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DISPLACEMENT-BASED SEISMIC DESIGN OF IN-PLAN ASYMMETRIC BUILDINGS WITH DAMAGE CONTROL

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Abstract

This paper presents a procedure for the displacement-based seismic design of in-plan asymmetric buildings with control of earthquake-induced damage. The method is based on the assumption that the maximum seismic response of a structure may be estimated for design purposes through a reference single degree of freedom system with inelastic behaviour, all this by applying basic equations of structural dynamics commonly used in current seismic design of buildings. The simultaneous bidirectional seismic demand is approximated by using smoothed design spectra as those proposed by currently used design regulations. To illustrate the steps required in the application of the design method proposed the displacement-based design of an in-plan asymmetric 15-storey reinforced concrete building subjected to a particular bidirectional seismic demand, representative of the horizontal acceleration records obtained at soft soils conditions as those of the lake-bed zone of Mexico City. The results obtained from this design method are compared with those obtained from the non-linear step by step dynamic analysis of the building when subjected to the same records for which was originally designed. Based on such comparison, the authors conclude that the method proposed allows a satisfactory design of this type of structures.

Keywords: Displacement-based design, asymmetric buildings, damage control, modal spectral analysis

1. Introduction

Most seismic design codes accept the use, with certain restrictions, of the static method or, without restrictions, dynamic methods (modal spectral or time history analyses) for the design of in-plan asymmetric buildings explicitly considering the effects of torsion and considering linear elastic behaviour, even for limit states involving seismically induced structural damage. In both cases, the existence of in-plan asymmetries are penalized by reductions of the seismic behaviour factor used for design despite that several studies have shown the limitations of this approach, since the distribution of forces in in-plan asymmetric that reach the non-linear range may differ significantly from that occurring in the elastic range. Moreover, the aforementioned scheme of penalizing seismic demands leads to a uniform alteration of the design strengths of all structural elements, hence, it is not particularly effective to consider appropriately the effects of structural irregularity in the design of inelastic structures, an issue that is essential to guarantee a target seismic performance under design conditions.

Of particular relevance to the seismic design of in-plan asymmetric buildings are the results of investigations of diverse authors stressing the limitations of the use of the force-based design procedures, where they have issued recommendations to consider explicitly the effect of strength distributions in the nonlinear torsional behaviour of in-plan asymmetric buildings and discussed their implications when used in performance-based seismic design of such structures. Along these lines, Paulay, [1], conducted research on the influence of torsional effects on the seismic behaviour of inelastic structures and concluded that suitable control of the distribution of stiffnesses and strengths is necessary to achieve desirable performance. Unfortunately, these investigations have been only partly continued, *e.g.* [2 and 3], making evident the need to further investigate this subject.

Based on such recommendations and on the recognition of the advantages of a displacement-based design approach over the conventional force-based design procedures recommended by most seismic design codes for building structures, in this paper, the authors propose a displacement-based seismic design procedure that allows the design of in-plan asymmetric buildings subjected to earthquake-induced bidirectional loading. The method proposed is based on the assumption that it is possible to approximate the maximum response of a multiple-degree of freedom structure by means of a reference single degree of freedom system (SDOF), whose properties are consistent with those of the fundamental mode of the structure. In this method, the design displacement of the reference SDOF system and the corresponding design displacement profile of the structure are a function of the design interstorey drift threshold prescribed for the ultimate limit state and of a proposed design damage distribution, accounting also for the structural geometry. The seismic design forces are calculated directly from such displacement profile.

Foremost, this paper shows an overview of the modal superposition method, as the design method proposed relies on its fundamentals. In the following section, this method is presented, along with a brief review of the reference SDOF system approach and a detailed step by step design procedure. Subsequently, the results of the application of the method proposed to design a 15-storey reinforced concrete building with in-plan asymmetric stiffness distributions, subjected to a bidirectional seismic demand characteristic of the soft soil sites in Mexico City, are presented, where it is shown that an acceptable correspondence between the actual and expected performance is attained. Finally, a brief discussion of the results and conclusions are given.

