

BRITISH COLUMBIA SMART INFRASTRUCTURE MONITORING SYSTEM (BCSIMS)

Y. Kaya⁽¹⁾, C. Ventura⁽²⁾

⁽¹⁾ Research Associate, The University of British Columbia, kayaya@mail.ubc.ca ⁽²⁾ Professor, The University of British Columbia, ventur@civil.ubc.ca

Abstract

The British Columbia Ministry of Transportation and Infrastructure and the University of British Columbia, Canada, have recently embarked on a program to instrument key structures to provide confirmation of seismic capacity, assist in focusing retrofit efforts, detect damage from any cause and provide rapid damage assessment of those structures following a seismic event. The instrumentation system installed at each structure is capable of remote configuration and can automatically upload data to a central server via the Internet. As part of this collaboration effective damage detection algorithms that provide reliable intelligence close to real time have been developed and implemented. This technology is used to i) detect, analyze and localize damage to structures; ii) transmit the data regarding these structures in real time via the internet; iii) display in animated and static web pages the data as appropriate for use by the Ministry and UBC. The alert systems and public access web pages can display real time seismic data from the BC Strong Motion Network to provide input for assessments by the Ministry of non-instrumented bridges. These systems may also provide other agencies, emergency responders and engineers with situational awareness. This Project will help transform the current practice of inspecting and evaluating all structures after an earthquake to a more rational and effective one that makes effective use of sensing technology with fast and efficient techniques for data analysis and interpretation. Inspections by the Ministry can then be focused and prioritized to maximize the effectiveness of scarce resources.

Keywords: Structural health monitoring; Strong motion; shake-map; real-time data processing; earthquake notification



1. Introduction

In order to mitigate this seismic risk in British Columbia, the Geologic Survey of Canada (GSC) through the Pacific Geoscience Centre (PGC) has maintained an urban Strong Motion Network (SMN) in BC since 2003. As part of this network, the GSC developed a strong motion Internet Accelerograph (IA) network, which is permanently connected to the Internet and records ground vibration data continuously. The instrument continuously computes a set of Strong Motion (SM) parameters, which characterize the intensity of shaking, and actively reports those values to the GSC's and the University of British Columbia's (UBC) data centers via 4 relays whenever ground shaking exceeds predefined threshold levels. Over the last several years, the British Columbia Ministry of Transportation (MoT) has been working with the PGC to expand the range of the network outside of urban centres and increase the number of stations in the network to the current 170 strong motion stations.

The MoT is responsible for 400 km of provincial disaster response routes in BC. The loss of any portion of these routes after an earthquake could significantly impact emergency response efforts and negatively affect public well-being. The MoT, moreover, maintains over 2500 bridges in the highest seismic zones, many of which are vulnerable to extensive damage in even a moderate quake and potential collapse in a major earthquake. The loss of the use of several structures would not only have immediate impact on public well-being and the ability of emergency vehicles to respond effectively, but would also cripple the economic recovery of the region. The better the information on these areas where structures and facilities are most vulnerable, the better the planning and the preparation can be done. By identifying those structures and facilities most susceptible to seismic forces through automatically generated shake-maps, decision-makers can do effective risk management. Fast and accurate field intelligence immediately following an earthquake can ensure the most effective deployment of vital services and mitigate damage to the built environment.

In a parallel effort, the MoT has also been instrumenting bridges and tunnels in collaboration with the Earthquake Engineering Research Facility (EERF) at the UBC since the late 1990's. The primary purpose of this legacy system was to capture the ground motion input and its effect on structures in the event of an earthquake. Building on these collaborations, the MoT and the UBC recently embarked on a program called the British Columbia Smart Infrastructure Monitoring System (BCSIMS), which integrates data from the instrumented structures (currently sixteen in total) and the SMN. The system organizes and processes real-time data in an efficient manner and delivers results and related reports to predefined recipients such as bridge inspectors at the MoT. This instrumentation program provides immediate notification after an event and incorporates remote Structural Health Monitoring (SHM) system. The goals of the system are: 1) to provide a real-time seismic structural response system to enable rapid deployment and prioritized inspections of the MoT's structures; 2) to develop and implement a structural health monitoring program to address the need for safe and cost-effective operation of structures in BC; and 3) to provide a real-time working platform (www.bcsims.ca) that can integrate many aspects of seismicity in BC.

