AN EXTENDED FINITE ELEMENT PROCEDURE FOR EARTHQUAKE ANALYSIS OF DAM-WATER-FOUNDATION ROCK SYSTEMS: VALIDATION OF METHOD FOR 2D PROBLEMS

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Abstract

An extended finite element analysis procedure (extended FEM) is presented for performing nonlinear earthquake analysis of concrete dams. The procedure overcomes the limitations of the "standard" FEM by applying wave-transmitting boundaries to simulate the semi-unbounded reservoir and foundation-rock, and specifying at these boundaries earthquake-induced tractions and pressure gradients determined from the design ground motion specified at a control point at the free surface. The procedure is validated numerically by computing the dynamic response of an idealized two-dimensional dam-water-foundation rock system and comparing against benchmark results obtained using the substructure method, in which the semi-unbounded fluid and foundation-rock domains are modeled rigorously. The results demonstrate that the extended FEM is an accurate, efficient and versatile analysis procedure that allows for modeling of arbitrary geometry and material behavior of dam-water-foundation rock systems without introducing unrealistic assumptions of massless rock or incompressible water sometimes used in engineering practice.

Keywords: Concrete dams; dam-water-foundation rock interaction; nonlinear analysis; wave-transmitting boundaries

1. Introduction

Evaluation of the seismic performance of dams requires dynamic analysis of two- or three-dimensional dam-water–foundation rock systems that recognize factors known to be significant in the earthquake response of dams [1]: (1) radiation damping due to the semi-unbounded size of the reservoir and foundation-rock domains; (2) dam-water interaction including water compressibility and wave absorption at the reservoir boundaries; (3) dam-foundation rock interaction including inertia effects of the rock; and (4) spatial variation of the ground motion. The state-of-practice however, often conducts finite element analysis using commercial software that ignores one or several of these factors in order to facilitate nonlinear dynamic analysis.

The objective of this paper is to present and validate an extended finite element analysis procedure (extended FEM) that overcomes the limitations of "standard" FEM by applying wave-transmitting boundaries to simulate the semi-unbounded fluid and foundation-rock domains, and specifying at these boundaries earthquake-induced tractions and pressure gradients determined from the design ground motion specified at a control point at the free surface.

2. Reasons why "standard" FEM is inadequate

The versatility of the FEM in modeling arbitrary geometries and variations of material properties makes it suitable for modeling a concrete dam. However, analysis of concrete dams is greatly complicated by interaction between the structure, the impounded reservoir and the deformable foundation rock that supports it, and the fact that the fluid and foundation-rock domains extend to large distances.

These interaction mechanisms may be included in a crude way by combining finite element models for a limited extent of the impounded water and foundation rock with a finite element model of the dam, thus reducing the "semi-unbounded" system to a finite-sized model with rigid boundaries, which, generally, do not exist at dam sites. Such a model does not allow for radiation of hydrodynamic pressure waves in the upstream direction or stress waves in the foundation rock because these waves are reflected back from the rigid boundaries, effectively trapping the energy in the bounded system. Thus, a significant energy loss mechanism, referred to as radiation damping, is not represented in the bounded models of the impounded water and foundation rock.

Because of the difficulties in modeling dam-water-foundation rock interaction and unbounded domains, the profession has often employed an expedient solution that includes in the finite element model a limited extent of foundation rock, assumed to have no mass, and approximates hydrodynamic effects by an added mass of water moving with the dam; the design ground motion – typically defined at a control point at the free surface – is applied at the bottom fixed boundary of the foundation-rock domain. This modeling approach has become popular in actual projects because it is easy to implement in commercial finite element software, such as ABAQUS, ANSYS, or SAP2000. However, such a model solves a problem that is very different from the real problem on two counts: (1) the assumptions of massless rock and incompressible water – implied by the added mass water model – are unrealistic, as research has demonstrated [1]; and (2) applying ground motion specified at a control point at the free surface to the bottom boundary of the finite element model contradicts recorded evidence that motions at depth generally differ significantly from surface motions.

3. Extended FEM

In this section we present an extended finite element procedure (extended FEM) for performing nonlinear earthquake analysis of concrete dams. The extended FEM builds on well-established procedures for soil-structure interaction analysis in the time domain [2], [3].

3.1. Statement of problem

The dam-water-foundation rock system (Figure 1a) consists of the dam with nonlinear material behavior; a bounded region of the foundation rock which may have inhomogeneities and nonlinear material behavior; a bounded region of the fluid, which may have irregular geometry and nonlinear behavior; and the remaining, unbounded parts of the foundation-rock and fluid domains, both assumed to be homogeneous and linear.

