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Cross-Coupling in Six-Degree-of-Freedom Shake Tables

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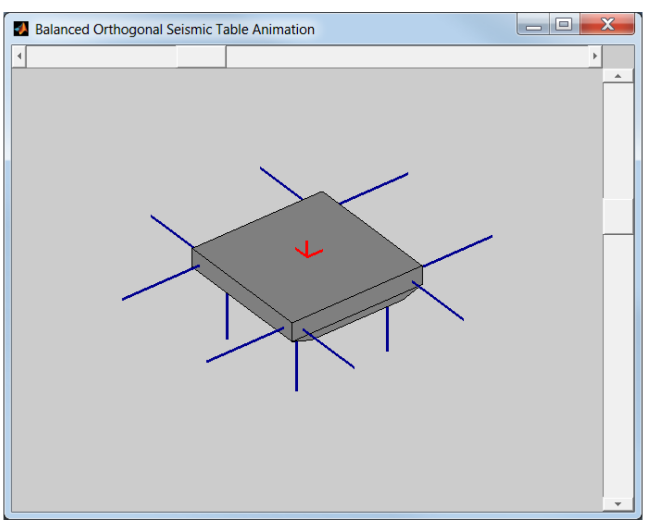
Abstract

Many considerations factor into the decision of what type of shake table to purchase: capacity, cost, operation, and fidelity. One aspect of fidelity that is seldom considered or understood is the dynamic cross-coupling that occurs between degrees-of-freedom (DOFs). A table with low cross-coupling has a natural tendency to move in the desired DOF without causing motion in the other DOFs. Cross-coupling is greatly influenced by actuator geometry. Most shake tables fall into one of five categories depending on their actuator configuration. In this paper, these five configurations are analyzed, compared, and ranked from a cross-coupling standpoint. Both for buyers of new shake tables and owners of existing tables, this analysis may promote a deeper understanding of fidelity limitations that arise due to actuator configuration.

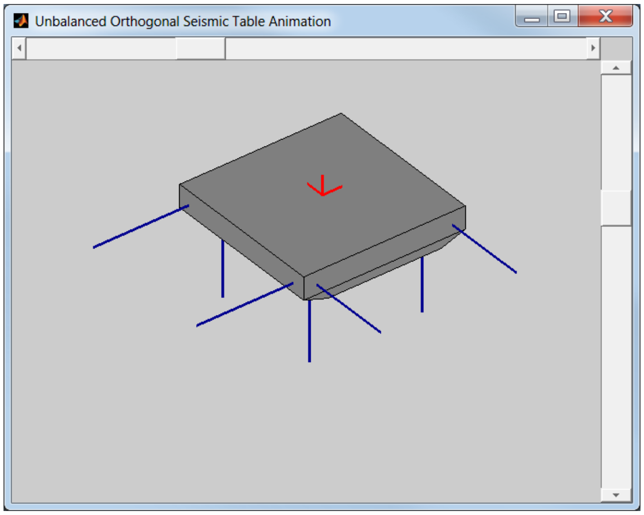
Keywords: shake table; degree-of-freedom; cross-coupling; MDOF dynamic systems; servo-hydraulic actuator

Actuator Configurations

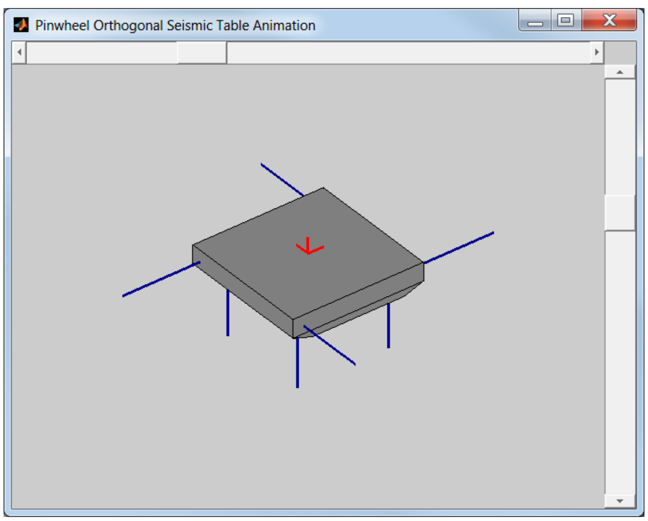
The five actuator configurations consist of three orthogonal configurations (*balanced*, *unbalanced*, and *pinwheel*) and two non-orthogonal configurations (*vee* and *stewart*):