The implementation of BCSIMS transforms the current practice of inspecting and evaluating all structures after an earthquake to a more rational and effective one that makes effective use of state-of-the-art sensing technology with fast and efficient techniques for data analysis and interpretation; therefore, the inspections can then be focused and prioritized to maximize the effective use of the scarce resources.

2. BCSIMS System Architecture

The development of the BCSIMS architecture has been a collaborative process between the UBC and MoT that started in early 2009. Fig.1 shows the overview of the BCSIMS architecture, and it consists of several subsystems: hardware and software, data acquisition system, data storage and processing tools, network communications, etc. In general, the BCSIMS network involves two main components: the SMN and the seismic SHM network. They are discussed in detail in the following subsections.



Fig. 1 – The architecture of the BCSIMS Network

2.1 Strong Motion Network

The SMN, developed and currently maintained by the GSC and the MoT, consists of 170 Internet Accelerometers (IA) stations [1], which are deployed across the province as shown in Fig.2. The IA stations are designed to detect and measure an earthquake in real-time, and send the recorded event data along with calculated seismic parameters over the Internet to a central location. This procedure promotes the idea of real-time response. The IA station has a digital recorder built internally that allows at least three days of raw data to be stored locally in a ring buffer. The IA stations are connected to the BCSIMS network via four relay serves, which are located at the UBC in Vancouver; the PGC in Sidney on the Vancouver Island; Kamloops in BC; and in Ottawa: this increases the reliability of the entire SM network.

The measured vibration data in the IA stations is filtered in real-time two times in parallel: 1-10 Hz bandpass and 0.1 Hz high-pass filters. The high-pass filtered data is utilized to calculate the important strong motion parameters in real-time such as the Peak Ground Acceleration (PGA), Peak Ground Velocity (PGV), Peak Ground Displacement (PGD) and various spectral intensity scales such as Katayama Spectral Intensity (kSI) [2]. The band-pass filtered data, on the other hand, is used to calculate the ratio of the Short-time average to Longtime average (STA-LTA). A smart algorithm embedded in the IA station detects the early onset of an earthquake. The algorithm is based on the combination of the threshold values for these seismic parameters. When a set of triggered parameters are exceeded, the sensor logs the earthquake records as an event file and later begins processing it after the earthquake is over. Event files are permanently stored on the sensor. After the analysis is done, the raw data along with the calculated seismic parameters are synchronized to the global database via one of the relay servers.



Fig. 2 – Strong Motion network

2.2 Shake-maps

An application is developed in Matlab [3] computing language to produce shake-maps for urban and regional scales. The core of the shake-map algorithm could be described in four steps as: 1) correction of the values of the PGA to rock site condition; 2) estimation of PGA amplitudes in virtual stations using bias-corrected attenuation relationships [4]; and 3) numerical interpolation between grids; and 4) amplification of the PGA values to original site condition. Peak horizontal accelerations at each IA station are used in the calculation of the PGA, and the peak values of the vertical components are not used because they are on average lower than the horizontal amplitudes. Ground Motion Prediction Equations (GMPE) [4, 5, 6] is used to estimate the peak horizontal amplitudes in between IA stations. The Instrumental Intensity (II) shake-map is based on a combined regression of PGA and PGV versus observed intensity for significant earthquakes [7]. Shake-maps are generated automatically following moderate and large earthquakes, and they represent the strong ground motion and shaking. These shake-maps are used by federal, provincial, and local organizations, both public and private, for post-earthquake response and recovery as well as for preparedness exercises and disaster planning. The notion of shake-maps in the BCSIMS network also provides these agencies with situational awareness.

