

STRUCTURAL DAMAGE MECHANISM REFLECTED IN THE TIME-FREQUENCY RESPONSE SPECTRA

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Abstract

Response spectrum relies on the intensity and the frequencies of ground motions to determine the seismic force in structural design. However, the absence of time makes it unable to track the damage development with time and further analyze the damage mechanism. So, 3D "time-frequency response spectrum (TFRS)" was proposed by some scholars as a complement to the response spectrum or a new method for structural analysis. Based on their studies, this paper took 3 typical historical earthquake records as examples to construct the 3D TFRS mesh and contour plots, and used them to analyze the characteristics of the TFRSs of these ground motions and predict the structural seismic behaviors. Then, time history analyses were conducted on a five-story building to validate the conclusions made by the TFRSs. Matlab / simulink was the analysis software used in this study, in which nonlinearity was considered by updating the state space matrix using a simplified nonlinear procedure. Results were illustrated by the number of crossings to different damage standards, response profiles, and hysteretic energy, etc. Results showed that, TFRS could clearly display the time and frequency characteristics of ground motions, with advantages outreached the traditional response spectra; TFRS could explain and predict different structural damage characteristics well in that it could tell what kinds of TFRS would cause what degree of damages, on structural, as well as non-structural components, etc. Thus TFRS is also a useful tool for researchers to choose proper ground motions in structural analysis and design.

Keywords: time-frequency response spectrum (TFRS); time effects of ground motions; structural damage mechanism.



o1.Introduction

The theory of response spectrum(RS) is one of the most important theories in the seismic design, but it has some limitations in practice, mainly in: 1) It is based on linear theory, and so the nonlinear characteristics of structures in actual earthquake response cannot be reflected; 2) Structural properties keep changing with time, while RS determined force is bonded to a fixed frequency, so RS is unable to do progressive analysis; 3) By RS, the most unfavorable seismic response of structures is the maximum response, so only the most intense section of the acceleration record of the ground motions are attached importance, while other dynamic response characteristics, such as the number or probability near the peak response, are not thought to be important contributions to structural damage[1].

Durations of ground motions cannot be ignored. For example, the PGA of the 1966 Parkfield earthquake in the U.S is as large as 150 cm/s², and the PGA of the 1972 Stonecanyon earthquake in the U.S is also up to 169 cm/s², but the damages near the recording sites were relatively slight; while the PGA of the 1962 Mexico earthquake is only 105 cm/s², but seismic damage in local and surrounding areas was serious. One reason for these is that the durations of the Parkfield and Stonecayon earthquakes are short and of the Mexico earthquake is long. Safak, E and Franke A [2] examined various measures of ground motions (e.g., peak values, intensities, durations, and effective frequency bands) and the elasto-plastic response spectra and concluded that, single measure and 1D model was not enough to explain the earthquake effects. Chai T.H.et.al[3] developed an energybased model for high-intensity seismic loading. Chai Y.H.[4,5] proposed duration-dependent nonlinear design spectra, trying to account for low-cycle fatigue and duration effects in the design base. Hancock J. and Bommer J.J.[6] reviewed the studies on the duration effects of strong motion and found that, studies employing damage measures related to cumulative energy usually find a positive correlation between duration and structural damage. Bommer J.J and Magenes G. et.al.[7] explored the correlations between the damage of 7 masonry structural models and a range of parameters of 500 strong motions and found that, the scatter in the correlation could partially explain the duration difference. However, studies regarding duration are still limited.

Nonlinear time history analysis could consider all factors. However, it requires complex modeling and tedious calculation, and it is case-variant. So its application is still limited so far. Moreover, even structures were designed based on a number of actual earthquake records and carefully constructed artificial records in the analysis, structures might still be unable to resist other earthquake records.

From 2000, Luo Q.F and his students [8-14] proposed and begin to work on the time-frequency response spectrum (TFRS), a 3D spectrum with time axis. It is the extension of the RS. Zhang X.Z. [8] calculated the TFRS of the El Centro earthquake, Luo Q. F. and Li S.D.[9], Li S.D. [10] calculated the TFRSs of some typical earthquakes, validated the TFRS using time history analysis on a SDOF model. Huang J.[11] proposed the nonlinear TFRS, and studied the influence of damping ratio and yielding displacement on the TFRSs. Che W. and Luo Q.F.[12] studied the damage mechanism of a synthesized rotational components of the ChiChi earthquake based on the translational records of the Wenchuan earthquake by TFRS. In his thesis and paper, Qiu Z.G [13,14] first used the contour plots of the TFRS to analyze the structural damage mechanism, and proposed a damage curve and the concept of potential damage energy in the study of TFRS.

