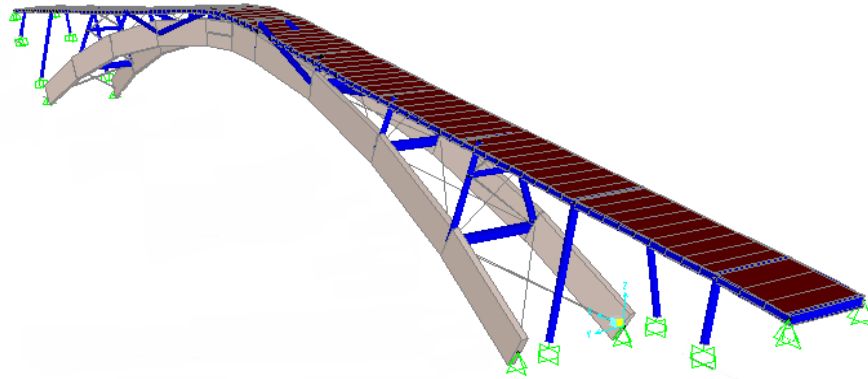


Fig. 6 – Footbridge model in SAP2000

Table 4 – Material Properties

	Laminated Timber	Steel	Traffic Slab
Young's Modulus (MPa)	112000	210000	5340
Poisson's Ratio	0.3	0.3	0.3
Weight (kg/m^3)	480	7800	1520

4.2 Model results

Modal analysis was performed to identify the modal properties of the structure. These dynamic properties were then used as response features to be compared with measurements on the real structure. Considering that, the experimental measurements were recorded with accelerometers attached to the footbridge traffic slab. The modes with higher participation of the traffic slab in the vertical direction were selected from the model and the corresponding modal frequencies and mode shapes are presented in Fig 7. The modal displacements calculated by the model at the locations where the sensors were attached in the real structure (A1 to A6) are presented in Table 5.

Table 5 – Modal displacements at the identified degrees-of-freedom

	ϕ_4	ϕ_{10}	ϕ_{13}	ϕ_{14}	ϕ_{15}	ϕ_{19}	ϕ_{22}	ϕ_{30}
A1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A2	1.42	1.24	-0.60	2.55	-0.15	1.01	0.80	-0.60
A3	1.25	0.83	-2.25	2.92	-1.00	0.63	0.41	-1.62

A4	-1.25	0.83	2.25	2.92	-1.00	-0.63	0.41	1.62
A5	-1.42	1.24	0.60	2.55	-0.15	-1.01	0.80	0.60
A6	-1.00	1.00	-1.00	1.00	1.00	-1.00	1.00	-1.00

