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DISPLACEMENT-BASED SEISMIC DESIGN OF FRAMES AT SOFT SOILS OF THE VALLEY OF MEXICO CONSIDERING P-DELTA EFFECTS

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Abstract

The amplification of demands of a structure subjected to lateral displacements due to the action of vertical loads over its deformed shape, the so-called second order effects or P-Delta, is an issue that has received considerable attention in earthquake engineering for many years, as it has been recognized that it may lead to undesirable structural behaviour and even collapse. To this date, the most extended approach followed in several national and international building codes to consider P-Delta effects in seismic design are based on either the amplification of demands via factors which are the product of extrapolation of elastic behavior under monotonic loading, or by the direct use of second order elastic analysis. Several studies show that such approach is not consistent with actual behavior of inelastic structures subjected to earthquake induced ground motions.

Moreover, as it is well known, several displacement based methods aimed at performance oriented design applications that rely on a more rational basis than code prescribed force-based design have been developed in the last two decades. However, few efforts have been carried out towards the development of criteria to account for P-Delta effects in this type of methods, and some of them rely on similar assumptions to that of the aforementioned approach. For this reason, the authors of this paper proposed a new design method that allows the design of structures with a P-Delta induced instability condition for either a near-collapse limit state, *i.e.*, exceedance of a code prescribed interstorey drift, or a sidesway-collapse limit state, *i.e.*, failure due to dynamic instability. The method proposed was presented in a previous paper where it was shown that it provided good results when used for the design of framed structures subjected to far-field ground motions of hard soil sites.

It is the purpose of the present paper to show the validity of the method proposed to design structures subjected to seismic demands characteristic of soft soil sites in the Valley of Mexico. It illustrates an overview of the response of unstable SDOF systems due to P-Delta effects subjected to such type of demands, and the results of design applications of the method proposed of 8-, 12-, 16- and 20-storey frames, along with the results of incremental dynamic analysis of such frames. The seismic demands considered were a set of 100 real earthquake ground motions recorded at a few soft soil sites of Mexico City. From the comparison of the expected performance *vs.* that attained from incremental dynamic analysis, the authors conclude that the method proposed allows the design of instability prone structures with severe P-Delta effects subjected to demands characteristic of the lake-bed zone of the Mexico Valley for either near-collapse or sidesway-collapse limit states.

Keywords: Displacement-based seismic design, P-Delta effects, dynamic instability, collapse, soft soil sites

1. Introduction

Second order (P-Delta) effects is the name given to the amplification of demands of a structure subjected to lateral displacements due to the action of vertical loads over its deformed shape. The most common approach followed in force based design procedures given by building codes, *e.g.*, the Mexico City Building Code, NTCS [1]; ASCE/SEI 7-10 [2]; to consider P-Delta effects in seismic design relies on elastic analysis; two options are usually given. The first one is to account for second order effects by modifying the flexural demands obtained from first order elastic analysis via amplification factors. The second option is to calculate P-Delta effects directly with second order analysis of the elastic structural model.

However, the displacement response of an inelastic structure with severe P-Delta effects under dynamic loading may be significantly different from that obtained via elastic analysis even if second order analysis is carried out, since the modification in stiffness due to gravity loads is more severe in the inelastic stage than in the elastic stage, hence, such approach is not appropriate. Even though building code procedures intend to provide control of gravity induced instability by limiting the deformations of the structures, such approach may not be effective [3, 4].

Several authors have provided criteria to account for P-Delta effects in displacement-based design methods [5, 6, 7]. The development of methods that rely on a displacement-based approach arose from the need of a more rational basis for performance oriented design than force-based procedures. However, such criteria to consider P-Delta effects rely on some assumptions that may lead to inadequate control of structural behaviour in certain cases and/or the validation of some of those proposals was limited to a few case studies.

For this reason, the authors of this paper developed an alternative displacement-based seismic design method that accounts for P-Delta induced instability [8]. The method proposed allows the design of regular framed structures for either a near-collapse limit state, i.e., exceedance of a code prescribed interstorey drift, or a sidesway-collapse limit state, i.e., failure due to dynamic instability. In such paper, the method was validated using a set of far field earthquake records given by FEMA [9] corresponding to stiff soil sites. It is the purpose of the present paper to show the validity of the method proposed to design structures subjected to seismic demands characteristic of soft soil sites in the Valley of Mexico.

Foremost, the paper provides an overview of the influence of P-Delta effects in structural response, followed by a succinct description of the response trends of unstable SDOF systems due to P-Delta effects subjected to seismic demands at soft soil sites of the Valley of Mexico. Afterwards, the design approach followed in the method proposed by the authors is described briefly, and a detailed step-by-step design procedure is presented. Subsequently, the validation of the method proposed via incremental dynamic analysis, IDA [10] of design applications of 8-, 12-, 16- & 20-storey frames subjected to 100 far-field earthquakes recorded at soft soil sites in the Valley of Mexico is presented. Finally, a discussion of the results and conclusions regarding this investigation are given.