The earthquake excitation is defined at a control point at the free surface by two components of ground acceleration in the cross-sectional plane of the dam: the horizontal component $a_g^x(t)$ transverse to the dam axis, and the vertical component $a_g^y(t)$.

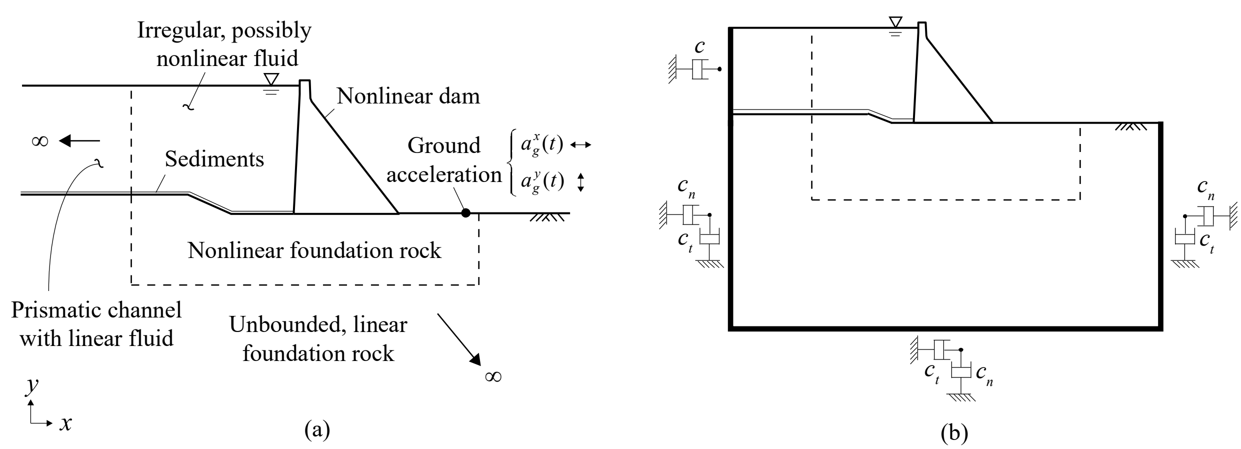


Figure 1: (a) Dam-water-foundation rock system with semi-unbounded fluid and foundation rock; (b) idealized model truncated by wave-transmitting boundaries.

3.2. Finite element idealization and definition of seismic input

The dam-water-foundation-rock system is idealized using standard solid elements for the dam and foundation rock, and acoustic elements for the fluid. Interface elements are employed to couple accelerations in the solid with hydrodynamic pressures in the acoustic fluid at the dam-water and water-foundation rock interfaces [4]. Wave absorption due to sediments at the reservoir-bottom is modeled by a one-dimensional boundary condition characterized by the reservoir-bottom reflection coefficient α [5].

Wave-transmitting boundaries in the form of standard viscous dampers [6] are introduced at two locations (Figure 1b): (1) the upstream end of the fluid domain to model its essentially infinite length; and (2) the bottom and side boundaries of the foundation-rock domain to model its semi-unbounded geometry. In discretized form, the distributed viscous dampers can be lumped to each nodal point on the boundary, resulting in dampers with coefficients c_n and c_t in the normal and tangential directions for the foundation-rock boundary:

$$c_n = A\rho_f v_p \ , \ c_t = A\rho_f v_s \tag{1a}$$

where A is the tributary area (or tributary length in a 2D model) of the nodal point; ρ_f is the density and v_s and v_p are the shear- and pressure wave velocities of the foundation medium at the boundary, respectively. For the acoustic fluid, the corresponding damper coefficient is [4]:

$$c = A / C_w \tag{1b}$$

where C_w is the speed of pressure waves in water.

The earthquake excitation is also more realistically defined in the extended FEM compared to the "standard" FEM. The free-field motion at the bottom and side boundaries of the foundation-rock domain is determined by deconvolution of the design ground motion, typically specified at a control point at the free surface (Figure 1a). The resulting spatially varying motions cannot be input directly at wave-transmitting boundaries. Instead, tractions and pressure gradients are determined from the free-field motion and applied at these boundaries.

Current deconvolution procedures assumes vertically propagating seismic waves [7]. This simplifies implementation of the extended FEM greatly, as the tractions and pressure gradients can then be determined from one-dimensional auxiliary analyses, as described in the following section.

3.3. Implementation of procedure

The extended finite element procedure can be implemented numerically through the following steps:

1. A finite element model of the dam-water-foundation rock system is developed using solid elements for the dam and foundation-rock domains and acoustic elements for the fluid domain; interface elements at the dam-water and water-foundation rock interfaces; line elements for sediments at the reservoir bottom; and viscous dampers to truncate the unbounded foundation-rock and fluid domains.
2. The earthquake motion at the base of the model is obtained from the design ground motion $a_g^k(t)$, $k = x, y$, specified at a control point at the free surface, by a conventional (one-dimensional) deconvolution analysis assuming vertically propagating seismic waves [7].