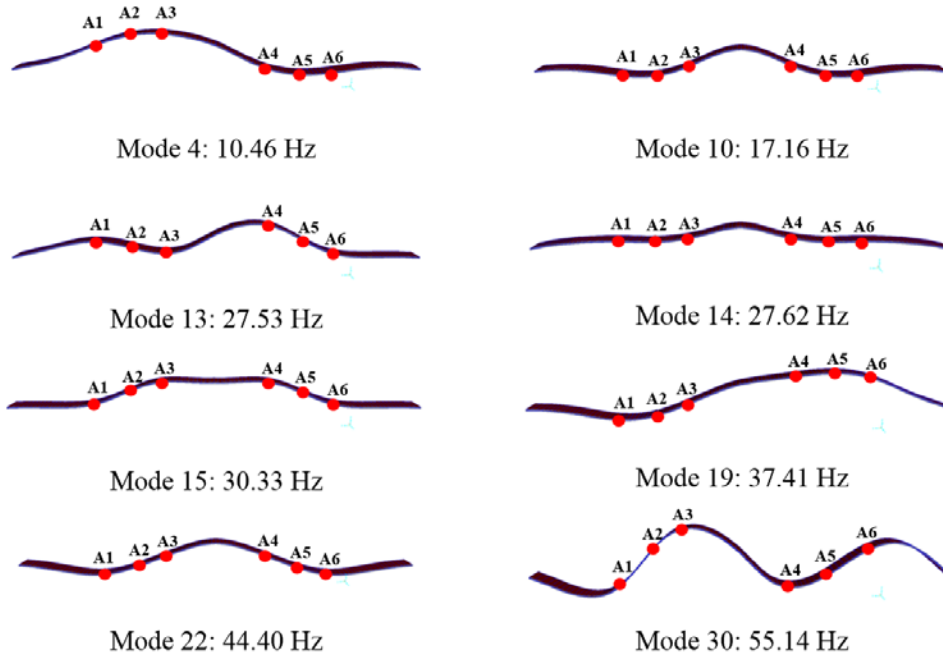


Fig. 7 – Results of the numerical model

4.3 Comparison between numerical and experimental model

The results obtained for first three modes were visually paired with the modes numerically predicted by our SAP2000 model. This comparison is presented in Fig 8.

The experimentally identified frequencies and those numerically computed are presented in Table 6. Even though, the model was constructed using the most appropriate information available, significant differences were observed between these two results that represent the same vibrating modes. This situation clearly illustrates the driving idea presented in this paper: Numerical models must be calibrated using experimental measurements if an accurate representation of the actual structures behavior is desired.

These differences can be related to misestimations of the actual material properties considered for the numeric model. For example, one of these properties is the timber Young's modulus that was obtained from Chilean Standard [7]. However, this property is sensitive to environmental conditions, such as, humidity and temperature, and it can be also affected by damage due to precedent earthquakes or other kind of extreme load. This estimation could be improved by performing experimental test on that material, but at the moment it was not possible to extract samples from the structure.

Table 6 –Experimental and numerical frequencies

Experimental frequency (Hz)	Numerical frequency (Hz)	Error
f1 = 3.53	f1 = 3.52	0.28%
f2 = 5.45	f4 = 5.00	8.26%
f3 = 7.31	f7 = 7.04	3.69%
Average error		4.08%

