16th World Conference on Earthquake Engineering, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N° 1141 (Abstract ID)

Registration Code: S-B1461947647

AN APPROXIMATE APPROACH FOR EFFICIENT STOCHASTIC INCREMENTAL DYNAMIC ANALYSIS

K. R. M. dos Santos[(1*)], I. A. Kougioumtzoglou[(1+)], A. T. Beck[(2)]

[(1*)] *Ph.D. Student, Department of Civil Engineering and Engineering Mechanics, Columbia University, kmd2191@columbia.edu*
[(1+)] *Assistant Professor, Department of Civil Engineering and Engineering Mechanics, Columbia University, iak2115@columbia.edu*
[(2)] *Associate Professor, Department of Structural Engineering, São Carlos School of Engineering, University of São Paulo, atbeck@usp.sc.br*

Abstract

Incremental dynamic analysis (IDA) has been a well-established methodology in earthquake engineering for assessing the performance of structural systems under a suite of ground motion records, each scaled to several levels of seismic intensity. Nevertheless, the need for performing nonlinear dynamic analyses both for various excitation magnitudes and for a large number of seismic records renders the IDA methodology potentially a computationally highly demanding task. In this paper, an efficient stochastic IDA methodology for nonlinear/hysteretic oscillators is developed by resorting to nonlinear stochastic dynamics concepts and tools such as stochastic averaging and statistical linearization. Specifically, modeling the excitation as a non-stationary stochastic process possessing an evolutionary power spectrum (EPS), an approximate closed-form expression is derived for the parameterized oscillator response amplitude probability density function (PDF) as a function of the excitation EPS intensity magnitude. In this regard, an IDA surface is determined providing the PDF of the engineering demand parameter (EDP) for a given intensity measure (IM) value. In contrast to an alternative Monte Carlo simulation (MCS) based determination of the IDA surface, the herein developed methodology determines the EDP PDF at minimal computational cost. Note that the technique can account for physically realistic excitation models possessing not only time-varying intensities but time-varying frequency contents as well. Numerical examples include a bilinear/hysteretic single-degree-of-freedom (SDOF) oscillator, whereas comparisons with pertinent MCS data demonstrate the reliability of the developed stochastic IDA methodology.

Keywords: incremental dynamic analysis; nonlinear system; stochastic dynamics; stochastic averaging; statistical linearization

1. Introduction

The emerging concept of performance based engineering (PBE) (e.g. [1]) advocates the assessment of the structural system performance in a comprehensive manner by properly accounting for the presence of uncertainties. Specifically, inherent in the philosophy of PBE is the definition of excitation related variables, known as intensity measures (IMs) (e.g. peak ground acceleration), and of system response related variables known as engineering demand parameters (EDPs) (e.g. inter-story drift ratio, deformation damage index, etc). Finally, the information provided via the functional relationship between the IMs and the EDPs, in conjunction with appropriately defined damage measures (DMs), is utilized for quantifying a selected decision variable (DV) (e.g. annual rate of loss, risk of financial loss, cost of failure, etc) [1-3] . Nevertheless, determining the functional relationship between the IMs and the EDPs constitutes typically a computationally cumbersome task, especially within a probabilistic framework where a large number of nonlinear dynamic analyses need to be typically performed in a Monte Carlo simulation (MCS) context.

In this regard, one of the methodologies applied in the field of earthquake engineering for estimating the functional relationship between the IMs (e.g. earthquake intensity, peak ground acceleration, 5% damped "first-mode" spectral acceleration Sa (T_1, 5%), etc) and the EDPs (e.g. interstory drift ratio, deformation damage index, etc) and, ultimately, for assessing the structural capacity of engineering systems, is the incremental dynamic analysis (IDA) [21]. IDA aims at assessing the performance of structural systems under a suite of ground motion records, each scaled to several levels of seismic intensity; thus, by performing a nonlinear dynamic analysis for each and every scaled record, a set of IDA curves is produced.

Clearly, performing IDA within a probabilistic framework, and depending on what kind of structural response statistical quantity is of interest, hundreds to thousands of IDA curves are typically required within a MCS context for a reliable statistical description of the EDP. Clearly, this can be a computationally prohibitive task. Indicatively, in [20] it is noted that the generation of a single IDA curve can last from thirty seconds to one hour, while in the case of a multi-record IDA, where thousands of curves are generated, the processing time can increase to weeks or even months. In this regard, several research efforts have focused on reducing the related computational cost by resorting to efficient MCS algorithms or by implementing, for instance, parallel computing strategies (e.g. [20]).