Based on above studies, this paper constructed the TFRSs of 3 historical earthquake records: 1976 Tianjin SN (station Tianjin People's Hospital), 2008 Wenchuan EW (station Wolong), and 1995 Kobe SN (station JMA). Then the time-frequency characteristics of the 3 records were analyzed by their respective TFRS, and further, the damage they might bring about on structures was predicted. As an example, a 5-story building was used to illustrate the damage mechanism reflected by the TFRSs. Structural and non-structural damage standards were set to show damage degrees. Results showed that, 3 key characteristics of ground motions, intensity, frequency, and time, could be thoroughly reflected in TFRSs. Also TFRS could reflect the structural damage mechanism progressed with time, so it is helpful in directing ground motion selection for more effective structural analysis.

2. 3D Time-Frequency Response Spectrum (TFRS)

2.1 Definition and formation

As is known, acceleration response spectrum is calculated by Eq.(1):



$$S_{a}(\xi,T) = \frac{2\pi}{T} \left| \int_{0}^{t} \ddot{y}(\tau) e^{-\xi \frac{2\pi}{T}(t-\tau)} \sin \frac{2\pi}{T} (t-\tau) d\tau \right|_{\text{max}}$$
(1)

where $S_a(\xi,T)$ is the acceleration response spectrum, ξ and T is natural period and damping of the SDOF system, respectively. TFRS is the absolute acceleration response time history for SDOF systems with same damping ratio (5% in this study) and different natural peirods under ground motions. It is the function of time and structual period, as shown in Eq.(2):

$$S_a(\xi,T) = \frac{2\pi}{T} \left| \int_0^t \ddot{y}(\tau) e^{-\xi \frac{2\pi}{T}(t-\tau)} \sin \frac{2\pi}{T} (t-\tau) d\tau \right|$$
(2)

Unlike response spectra(RS), TFRSs do not maximize the response with respect to time. TFRS are 3D spacial spectra of all 3 key elements of ground motions: intensity, frequency, and duration. Luo Q. F. summarized 3 advantages of the TFRS over the response spectrum[14]: 1) It retains the basic characteristics of the RS and thus are easily accessible; 2) It can analyze both the time-frequency characteristics of ground motions and the damage mechanisms of structures; 3) It can analyze the cumulative damages of structures.

Eq.(2) gives the expression of the acceleration TFRS, which governs the responses of accelerationsensitive elements, such as non-structural attachments and building contents. Acceleration damages are inertial damages, such as overturning, sudden moving, falling, and colliding of free-standing items, and connection damage of anchored items. Similarly, displacement TFRS can be constructed too. It governs the responses of displacement-sensitive elements, such as walls, columns, braces, pipelines, veneers, etc. Deformation damages include excessive drift, distortion, collapse, etc.

2.2 Typical TFRSs of historical earthquake records

In this paper, 3 earthquake ground motions, 1976 Tianjin SN, 2008 Wenchuan EW, and 1995 Kobe EW (KJM), were selected to illustrate the plots of the TFRSs and their connotations. Time histories of the 3 earthquakes are shown in Fig. 1. It can be seen that, the Tianjin wave contains more low-frequency components; the Wenchuan wave contains more high-frequency components; the Kobe and Wenchuan waves have a few intense sections of spindle shapes; the accelerations of the Tianjin and Kobe waves both increase suddenly from second 7 or so; etc. These characteristics, and more others, will be reflected in the plots of TFRSs, too, later.



Fig. 1 - Time histories of Tianjin, Kobe & Wenchuan earthquakes

Not to loss generality, PGAs of the 3 records will be scaled to the same 150cm/s² in TFRS calculation, and structural damping ratio is assumed to be 5%. With SDOF mass-spring oscillators and formula (2), TFRSs are then obtained. Fig.(2) shows the contour maps of the acceleration TFRSs, and Fig.(3) shows the contour maps of the displacement TFRSs, where x- and y- axes are time and period, respectively, and z-axis is implicitly shown in the contour lines of different colors, which indicate the response values.

The blank margin at the 7th second in the Tianjin and Kobe TFRS contour maps, and the two balls of bright colors in the Wenchuan TFRS contour maps (Fig. (2) & Fig.(3)) show how the earthquake energy distributes with time, which agree with the observations from Fig. (1).