2.3 Strong Motion Event Report

Upon creating the shake-maps, a strong motion event report is automatically generated at the main server and emailed to predefined subscribers list such as bridge inspection engineers of the MoT. The report includes the snapshot of the earthquake including typical parameters such as location, magnitude, depth, etc. It also lists the strong motion stations that are triggered by that earthquake and peak responses. The data recorded by each triggered station is automatically processed, and the results are included in this report. Processed data includes



recorded acceleration, calculated velocity and displacement time histories as well as the acceleration response spectrum and the smoothed Fourier Amplitude Spectrum [8].

2.4 BCSIMS Web Interface

The www.bcsims.ca website is the gateway for user interaction and operational management of the BCSIMS network. Fig.2 shows a screen shot of the homepage of the website. The amount of information accessible on the webpage depends on user type. Public users have access to strong motion network and the shake-maps generated after an event, whereas authorized users are allowed to access further processed information such as automatically generated Strong Motion Event Reports and Structural Event Reports. The structure stations and the SMN are displayed as different icons on a digital geomap: the circles represent the SMN stations and the squares are structural stations. The interactive map allows zooming in/out to focus on a particular station or area. By clicking on a station on the map, additional metadata is provided such as geographical location, picture of the station, and live links to webcams if available. A link in the metadata allows users to view the event data recorded by that station. This includes recorded raw acceleration, calculated velocity and displacement, Fourier amplitude spectrum, and the acceleration response spectrum. An option to view or download all of the recorded data is available in the website for advanced users to further investigate the recorded event data. Such event data is very useful for those bridges that are non-instrumented and located near an IA station. The volume data could then be used to quickly assess the shaking intensity at or near a non-instrumented bridge.



Fig. 3 - Earthquake Simulation in the BCSIMS website

2.5 Earthquake Simulation

The earthquake simulation tool, on the other hand, enables users either to simulate an earthquake on the website. Simulation can be done with different earthquake magnitude, epicenter location, and earthquake type (e.g., subduction or crustal earthquake). The two growing yellow and red circles as depicted in Fig.3 indicate the p-and the s-waves propagations. The color of all of the IA stations on the map is blue by default: none of them are



triggered yet. Once the user starts the simulation, these two circles start growing on the map, and the colour of the IA stations will change from blue to green in real-time. This means that the yellow circle reached the IA station, and as result of that, the p-wave triggered that IA station. When the red circle (s-wave) reaches to the IA station, the colour of that station will change to red: the IA station started recording the vibration data. The simulation stops when the s-wave reaches the user defined distance from the epicenter. After the simulation is over, the final size and the colour of the IA stations on the map indicate the expected PGA and the kSI value for each station, respectively. The expected PGA is calculated based on the GMPE proposed by [4,5,6].

Simulated shake-maps provide emergency responders, bridge maintenance engineers, and other public and governmental agencies with an expected shaking intensity across the urban areas and at the location of critical infrastructures before the simulated earthquake occurs in that area.

2.6 Structural Health Monitoring (SHM)

In parallel to recent developments in sensor and recording technologies, a large number of bridges and tunnels in the BCSIMS network are now installed with SHM systems, which involves continuous monitoring of the dynamic characteristics of structures by digital instruments (e.g., accelerometers, displacement transducers, environmental sensors, etc.). The main objective in the SHM is to track the changes in the dynamic characteristics of the structure in order to detect and locate damage, and to make automatic decisions on the danger level in the structure and the actions that need to be taken. All of the seismic SHM system installed in BCSIMS network significantly mitigated the seismic risk and the overall maintenance costs of a structure, especially for large structures (e.g., bridges and tunnels).