In this paper, an efficient stochastic IDA methodology for nonlinear/hysteretic oscillators is developed by resorting to nonlinear stochastic dynamics concepts and tools such as stochastic averaging and statistical linearization [12]. Specifically, modeling the excitation as a non-stationary stochastic process possessing an evolutionary power spectrum (EPS), and scaling appropriately the intensity of the excitation EPS, an approximate closed-form expression is derived for the parameterized oscillator response amplitude PDF. In this regard, an IDA surface is determined providing the PDF of the EDP for a given IM value. In contrast to an alternative MCS based determination of the IDA surface, the methodology developed herein determines the EDP PDF at minimal computational cost. Note that the technique can account for physically realistic excitation models possessing not only time-varying intensities but time-varying frequency contents as well. Numerical examples include a bilinear/hysteretic single-degree-of-freedom (SDOF) oscillator, whereas comparisons with pertinent MCS data demonstrate the accuracy of the developed stochastic IDA methodology.

2. Stochastic Averaging

Consider a nonlinear SDOF oscillator whose motion is governed by the following stochastic differential equation (SDE)

$$\ddot{x} + \beta_0\dot{x} + z(t, x, \dot{x}) = w(t) \tag{1}$$

where a dot over the variables denotes differentiation with respect to time t; x is the displacement; $z(t, x, \dot{x})$ is the restoring force that depends on the values of x and $\dot{x}$; $\beta_0 = 2\omega_0\xi_0$ is the linear damping coefficient; ω_0 is the natural frequency of the linear oscillator (i.e. $z(t, x, \dot{x}) = {\omega_0}^2 x$); ξ_0 is the ratio of critical damping; and $w(t)$

represents a Gaussian, zero-mean non-stationary stochastic process possessing an evolutionary broad-band power spectrum $S_w(\omega, t)$ [14].

Relying primarily on the assumption of light damping, a combination of statistical linearization and deterministic/stochastic averaging is performed for approximating the second-order stochastic differential equation (SDE) (Eq. (1)), first, by a linear time-variant oscillator, and second, by a first-order SDE governing the response amplitude process $A(t)$. In this regard, a linearized version of Eq. (1) is given by

$$\ddot{x} + \beta_{eq}(t)\dot{x} + \omega_{eq}^2(t)x = w(t) \tag{2}$$

where $\beta_{eq}(t)$ is the time-dependent equivalent damping element and $\omega_{eq}^2(t)$ is the time-dependent equivalent squared natural frequency. These parameters are obtained via a mean square minimization procedure of the error between Eq. (1) and Eq. (2) yielding

$$\beta_{eq}(t) = \beta_0 + E[\beta(A)] = \int_0^\infty \left(\frac{1}{\pi}\int_0^{2\pi} \frac{\cos[\psi]z[t, A\cos(\psi), -\omega(A)A\sin(\psi)]}{A\ \omega(A)} d\psi\right) p(A,t)dA \tag{3}$$

and

$$\omega_{eq}^2(t) = E[\omega^2(A)] = \int_0^\infty \left(-\frac{1}{\pi}\int_0^{2\pi} \frac{\sin[\psi]z[t, A\cos(\psi), -\omega(A)A\sin(\psi)]d\psi}{A} d\psi\right) p(A,t)dA \tag{4}$$

In Eqs. (3-4) the non-stationary response amplitude PDF $p(A,t)$ is modeled as a time-dependent Rayleigh PDF in the form [12,18]

$$p(A,t) = \frac{A}{c(t)} e^{-\frac{A^2}{2c(t)}} \tag{5}$$

Considering next the oscillator initially at rest, i.e., $p(A, t = 0) = \delta(A)$, where $\delta(.)$ is the Dirac delta function, and defining the slowly varying with time response amplitude $A(t)$ as $A^2(t) = x^2(t) + \left[\dot{x}(t)/\omega_{eq}(t)\right]^2$, substituting Eq. (5) into the Fokker-Planck partial differential equation associated with the first order SDE governing the evolution of the response amplitude process PDF, the following first-order nonlinear deterministic differential equation can be obtained after some tedious mathematical manipulations, i.e.

$$\dot{c}(t) = -\beta_{eq}\big(c(t)\big)c(t) + \frac{\pi S_w\big(\omega_{eq}(c(t)), t\big)}{\omega_{eq}^2(c(t))} \tag{6}$$

Eq. (6) constitutes a first-order nonlinear ODE for the time-dependent parameter $c(t)$, which can be solved by standard numerical schemes such as the Runge-Kutta. Once $c(t)$ is determined, both the non-stationary response amplitude PDF of Eq. (5) and the time-dependent equivalent linear elements of Eq.(3) and Eq.(4) can be evaluated. A detailed presentation of the derivation of Eq. (6) can be found in [12,18].