2. Modal superposition method

The dynamic equilibrium equation for a three-dimensional multi degree of freedom structure subjected to a bidirectional seismic demand at its base is given by the following equation:

$$[\mathrm{M}]\{\ddot{\mathrm{u}}^{\mathrm{t}}\} + [\mathrm{C}]\{\dot{\mathrm{u}}^{t}\} + [\mathrm{K}]\{\mathrm{u}^{t}\} = \{0\} \qquad (1)$$

where:

$$\{\ddot{\mathrm{u}}^{\mathrm{t}}\} = \{\ddot{\mathrm{u}}\} + \{\ddot{\mathrm{u}}_{\mathrm{g}}\} \qquad (2)$$

$[\mathrm{M}]$: Mass matrix

$[\mathrm{C}]$: Damping matrix

$[\mathrm{K}]$: Stiffness matrix

$\{\mathrm{u}^{t}\}$: Total displacement vector

$\{\dot{\mathrm{u}}\}, \{\dot{\mathrm{u}}\}, \{\ddot{\mathrm{u}}\}$: Displacement velocity and acceleration vectors, respectively

$\{\ddot{\mathrm{u}}_{\mathrm{g}}\}$: Ground acceleration vector

Substituting Eq. 2 in 1 leads to:

$$[\mathrm{M}]\{\ddot{\mathrm{u}}\} + [\mathrm{C}]\{\dot{\mathrm{u}}\} + [\mathrm{K}]\{\mathrm{u}\} = -[\mathrm{M}]\{\ddot{\mathrm{u}}_{\mathrm{g}}\} \qquad (3)$$

Transforming the relative to base displacement, velocity and acceleration vectors to modal, $\{\mathrm{v}\}$, :

$$\begin{aligned} \{\mathrm{u}\} &= [\Phi]\{\mathrm{v}\} \\ \{\dot{\mathrm{u}}\} &= [\Phi]\{\dot{\mathrm{v}}\} \\ \{\ddot{\mathrm{u}}\} &= [\Phi]\{\ddot{\mathrm{v}}\} \end{aligned} \qquad (4)$$

where $[\Phi]$ is the mass normalized modal shape matrix

Substituting this transformation in the dynamic equilibrium equation, Eq. 1, and pre-multiplying this equation by the transpose of the modal shapes matrix, $[\Phi]^{\mathrm{T}}$, the following system of uncoupled dynamic equilibrium equations in terms of the modal coordinates is obtained:

$$[\Phi]^{\mathrm{T}}[\mathrm{M}][\Phi]\{\ddot{\mathrm{v}}\} + [\Phi]^{\mathrm{T}}[\mathrm{C}][\Phi]\{\dot{\mathrm{v}}\} + [\Phi]^{\mathrm{T}}[\mathrm{K}][\Phi]\{\mathrm{v}\} = -[\Phi]^{\mathrm{T}}[\mathrm{M}]\{\ddot{\mathrm{u}}_{\mathrm{g}}\} \qquad (5)$$

where

$$[\Phi]^{\mathrm{T}}[\mathrm{M}][\Phi] = [\mathrm{I}] \qquad (6)$$

$$[\Phi]^{T}[C][\Phi] = [2\zeta\omega] \qquad (7)$$

$$[\Phi]^{\mathrm{T}}[\mathrm{K}][\Phi] = [\omega^{2}] \qquad (2)$$

$$[\Phi]^{\mathrm{T}}[\mathrm{M}] = [\Gamma] \qquad (3)$$

and $[I]$, $[2\zeta\omega], [\omega^2]\ y\ [\Gamma]$ are the unit, the damping, the modal frequencies and the participation factors matrices respectively, *i.e.*,

$$\{\ddot{v}\} + [2\zeta\omega]\{\dot{v}\} + [\omega^2]\{v\} = -[\Gamma]\,\{\ddot{u_g}\} \qquad (40)$$

From this the dynamic equilibrium equation in terms and corresponding to the i^{th} modal coordinate of a structure subjected to a bidirectional demand is:

$$\ddot{v}_i + 2\zeta\omega_i\dot{v}_i + \omega_i^2 v_i = -\Gamma_{xi}\ddot{u}_{gx} - \Gamma_{yi}\ddot{u}_{gy} \qquad (11)$$