3. From the deconvolved ground motion, tractions at the wave-transmitting boundary at the base of the foundation-rock model is determined from [8]

$$\tau(t) = 2\rho_f v_s \, \dot{u}_{Ix}(t) \tag{2a}$$

$$\sigma(t) = 2\rho_f v_p \, \dot{u}_{Iy}(t) \tag{2b}$$

where $\dot{u}_{Ik}(t)$ is the velocity time history for the incident (upwards propagating) wave train in the *k*-direction. The incident wave train is computed as 1/2 the *outcrop motion* at the same location [9], obtained from the deconvolution analysis.

4. Along the vertical boundaries on the two sides of the foundation-rock domain, free-field conditions are enforced by (conceptually) attaching one-dimensional "free-field columns" to the main mesh [3]. This can be implemented either by including one-dimensional boundary elements solved together with the main mesh [10], or by directly applying normal and tangential tractions to the vertical boundaries [2]:

$$\sigma(t) = \sigma^0(t) + \rho_f v_p \, \dot{u}_x^0(t) \tag{3a}$$

$$\tau(t) = \tau^0(t) + \rho_f v_s \, \dot{u}_y^0(t) \tag{3b}$$

where the free-field tractions $\sigma^0(t)$, $\tau^0(t)$ and velocities $\dot{u}_k^0(t)$ are recorded from auxiliary analysis of a one-dimensional foundation-rock column (Figure 2a and 2b) subjected to the forces of Equation (2) at its base.

5. Along the upstream end of the fluid domain, free-field conditions are enforced by applying pressure gradients. These will be non-zero only for vertical excitation because the upstream channel is assumed prismatic. For vertical excitation, pressure gradients are computed from

$$\frac{\partial p}{\partial n} = \frac{1}{C_w} \dot{p}^0(t) \tag{3c}$$

where the time derivative of the free-field pressures, $\dot{p}^0(t)$, are recorded from an auxiliary analysis of a one-dimensional water column (Figure 2c) subjected to vertical accelerations $a_g^y(t)$ at its base. In deriving Equation (3c) it has been utilized that the free-field pressure gradient $\partial p^0 / \partial n \equiv 0$ at the upstream end of the fluid domain due to the uniform cross section in the prismatic channel.

6. Nodal forces are determined from the continuous tractions and pressure gradients of Equations (2) and (3), and the prescribed finite element shape functions for the elements adjacent to the boundary.

   As an alternative to Steps 3-6, the Domain Reduction Method (DRM) [11] may be used to apply the seismic excitation via a single layer of elements adjacent to the wave-transmitting boundaries. The free-field motion necessary in determining the effective seismic forces in the DRM can be recorded from auxiliary analyses of the one-dimensional foundation-rock and fluid columns (Figure (2).

7. The nonlinear dynamic response of the discretized dam-water-foundation rock system due to the boundary forces computed in Steps 3-6 is determined. If static loads – such as gravity loads or hydrostatic pressures – are to be included, these must be obtained in a separate initial static analysis where the static state of the dam, including static reaction forces along the foundation-rock boundaries, is recorded. The static state of the dam is taken as the initial state in the nonlinear dynamic analysis.

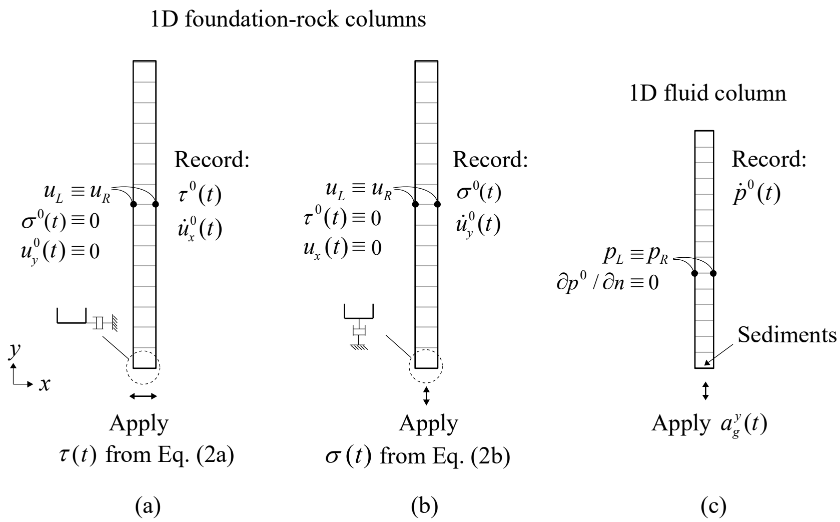


Figure 2: One-dimensional auxiliary systems to determine tractions and pressure gradients; (a)-(b) foundation-rock columns; (c) fluid column.