Fig. 4 - Location of 2594 bridges in the BCSIMS website



Fig.4 depicts the locations of the bridge structures in the BCSIMS network, and the Table 1 makes a list of the structures that have seismic SHM system installed and are currently being monitored in real-time in the BCSIMS network. A sensor in the Table 1 means a single channel of a data acquisition system that measures a physical response quantity of a structure such as acceleration or displacement. The instrumentation system installed at each structure is capable of remote configuration and can automatically upload data to multiple remote servers via the Internet. Each instrumented structure in the BCSIMS has a Data Acquisition (DAQ) unit to retrieve data from each sensor on the structure. The type and the number of the sensor used in a seismic SHM system always depends on the expected failure mechanism of the structure and the intentions of the SHM system installed.

No	Structure Name	Total Length (m)	Inst. Year	Number of channel	Type of Sensor
1	French Creek (FC)	200	1997	12	А
2	George Massey Tunnel (GMT)	660	1996	11	A P
3	Queensborough (QB)	914	1996	12	A P
4	Ironworkers Memorial Second Narrows Crossing (IMSNC)	1290	2011	122	A S W T
5	Pitt River (PR)	380	2009	46	AW
6	William R. Bennett (WRB)	1077	2008	12	А
7	Portage Creek Bridge (PCB)	129	1983	41	A S
8	Port Mann (PM)	850	2013	336	ASWDTHP
9	176th Underpass (176B)	75	2013	26	ATH
10	Gaglardi Way Underpass (GWU)	65	2013	22	ATH
11	Kensington Avenue Underpass (KAU)	75	2013	30	ATH
12	Fraser Heights - Wetlands (FHW)	476	2013	20	ATH
13	8264 BNSP Sunbury	68	2014	36	A H W D
14	8270 BNSF Viaduct East Mill Access	195	2014	84	A H W D
15	8313 Hwy-17 Deltaport	133	2014	36	A H W D
16	Earthquake Engineering Research Facility (EERF) at UBC	-	2013	16	А

Table 1 – List of intrumented structurs in the BCSIMS network

A: Acceleration, P: Piezometer, S: Strain gauge, W: Wind, T: Temperature, D: Displacement, H: Humidity

The collected data from each structure is archived and processed in real-time on the Data Processing Server (Fig.1). The data processing includes drift analysis [8], modal identification [9], and the calculation of the important statistics of each channel such as mean, root-mean-square (RMS) and standard deviation etc. [10, 11]. The finite element model updating, the damage detection, and the structural event reports are other important components that have been developed and implemented in the BCSIMS. The results of all of these analyses are permanently stored on the global database in the main server. The entire SHM network is run by the UBC with at least one backup server located at the PGC. In the future an additional out-of-province backup server will be set up. As a physical complement to the web based monitoring network, a control room is established at the UBC and now is in operation as of 2014. As seen in Fig.5, the control room is envisioned as a situation-room



with all the necessary skilled human resources and enabling technologies available, and it ensures a continuous watch and attendance.



Fig. 5- Seismic monitoring control room at the UBC

2.7 Data archiving

The UBC has developed its own data archiving standards and protocols. The collected data at the DAQ is streamed via Internet to the data processing server at the UBC where they are stored in 5-minute lengths of VIF files. The VIF file is a binary file that contains raw data from all of the sensors for a given monitored structure. The length of the VIF file is set to 5-minute by default, but the system administrator can change it in order to manage the entire SHM network much efficiently. As soon as a new VIF file becomes available in the data processing server, it is further compressed so as to minimize the disk space. The compressed data is then achieved in a two-week length of ring buffer in a designated folder on the data processing server. The length of the ring buffer is scalable based on the available disk space on the server.