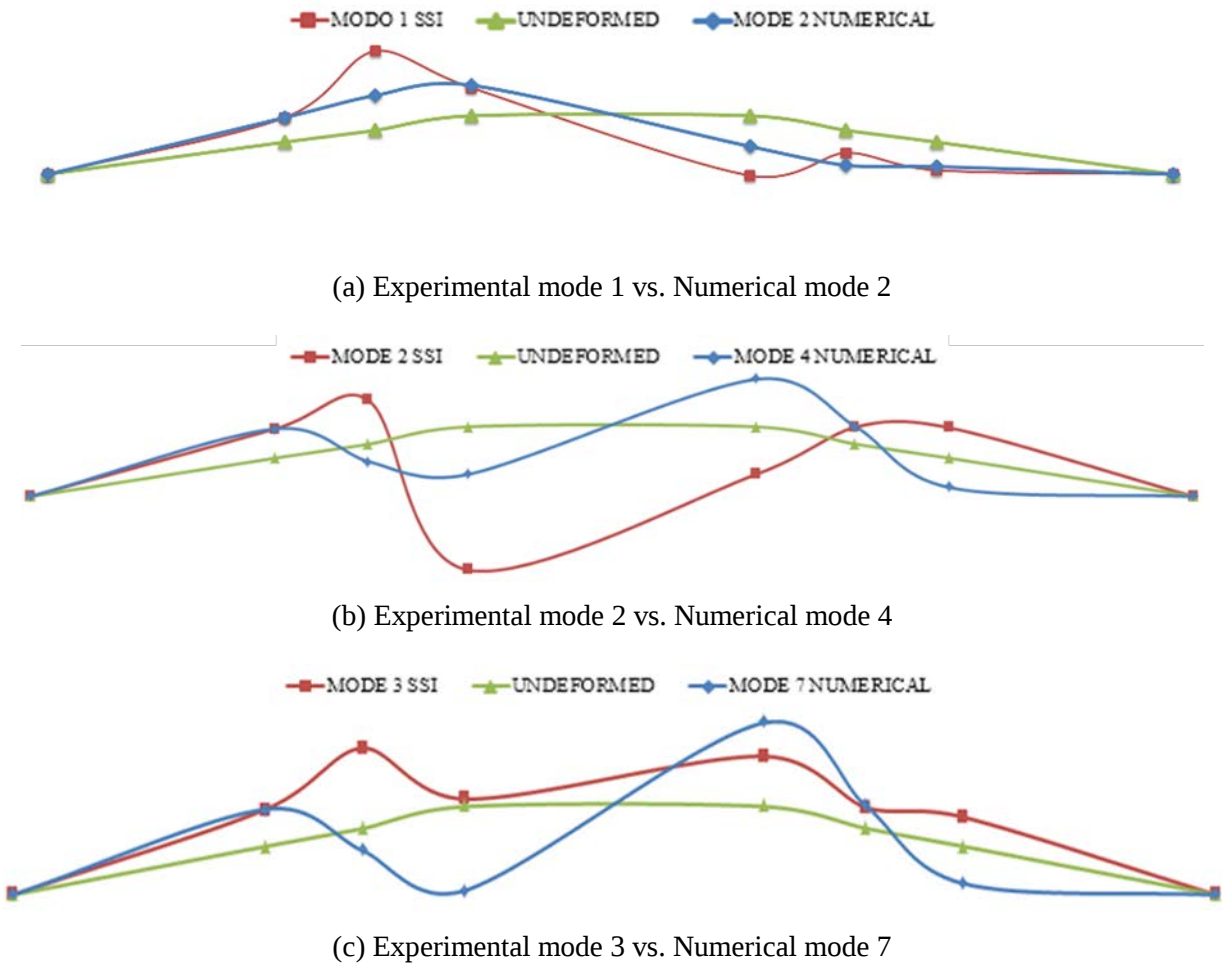


Fig. 8 – Comparison of experimental and numerical mode shapes

5. Model calibration

5.1 Selection of the calibration parameters

The calibration parameters should be selected based on their uncertainty and sensitivity. Efforts in adjusting a model by modifying parameters with well-known properties (low uncertainty) would be a waste of resources. Similarly, an attempt of calibrating a model by varying parameters that do not significantly affect structural response (low sensitivity) can produce physically anomalous results in these calibration.

The process for selecting the calibration parameters for this model updating process was based on the PIRT method. PIRT is a tool used to rank the inputs and assumptions considered to generate a model based on the effect that these calibration parameters have on the structural response [8]. The PIRT table contains the levels of uncertainty and sensitivity associated with each considered parameter and these are classified in low, medium or high. The product of sensitivity and uncertainty is calculated using the combination rule defined in Table 7 that gives greater weight to sensitivity than to uncertainty.

Table 7 – PIRT Combination Rules

Uncertainty	Sensitivity	Combined effect
High	High	High
High	Medium	High
High	Low	Medium



Medium	High	High
Medium	Medium	Medium
Medium	Low	Medium
Low	High	Medium
Low	Medium	Medium
Low	Low	Low

The material properties and geometric parameters that are believed to affect the dynamic response of the footbridge are shown in Table 8.

Steel material parameters were assumed as low uncertainty, because steel is a material which production and quality is well standardized in Chile. Material parameters associated to timber was considered as high uncertainty. Mainly, because the variability timber properties depended on the structural grade of timber and its moisture. For example, according to the Chilean standard [7], the Young's modulus of timber varies from 8900 MPa to 10500 MPa depending on their structural grade. The structural grade was determined based on a visual inspection of timber and defects identification. Hence, this classification depends on the ability of the observer. In this case, the timber of traffic slab was classified as Structural Grade G2. In addition, timber Young's modulus must be corrected depending on the timber moisture. In general, a normal moisture in timber is 12%, but in Concepción city these moisture range from 10% to 20%, due to ambient conditions. Consequently, Young's modulus has a 30% variability only due to moisture content.

The numerical model presented a high dependence on the properties assigned to the timber of the traffic slab, while the material properties assigned to the laminated timber arc beams and steel frame had a less significant effect on structural response. The variation of modal frequencies due to variation in the Young's modulus of the timber assigned to the traffic slab is shown in Fig 9 and it can be seen that Mode 7 is the most sensitive to that parameter.

Finally, it can be seen in Table 8 that the parameter with highest PIRT rank was the Young's modulus of timber traffic slab and, consequently, it was selected as calibration variable.