4. Validation of procedure

The extended FEM is validated numerically by computing the dynamic response of the idealized two-dimensional dam-water-foundation rock system shown in Figure 3 and comparing against benchmark results obtained using the computer program EAGD84 [12]. EAGD84 utilizes the substructure method [5] to model the semi-unbounded fluid and foundation-rock domains without discretization and specifies the earthquake excitation directly at the dam-foundation interface; it therefore avoids artificial model truncations and deconvolution of the earthquake motion. The substructure method as implemented in EAGD84 does not allow for nonlinear material behavior. The procedure is therefore validated assuming linear behavior of the dam, foundation rock and fluid.

4.1. Numerical model

The idealized dam in Figure 3a has a triangular cross-section of height H_s = 120 m, a vertical upstream face and a downstream face with a slope of 0.8 to 1. The concrete in the dam is assumed to be homogeneous, isotropic and linear elastic, in a state of plane strain, with modulus of elasticity E_s = 22.4 GPa, density ρ_s = 2483 kg/m^3, and Poisson's ratio ν_s = 0.20. The foundation rock is assumed to be homogeneous, isotropic and linear elastic, in a state of plane strain, with modulus of elasticity E_f = 22.4 GPa (i.e. E_f / E_s = 1) density ρ_f = 2643 kg/m^3, and Poisson's ratio ν_f = 0.33. The impounded water has the same depth as the height of the dam, H = H_s, density ρ_w = 1000 kg/m^3, and pressure wave velocity C_w = 1440 m/s. The reservoir bottom reflection coefficient is selected as α = 0.75.

Material damping in the dam and foundation rock is modeled by mass- and stiffness proportional Rayleigh damping. Since EAGD84, used to obtain the benchmark results, employs a (frequency independent) constant hysteretic damping model, the Rayleigh coefficients for the dam and foundation rock is – to the extent possible – selected to be consistent with such a model. The Rayleigh coefficients a_0 and a_1 are obtained by specifying the damping ratios ζ_s and ζ_f, for the dam concrete and foundation rock, respectively, at the following frequencies: (a) the first and second natural frequencies of the dam-water-foundation rock system when computing the response to a single earthquake excitation; or (b) the excitation frequency $f_j = \omega_j / 2\pi$ and f = 1 Hz when computing the frequency response function for a single excitation frequency ω_j. Broad numerical testing confirmed this approach to give excellent agreement with the constant hysteretic damping model in EAGD84.

The dam-water-foundation rock system is modeled in the finite element program OpenSees [13]. The FE mesh for the dam has 15 elements along the base and 29 elements along the height (Figure 3a). The FE meshes of the dam, foundation-rock and fluid domains match at their respective interfaces, with a gradually increasing element size towards the outer edges of the model. The maximum element size is limited to less than one-tenth of the shortest wavelength considered in the analysis to ensure satisfactory wave propagation in the mesh [14]. The total width and height of the foundation-rock region is selected as 10*B* and 3*B*, respectively where *B* is the width of the dam at the base. These dimensions are considered sufficiently large to minimize unwanted reflections from the wave-transmitting boundaries for the system parameters considered.

Frequency response functions are obtained by time domain analysis of the FE system in Figure 3b subjected to tractions and pressure gradients determined from deconvolution of a harmonic motion $a_g^k(t)$ of unit amplitude for a wide range of excitation frequencies. The response of the dam is computed by solving the equations of motion for the FE system for long enough time to determine the steady-state amplitude of the acceleration at the crest of the dam. Results are plotted against the normalized frequency ω / ω_1, where ω_1 is the fundamental frequency of the dam on rigid foundation with empty reservoir.

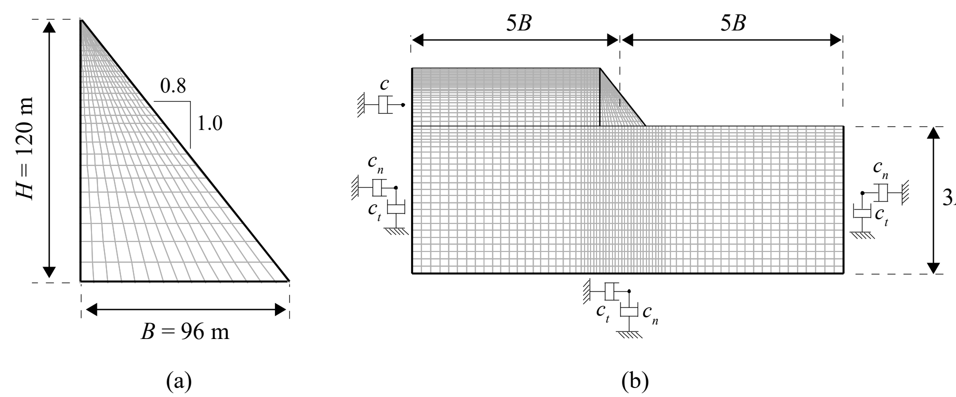


Figure 3: (a) Geometry and finite element mesh for triangular dam cross section; (b) overall dam-water-foundation rock model with characteristic dimensions.