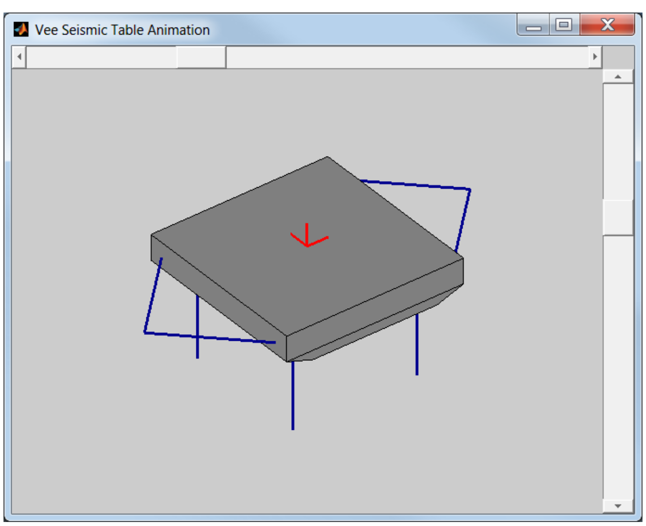
The ***balanced orthogonal*** configuration is orthogonal in the X, Y, and Z directions and is symmetric with respect to all three Cartesian axes, hence the name "balanced". This configuration is used in high force/acceleration testing, which requires that the large forces generated be distributed among many horizontal actuators. Its primary disadvantage is its large footprint, consuming a large amount of laboratory floor space. Due to its orthogonality and symmetry, this configuration is believed to have minimal cross-coupling.



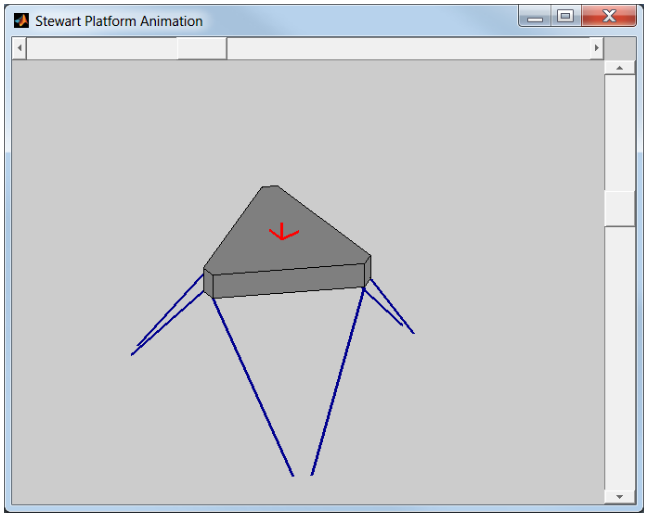
The ***unbalanced orthogonal*** configuration has a smaller footprint than the *balanced orthogonal* configuration and is used when lab floor space is limited. It is orthogonal but not symmetric due to the lack of actuators on two sides of the table, hence the name "unbalanced". The lack of actuators on two sides offers lab personnel better access for mounting and instrumenting the specimen. However, it also results in unequal distribution of horizontal actuator mass among the table corners, so that translational motion generates reaction moments that result in cross-coupling into the rotational DOFs.



The ***pinwheel orthogonal*** configuration is orthogonal and antisymmetric with respect to X and Y axes. Unlike the *unbalanced orthogonal* configuration, its horizontal actuator mass is equally distributed at the table corners, so that translational motion does not generate reaction moments that result in rotational cross-coupling. The number of horizontal actuators is half that of the *balanced orthogonal* configuration, reducing complexity and cost. Its primary disadvantage is its large footprint.