2.8 Statistics

The one of the key requirements in real-time seismic SHM is that the recording, processing, and analysis of the data should all be done in real-time. The seismic SHM system can track slow changes in the characteristics of the structure, such as those due to aging, change of usage, traffic, and other environmental factors (e.g., temperature, wind, rain, etc.). The change of modal properties of the structure especially due to environmental conditions can be larger than the change due to the damage; therefore, the effect of environmental conditions (e.g., traffic and temperature load in case of bridge) on modal properties of structure must be accounted for as they can completely mask the change of modal properties caused by real damage [12]. Due to the continuous recording in the SHM, the statistical characteristics of the structural changes and their correlation with the factors that might cause such changes are established using the statistical parameters (e.g., mean, standard deviation, root-mean-square, etc.). These parameters help to better understand the dynamic behavior of the bridge under different loading conditions, such as seasonal temperature change, daily traffic loads on bridge, etc. This also enables building up a statistical history of the correlation between the dynamic response of the



bridge and the environmental factors so that the effect of environment can be removed from the recorded data. This information is very helpful to decide whether the identified change in a structural parameter actually represents damage or not [12, 13].

2. 9 Drift analysis

Structural damage at bridge piers are often directly related to displacement demands; therefore, it is usually controlled most efficiently in many seismic bridge codes by imposing displacement (or drift) limits on these structural members. The drift at a bridge pier is defined as the relative displacement of the pier top with respect to its base; therefore, in order to calculate the drift, the displacements at these two locations need to be estimated, if not measured, and this is done using the recorded acceleration data at the top and the base of the pier. The common practice to calculate displacements from acceleration data is to take the double integration of the recorded accelerations; however, the integration operation on raw data significantly increases the noise amplitudes in the integrated signal [8]. New drift calculation tool developed in the BCSIMS minimizes such noise increase. The new tool band-pass filters the recorded data around the resonant frequencies of the bridge pier, and later combines them to obtain the total displacement or drift for each pier. These resonant components have much higher signal-to-noise ratio and therefore the noise influence during the integration and differentiation is minimum.

The administrator of the BCSIMS network specifies the drift pairs for each of the instrumented piers on a bridge, and the system computes the drift from the integrated displacement values. The peak values during an event are stored in the global database, and any drift value exceeding predefined threshold value will indicate a possible damage in the structure, which initiates an e-mail notification being sent out to predefined list of users including bridge engineers and inspectors at the MoT. The drift thresholds for each pier are determined based on the Canadian Highway Bridge Design Code, 2015 and detailed nonlinear finite element model analysis.

2.10 Modal Identification

The SHM system in the BCSIMS estimates the modal parameters of the structure (e.g., modal frequencies, modal damping ratios, and mode shapes) from output-only data (e.g., recorded bridge vibration data only) using the Stochastic Subspace Identification (SSI) [9, 14, 15, 16] method. The SSI method was developed in 1991, and the first application to modal identification problems papered in 1995 [17, 15]. It has many advantages over other modal identification methods, the most important of which, among many others, is that the method could be fully automated so that no human interaction would be necessary.

The modal parameters involve very important information about the dynamic characteristics of the structure; therefore, as soon as new vibration data from any structure in the BCSIMS network becomes available in the data processing server, the modal properties of that structure are automatically estimated in near real-time using the SSI method, and the results are stored in the main server (Fig.1). The estimated modal properties of each structure are then posted on a control chart, which enables to track them against time. The identified modal properties are further used in finite element model updating and damage detection.

2.11 Finite element model updating and damage detection

As part of the BCSIMS system, an initial Finite Element (FE) model is created for each instrumented structure, and two steps of FE model updating is done on the initial FE model. In the first step, the FE model updating is done based on the on-site detailed Ambient Vibration Test (AVT), which is carried out for each structure in the BCSIMS network. This updated FE model represents the undamaged structure. The updated model is then used for several different purposes such as detailed seismic assessment of the bridge. In the second step, the FE model is further updated automatically using the identified modal properties of the bridge after a severe event (e.g., a seismic event) where the modal characteristics of the FE model is calibrated/updated by ensuring that the updated FE model better reflects the measured/recorded data than the initial FE model. The updated FE model is then used to detect the damage on the structure by comparing the updated FE model to the undamaged FE model in the first step. This method is based on the fundamental idea that damage will cause detectable changes in modal properties of the structure [18, 19]. This process has been fully automated in the BCSIM network.