Fig. 2 also shows that, the TFRS of the Tianjin wave has a bunch of closely spaced crests/spines with similar amplitudes aligned in parallel in a wide range of structural periods (0s-1.9s) and duration (7.0s-19.19s). It



can be imagined that, from the 7.0th sec, structures having a period within 0s-1.9s would be subjected to the continuous attack of the primary crests and experience extensive damage. Both Fig. (2) and Fig.(3) also show that, there are some secondary crests in the up right direction(later time, longer period), which means that, even if the structure were only slightly damaged in the first round of strikes, the resulting stiffness degradation, or lengthened period, would push it to the second round of strikes on the right side, which is unfavorable for flexible structures, and thus would deteriorate the situation. So this earthquake is unfavorable to both structures and their attachments, because it could cause both large displacement and acceleration in a wide low-period range.



Fig. 2 - Contour plots of acceleration TFRSs

The strong effects of the Kobe earthquake focus on the range of periods of 0s-1.3s and duration of 7.0s-20s(Fig.(2)&(3)). Out of the time range, the accelerations decrease quickly (Fig.(2)), but displacements increase for structures with longer periods (Fig.(3)). So it can be foreseen that, Kobe earthquake is most devastating to building structures with periods of 0s-2.3s and non-structures with period of 0s-1.3s within time of 7.0s-20s(Fig.(2)&(3)).



Fig. 3 – Contour plots of displacement TFRSs

The Wenchuan wave has 3 acceleration crests along time axis: 8.0s-16.0s, 29.0s-38.0s, and 64.5s-65.5s for periods 0s-0.7s. So, structures with periods of 0s-0.7s would be attacked by three rounds of excitations, especially the first two. Structures with periods greater than 0.7s, and displacement-sensitive structures, the damaging effects are minor. Also notice that, under the same intensity of ground motions, the acceleration responses to Wenchuan earthquake are much smaller than the other two. This earthquake is only unfavorable to non-ductile structures with less resilience.

Fig.(2) & Fig.(3) also show that, if a line is drawn to connect the points of max responses at different time and frequency, then the projection of this line on the frequency axis is exactly the response spectrum(it would be clearer if the contour plots are shown in 3D mesh graphs). That's why the RS could not consider the time effects. Imagine to cut a plan(either by a straight line or a curved line) from the 3D TFRS without projection on either axis, a spacial plan spectrum would be formed. With it, pushover or IDA (Incremental Dynamic Analysis) methods that consider all 3 key elements of ground motions would be easily embedded in.



3. Building information & simulation methods

The objective of this study is to examine the building displacement and acceleration responses with time in light of TFRS, so building type doesn't matter. What matters is the building natural period and its stiffness change with deformation. Because $5\sim7$ story buildings with fundamental natural period of $0.5\sim0.7$ s are common for both residential and public uses, and due to page limit, a 5-story RC frame building with natural period of 0.551s is adopted to validate the TFRS analyses here. It is simplified as a spring-mass model. Table 1 shows the story masses and stiffness in its E-W direction.

Table 1	- Story	masses	and	stiffness
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Story	1	2	3	4	5
Mass, m (kg)	986400	758900	758900	768700	389200
Stiffness, k (kN/m)	0.920×10 ⁶	1.066×10^{6}	1.066×10^{6}	1.066×10^{6}	0.985×10^{6}

The dynamic equation of motion is:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathbf{M}\mathbf{1}\ddot{\mathbf{x}}_{g} \tag{1}$$

where **u** is displacement with respective to the ground, **M** is the diagonal mass matrix, K is the stiffness matrix, **C** is Rayleigh damping matrix using 5% in the 1st and 3rd mode, **1** is the earthquake influence vector, and \ddot{x}_g is ground earthquake acceleration.

Rewrite Eq.(1) into state-space form in Eq. (2a), 2(b) & 2(c):

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{B}\ddot{\mathbf{x}}_{g} \tag{2a}$$

$$\mathbf{y} = \mathbf{C}\mathbf{z} + \mathbf{D}\ddot{\mathbf{x}}_{g} \tag{2b}$$

$$\mathbf{z} = \begin{bmatrix} \mathbf{u} \\ \dot{\mathbf{u}} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{1} \end{bmatrix}$$
(2c)

where **C** and **D** are up to the specified case of outputs.