The ***vee*** configuration is symmetric but not orthogonal. The mechanical advantage provided by the angled horizontal actuators reduces actuator stroke and velocity requirements by 30%. The shorter stroke results in shorter actuators, which mitigates actuator bowstring resonance problems, and results in a smaller footprint. The presence of two unobstructed sides improves table access, and makes it possible to join two tables together into a larger one. A disadvantage is that because of angled actuators, X and Y capacities are coupled and cannot be dissimilar, unlike the orthogonal configurations.



The ***stewart*** (also called *hexapod*) configuration is an old design that has been used in motion simulators for many years, but has not been used much in seismic applications due to a prevailing belief that (rightly or wrongly) it suffers from high cross-coupling. It is neither orthogonal nor symmetric. It can be mounted to a strong-floor instead of a more expensive foundation pit, but the elevated table surface makes access more difficult. Because of the lack of orthogonality, X, Y, Z capacities are coupled and it is difficult to design a table with different capacities, unlike the orthogonal configurations.

In the following sections, each of the five actuator configurations is represented by a hypothetical table design that is compared against the others for cross-coupling under a variety of test conditions.

Factors That Affect Cross-coupling

Cross-coupling is influenced by factors other than actuator geometry: table dynamics, offset from home position, location of the center of gravity (CG), specimen dynamics, and closed-loop control also play significant roles.

To facilitate a meaningful comparison of the five actuator configurations, key physical properties such as natural frequency, maximum displacement, maximum velocity, and maximum force are matched to the best extent possible:

Natural Frequency

	Balanced	Unbalanced	Pinwheel	Vee	Stewart	
X	13	13	13	13	13	Hz
Y	13	13	13	13	13	Hz
Z	23	23	23	23	22	Hz
Roll	21	21	21	21	21	Hz
Pitch	21	21	21	21	21	Hz
Yaw	21	21	21	21	21	Hz

Maximum Displacement

	Balanced	Unbalanced	Pinwheel	Vee	Stewart	
X	0.30	0.30	0.30	0.41	0.73	m
Y	0.30	0.30	0.30	0.41	0.67	m
Z	0.15	0.15	0.15	0.15	0.58	m
Roll	4.85	4.85	4.85	4.85	24.16	deg
Pitch	4.85	4.85	4.85	4.85	21.24	deg
Yaw	7.24	7.24	10.72	5.27	25.80	deg

Maximum Velocity

	Balanced	Unbalanced	Pinwheel	Vee	Stewart	
X	2.3	2.3	2.3	2.3	2.3	m/s
Y	2.3	2.3	2.3	2.3	2.1	m/s
Z	1.8	1.8	1.8	1.8	1.7	m/s
Roll	60	60	60	60	68	deg/s
Pitch	60	60	60	60	65	deg/s
Yaw	63	63	63	29	78	deg/s

Maximum Force

	Balanced	Unbalanced	Pinwheel	Vee	Stewart	
X	814	814	814	814	838	kN
Y	814	814	814	814	797	kN
Z	1279	1279	1279	1279	1922	kN
Roll	2239	2239	2239	2239	1639	kN-m
Pitch	2239	2239	2239	2239	1452	kN-m
Yaw	3418	3418	3418	3743	2425	kN-m

Each table's displacement controllers are tuned for reasonable flatness and a magnitude peak of 1.10, using only displacement error and force feedback stabilization gains. Resulting closed-loop bandwidths are remarkably similar:

Closed-Loop -3dB Bandwidth

	Balanced	Unbalanced	Pinwheel	Vee	Stewart	
X	12	12	12	13	11	Hz
Y	12	12	12	13	12	Hz
Z	22	22	22	22	21	Hz
Roll	20	20	20	20	21	Hz
Pitch	20	20	20	20	21	Hz
Yaw	22	22	22	22	20	Hz

With regard to the influences of offset from home position, CG location, and specimen dynamics, there are countless cases that could potentially be examined. Of these, five cases are chosen as instructive:

	Position offset {X Y Z}	CG offset {X Y Z}
Case 1: Home position, minimal OTM	{0 0 0}	{0 0 0}
Case 2: Home position, significant OTM	{0 0 0}	{0 0 1}
Case 3: X-only position offset	{0.2 0 0}	{0 0 0}
Case 4: X, Y, and Z position offsets	{0.2 0.2 0.1}	{0 0 0}
Case 5: Lightly-damped resonant specimen (100% of table mass, 5 Hz, 5% damping, 1m CG)		

The nonzero offsets above have units of meters, and represent 67% of displacement capacity for each respective translational DOF of the orthogonal tables, less for the non-orthogonal tables. The CG location of each table, to which the CG offset is added, is chosen to be the "sweet spot", i.e., the location that minimizes overturning-moment (OTM) effects. For the *balanced*, *unbalanced*, *pinwheel*, and *vee* configurations, this location lies in the plane of the attachment points of the horizontal actuators, and results in identically zero OTM. For the *stewart* configuration, there is no location that results in identically zero OTM, but there is a location where OTM is minimized.

These five cases were analyzed with the tables under closed-loop control. However, the amount of cross-coupling in a closed-loop system depends greatly on the particular system tuning, and because there are countless possible tunings, comparisons require judgment.

Cross-coupling plots for each of the five cases are shown on the following pages. The plots are velocity frequency response functions (FRFs) from the on-diagonal DOF *output* to each off-diagonal DOF *output*. Note that the on-diagonal DOF *output* is used as the reference rather than the *input*, the reason being that relative output motion is a better indicator of cross-coupling than absolute output motion. With this reference convention, on-diagonal FRFs are of course unity for all frequencies.

The units of the velocity FRF plots are *fraction of full-scale* so that cross-coupling percentage values can be determined directly by inspection. For example, suppose that the X-to-Pitch FRF peaks at a value of 0.5. This means that for X motion at 100% of maximum X velocity, the peak cross-coupling on pitch will be 50% of maximum pitch velocity. Because the FRFs are scaled according to DOF maxima, and to facilitate comparison, care is taken to assure that all five actuator configurations have approximately the same maximum velocity.

To reduce visual clutter, only FRFs that have at least one curve above 5% cross-coupling are shown. Also note that in the plots, *Rx*, *Ry*, and *Rz* are shorthand for *roll*, *pitch*, and *yaw*, respectively.

Mathematics

A full treatment of the mathematics used to compute the FRFs is beyond the scope of this paper. A brief description is as follows: Actuator geometry, stiffness, and damping are combined to form the DOF stiffness matrix $\mathbf{K}_T$ and damping matrix $\mathbf{C}_T$, which along with the mass matrix $\mathbf{M}_T$ form the equation describing the motion state $\mathbf{x}_T$ in response to force $\mathbf{f}_T$

$$\mathbf{M}_T\ddot{\mathbf{x}}_T + \mathbf{C}_T\dot{\mathbf{x}}_T + \mathbf{K}_T\mathbf{x}_T = \mathbf{f}_T$$

The closed-loop controller equation describing the relationship between command $\mathbf{r}$, feedback $\mathbf{y}$, and controller output $\mathbf{u}$ given gain matrix $\mathbf{K}_C$ is given by

$$\mathbf{u} = \mathbf{r} - \mathbf{K}_C\mathbf{y}$$

The motion and controller equations are combined and converted to state-space form using lengthy but straightforward analysis (details omitted)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

The state-space equations are then transformed into a matrix of Laplace transfer function polynomials using MATLAB function *ss2tf*, and evaluated on a frequency grid using MATLAB function *freqs*.