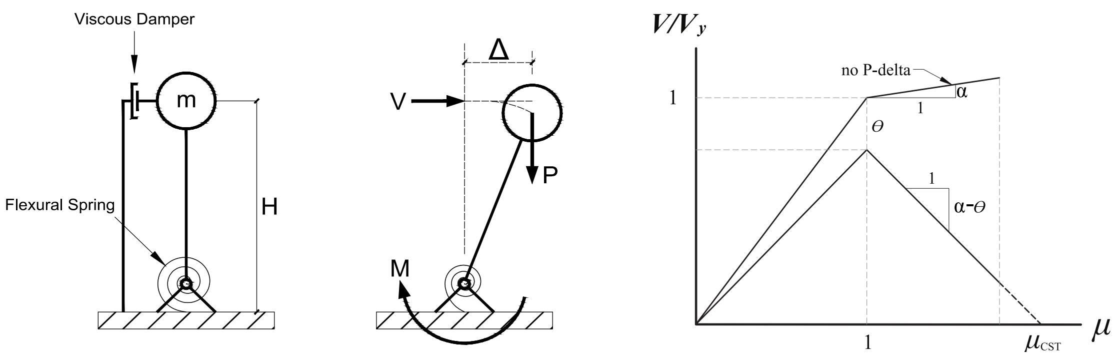
2. P-Delta effects in structures

To illustrate P-Delta effects in structures consider the simple case of the SDOF system shown in Fig. 1. The system consists of a rigid column of height H, attached at its base to a flexural spring, whose behaviour is defined by a bilinear backbone of elastic stiffness K^E and post-yield stiffness, $K^D=\alpha K^E$; a mass, m, and a viscous damper with a damping coefficient c. Such system is subjected to a lateral load V and a vertical load P (Fig. 1.a).

As it may be inferred from the aforementioned figure, the vertical load generates an additional moment at the base of the system due to its displacement. The increase in flexural demand with respect to the first order response can be interpreted as a decrease of the system´s lateral stiffness and strength, which can be characterized by a geometric transformation. The parameter commonly used to quantify the influence of P-Delta effects in the lateral stiffness of a structure is the so-called stability coefficient θ, defined as the ratio of stiffness decrease to first order elastic stiffness, K^E. For the SDOF system shown in Fig. 1, such parameter, which is the same in any state of the structure, is given by:

$$\theta = \frac{K^t - K^{t\prime}}{K^E} \tag{1}$$

This decrease in strength and stiffness of the SDOF system due to gravity loads may be interpreted as “shearing” of its load-deformation capacity relationship (backbone curve) in function of the stability coefficient as shown in Fig.1.b. Evidently, this “shearing” effect of the load displacement relationship affects the response of the SDOF system under dynamic loading. The decrease of stiffness due to gravity loading leads to a lengthening of the initial period of the system with respect to the first order period. In general, this period shift produces a small effect in the dynamic response of structures in their elastic stage.



a) b)

Fig. 1 – SDOF system subjected to lateral and vertical forces: a) model and free body diagram; b) shearing of load-displacement relationship (displacement ductility, μ, *vs* base shear normalized to yield base shear, V/V_y)

However, the response in the inelastic stage may be severely affected by P-Delta effects as the first order stiffness in some lapses of time is significantly reduced due to damage, hence, leading to a low effective stiffness value and maybe even to a condition of static instability, *i.e.*, a negative stiffness in a segment of the load-displacement relationship, in such instances. This instability condition does not necessarily imply failure of the system under dynamic loads as the inertial and damping forces provide a stabilizing effect on the response [3].

The dynamic response of unstable systems is characterized by the progressive increase of displacements in a single direction throughout successive load cycles, *i.e.*, the cyclic response tends to be non-reversible, as a consequence of the stiffness decrease within each load cycle. This effect, referred to as “ratcheting” or “crawling” of structural response [11, 12], leads to larger residual displacements and, in the extreme case, to failure due to sidesway-collapse. This type of failure is a consequence of dynamic instability, which may be defined as the disproportionate response of a system with “negative stiffness” subjected to dynamic loading for a relatively small variation of its intensity in a lapse of time [3, 8].

As in any system subjected to dynamic loading, the response of instability prone systems due to P-Delta effects strongly depends on the characteristics of the demand. Particularly, the occurrence of dynamic instability is highly dependent to the intensity and frequency content of the input, as well as its duration. For this reason, non-linear dynamic analysis is required to identify sidesway-collapse for a particular loading.

The degree of influence of P-Delta effects in structural response depends also on the properties of the backbone curve. For an SDOF system with a given yield strength, as the stability coefficient is larger, second order effects increase, thus, the stability coefficient can be used as a measure of the dynamic instability potential of the system. In fact, the upper bound of stable response under dynamic loading is the static collapse ductility, μ_{cst}, given by Eq. 2 [13], which is a function of the stability coefficient and the first order post-yield stiffness. Moreover, the yield strength of the system also plays a significant role in the amplification of response; for fixed values of elastic and effective post-yield stiffness ratio, as the strength of the system is larger, the influence of P-Delta effects is more significant, which implies that for larger ductility values the amplification of response is more severe.

$$\mu_{cst} = \frac{1-\alpha}{\theta-\alpha} \tag{2}$$

Furthermore, the influence of P-Delta effects in structural response and, thus, the occurrence of dynamic instability, depend on the characteristics of the hysteresis model that rules the response of the system. Several studies show that systems with bilinear hysteretic behavior (non-degrading) are more susceptible to dynamic instability than peak-oriented models [6, 13, 14]. According to such studies, this is due to the fact that the response of bilinear systems falls within a negative stiffness segment in larger and more lapses of time, thus, the displacement increase in a single direction is more severe than that of systems with peak-oriented behaviour. Therefore, it can be accepted that steel structures are more susceptible to P-Delta effects than concrete structures, not only owing to their flexibility, but also to the nature of their cyclic response.