Decomposing v_i, $\dot{v}_i$ y $\ddot{v}_i$ in terms of two components each corresponding to each of the bidirectional seismic demand terms in Eq. 11

$$v_i = v_{ix} + v_{iy} \qquad (12)$$

Using Eq. 12, Eq. 11 may be separated in two uncoupled dynamic equilibrium equations:

$$\ddot{v}_{ix} + 2\zeta_i\omega_i\dot{v}_{ix} + \omega_i^2 v_{ix} = -\Gamma_{xi}\ddot{u}_{gx} \qquad (13)$$

$$\ddot{v}_{iy} + 2\zeta_i\omega_i\dot{v}_{iy} + \omega_i^2 v_{iy} = -\Gamma_{yi}\ddot{u}_{gy} \qquad (14)$$

The dynamic equilibrium equations for two single degree of freedom, SDOF, oscillators, each subjected to the respective $\ddot{u}_{gx}$ and $\ddot{u}_{gy}$ demands are:

$$\ddot{D}_{ix} + 2\zeta\omega_i\dot{D}_{ix} + \omega_i^2 D_{ix} = -\ddot{u}_{gx} \qquad (55)$$

$$\ddot{D}_{iy} + 2\zeta\omega_i\dot{D}_{iy} + \omega_i^2 D_{iy} = -\ddot{u}_{gy} \qquad (16)$$

where D_{ix} y D_{iy} are the respective displacements produced by the demands $\ddot{u}_{gx}$ y $\ddot{u}_{gy}$.

Comparing, one to one, Eqs. 13 and 14 with Eqs. 15 and 16:

$$v_{ix} = \Gamma_{xi}D_{ix} \qquad (17)$$

$$v_{iy} = \Gamma_{yi}D_{iy} \qquad (18)$$

Substituting Eqs. 17 and 18 in Eq. 12:

$$v_i = \Gamma_{xi}D_{ix} + \Gamma_{yi}D_{iy} \qquad (19)$$

and substituting this equation in Eq. 4 the displacements for the j building level associated to mode i may be written as:

$$u_{ijx} = \phi_{ijx}\left(\Gamma_{xi}D_{ix} + \Gamma_{yi}D_{iy}\right) \qquad (6)$$

$$u_{ijy} = \phi_{ijy}\left(\Gamma_{xi}D_{ix} + \Gamma_{yi}D_{iy}\right)$$

$$u_{ij\theta} = \phi_{ij\theta}\left(\Gamma_{xi}D_{ix} + \Gamma_{yi}D_{iy}\right)$$

In structural design practice, where modal spectral analyses are used to estimate maximum responses under bidirectional seismic action, a correlation factor between the responses to the demands applied separately in each direction, β, is required. In this case, if the maximum displacement for each demand component may be calculated, this maximum values, occurring at different times, may be combined to calculate an upper bound approximation of the maximum displacement under the demands acting simultaneously in both horizontal directions. Thus, the maximum roof displacement associated to mode i, Δ_{irx} and the maximum modal displacement, D_{ix}, can be estimated by the following equations:

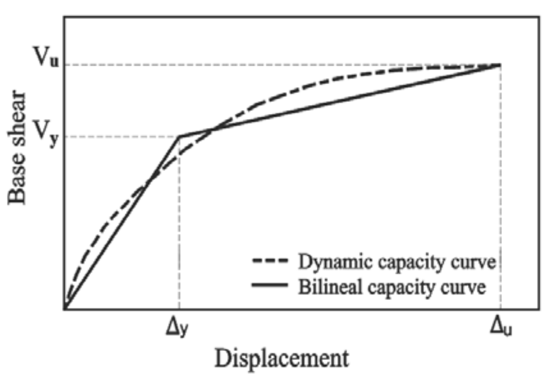
$$\Delta_{irx} = \phi_{irx} D_{ix} \left(\Gamma_{xi} + \beta \Gamma_{yi} \right) \quad (21a)$$

$$D_{ix} = \frac{\Delta_{irx}}{\phi_{irx} \left(\Gamma_{xi} + \beta \Gamma_{yi} \right)} \quad (21b)$$

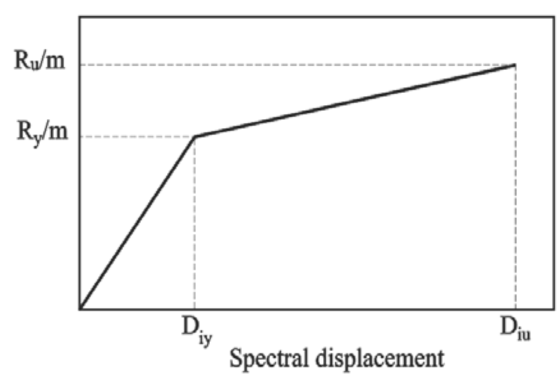
The method proposed by the authors, which is based on the principles of the modal superposition method, relies on the latter equation to define a design displacement.