4.2 Dam-foundation rock interaction

The frequency response function for the dynamic response of the dam on flexible foundation rock with empty reservoir is shown in Figure 4. It is apparent that the results computed by the extended FEM closely match the benchmark results from the substructure method. The close agreement demonstrates that (1) the truncated foundation-rock model in the extended FEM is able to accurately represent the effects of dam-foundation rock interaction and radiation damping in the semi-unbounded foundation rock; (2) the seismic input is appropriately defined by specifying tractions determined from an auxiliary analysis of a 1D foundation-rock column using the deconvolved ground motion; and (3) the size chosen for the foundation-rock domain is sufficiently large to minimize unwanted reflections from the wave-transmitting boundaries.

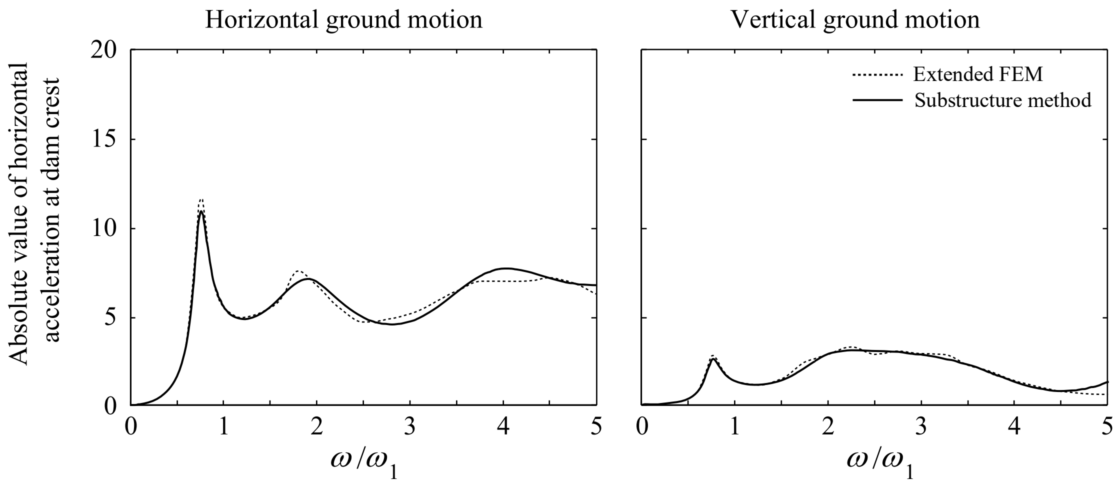


Figure 4: Comparison of frequency response functions for dam on flexible foundation rock with empty reservoir computed by the extended FEM and the substructure method; $\zeta_s = \zeta_f = 2\%$; $E_f / E_s = 1.0$.

4.3. Dam-water interaction

The frequency response function for the dynamic response of the dam on rigid foundation with full reservoir is shown in Figure 5. The results computed by the extended FEM closely match the benchmark results from the substructure method. The close agreement demonstrates that (1) the truncated fluid model used in the extended FEM is able to accurately represent the effects of dam-water interaction, reservoir bottom absorption and radiation damping in the semi-unbounded fluid domain; (2) free-field conditions are appropriately enforced by applying pressure gradients determined from an auxiliary analysis of a 1D fluid column for vertical excitation; and (3) the size chosen for the fluid domain is sufficiently large to minimize unwanted reflections from the wave-transmitting boundary.