3. Influence of P-Delta effects in displacement response of structures at soft soil sites in the Valley of Mexico

In this study, a comprehensive analysis of the response of unstable SDOF systems due to P-Delta effects subjected to characteristic earthquake ground motions of soft soil sites in the Valley of Mexico was carried out. The vast majority of the previous studies on the subject considered seismic demands corresponding to hard soil sites, *e.g.*, Bernal [3], Petinga and Priestley [6] Wei *et al.* [7]. A noticeable exception is the work of Fenwick *et al.* [15], in which amplification factors of dynamic response were calculated for an accelerogram of the 1985 Mexico City earthquake recorded at a flexible soil site. However, that was the only record of such characteristics used in their investigation.

The seismic demands considered in this study were a set of 100 real earthquake accelerograms of far-field seismic events recorded at soft soil sites in the Valley of Mexico, compiled by Miranda and Ruiz Garcia [16]. Such demands are denoted as “VM set” in the remainder of this paper. Response spectra, constant ductility and collapse response, of SDOF systems with elasto-plastic hysteretic behavior exhibiting P-Delta induced instability were calculated for each of the records via Incremental Dynamic Analysis, IDA [10].

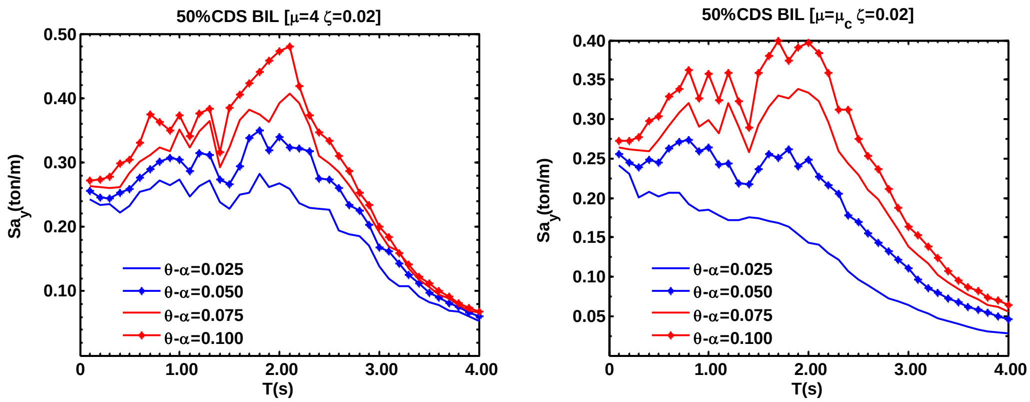
The parameters that define such type of spectra are an effective post-yield stiffness ratio, *i.e.*, the difference of stability coefficient and hardening ratio, $\theta - \alpha$; a damping ratio, ζ; and a hysteresis rule [13]. The abscissas of such spectra are the first order periods, T, and the ordinates may be yield-pseudo-acceleration, Sd_y ultimate displacement, Sd_u, yield reduction factor, *i.e.*, relative intensity, R, or inelastic displacement ratio C_d. Even though it is more appropriate to define the abscissa of spectra of soft-soil site demands in terms of the ratio of structural period to dominant period of the ground motion, T/T_g, this definition was not employed in this study as the abscissas of the design spectra given by the NTCS [1] are set in terms of T.

The damping ratios considered were 2% and 5%. As the former corresponds to the value accepted for design of steel structures, whose hysteretic response can be characterized by an elastoplastic model which is the most susceptible to P-Delta effects, the spectra built for such value are shown in the following. The effective post-yield stiffness ratios considered were 0.025, 0.05, 0.075 and 0.10, which are representative values of actual structures [17]. Constant ductility spectra were built for values of 3 to 8 in increments of one.

In order to characterize the response of the set in statistical terms, 16%, 50% and 84% response spectra were estimated, under the assumption that the set of responses for different records follows a lognormal distribution [13]. The spectra were calculated considering aleatory uncertainty *i.e.*, record to record variability, only. Figs. 2 and 3 depict sets of collapse spectra ($\mu=\mu_{cst}$) and constant ductility spectra ($\mu=4$) of the aforementioned ensemble of earthquake records in terms of median yield pseudo-acceleration and median ultimate displacement for various $\theta - \alpha$ values and $\zeta = 0.02$. $\mu = 4$ is the maximum ductility allowed for the design of ductile structures such as steel moment resisting frames by the NTCS [1].