3. Stochastic Incremental Dynamic Analysis

In this section a novel stochastic IDA framework based on the nonlinear oscillator stochastic averaging/linearization treatment delineated in the previous section is developed.

Considering the ground motion records to be realizations compatible with a stochastic process EPS, and exploiting the closed form expressions for the nonlinear oscillator response amplitude PDF, analytical expressions are derived for the parameterized EDP statistics as functions of the excitation EPS intensity magnitude. In comparison with a standard IDA framework, the complete statistical characterization (i.e. PDF determination) of the EDP quantity is achieved at minimal computational cost; thus, circumventing a computationally demanding MCS kind statistical treatment [20] that requires the, computationally prohibitive in many cases, derivation of a large number of IDA curves.

In general, in the earthquake engineering field several EPS $S_w(\omega, t)$ forms have been proposed for describing the ground motion time-varying intensity and frequency content (e.g. [11, 6]). In the following, a parameterized EPS is introduced in the form

$$S_w(\omega, t; S_0) = S_0 \cdot S(\omega, t) \tag{7}$$

where $S_0 \in (0, \infty)$ represents the EPS $S(\omega, t)$ intensity magnitude, and $S_w(\omega, t; S_0)$ is the ground acceleration EPS for a given S_0. In a performance based engineering context, the IM can be represented by S_0, whereas the EDP considered herein is the maximum in time response displacement amplitude A. Clearly, based on the time-dependent Rayleigh PDF (Eq. (5)) the maximum response displacement amplitude A occurs also when the system response variance $c(t)$ attains its maximum value in time $c_{max}(S_0)$ for a given value of S_0 as in Fig. 1. Thus, for various values of S_0, a curve of maxima (Fig. 2) has the functional form $c_{max}(S_0)$. In this regard, the EDP PDF can be obtained in the form

$$p(A; S_0) = \frac{A}{c_{max}(S_0)} e^{-\frac{A^2}{2c_{max}(S_0)}} \tag{8}$$

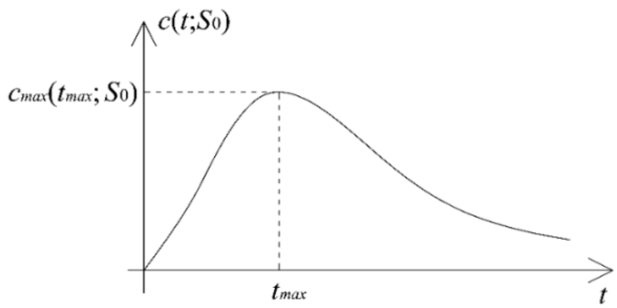

Fig. 1 – Parameterized system response variance for a given EPS intensity magnitude S_0.

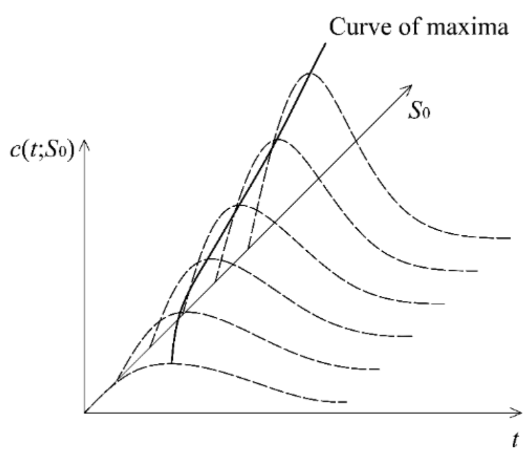

Fig. 2 – Curve of system response variance maxima for different EPS intensity magnitude S_0.

Clearly, the complete statistical description of the EDP is provided by the PDF of Eq. (8), whereas the derivation of other EDP related statistical quantities, such as the mean and the standard deviation, is a trivial task.

4. Numerical example

In this section, numerical examples are included demonstrating the efficiency and accuracy of the herein developed stochastic IDA. To this aim, the bilinear hysteretic oscillator [4], widely used in earthquake engineering applications (e.g. [5, 8]), is considered in detail.

Focusing on the bilinear hysteretic oscillator, its equation of motion is given by [15]

$$\ddot{x} + \beta_0 \dot{x} + \alpha \omega_0^2 x + (1-\alpha)\omega_0^2 x_y z = w(t) \tag{9}$$

where the internal variable $z(t, x, \dot{x})$ is given by the differential equation

$$x_y \dot{z} = \dot{x}[1 - H(\dot{x})H(z-1) - H(-\dot{x})H(-z-1)] \tag{10}$$

In Eq. (10) $H(.)$ denotes the Heaviside step function defined as $H(n) = 0$ if $n < 0$, and $H(n) = 1$, otherwise.