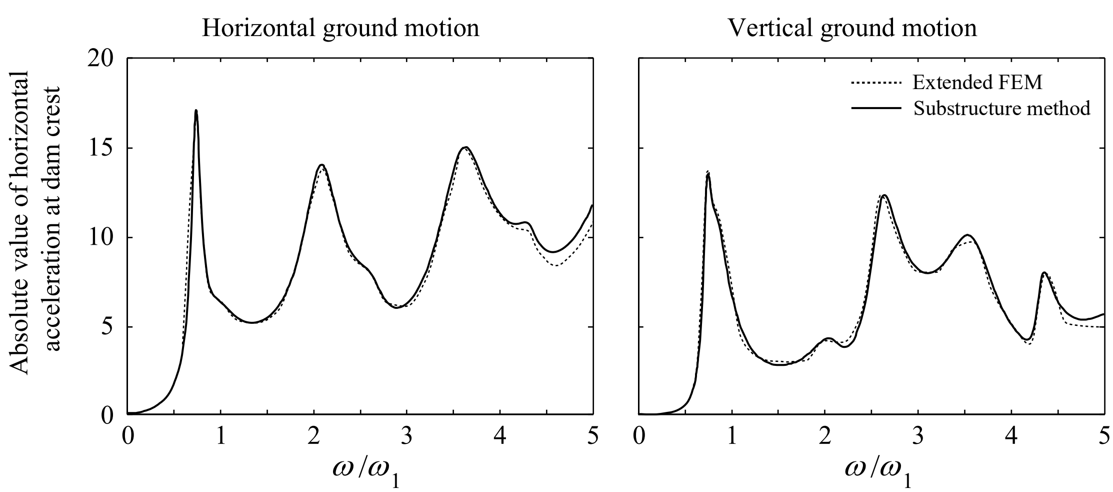


Figure 5: Comparison of frequency response functions for dam on rigid foundation with full reservoir computed by the extended FEM and the substructure method; $\zeta_s = 5\%$; $\alpha = 0.75$.

4.4. Dam-water-foundation rock interaction

The frequency response function for the dynamic response of the dam on flexible foundation rock with full reservoir is shown in Figure 6, where they are compared to benchmark results from the substructure method. Since the computer program EAGD84 – used to compute the benchmark results – does not consider interaction between the water and the foundation rock, water-foundation rock interaction is also neglected in the extended FEM for subsequent analyses.

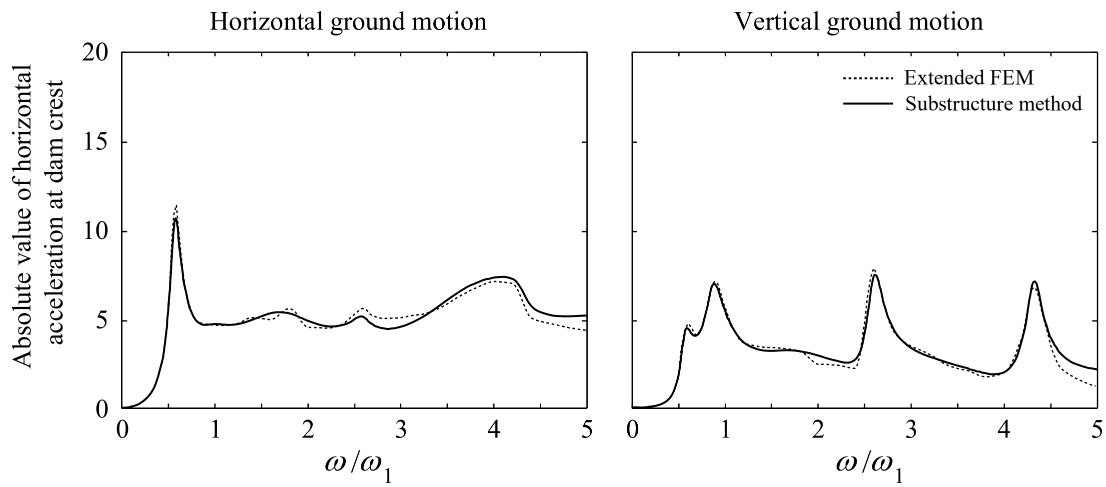


Figure 6: Comparison of frequency response functions for dam on flexible foundation rock with full reservoir computed by the extended FEM and the substructure method; $\zeta_s = \zeta_f = 2\%$; $E_f / E_s = 1.0$; $\alpha = 0.75$.

The close agreement between the extended FEM and benchmark results demonstrates the validity of the procedure, more specifically that (1) the truncated foundation-rock and fluid models used in the extended FEM are able to accurately represent the effects of dam-water-foundation rock interaction, reservoir bottom absorption and radiation damping in the semi-unbounded foundation-rock and fluid domains; (2) the seismic input is appropriately defined by specifying tractions and pressure gradients at the foundation-rock and fluid boundaries determined from auxiliary analyses of 1D foundation-rock and fluid columns using the deconvolved ground motion; and (3) the size chosen for the foundation-rock and fluid domains is sufficiently large to minimize unwanted reflections from the wave-transmitting boundaries.

4.5. Response to transient motion

The response of the dam-water-foundation rock system to the S69E and vertical components of the ground motion recorded at Taft Lincoln School Tunnel during the Kern County, California, earthquake July 21st 1952 is presented in Figure 7. As expected, the crest displacements and accelerations computed by the extended FEM are practically indistinguishable from the substructure method results. The envelope values of maximum principal stresses in the dam due to the S69E component of the Taft ground motion is presented in Figure 8. Again, the response computed by the extended FEM closely matches that of the substructure method.