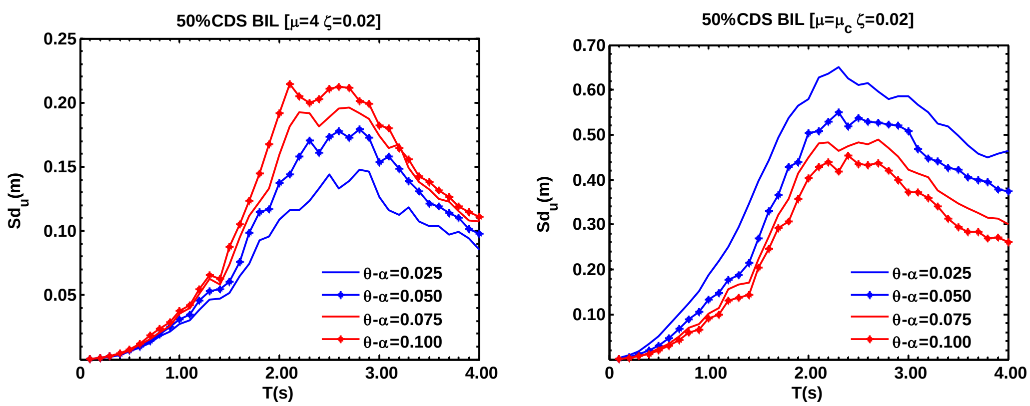
As can be observed in the aforementioned figures, the shapes of the spectra are characteristic of soft soil demands. The maximum 50%Sa_y is found at approximate 2 seconds which implies that the predominant periods of the ground motion set are considerably large. Moreover, trends of behaviour of unstable systems can be readily identified: the smaller the effective post-yield stiffness ratio the larger the strength required to reach the considered ductility value, the larger the ductility values the smaller the yield strength.

Furthermore, for the purpose of attaining insight into the level of amplification of dynamic response due to second order effects, amplification factor spectra were also calculated for the aforementioned ductility values. The definition of the amplification factor considered in this work is the ratio of the first order strength required by the unstable system to develop a given ductility to that of the corresponding first order elastoplastic system with the same ductility [18].



a) b)

Fig. 2 – Median response spectra in terms of yield pseudo-acceleration of SDOF systems with P-Delta negative post-yield stiffness subjected to the VM record set for various θ-α values: a) constant ductility (μ=4); b) collapse (μ=μ_{cst})



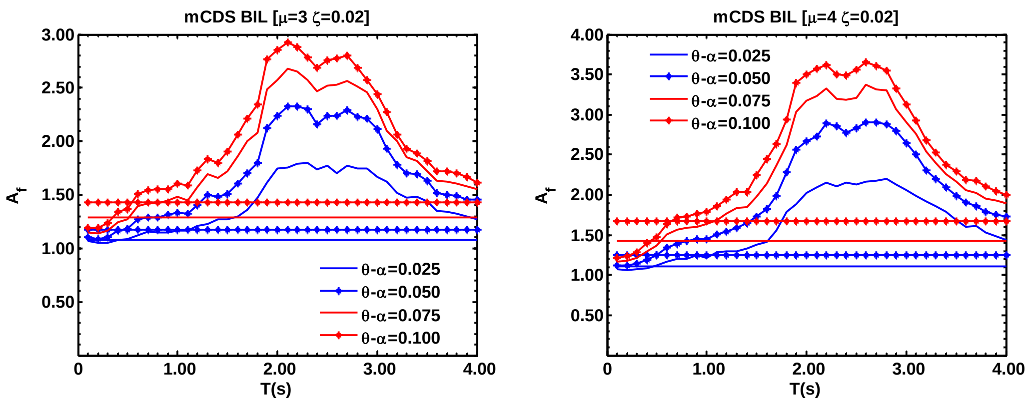
a) b)

Fig. 3 – Median response spectra in terms of ultimate displacement of SDOF systems with P-Delta negative post-yield stiffness subjected to the VM record set for various θ-α values: a) constant ductility (μ=4); b) collapse (μ=μ_{cst})

Mean amplification factor spectra calculated for fixed ductilities of 3 and 4 for several θ–α values are depicted in Fig. 4. μ=3 is the other ductility value prescribed in the NTCS [1] for ductile structures. Furthermore, the corresponding period independent amplification factors prescribed by the NTCS [1], which is defined as the inverse of 1-$\mu\cdot\theta$, are depicted as straight lines with the same line-style and color as that of the calculated amplification factor spectrum.

As it can be readily observed in such figures, the mean amplification factors increase progressively from the acceleration-dominant region of the spectrum to the velocity region where their maximums, 2.95 and 3.60 for ductilities 3 and 4, respectively, occur at periods slightly larger than 2 s, from where the level of amplification decreases afterwards. The corresponding values of coefficient of variation lie between 0 to 0.50 and 0 to 0.60, respectively. Furthermore, it is important to note the overly large differences between the actual response amplification and that estimated with the prescribed equation.

Therefore, the approached followed in the NTCS [1], based on monotonic amplification of demand, is not appropriate for the design at soft soils sites in the Mexico Valley as it does not take into account the period dependency of dynamic response amplification. Moreover, the criteria to account for P-Delta effects in displacement-based design given by authors such as Pettinga and Priestley [6] and Wei *et al.* [7], is also not appropriate for the design of steel structures as they consider period-independent amplification factors. Nonetheless, the approach followed in the displacement-based seismic design method proposed by the authors of this paper does not present such limitation as the seismic demands employed are given by spectra of the type shown in Figs. 2 and 3.