A typical force-displacement diagram is plotted in Fig. 3, with k being the pre-yield stiffness, k_p the post-yield stiffness, $\alpha = k_p/k$, x_y the yield displacement, and F_y denoting the yield force, and $z(t, x, \dot{x}) = \alpha k + (1-\alpha)k x_y z/m$.

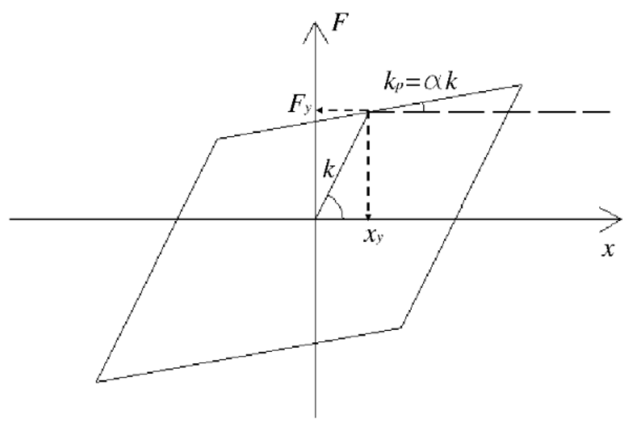

Fig. 3 – Force-displacement diagram of a bilinear hysteretic oscillator.

Next, $\beta_{eq}(t)$ and $\omega_{eq}^2(t)$ can be obtained analytically by employing Eq. (3) and Eq. (4), yielding [19]

$$\beta_{eq}\big(c(t)\big) = 2\omega_0 \xi + \frac{4 x_y (1-\alpha)\omega_0^2}{\pi c(t)} \int_{x_y}^{\infty} \frac{1}{\omega(A)} \left(1 - \frac{x_y}{A}\right) \exp\left(-\frac{A^2}{2c(t)}\right) dA \tag{11}$$

and

$$\omega_{eq}^2\big(c(t)\big) = \omega_0^2 \left\{\alpha + (1-\alpha)\left[1 - \exp\left(-\frac{x_y}{2c(t)}\right) + \frac{1}{\pi c(t)} \int_{x_y}^{\infty} \left(\Lambda - \frac{1}{2}\sin(2\Lambda)\right) A \exp\left(-\frac{A^2}{2c(t)}\right) dA\right]\right\} \tag{12}$$

The system response variance $c(t)$ is obtained numerically by solving Eq. (6) via standard integration schemes such as the Runge-Kutta. In the ensuing example, the values $\xi_0 = 0.05, T_0 = \frac{2\pi}{\omega_0} = 0.5s, x_y = 0.016m$ are used, whereas the oscillator is assumed to be initially at rest. Further, a time-modulated excitation EPS is utilized to model a non-stationary stochastic process characterizing the ground motion. Results obtained by the developed stochastic IDA are compared against pertinent MCS data (10,000 realizations) generated via a spectral representation approach [16].

In this example the ground motion excitation is modeled as a non-stationary stochastic process, i.e. $w(t) = g(t)v(t)$, where $g(t)$ is a deterministic time-modulating function, and $v(t)$ is a stationary stochastic process. The EPS of $w(t)$ is given by

$$S_w(\omega, t|S_0) = |g(t)|^2 S_v(\omega|S_0) \tag{13}$$

where the time-modulating function $g(t)$ is given by

$$g(t) = Ct \exp\left(\frac{-bt}{2}\right) \tag{14}$$

In Eq. (14) C and b are constants related to the ground motion intensity and duration, respectively. Further, the stationary process power spectrum $S_v(\omega)$ is modeled as the widely used in earthquake engineering double-sided Kanai-Tajimi spectrum modified by Clough and Penzien (C-P) [6] in the form

$$S_v(\omega|S_0) = S_0 \frac{\omega_g^2 + 4\xi_g^2\omega^2\omega_g^2}{\left(\omega_g^2 - \omega^2\right)^2 + 4\xi_g^2\omega^2\omega_g^2} \frac{\omega^2}{(\omega_f^2 - \omega^2)^2 + 4\xi_f^2\omega^2\omega_f^2}. \tag{15}$$

It is worth noting that several methodologies have been developed for determining the parameters of the C-P power spectrum, so that the resulting EPS is compatible with a prescribed design spectrum, as provided by the structural design codes [9,7,17]. For instance, an inverse optimization problem was formulated and solved in [9] for computing the EPS parameters of Eq. (15), so that the time-modulated C-P spectrum is compatible with the design spectrum provided by the European aseismic code - EC8. In this regard, the parameter values for soil B and peak ground acceleration (PGA) equal to 0.36g ($S_0 = 1\, m^2/s^3$) were evaluated in [9] to be $C = 17.76\, cm/s^{2.5}$; $b = 0.58\, s^{-1}$; $\xi_g = 0.78$; $\omega_g = 10.73\, rad/s$; $\xi_f = 0.90$; $\omega_f = 2.33\, rad/s$.