To account for the nonlinearity in the Eq. (2a), the restore force f_i at step i is calculated after step i-1 by the Bouc-Wen model in Eqs. (3):

$$\mathbf{f}_{i} = \alpha \mathbf{k} \mathbf{u}_{i-1} + (\mathbf{1} - \alpha) \mathbf{k} \mathbf{z}_{i-1} , \quad \dot{\mathbf{z}} = \mathbf{A} \dot{\mathbf{u}} - \beta \dot{\mathbf{u}} |\mathbf{z}| - \gamma |\dot{\mathbf{u}}| \mathbf{z}$$
(3)

where z is an evolutionary variable that accounts for the history dependence of the response. A = 1, $\alpha = 0.85$, $\beta = 0.2$, and $\gamma = 0.2$ in this study.

Then, the restore force is replaced by the equivalent linear force after each step and thus updated the Eq. (2a). To account for the hysteretic effects, the equivalent stiffness $k_{eq,i}$ will take the value of the last step if the structure did not yield (ductility ratio≤1). With $k_{eq,i}$, the hysteretic energy E_h can then be estimated as in Eq. (4):

$$\mathbf{f}_{i} = \begin{cases} \mathbf{k}_{eq,i} \left(\mathbf{u}_{i-1} + \mathbf{z}_{i-1} \right) & \mu_{i} > 1 \\ \mathbf{k}_{eq,i-1} \left(\mathbf{u}_{i-1} + \mathbf{z}_{i-1} \right) & \mu_{i} \le 1 \end{cases}, \text{ and } \mathbf{E}_{h} = \mathbf{K}_{eq} \mathbf{z}^{2} / 2$$
(4)

where $\mu_i = u_{i,max}/u_y$. This iteration may degrade too quickly, but it helps to explain the phenomena of TFRSs.

Consider a hospital, some damage standards for structures and non-structures [15] are shown in Table 2:

Standards	structure		non-structure					
	Yield	Light damage	Water pipes	X-ray machine	Switch box	Laboratory equipment		
drift ratio	1/550	1/250	1/200					
acceleration, m/s ²				4.9	2.0	1.47		

Table 2 – Damage standards



4. Result analyses

Here, 2 levels of earthquakes, 147cm/s^2 and 220cm/s^2 , were used in simulation. They correspond to moderate and major design levels, respectively, for the fortification design standard of intensity 7(Chinese code). To show how damage accumulates with time, responses of linear and nonlinear models were both listed for comparison. Table 3 showed the ductility ratio μ_{max} , period T_{max} , and number of crossings to the drift standards and acceleration standards. Drift standards, yield (YD) and light damage (LD), are meaningful for structural and displacement-sensitive non-structures, such as water pipes and partition walls, while acceleration standards are meaningful for acceleration-sensitive non-structures, such as X-ray machine(X-ray) and switch boxes (sw box).

	Earthquake	T_{max}	μ_{max}	# YD	# LD	a _{max}	#X-ray	# sw box
Linear,	Tianjin	0.55	2.05	68	0	5.02	4	162
Moderate, $147 \text{ sets} / s^2$	Kobe	0.55	1.53	25	0	4.25	0	108
PGA=14/ cm/s	Wenchuan	0.55	1.21	16	0	3.36	0	69
Linear,	Tianjin	<mark>0.55</mark>	<mark>3.07</mark>	<mark>395</mark>	<mark>45</mark>	<mark>7.51</mark>	<mark>62</mark>	<mark>509</mark>
Major, PGA=220 cm/s ²	<mark>Kobe</mark>	<mark>0.55</mark>	<mark>2.30</mark>	<mark>223</mark>	7	<mark>6.36</mark>	<mark>17</mark>	<mark>298</mark>
	Wenchuan	<mark>0.55</mark>	<mark>1.60</mark>	<mark>128</mark>	<mark>0</mark>	<mark>5.02</mark>	<mark>2</mark>	<mark>295</mark>
Nonlinear, Moderate, PGA=147 cm/s ²	Tianjin	0.88	4.52	324	114	4.59	0	149
	Kobe	0.88	3.46	178	19	3.52	0	46
	Wenchuan	0.66	1.04	6	0	3.36	0	18
Nonlinear, Major, PGA=220 cm/s ²	Tianjin	<mark>0.90</mark>	<mark>6.74</mark>	<mark>594</mark>	<mark>241</mark>	<mark>7.00</mark>	<mark>68</mark>	<mark>323</mark>
	Kobe	<mark>0.90</mark>	<mark>5.39</mark>	<mark>239</mark>	<mark>89</mark>	<mark>5.95</mark>	<mark>11</mark>	<mark>142</mark>
	Wenchuan	<mark>0.66</mark>	<mark>1.64</mark>	<mark>388</mark>	<mark>87</mark>	<mark>2.82</mark>	<mark>0</mark>	<mark>67</mark>