3. Displacement-based seismic design method with damage control

The displacement-based seismic design method proposed in this paper relies on the assumption that it is possible to approximate the maximum response of a multiple degree of freedom structure by means of a reference SDOF oscillator with characteristics consistent with those corresponding to the fundamental mode of vibration of the structure. This oscillator is called the reference SDOF system. The fundamental principle of this method is that, it is possible to approximate the capacity curve of a multi degree of freedom structure by a bilinear curve (Fig. 1), and from it, using the modal coordinates associated with it, is also possible to approximately calculate the seismic performance of the structure by constructing the behaviour curve of a reference SDOF system expressed in coordinates of pseudo-acceleration (strength per unit mass) *vs.* spectral displacement. The slope of the first branch of this behaviour curve represents the stiffness properties of this system behaving in the elastic range and the slope of the second branch represents the properties corresponding to the structure in the post-yield range of behaviour. The characteristics of this second branch are defined from a structure with a distribution of damage proposed by the designer. The strength per unit mass (R_y/m), corresponding to the crossing point of these branches is associated with the level of demand for the structural elements which will exhibit damage when subjected to the design demands. The ultimate strength per unit mass (R_u/m) is related to the level of demand of structural elements that are assumed to remain undamaged under design demands.



a) Capacity curve, real and its bilinear approximation



b) Approximate behaviour curve in the ADRS format

Fig. 1 Illustration of the capacity curve of a structure and the behaviour curve of the corresponding reference SDOF system as those used by the design method proposed.

Based on the above fundamentals, the detailed steps required for the application of the method proposed for the displacement-based design of buildings with in plan asymmetries and subjected to bidirectional seismic demands can be summarized in what follows:

1. Construction of the structural model of the pre-designed undamaged building. The pre-design of the building for the considered demands may be obtained in accordance with current design regulations based on forces either through a static analysis under gravity and equivalent lateral loads or by a spectral modal analysis.

2. Definition of the performance index associated the performance objective. Generally, the target performance index used for the displacement-based design of buildings is the maximum interstorey drift.

3. Definition of an acceptable configuration of structural damage under design conditions. This configuration is defined by selecting the structural members and sections in which damage under design conditions is accepted to occur. This damage is introduce in the structural model, here referred as "damaged", in accordance with the characteristics of the level of analysis to be carried out.

4. Determination of the dynamic characteristics of the undamaged and damaged models. Eigen value analysis of both models, aimed to obtain periods and mode shapes are carried out and, are performed. From these analyses, the contribution of mode that most influence the response of the structure is defined and the frequencies associated to this mode define the branches of the behaviour curve of the reference SDOF; one frequency for the elastic branch, ω_e, and the other for the inelastic branch, $\alpha\omega_e$, Fig. 2.

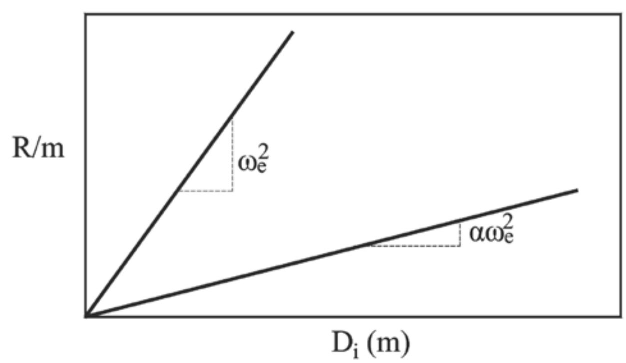


Fig. 2 Branches of the behaviour curve of the reference SDOF system.

5. Calculation of the yield displacement, $\overline{D_{11}}$, of the behaviour curve of the reference SDOF system define from the yield of the target interstorey drift of the structure, using equations in terms of the geometry and properties of the materials of the structure [4, 5].