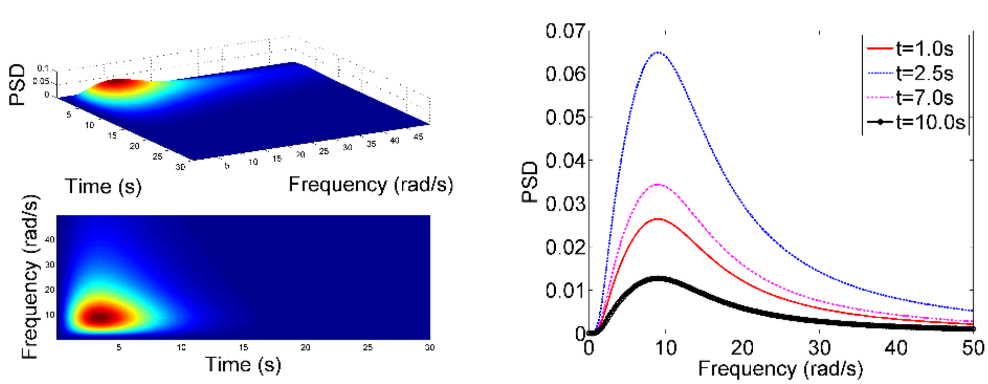

Fig. 4 – Time-modulated Clough-Penzien excitation EPS, surface a), and sections b).

In the following example, the above parameter values are used, while a functional relationship is established between the EPS intensity magnitude S_0 and the spectral acceleration $S_a(T_1, 5\%)$. This is motivated by the trend of earthquake engineering applications, where the spectral acceleration is chosen as the IM. To this aim, for a given value of S_0, realizations compatible with Eq. (13) are generated (500 realizations) via the spectral representation approach [16]. Next, the equation of motion of a 5% damped linear SDOF oscillator with a natural period $T_1 = 2\pi/\omega_0 = 0.5\,s$ is solved, by utilizing a fourth order Runge-Kutta numerical integration scheme. Finally, the spectral acceleration for a given S_0 value is estimated as the ensemble average of the response realizations. The process is then repeated for a range of S_0 values. The functional relationship S_0 - $S_a(T_1 = 0.5\,s, 5\%)$ is depicted in Fig. 5. Further, the functional form

$$S_a(T_1, 5\%) = P_1(S_0)^{P_2} \tag{14}$$

is fitted to the MCS data for convenience purposes, whereas a least-squares minimization scheme yields the parameters values $P_1 = 1.065$ and $P_2 = 0.5$.

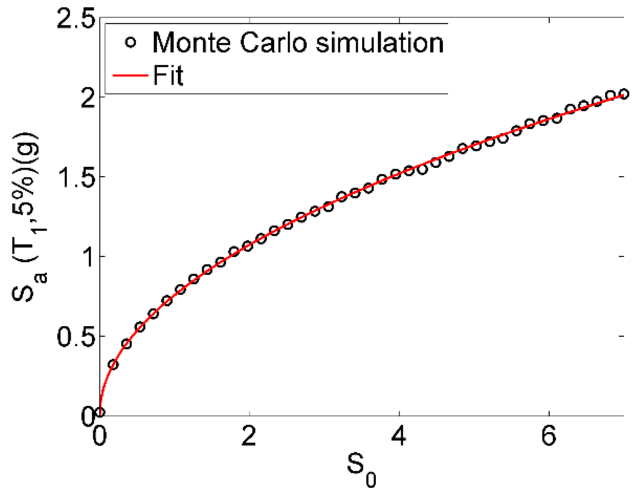

Fig. 5 – Functional relationship between the spectral acceleration $S_a(T_1 = 0.5\,s, 5\%)$ and S_0 for a time-modulated Clough-Penzien excitation EPS.