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Table 3 showed that, for this building, 1) Of the 3 earthquakes, Tianjin was the one that caused the maximum drift, acceleration, number of crossings to all damage standards for both linear and nonlinear cases, because the resultant periods have been within the highlight areas of the TFRSs. 2) Wenchuan causes the minimum responses among the 3 earthquakes. So it was not devastating to structures with this natural period. The number of crossings to the acceleration standards is also the least, especially for the nonlinear case, showing that the accelerations decay quickly. 3) The drift and acceleration responses to Kobe were moderate, but drift decay quickly for nonlinear cases (with less crossings) when periods were lengthened to 0.9s (beyond the highlight area of 0s~0.7s). All these phenomena perfectly agree with the TFRS' predictions. Also shown in Table 3 is that, nonlinear drift responses to major earthquakes would result in serious structural damage, while linear major earthquake would result in serious non-structural damage, and among which, switch box would suffer more serious damage than medical equipment, such as X-ray machines.

The details can be seen in the linear and nonlinear time history responses of Fig.4 with PGA=220 cm/s².



Fig. 4 – Linear & nonlinear time history responses to the major earthquake



In Fig.4, the 2 groups of horizontal lines represent the two damage standard levels. Clearly, earthquakes with same PGA cause very different responses on the structure. Tianjin earthquake causes the maximum story drifts and accelerations, both in linear case (T = 0.551s) and in nonlinear case (T = 0.90s), because structural periods are within sensitive areas of displacement and acceleration TFRSs. So it is unfavorable to both structures and non-structural attachments. Kobe earthquake is similar to Tianjin, except that in the nonlinear case when T grows longer (T = 0.90s), the acceleration responses decay quickly with time, and so less X-ray machines and switch box are in danger than in the linear case when T = 0.551s. For Wenchuan earthquake, displacement responses are only a little bit larger in the nonlinear case than the linear case, while accelerations decrease for the nonlinear case, because T = 0.66 is nearly outside of the acceleration highlight area of 0s~0.7s. So in Wenchuan earthquake, the damage would occur most on pharmacy room cabinet, laboratory equipment, and switch box on the wall (especially for the linear case after 13s), but X-ray machines and operation room equipment are safe. The results coincide with TFRSs.

Fig. 5 shows μ and E_h of the 1st story with time. It can be seen that, larger earthquake intensity does not necessarily result in larger E_h (deformation related damage). For example, E_h did not change much in Wenchuan. It even decreases in larger intensity case after 12s in Kobe. These agree well with the time history analyses, and have been shown in TFRS, too.



Fig. 5 Ductility ratio & hysteretic energy with time by the simplified iteration method

Fig.6 showed the response profile of the structure to the moderate level and major level earthquakes, respectively. The plastic deformation development can be seen from the comparison. It can be seen that, with earthquake intensities increase, drifts barely change for the wenchuan earthquake, and acceleration even decrease; while Tianjin and Kobe earthquakes both have drifts and acceleration increase, in which the drifts of the Tianjin earthquake increase up to the 4th floor (even the 4th floor would experience some moments of yielding), and drifts in Kobe earthquake increase evenly and steadily.



Fig. 6 - Response profiles to the moderate and major earthquakes



The above results show that, TFRS is very effective in predicting the structural behavior and damage pattern. It is an effective tool in analyzing ground motion characteristics. With it, proper ground motions could be found for the structural analysis.

5. Conclusions

Time-frequency response spectrum (TFRS) is a 3D spacial spectrum composed of all 3 key elements of ground motions: intensity, frequency, and duration. This study selected 3 typical earthquake records, constructed their TFRSs, analyzed their peak value distributions in time and frequency domains of TFRS's, and inferred their different influence on the structural seismic responses. The conclusions from the TFRSs and the effectiveness of the TFRSs were validated by analyzing the structural damage mechanisms of a five-story structure using linear and nonlinear time history analyses. Results show that, TFRS is effective for both ground motion analyses and structural damage mechanism analyses; it is of strong application value in selecting right ground motions when doing structural analyses, as well as potential theoretical value when it is required to examine the time effects of earthquakes to explore and establish new structural design methods.

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