A second damage detection algorithm, which is based on the state-space formulation and its residuals [20, 21], has been developed to detect the damage on structures. The method uses the covariance of the recorded vibrations and does not require the FE model of the structure. One desirable feature for this method is its ability to detect damage even in the presence of noise and common environmental effects such as wind speed & direction, temperature, and traffic load changes; therefore, it is robust to any changes due to ambient conditions. The method makes use of clustering in order to increase the speed of the algorithm and the accuracy of the damage detection. A baseline model is determined using the measurements from the undamaged structure, the length of which differs for each structure. A statistical-based threshold value is automatically calculated using the baseline model, but such threshold value is also user selectable. As soon as a new set of measurement data becomes available in the data processing server, a damage index is calculated for that set of data and stored permanently in the global database. If the system is triggered for any reason, such damage index is calculated automatically. Calculated damage indexes are then posted in a control chart that can take a user-definable set of damage indicators and unify them to a single control value, which is then compared with the statistical threshold value calculated earlier [20].

Both of the damage detection algorithms implemented in the BCSIMS system provide reliable intelligence and run simultaneously in near real-time for each instrumented structure, and a notification is issued via e-mail to predefined subscribers list when damage is detected by either of the damage detection methods.

2.12 Structural event report

As soon as an event is detected in BCSIMS, it initiates an event recording on the Data Processing Server, and the recorded event data is permanently stored in a designated secure directory on the Data Processing Server after the seismic event is over. The Structural Event Report, which is issued approximately 5-miutes after the seismic event is over, will be automatically generated, e-mailed to predefined subscribers list, and published on the BCSISM website. This report provides immense information on the latest status of the bridge after an earthquake, and one report will be issues for each structure. This report may also be issued for several reasons such as impact, over loading, wind, ship collision with bridge, or simply for a scheduled health assessment.

3. Summary and conclusion

The British Columbia Ministry of Transportation and the University of British Columbia, in collaboration with Structural Vibration Solutions, have embarked on a program, which is called the British Columbia Smart Infrastructure Monitoring System (BCSIMS). The system integrated data both from the instrumented structures and the strong motion network. It then organizes and processes this information in an efficient manner and deliver the collected information along with processed data to the appropriate parties such as bridge engineers in the Ministry of Transportation. The strong motion event report and the structural event report provided the MoT and other public and government agencies with additional level information so that they can manage their scarce resources effectively in the event of an earthquake.

The tools and techniques that have been developed in the BCSIMS have been tested by several small to moderate earthquakes since its first installation in 2009. The results showed that the BCSIMS system has successfully responded to these earthquakes as it was designed to and provided the immense information needed for the MoT engineers.

4. Acknowledgement

The BCSIMS project has fully funded by the Ministry of Transportation (MoT) BC since 2009. Some of the tools and techniques in BCSIMS project have been jointly developed and programed by Structural Vibration Solutions (SVS) in Aalborg in Denmark and French Institute for Research in Computer Science and Automation (INRIA) in Rennes in France. Authors greatly acknowledge the MoT, SVS, and INRIA for their valuable contributions.