To assess the efficiency of the proposed approach a range of $S_a = S_a(T_1, 5\%)$ from $0.001g$ to $10.0g$, where $g = 9.81 m/s^2$, is considered. First, the linear oscillator is studied, i.e., $\alpha = 1$, and the response amplitude PDF obtained via the stochastic averaging/linearization approach (Fig. 6a) is compared with the response amplitude PDF obtained by MCS data (Fig. 6b). Similarly, for a nonlinear oscillator with $\alpha = 0.2$, in Figures 7a,b the response amplitude PDFs obtained by a stochastic averaging/linearization approach and by MCS data are plotted, respectively. It can be readily seen that the accuracy level is quite satisfactory, given the approximations involved in the proposed approach.

In Fig. 8 the response amplitude PDFs obtained via the proposed approach, considering both α=1 (Fig. 8a) and α=0.2 (Fig. 8b), are compared with MCS based PDF for various levels of spectral acceleration.

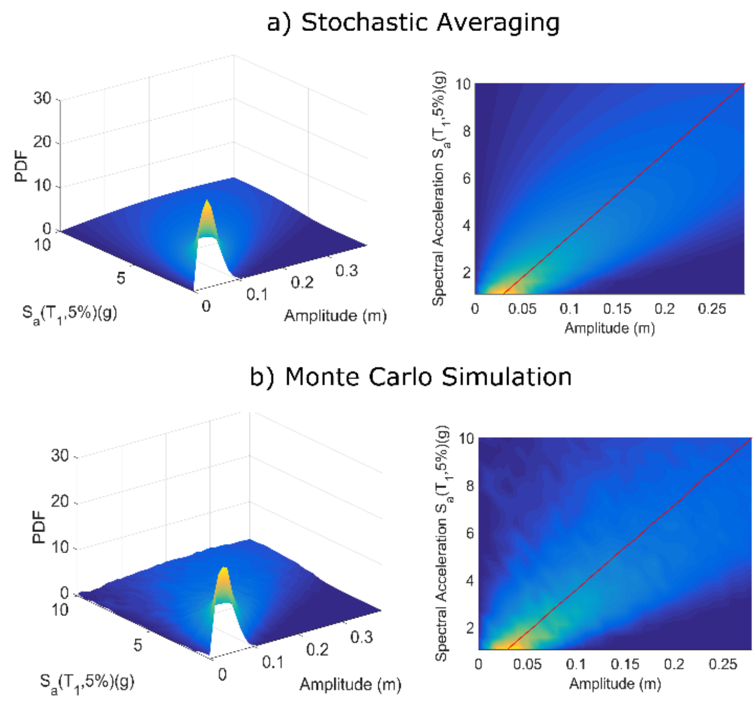

Fig. 6 – Response amplitude PDF (α = 1, T=0.5s) obtained by a) stochastic averaging/linearization and by b) Monte Carlo simulation (10,000 realizations). The solid red line denotes the EDP mode.

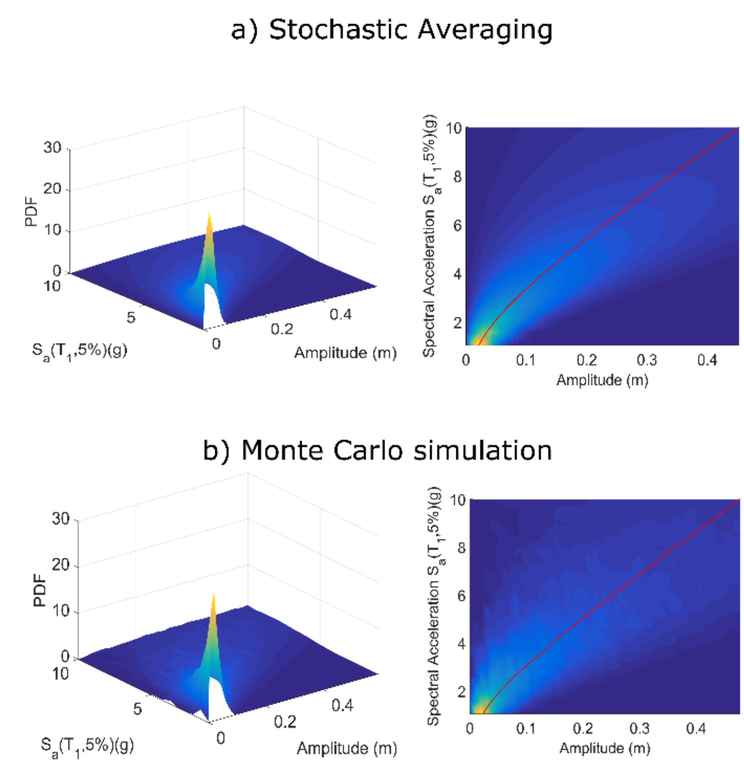

Fig. 7 – Response amplitude PDF (α = 0.2, T=0.5s) obtained by a) stochastic averaging/linearization and by b) Monte Carlo simulation (10,000 realizations). The solid red line denotes the EDP mode.