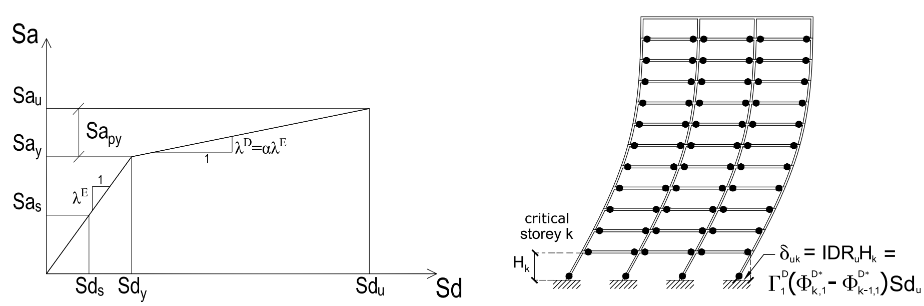
a) b)

Fig. 4 – Mean response amplification factors due to P-Delta effects of SDOF systems subjected to the VM record set for various θ-α values: a) μ=3); b) μ=4. Straight lines indicate NTCS [1] specified amplification factor

4. Displacement based-seismic design method of instability prone frames due to P-Delta effects

4.1. Design approach: reference SDOF system and design behaviour curve.

The method proposed by López *et al.* [8] is based on the approximation of maximum inelastic response of a MDOF structure by means of an inelastic SDOF system with bilinear backbone whose properties are congruent with the first mode properties of the former. Such oscillator is referred to as reference SDOF system and its spectral displacement *vs.* spectral pseudo-acceleration plot is called behaviour curve. The design approach consists on the definition of a “design behaviour curve”, whose properties are such that the considered structure shall satisfy a given performance objective, PO.



a) b)

Fig. 5 – Fundamental concepts of the method proposed by Lopez *et al.* [8]: a) design behaviour curve of RSDOF system; b) critical storey drift associated with design damage state.

For such purpose, the yield point of such curve is determined from the material properties and the geometry of the structure. The slope of the second branch of the curve, in terms of the post-yield stiffness ratio, is defined in accordance with a proposed damage state, *e.g*, strong column-weak beam. The ultimate displacement is set in such a way that the maximum interstorey drift of the structure shall not exceed a given threshold and/or structural instability is reached under design demands associated with the ultimate limit state; this displacement is defined taking into account the modal shapes of the structure under the damage state proposed in the design process.

4.2. Design procedure

The application of the method proposed to design an unstable frame due to P-Delta effects aimed to the fulfilment of an ULS, either near-collapse or sidesway-collapse, can be summarized in the following steps.

1. Pre-dimensioning of the structure based on the designer's experience and construction of an elastic model in a structural analysis program.

2. First and second order modal analysis of the elastic model from which the corresponding dynamic properties are attained and the elastic stability coefficient, θ^E, is calculated by means of Eq. (3),

$$\theta^E = \frac{MPR^E\lambda^E - MPR^{E\prime}\lambda^{E\prime}}{MPR^E\lambda^E} \tag{3}$$

   where $\lambda^{E\prime}$ and $MPR^{E\prime}$, λ^E, and MPR^E, are the elastic fundamental eigen-values and -modal mass participation ratios with and without P-Delta effects, respectively.

3. Definition of the design damage distribution for the ULS, *e.g.*, strong column-weak beam behaviour with inelastic action at first storey column bases and, construction of the "damage model", a replica of the elastic model in which the design damage state is characterized by rotational springs with reduced stiffness values consistent with rational values of post-yield stiffnesses of structural elements.

4. Second order modal analysis of the so called "damaged model" from which the corresponding dynamic properties are attained. If the resulting eigen-value is negative, a first order modal analysis is also carried out. Subsequently, the first order post-yield stiffness ratio, α, the inelastic stability coefficient, θ^I, and the auxiliary stability coefficient, θ_{aux}, are calculated with the following equations:

$$\alpha_j = \frac{MPR_j^D\lambda_j^D}{MPR_j^E\lambda_j^E} \tag{4}$$

$$\theta^I = \frac{MPR^D\lambda^D - MPR^{D\prime}\lambda^{D\prime}}{MPR^E\lambda^E} \tag{5}$$

$$T_{AUX} = T_1\sqrt{\frac{1-\alpha}{1-\alpha-\theta^E+\theta^I}} \tag{6}$$

$$\theta_{aux} = \frac{\theta^I - \alpha\theta^E}{1-\alpha-\theta^E+\theta^I} \tag{7}$$

   where $\lambda^{D\prime}$ and $MPR^{D\prime}$, λ^D, and MPR^D are the inelastic fundamental eigen-values and -modal mass participation ratios of the structure with and without P-Delta effects, respectively.

5. Calculation of yield and ultimate displacements and, consequently, the design ductility, μ, via the following equations:

$$Sd_y = \frac{IDR_y H_k}{\Gamma_1^{E\prime}(\phi_{k1}^{E}{}' - \phi_{k-11}^{E}{}')} \tag{8}$$

$$\phi_{i\,1}^{D*\prime} = \frac{1}{\mu}\left[\frac{\Gamma_1^{E\prime}}{\Gamma_1^{D\prime}}\phi_{i\,1}^{E}{}' + (\mu-1)\phi_{i\,1}^{D}{}'\right] \tag{9}$$

$$Sd_u = \frac{IDR_u H_k}{\Gamma_1^{D\prime}(\phi_{k\,1}^{D*}{}' - \phi_{k-1\,1}^{D*}{}')} \tag{10}$$