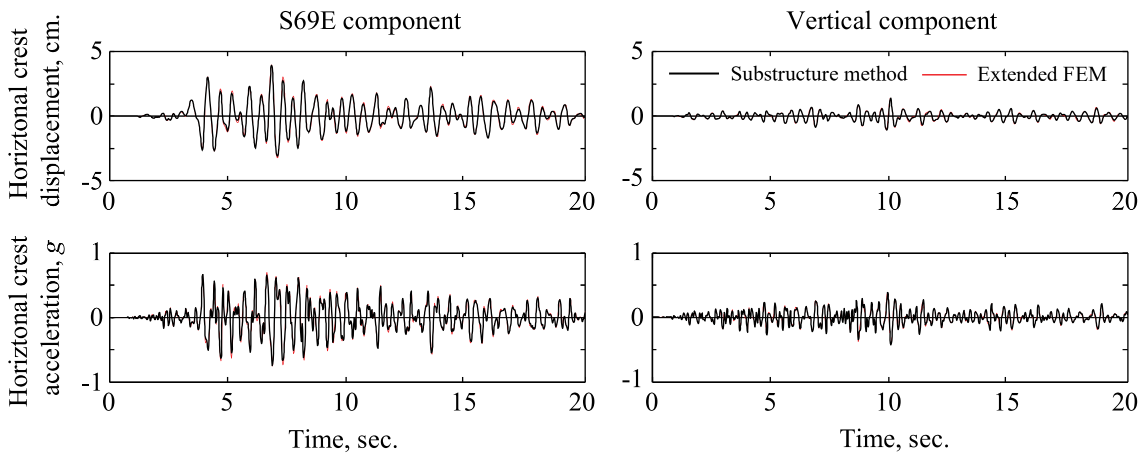
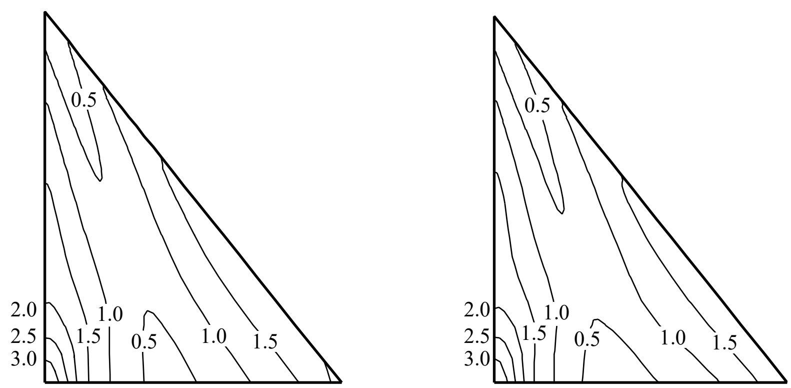


Figure 7: Response of dam on flexible foundation rock with full reservoir subjected to the S69E and vertical components, separately, of Taft ground motion; $\zeta_s = \zeta_f = 2\%$. $E_f / E_s = 1.0$; $\alpha = 0.75$.



(a) Extended FEM (b) Substructure method

Figure 8: Envelope values of maximum principal stresses, in MPa, in dam on flexible foundation rock with full reservoir due to S69E component of Taft ground motion; initial static stresses are excluded; $\zeta_s = \zeta_f = 2\%$. $E_f / E_s = 1.0$; $\alpha = 0.75$.

Although coupling between neighboring solid and fluid elements leads to an unsymmetric system of global equations, the use of viscous dampers as wave-transmitting boundaries makes the procedure computationally very efficient. For example, the total analysis time for implicit analysis of the finite element model in Figure 3 with approx. 3500 elements took just 2 minutes (128 seconds) for 2000 time steps of the Taft ground motion on a typical single-core laptop without parallel computing capabilities. This includes CPU-time for deconvolution of the ground motion, dynamic analyses of 1D auxiliary foundation-rock and fluid columns, and 2D dynamic analysis of the dam-water-foundation rock system.

The preceding results demonstrate that the extended FEM is an accurate, efficient and versatile analysis procedure that allows for modeling of arbitrary geometry and material behavior of dam-water-foundation rock systems without introducing the unrealistic assumptions of massless rock or incompressible water sometimes used in engineering practice.