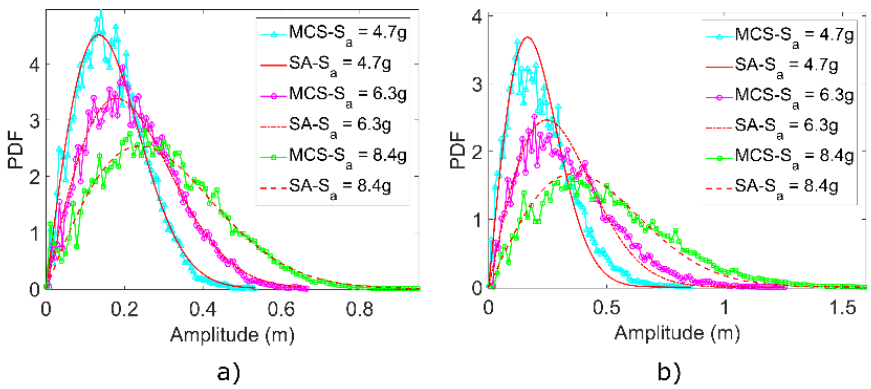

Fig. 8 –Response amplitude PDF for various excitation/spectral acceleration levels for (T=0.5s) a) α = 1 and b) α = 0.2; comparisons with pertinent MCS data (10,000 realizations).

Further, based on the estimated response amplitude PDF, various other related statistics can be readily determined. For instance, in Fig. 9 the 16%, 50% and 84% fractiles are plotted, both for the linear ($\alpha = 1$) and the nonlinear ($\alpha = 0.2$) oscillators. Comparisons with MCS data demonstrate a satisfactory degree of accuracy, even for the nonlinear case.

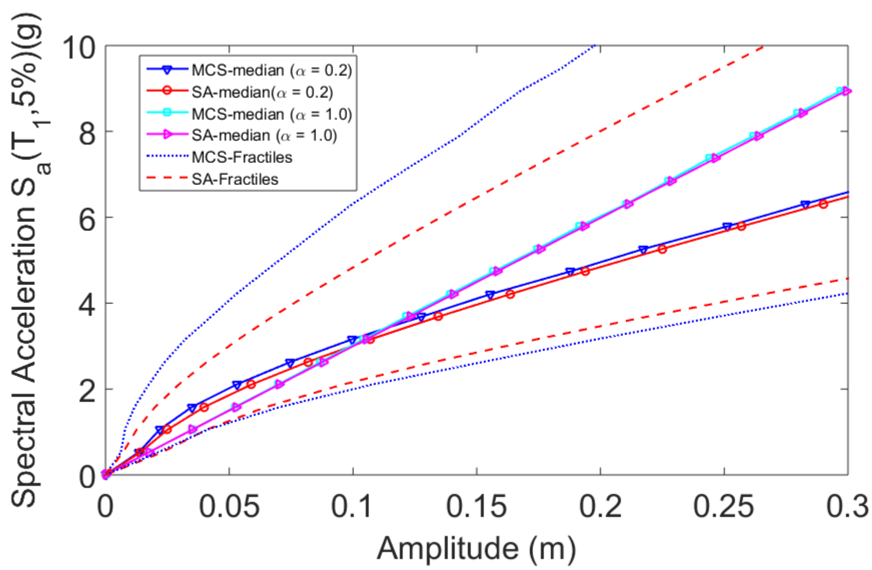

Fig. 9 – Stochastic IDA curves (16%, 50% and 84% fractiles) (T=0.5s); comparisons with MCS data (10,000 realizations).

Regarding the computational performance of the proposed approach, it is worth noting that using a PC with standard configurations, the stochastic averaging based approach takes approximately 2-3 s to generate the data for Fig. 7[a], whereas a MCS based approach utilizing 10,000 realizations takes approximately 20 h. Considering 30 to 300 excitation records per scaling level, as it is common in the standard IDA implementation, the related cost would be approximately 30 min for a SDOF bilinear oscillator. Note, however, that a significantly higher number of records is needed, in general, for estimating reliably higher order statistics (or even the PDF).