6. Determination of the target displacement of the reference SDOF system. This displacement is defined in terms of the maximum permissible value of interstorey displacement associated to the design performance level. The calculation of the target displacement of the behaviour curve starts from:

$$\delta_{dj} = \delta_{1dj} + \delta_{2dj} \quad (22)$$

where δ_{1dj} and δ_{2dj} are the interstorey displacements at level j where the maximum occurs, subindex 1 represents the elastic stage and subindex 2 the pos-yield stage.

The contribution of the i^{th} mode for each stage is defined by the following equations:

$$\delta_{1dji} = \rho_1 \delta_{1j} \quad (23)$$

$$\delta_{2j} = \delta_{disj} - \delta_{1j} \quad (24)$$

$$\delta_{2dji} = \rho_2 (\delta_{disj} - \delta_{1j}) \quad (7)$$

where:

δ_{disj} is the target interstorey,

$$\rho_1 = \frac{\delta_{1ji}}{\sum_{i=1}^{n} \delta_{1ji}} \quad (26)$$

and

$$\rho_2 = \frac{\delta_{2ji}}{\sum_{i=1}^{n} \delta_{2ji}} \quad (87)$$

To calculate the roof displacements corresponding to the mode with the highest contribution to response, *i.e.*, Δ_{r1i} y Δ_{r2i}, corresponding to each of the performance stages the following equations are used:

$$\Delta_{r1i} = \frac{\delta_{1dji}}{\delta_{1ji}} \quad (28)$$

$$\Delta_{r2i} = \frac{\delta_{2dji}}{\delta_{2ji}} \quad (29)$$

Substituting Eqs. 28 and 29 in Eq. 21, the displacements of mode *i* (*e.g.*, the X projection) for each stage, are calculated:

$$\overline{D_{\imath 1x}} = \frac{\Delta_{r1i}}{\phi_{irx}(\Gamma_{xi} + \beta \Gamma_{yi})} \quad (30)$$

$$\overline{D_{\imath 2x}} = \frac{\Delta_{r2i}}{\phi_{irx}(\Gamma_{xi} + \beta \Gamma_{yi})} \quad (31)$$

7. Determination of the ductility corresponding to the target design displacement. The ductility, μ, is calculated from the displacements defined in the previous step,

$$\mu = \frac{\overline{D_{\imath 2x}}}{\overline{D_{\imath 1x}}} \quad (32)$$

8. Calculation of the displacement of the reference SDOF system. From an inelastic displacement spectrum, associated with ductility, μ, calculated in the previous step and the value of α defined in Fig.

2, the spectral interstorey displacement (D_{ux}) corresponding to the fundamental period of the elastic model, T1, is obtained(Fig. 3). This displacement is compared with the target displacement ($\overline{D_{12x}}$); if they are equal, the design process continues; if not, the initial structure and/or the distribution of damage proposed is modified and the analysis reinitiated at step 2 or 3, depending on what was modified, all this in an iterative fashion until the closeness of such displacements is reached.

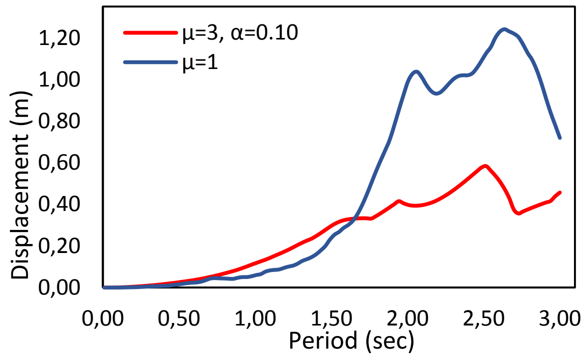


Fig. 3 Elastic and inelastic displacement spectra.

9. Construction of the behaviour curve of the reference SDOF system. With the parameters obtained in the previous steps, *i.e.*, yield displacement, ultimate displacement, yield strength, ultimate strength, initial stiffness, post-yield stiffness, ductility and initial to post-yield stiffness ratio; the behaviour curve of the reference SDOF is built (Fig. 4).

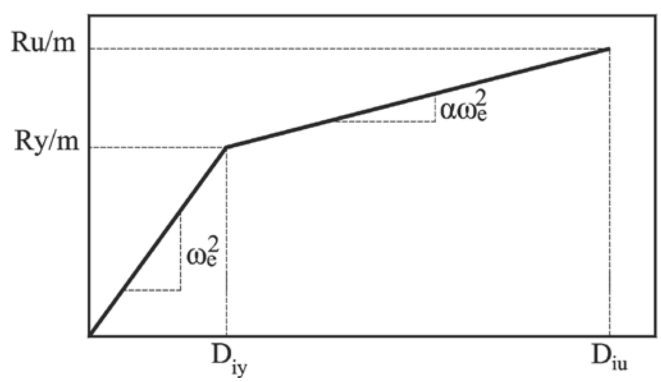


Fig. 4 Behaviour curve of a reference SDOF system.