5. References

- [1] Rosenberger, A., Beverley, K., and Rogers, G., 2004. The new strong motion seismic network in southern British Columbia, CANADA, 13th World Conference on Earthquake Engineering Vancouver, B.C., Canada. Paper No. 3373
- [2] Katayama, T., Sato, N. and Saito, K. 1998. SI-Sensor for the Identification of Destructive Ground Motion, Proc. Ninth World Conference of Earthquake Engineering, Tokyo-Kyoto, VII p. 667-672
- [3] MATLAB Commercial Integrating Technical Computing Program, The MathWorks Inc., Natick, MA, USA, 2000, <u>www.mathworks.com</u>
- [4] Boore, D.M. and Atkinson, G.M., 2008. Ground-Motion Prediction Equations for the Average Horizontal Component of PGA, PGV, and 5%-Damped PSA at Spectral Periods between 0.01 s and 10.0 s, *Earthquake Spectra*, Volume 24, No. 1, pages 99–138, February 2008
- [5] Atkinson, G.M. 2005. Ground Motions for Earthquakes in Southwestern British Columbia and Northwestren Washington: Crustal, In-Slab, and Offshore Events, Bulletin of Seismological Society of America, v. 95, no. 3, p. 1027-1044
- [6] Atkinson, G.M., and Boore, D.M. 2011. Modifications to Existing Ground-Motion Prediction Equations in Light of New Data, Bulletin of Seismological Society of America, v. 101, no. 3, p. 1121-1135
- [7] Wald, D.J., Quitoriano, V., Heaton, T.H., Kanamori, H. 1999. Relationship between Peak Ground Acceleration, Peak Ground Velocity, and Modified Mercalli Intensity in California, Earthquake Spectra, v. 15, no. 3
- [8] Kaya, Y. and Safak, E., 2014. Real-time analysis and interpretation of continuous data from structural health monitoring (SHM) systems, *Bulletin of Earthquake Engineering*, DOI 10.1007/s10518-014-9642-9
- [9] Overschee, P.V. and Moor, B.D., 2011. Subspace Identification for Linear Systems: Theory, Implementation and Applications, Kluwer Academic Publishers, Dordrecht, the Netherlands
- [10] Hoehn, L. and Niven, I., 1985. Averages on the Move, Mathematics Magazine v. 58, No 3, p. 151-156
- [11] Hastie T., Tibshirani R. and Friedman J. 2011. The Elements of Statistical Learning: Data Mining, Inference, and Prediction, *Springer Series in Statistics*
- [12] Kaya, Y., Turek, M., Ventura, C., 2013. Temperature and Traffic Load Effects on Modal Frequency for a Permanently Monitored Bridge, Conference Proceedings of the Society for Experimental Mechanics Series 38, DOI 10.1007/978-1-4614-6519-5_6
- [13] Peeters, B. and Roeck, G.D. 2011. One-year monitoring of the Z24-Bridge: environmental effects versus damage events, Earthquake Engineering and Structural Dynamics, 2001; 30:149-171
- [14] Overschee, P.V. and Moor, B.D.1991. Subspace algorithms for the stochastic identification problem. In Proceedings of the 30th IEEE Conference on Decision and Control, Brighton, UK; 321{1326.
- [15] Peeters, B. and Roeck, G.D., 1999. Reference-based stochastic subspace identification for output-only modal analysis. Mechanical Systems and Signal Processing 1999; 13(6): p. 855-878.
- [16] Ljung L., System Identification: Theory for the User (2nd Edition), Prentice Hall PTR, 1999
- [17] Peeters, B., Roeck, G.D., Pollet, T., and Schueremans, L. 1995. Stochastic subspace techniques applied to parameter identification of civil engineering structures. In Proceedings of New Advances in Modal Synthesis of Large Structures: Nonlinear, Damped and Nondeterministic Cases, Lyon, France, p. 151-162
- [18] Fan, W. and Qiao, P. 2011. Vibration-based damage identification methods: a review and comparative study. Structural Health Monitoring 10(1):83–111
- [19] Dohler, M., Mevel, L. and Hille, F. 2013. Subspace-based damage detection under changes in the ambient excitation statistics, *Mechanical Systems and Signal Processing*, v45, p 207-227
- [20] Basseville, M., Mevel, L. and Goursat, M. 2004. Statistical model-based damage detection and localization: subspacebased residual and damage-to-noise sensitivity ratios, *Journal of Sound and vibration*, v 275, p 769-794