$$\mu = \frac{Sd_u}{Sd_y} \tag{11}$$

where IDR_y is the yield interstorey drift, which can be estimated by approximate expressions such as those given by Priestley [19]; k is the critical storey in the elastic stage; H_k is the height of the critical storey; $\Phi^{E}{}_{k\,1}{}'$ and $\Phi^{E}{}_{k-1\,1}{}'$ are the fundamental modal coordinates of the critical storey and preceding storey obtained from modal analysis (second order) of the elastic model, respectively; $\Gamma^{E}{}_{1}{}'$ is the second order fundamental modal participation factor. IDR_u denotes the interstorey drift ratio threshold of the ULS; k, is the critical storey in the inelastic stage, which is not necessarily the same as that of the elastic stage considered in Eq. (8); H_k is the height of the critical storey; $\Gamma^{E}{}_{1}{}'$ and $\Gamma^{D}{}_{1}{}'$ identify the modal participation factor of the first mode attained from the second order modal analyses of the elastic and inelastic model, respectively; $\Phi^{E}{}_{i\,1}{}'$ and $\Phi^{D}{}_{i\,1}{}'$ are the corresponding elastic and inelastic modal shapes; $\Phi^{D^*}{}_{i\,1}{}'$ identifies the design modal shape for the ULS; $\Phi^{D^*}{}_{k\,1}{}'$ and $\Phi^{D^*}{}_{k-1\,1}{}'$ denote the coordinates of the modal shape corresponding to the critical storey and the preceding storey, respectively.

6. From the ULS design ultimate displacement spectrum (Fig. 3) corresponding to μ and θ_{aux}-α, the required auxiliary period for such limit state, T_{req}, is obtained. If the latter is the smallest and is significantly different than the T_{aux} value estimated in step 4 recalculate the effective negative post-yield stiffness, θ_{aux}–α, taking the first mode eigen-value calculated in step 4, and repeat steps 5 to 6 until a sufficient approximation of T_{req} is attained.

7. Modal spectral analysis of both, the elastic and damaged, models. The first one is carried out using directly the design yield pseudo-acceleration spectrum (Fig. 2). The analysis of the second model is performed defining the modal demands from the post-yield strength, Sa_{py}, which is given by the following equation:

$$Sa'_{pyj} = Sa'_{yj} \propto '_{j}(\mu - 1) \tag{12}$$

   where α'_j is the second order post-yield stiffness ratio calculated with Eq. (4) considering the second order modal properties, $\lambda^{E\prime}$, $MPR^{E}{}_{1}{}'$, $\lambda^{D\prime}$ and $MPR^{D}{}_{1}{}'$. In this step, if it is deemed necessary to account for the higher mode contribution to ultimate displacement, modal displacements of higher modes can be calculated with Eqs. (9) and (10) considering the corresponding participation factors and modal shapes, from which the final design displacements can be calculated using a conventional modal combination rule such as the square root of sum of squares, SRSS, rule.

8. Calculation of modal design forces of structural elements. If the SRSS rule is used, they can be estimated via the following equation:

$$F_k = \sqrt{\sum_{j}^{n}\left(F_{kj}^{E\prime} + F_{kj}^{D\prime}\right)^2} \tag{13}$$

   In this equation, $F^{E}{}_{k\,j}{}'$ denotes the demand of element k corresponding to mode j obtained from the modal spectral analysis of the elastic model; $F^{D}{}_{k\,j}$ is the demand of element k associated with mode j attained from the modal spectral analysis of the damaged model (both second order analyses); and n is the number of modes considered. Finally, with the calculated forces, the design and detailing of structural elements is carried out considering appropriate criteria regarding the behaviour of materials and structural types according to building codes or other accepted design provisions.

5. Design applications of the method proposed and validation via IDA

Design applications aimed at theoretical P-Delta induced collapse or near-collapse, were carried out for 8-, 12-, 16- and 20-storey non-deteriorating generic frames, regular in elevation (Fig. 6). The seismic demands considered were the aforementioned VM set of real earthquake records. Each frame was designed for different levels of axial load corresponding to θ_{aux}–α values equal to 0.025 to 0.10 in increments of 0.025. To allow flexibility in the validation of the method proposed for various levels of axial load and ductility values, the design was carried out considering a fixed fundamental period for each frame and the strength of structural components was provided according to the design targets of the ULS via steps 7 to 8 of the design procedure. The period values considered were 1.40 s, 1.90 s, 2.30 s and 2.80 s, for the 8-,12-,16- and 20-storey frame, respectively.

Two types of design applications were performed: one oriented to actual sidesway-collapse and the other aimed at near-collapse considering an interstorey drift threshold associated with μ=4. The goal of the design applications was that dynamic instability occurs or that the interstorey drift threshold is exceeded in any floor, respectively, for 50% of the record set at the design target intensity. The median yield pseudo-acceleration spectra and median ultimate-displacement spectra shown in Figs. 2 and 3 are employed to design these case studies. Such spectra were scaled in each application in such a way that the intensity of each frame matches the spectral pseudo-acceleration, Sa_e, value at period T_{aux} of the response spectrum of the E-W component of the Michoacán Earthquake of 1985, recorded at the SCT station in Mexico City. Such intensity is denoted in the following as Sa_{tar}.

The validation of the design applications was carried out using incremental dynamic analysis, IDA, [10]; the intensity measured considered was the elastic pseudo-acceleration, $Sa_{e,}$ of the corresponding linear system. The series of nonlinear dynamic analyses were performed with OpenSees [20]. The details of the non-linear dynamic analyses are the same those in the study of Lopez [8]. IDA was performed using increments between 0.05 to 0.20 m/s^2 up to the attainment of numerical instability.