10. Calculation of the design forces. For the design forces of the structural elements, two modal analysis over time are carried out, one for the undamaged model, including the gravitational loads using as seismic demands the records scaled by the λ1 factor defined as the ratio of the yield displacement of the oscillator to the maximum elastic displacement and the other for the model with damage included using as seismic demand the same records but scaled by the λ2 factor defined as the ratio between the target displacement and maximum displacement corresponding to stage 2. The design forces for the structural elements are obtained by adding the forces of the two aforementioned modal analyses.

$$\lambda_1 = \frac{\overline{D_{11x}}}{\overline{D_{1x}}} \tag{33}$$

$$\lambda_2 = \frac{\overline{D_{12x}}}{\overline{D_{2x}}} \tag{34}$$

4. Application of the design method proposed

4.1 Description of the building used for illustration

To illustrate the application of the design method proposed a 15-storey reinforced concrete building was designed using as seismic demand for comparative purposes of the results expected using the design method proposed under the records of the horizontal components of a real earthquake. The performance results obtained from the designed building were compared with those obtained from a nonlinear dynamic step by step analysis of the building subjected to the same seismic demand used in its design. For the design of the structure used in this paper the following considerations were taken:

1. Floor diaphragms are considered infinitely rigid in its plane, *i.e.*, there are three degrees of freedom per floor, two horizontal displacements and a rotation around the vertical axis and the beams are considered as infinitely rigid for axial deformation effects.

2. No damage is considered to occur at walls, at the base of the columns of the first level and on the beams of the 4 upper levels (12-15).

The building has a rectangular plan with three bays 7 m long in X direction and four bays 8 m long in Y direction. The X1 and Y1 axis are formed by reinforced concrete frames with walls 0.2 m thick. The slabs were 0.12 m thick and the interstoreys 3.3 m high. Figure 5 illustrates the building plan and elevation and the frames with walls.

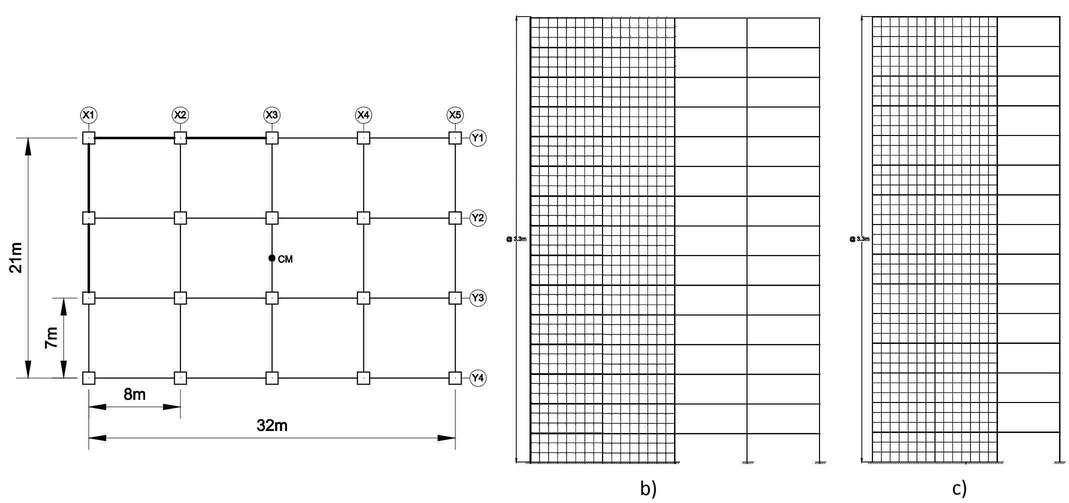


Fig. 5 Building example, a) Building plan, b) Frame Y1 and c) Frame X1**.**

The properties of materials were, for concrete: strength: f'c=250Kg/cm^2, Young modulus Ec = 221359Kg/cm^2 and weight density WD=2400Kg/m^3 and for steel: fy = 4200Kg/cm^2.