5. Conclusions

Presented in this paper is an extended finite element procedure that overcomes the limitations of "standard" FEM by introducing wave-transmitting boundaries at two locations: (1) the upstream end of the fluid domain to model its essentially infinite length; and (2) the bottom and side boundaries of the foundation-rock domain to model its semi-unbounded geometry. The earthquake excitation is also more realistically defined in the extended FEM compared to the "standard" FEM. The seismic input is specified by earthquake-induced tractions and pressure gradients at the foundation-rock and fluid boundaries of the truncated finite element model, determined from the free-field motion obtained by deconvolution of the design ground motion specified at a control point at the free surface. The finite element model of the fluid includes water compressibility and reservoir bottom sediments, and the finite element model of the foundation-rock includes mass, stiffness, and material damping appropriate for rock. Thus, the unrealistic assumptions of massless rock and incompressible water are eliminated.

Validation of the extended FEM by computing the dynamic response of an idealized dam-water-foundation rock system and comparing against results obtained using the substructure method demonstrates that (1) the truncated foundation-rock and fluid models are able to accurately represent the effects of dam-water-foundation rock interaction, reservoir bottom absorption, and radiation damping in the semi-unbounded foundation-rock and fluid domains; (2) the seismic input is appropriately defined by specifying tractions and pressure gradients determined from the free-field motion, which in turn is obtained from auxiliary analyses of 1D foundation-rock and fluid columns subjected to the deconvolved ground motion; and (3) by choosing the size of the foundation-rock and fluid domains sufficiently large, unwanted reflections from the wave-transmitting boundaries can be minimized.

The extended FEM is applicable to nonlinear systems, thus permitting modeling of concrete cracking, as well as sliding and separation at construction joints, lift joints, and concrete-rock interfaces. Implementation of the procedure is facilitated by commercial finite element software – with its user-friendly interfaces – that can model simple wave-transmitting boundaries and permit specification of earthquake-induced tractions and pressure gradients at these boundaries.

6. Acknowledgements

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7. References

[1] A. K. Chopra, “Earthquake analysis of arch dams: Factors to be considered,” *J. Struct. Eng.*, vol. 138, no. 2, pp. 205–214, 2012.

[2] J. P. Wolf, *Soil-structure interaction in time domain*. Englewood Cliffs, NJ: Prentice Hall, 1988.

[3] O. C. Zienkiewicz, N. Bicanic, and F. Q. Shen, “Earthquake input definition and the transmitting boundary conditions,” in *Advances in Computational Nonlinear Mechanics*, Springer, 1989, pp. 109–138.

[4] O. C. Zienkiewicz and P. Bettess, “Fluid-structure dynamic interaction and wave forces. An introduction to numerical treatment,” *Int. J. Numer. Methods Eng.*, vol. 13, pp. 1–16, 1978.

[5] G. Fenves and A. K. Chopra, “Earthquake analysis of concrete gravity dams including reservoir bottom absorption and dam-water-foundation rock interaction,” *Earthq. Eng. Struct. Dyn.*, vol. 12, no. 5, pp. 663–680, 1984.

[6] J. Lysmer and R. L. Kuhlemeyer, "Finite dynamic model for infinite media," *J. Eng. Mech. Div.*, vol. 95, no. 4, pp. 859–878, 1969.

[7] P. B. Schnabel, J. Lysmer, and H. B. Seed, "SHAKE: A computer program for earthquake response analysis of horizontally layered sites," *Report No. UCB/EERC-72/12*. Earthquake Engineering Research Center, University of California, Berkeley, 1972.

[8] W. B. Joyner and A. T. F. Chen, "Calculation of nonlinear ground response in earthquakes," *Bull. Seismol. Soc. Am.*, vol. 65, no. 5, pp. 1315–1336, 1975.

[9] L. H. Mejia and E. M. Dawson, "Earthquake deconvolution for FLAC," in *Proc. of the 4th International FLAC Symposium on Numerical Modeling in Geomechanics*, 2006.

[10] A. H. Nielsen, "Absorbing boundary conditions for seismic analysis in ABAQUS," in *ABAQUS Users' Conference*, 2006, pp. 359–376.

[11] J. Bielak, K. Loukakis, Y. Hisada, and C. Yoshimura, "Domain reduction method for three-dimensional earthquake modeling in localized regions, Part I: Theory," *Bull. Seismol. Soc. Am.*, vol. 93, no. 2, pp. 817–824, 2003.

[12] G. Fenves and A. K. Chopra, "EAGD-84: A computer program for earthquake analysis of concrete gravity dams," *Report No. UCB/EERC-84/11*. Earthquake Engineering Research Center, University of California, Berkeley, 1984.

[13] F. McKenna, "OpenSees: A framework for earthquake engineering simulation," *Comput. Sci. Eng.*, vol. 13, no. 4, pp. 58–66, 2011.

[14] R. L. Kuhlemeyer and J. Lysmer, "Finite element method accuracy for wave propagation problems," *J. Soil Mech. Found. Div.*, vol. 99, no. 5, pp. 421–427, 1973.