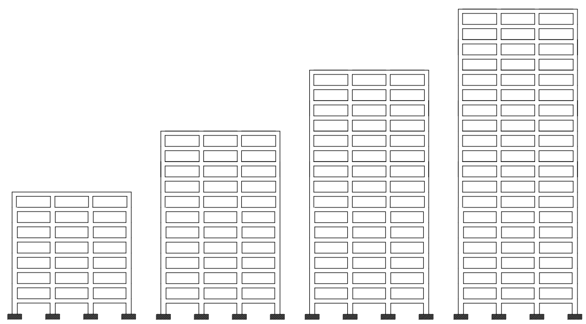


Fig. 6 – Case studies considered: 8-, 12-, 16- and 20-storey frames

The assessment of structural performance was carried out via the IM-approach [10, 21]. From IDA of the designed frames, the intensity steps corresponding to dynamic instability or the exceedance of the design interstorey drift associated with μ=4, as applicable, were identified for each record, from which the 16%, 50% and 84% intensities, denoted as Sa_{ana}, were calculated via counted statistics. In order to quantify the uncertainty of Sa_{ana}, confidence intervals associated with a 0.95 confidence level were estimated using the bootstrap method [22], generating 3000 bootstrap samples. Moreover, for the purpose of investigating if the method proposed is able to approximate structural response associated with other ductility values, the same scheme was carried out in the sidesway-collapse design applications for ductility values ranging from 3 to 8.

Individual comparisons between Sa_{tar} and Sa_{ana} for all percentiles and ductilities were carried out. In general, a good approximation of the target intensities was attained in all case studies considered and, in most of the cases, the target intensities fell within the confidence interval. Fig. 7 depicts the IDA curves, in terms of maximum interstorey drift, IDR_{max} *vs.* Sa_e, of the 16-storey frame with θ_{aux}-α=0.05, along with the computed (blue) and target (red) 50% intensities associated with both the non-exceedance of μ=4 and the onset of dynamic instability, along with the corresponding confidence interval [LCL, UCL] where a good approximation can be observed.

To attain a global perspective of the good approximations attained with the method proposed, a statistical analysis of the errors between target and analysis percentile intensities was carried out for all ductility values. Fig. 8 illustrates the results of such analysis for the 50% and 84% intensities. From left to right, the first subplot depicts the mean (m) and standard deviation (σ) of the relative errors (*E*r) between Sa_{tar} and Sa_{ana}, in colours blue and red, respectively. The second subplot illustrates the relative frequency (f_r) of the location of the design intensity with respect to the confidence interval, in which *BCI*, *WCI* and *ACI* denote “below”, “within” and “above” the confidence interval, respectively.

As can be readily observed in Fig. 8, the means and standard deviations of the relative errors between target and analysis percentile collapse intensities are low, thus, indicating that, in general, the correspondence was good in all of the case studies considered. Moreover, the target intensities fell within the confidence interval in most of the cases. Furthermore, since the method proposed relies on a displacement-based approach, individual comparisons between the target displacement and the interstorey drift profiles (Eq.(9)) and the actual percentile profiles corresponding to the last non-collapse intensity step attained from IDA, were carried out. The profiles corresponding to actual collapse design applications were estimated considering $\mu=\mu_{cst}$. Fig. 9 shows the envelope profile comparisons of the 16-storey frame with θ_{aux}-α=0.05. In such figure, *IDR* denotes the interstorey drift ratio, *NDR* denotes the displacement normalized to the total height of the structure. As can be readily observed, good correspondence between the design displacement profile and those calculated via IDA was achieved in such case study.

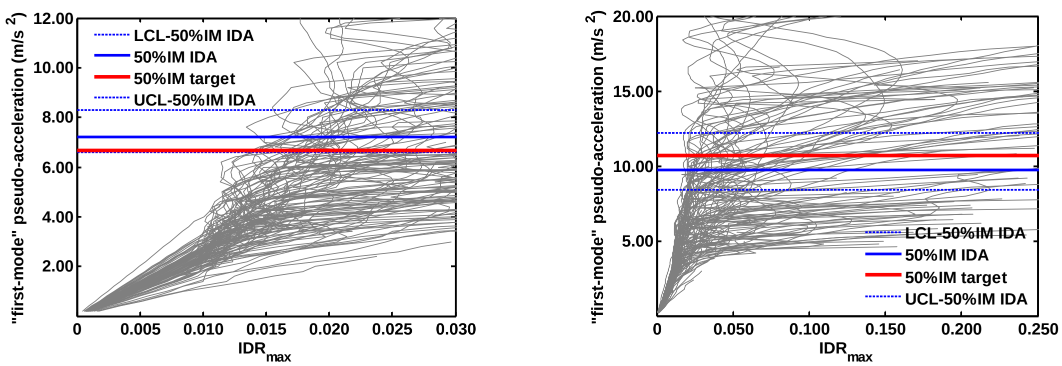
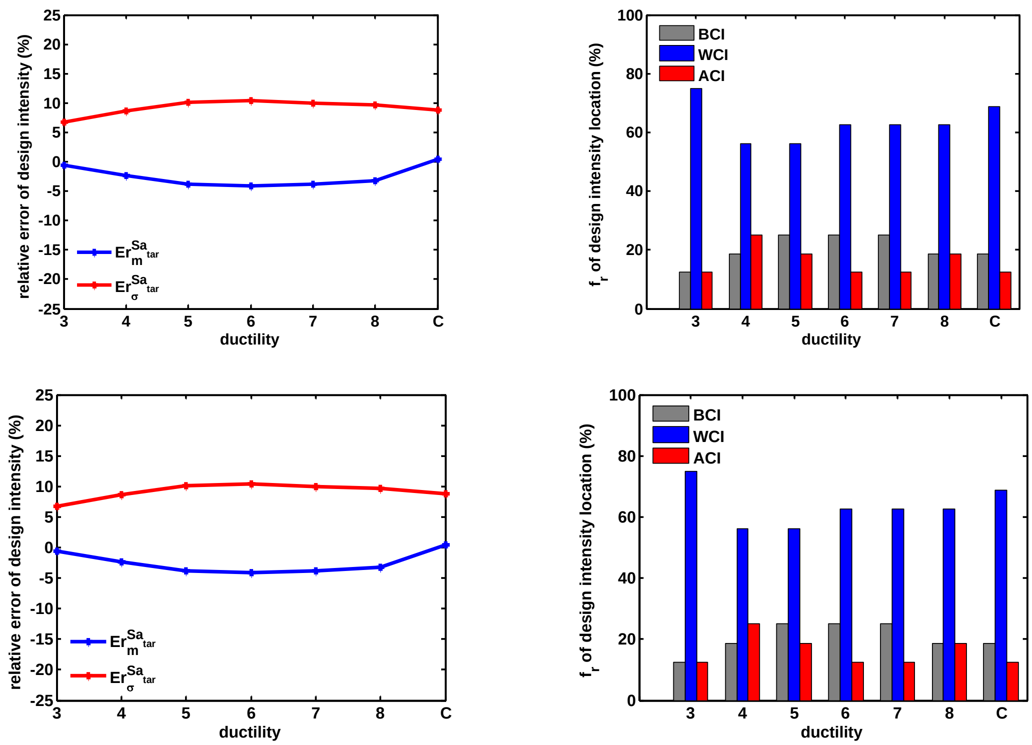