For this example performance index used as target was a 0.02 maximum interstorey drift recommended for the ultimate limit state, according to the Building Code of the Federal District and its Complementary Technical Standards NTC RCDF [6].

4.2 Design seismic demands

For the design of the building the records of the NS and EW components of the September 19, 1985 earthquake obtained at the SCT station in Mexico City were used as design demands. The reference SDOF system used in the application of the design method corresponded to the fundamental mode of the structure. Modal analysis on time were performed with the commercial structural analysis program SAP2000 [7] and for dynamic nonlinear step by step analysis the PERFORM 3D V5 program was used [8].

4.3 Analysis of results

To demonstrate the validity and scope of the design method proposed the performance objective and other results of the method are compared with those calculated with a nonlinear dynamic step by step analysis under the same design demands.

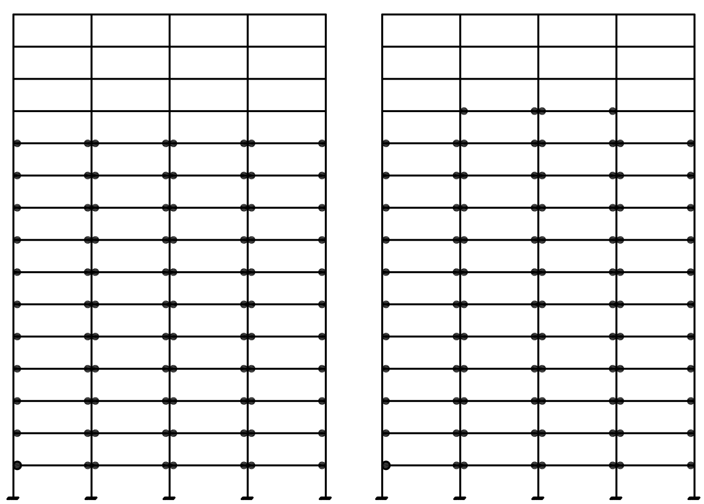
The properties of the behaviour curve of this structure are the following:

Yield displacement (D_y=0.15 cm), strength per unit mass at yield (S_{ay}=1.79 m/s2), ultimate displacement (D_u=0.30 cm), ultimate strength per unit mass (S_{au}=1.86 m/s^2), ratio of post-yield to elastic stiffness (α=10%) and ductility = 2.0.

The design forces of the structural elements were calculated using 2 modal analysis in time, the first for the undamaged structure (factor, $\lambda 1$ = 0.24) and the second for the damaged structure (factor, $\lambda 2$ = 0.6).

Figure 6 shows a comparison between the damage distributions in frame X5 used for the application of the design method proposed and the corresponding obtained from the nonlinear dynamic step by step analysis. It is noted that the frame designed has a damage distribution similar to that proposed, so it may be concluded that damage control was satisfactory.

Figures 7 and 8 show the comparisons between displacements and interstorey drifts corresponding to the proposed method and calculated in a nonlinear dynamic step by step analysis along the intersection of frames X5 and Y1. It is noted that the maximum interstorey drifts in the Y direction are similar. However, these figures also show that the displacement profiles in X and Y directions and the interstorey drifts in the X direction obtained with the method proposed are smaller but not necessarily close to the interstorey drift used as target, which in this case was the maximum interstorey drift in Y direction.



a) b)

Fig. 6 Damage distribution in frame X5, a) Method proposed and b) Step-by-step analysis