5. Conclusions

A novel efficient stochastic incremental dynamic analysis (IDA) methodology for nonlinear/hysteretic oscillators has been developed, by resorting to nonlinear stochastic dynamics concepts and tools such as stochastic averaging and statistical linearization. Specifically, considering the ground motion records to be realizations compatible with a stochastic process EPS, closed form expressions have been derived for the chosen parameterized EDP PDF. This is done at minimal computational cost; thus, circumventing a computationally demanding MCS kind statistical treatment [20] that requires the, computationally prohibitive in many cases, derivation of a large number of IDA curves. The numerical example has included a bilinear/hysteretic SDOF oscillator, whereas comparisons with pertinent MCS data have demonstrated the accuracy of the developed stochastic IDA methodology. Hopefully, due to its versatility, the developed stochastic IDA methodology can be used for preliminary structural system design applications within a PBE framework in various engineering fields such as earthquake and wind engineering.

5. References

[1] Augusti G, ciampoli M (2008): Performance-Based Design in risk assessment and reduction. *Probabilistic Engineering Mechanics*, **23**, 496-508.

[2] Barbato M, Petrini F, Unnikrishnan VU, Ciampoli, M (2013): Performance-based hurricane engineering (PBHE) framework. *Structural Safety*, **5**, 187-203.

[3] Beck AT, Kougioumtzoglou IA, Dos Santos KRM (2014) Optimal performance-based design of non-linear stochastic dynamical RC structures subject to stationary wind excitation. *Engineering Structures*, **78**, 145-153.

[4] Caughey TK, (1960): Sinusoidal excitation of a system with bilinear hysteresis. *Journal of Applied Mechanics*, **27**, 640-643.

[5] Chopra AK, Chintanapakdee C (2004): Inelastic Deformation Ratios for Design and Evaluation of Structures: Single-Degree-of-Freedom Bilinear Systems. *Journal of Structural Engineering*, **130**, 1309-1319.

[6] Clough RW, Penzien J (1993): *Dynamics of Structures*, McGraw-Hill.

[7] Giaralis A, Spanos PD (2009): Wavelet-based response spectrum compatible synthesis of accelerograms-Eurocode application (EC8). *Soil Dynamics and Earthquake Engineering*, **29**, 219-235.

[8] Giaralis A, Spanos PD (2010): Effective linear damping and stiffness coefficients of nonlinear systems for design spectrum based analysis. *Soil Dynamics and Earthquake Engineering*, **30**, 798-810.

[9] Giaralis A, Spanos PD (2012): Derivation of response spectrum compatible non-stationary stochastic processes relying on Monte Carlo-based peak factor estimation. *Earthquakes and Structures*, **3**, 719-747.

[10] Iwan WD, Paparizos LG (1988): The stochastic response of strongly yielding systems. *Probabilistic Engineering Mechanics*, **3**, 75-82.

[11] Kanai K (1957): Semi-empirical formula for the seismic characteristics of the ground. Earthquake Research Institute.

[12] Kougioumtzoglou IA, Spanos PD (2009): An approximate approach for nonlinear system response determination under evolutionary stochastic excitation. *Current Science*, **97**, 1203-1211.

[13] Preumont A (1985): The Generation of Non-Separable Artificial Earthquake Accelerograms for the Design of Nuclear-Power Plants. *Nuclear Engineering and Design*, **88**, 59-67.

[14] Priestley MB (1965): Evolutionary Spectra and Non-Stationary Processes. *Journal of the Royal Statistical Society Series B-Statistical Methodology*, **27**, 204-237.

[15] Roberts JB, Spanos PD (2003): *Random Vibration and Statistical Linearization*, Dover.

[16] Shinozuka M, Deodatis G (1991): Simulation of stochastic processes by spectral representation. *Applied Mechanics Reviews*, **44**, 191-204.

[17] Spanos PD, Giaralis A, Jie L (2009): Synthesis of accelerograms compatible with the Chinese GB 50011-2001 design spectrum via harmonic wavelets: artificial and historic records. *Earthquake Engineering and Engineering Vibrations*, **8**, 189-206.

[18] Spanos PD, Kougioumtzoglou IA (2014): Survival Probability Determination of Nonlinear Oscillators Subject to Evolutionary Stochastic Excitation. *Journal of Applied Mechanics-Transactions of the Asme*, **81**.

[19] Tubaldi E, Kougioumtzoglou IA (2015): Nonstationary stochastic response of structural systems equipped with nonlinear viscous dampers under seismic excitation. *Earthquake Engineering & Structural Dynamics*, **44**, 121-138.

[20] Vamvatsikos D (2011): Performing incremental dynamic analysis in parallel. *Computers & Structures*, **89**, 170-180.

[21] Vamvatsikos D, Cornell CA (2002): Incremental dynamic analysis. *Earthquake Engineering & Structural Dynamics*, **31** (3), 491-514.