Table 8 – PIRT for the footbridge model

Model Parameter	Uncertainty	Sensitivity	Combined effect	Action
Young's Modulus of timber traffic slab	High	High	High	Calibrate
Young's Modulus of laminated timber beams	Medium	Low	Medium	No calibrate
Young's Modulus of steel frame	Low	Low	Low	No calibrate
Density of timber traffic slab	Medium	Low	Medium	No calibrate
Density of laminated timber beams	Medium	Low	Medium	No calibrate
Density of steel	Low	Low	Low	No calibrate
Thickness of steel sections	Low	Low	Low	No calibrate

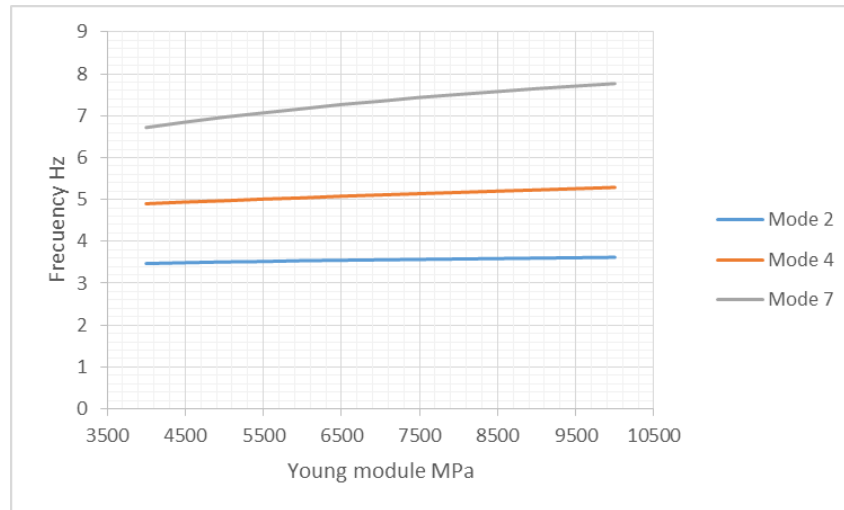


Fig. 9 – Frequency variation due to changes in Young's modulus of traffic slab timber

5.2 Model Updating

The model updating was performed by an iterative process which objective was adjusting the value obtained for frequencies of numerical mode 1, 4 and 7 with the three experimentally identified frequencies. This updating process was made by varying the selected calibration parameter (Young's Modulus of timber traffic slab) and minimizing the average error obtained for these three pairs of frequencies. The final value assigned to the Young's Modulus of timber traffic slab was 6300 MPa, which is 18% higher than the initial value.

Table 9 show the result of this process and it can be verified that the average error was reduce in 1/3 of its original value presented in Table 6.

Table 9 –Experimental and numerical calibrated frequencies after calibration

Experimental frequency (Hz)	Numerical frequency (Hz)	Error
f1 = 3.53	f1 = 3.54	0.28%
f2 = 5.45	f4 = 5.07	6.97%
f3 = 7.31	f7 = 7.23	1.09%
Average error		2.78%

6. Conclusions

Operational modal analysis was performed on an arched footbridge in Concepcion, Chile. Ambient vibrations due to pedestrian and vehicular traffic were considered as source of excitations. SSI and FDD methods were successfully employed to extract the modal properties of the structure.

SSI and FDD methods were able of identifying nine modal frequencies from Test 2, while in the other tests a few modes were missing. The results of both system identification techniques are coincident. As long both techniques are completely independent and based on different numerical approaches, the coincident results confirm that the identified modes corresponded to actual vibration modes and not only to numerical artefacts.

The deficient results in MAC obtained when Test 3 is compared to the others may be interpreted as a dissimilar mode identification in this test. This can be attributed to the difference in the time frame considered for



performing the data recording. Test 1 and 2 were performed one hour earlier than Test 3 and that may implied differences in source of excitation.

The low dispersion observed in the detected frequencies and mode shapes demonstrates an accurate identification of modal parameters. These parameters were considered as target values for model calibration.

The average error of the first three modal frequencies was reduced in one third by adjusting the value of the timber traffic slab Young's Modulus. The final value of this parameter was 6300 MPa which was within the range of realistic values for this parameter. This difference between the initial and final value of the calibration parameter can be attributed to an erroneous and conservative visual classification of timber structural grade at the initial stage of this study.

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