Fig. 7 IDA curves and %50 collapse intensities of 16-storey frame with θ_{AUX}-α=0.05: a) constant ductility (μ = 4); b) collapse ductility ($\mu = \mu_{cst}$)



a) b)

Fig. 8 Comparison of analysis vs. target 50% and 84% collapse intensities of frames designed for the VM set of records: a) means and standard deviations of relative error; b) location of design intensity with respect to the confidence interval [LCL, UCL]

The degree of correspondence between target and analysis profiles was measured via the modal assurance criterion (MAC), given by Eq. (14). In such equation, *VEC* denotes the response vector, *e.g*, *IDR*, *NDR*, sub-indices *dem* and *tar* indicate demand and target, respectively. A MAC value of 1 implies that the compared shapes are equal.

$$MAC = \frac{(\{VEC_{dem}\} \cdot \{VEC_{tar}\})^2}{(\{VEC_{dem}\} \cdot \{VEC_{dem}\})(\{VEC_{tar}\} \cdot \{VEC_{tar}\})} \quad (14)$$

Statistical analysis of the MAC results shows good correspondence between the target shapes and the percentile shapes attained from IDA. For all percentile and ductility values considered, the median MAC values were in the range of 0.98 to 1.00 for the displacements, and 0.90 to 1.00 for interstorey drifts: the associated standard deviations values were in the range of 0 to 0.02, and 0.03 to 0.08, respectively. Evidently, the good agreement between the target and analysis percentile intensities is related to the good correspondence between expected and obtained shapes.

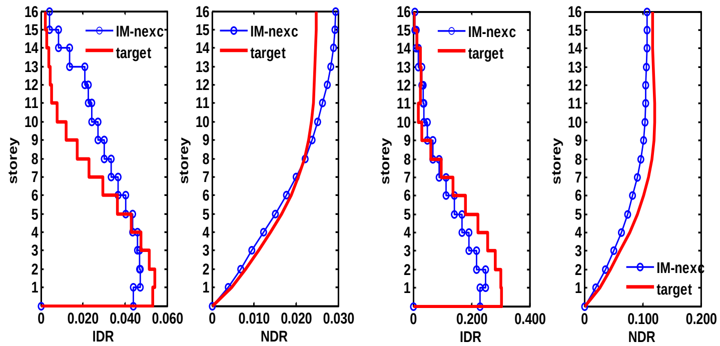


Fig. 9 – 50%Displacements and 50% interstorey drift profiles of 16-storey frame with T_1=2 s and θ_{aux}-α=0.05; left: near-collapse (μ=4); right: sidesway-collapse (μ= μ_{cst})

6. Conclusions

The results obtained in this investigation demonstrate that the method proposed allows to approximate sufficiently the seismic performance of framed structures that exhibit P-Delta induced negative stiffness when subjected to seismic loading characteristic of soft soil types. Nonetheless, it is necessary to carry out an investigation regarding the development of a criterion to consider soil-structure interaction, as, in soft soils sites, such issue may influence significantly structural response.

The application of the method requires the use of elastic analysis and a set of design spectra corresponding to SDOF systems with a P-Delta induced negative post-yield stiffness, hence, it does not require non-linear dynamic analysis and can be carried out using commercial software that performs modal spectral analysis.

Up to this point, the method proposed has been extensively validated using regular planar frames. The results obtained encourage the continued development of this displacement based approach for both assessment and design purposes, thus, applications of the method proposed in regular and irregular 3-D buildings is currently underway.

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