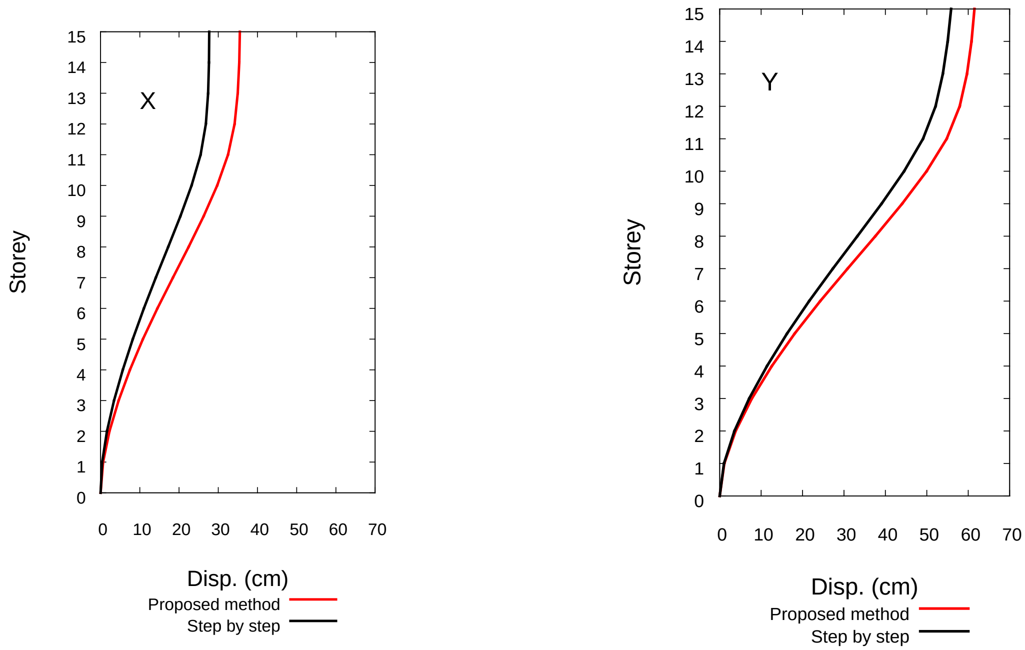


Fig. 7 Comparison of the profile of maximum horizontal displacements of frame X5

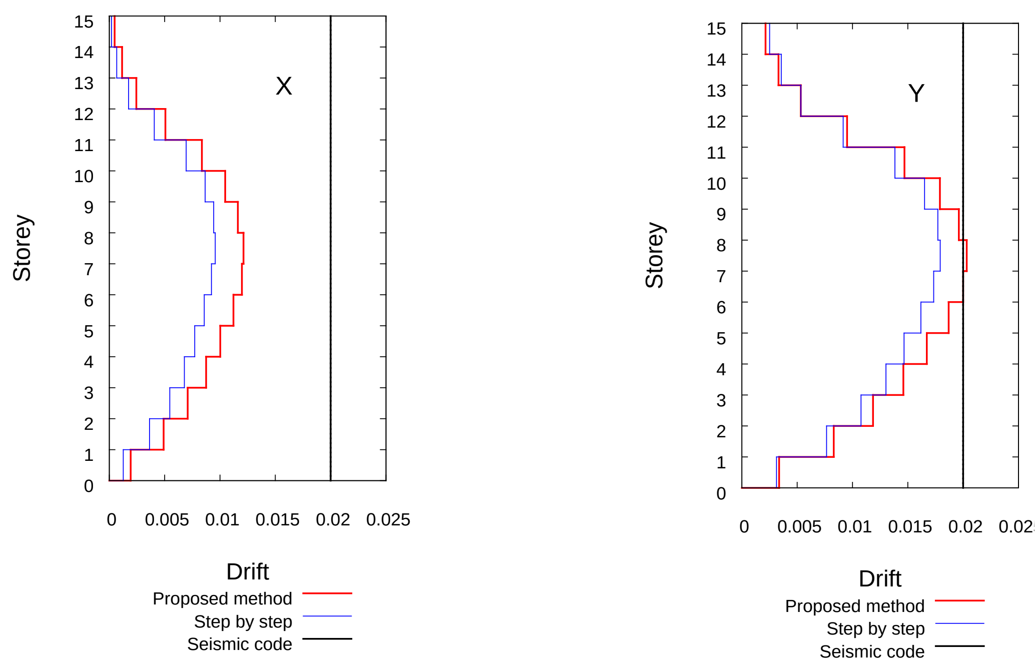


Fig. 8 Comparison of the interstorey drifts for frame X5

5. Conclusions

From the results attained from this investigation the following conclusions may be derived:

1. The differences between the proposed design distribution of structural damage and that obtained from nonlinear dynamic step by step analysis to the same seismic demand for which was designed, were not significant.

2. The maximum interstorey drift attained from the nonlinear step by step dynamic analysis closely matched the design interstorey drift threshold, hence, adequate control of structural behaviour was achieved.

3. Therefore, it can be generally concluded that the method proposed allows the design of buildings with in-plan irregularities that satisfy an ultimate limit state.

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