Observations

As can be seen in the plots on the following pages, all actuator configurations experience cross-coupling, differing only in amount, bandwidth, and test conditions in which it manifests itself. It is useful to determine in general which configuration has the least cross-coupling and which one has the most, and which ones lie in-between. This can be accomplished by ranking the actuator configurations for each test case, then tallying how many times a particular configuration appears at a given rank. Doing so results in the following ranking:

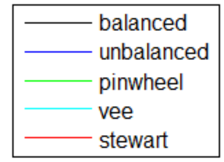
Ranking (from least to most cross-coupling)

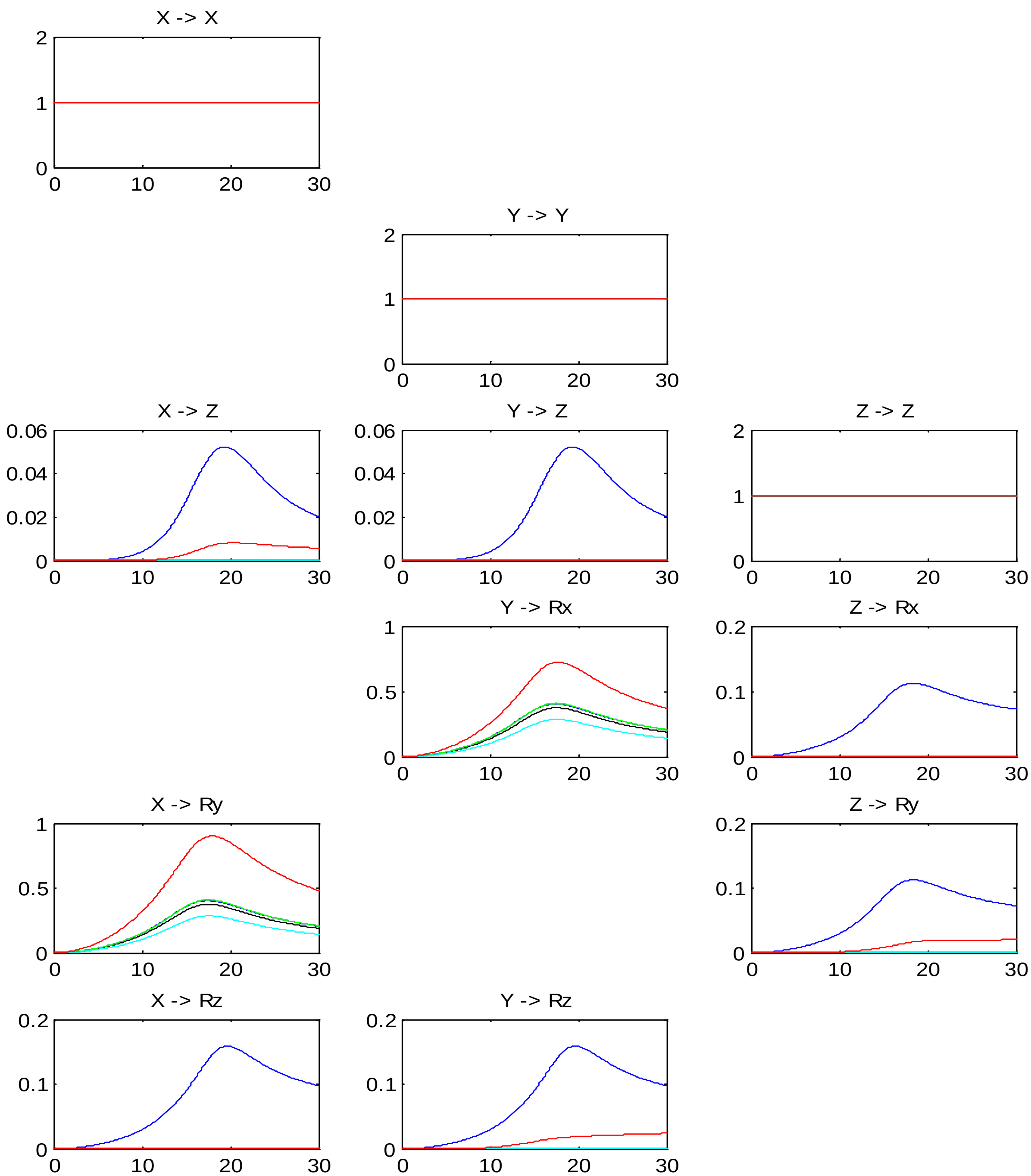
#1 (in 3 out of 5 test cases): *vee*
#2 (in 4 out of 5 test cases): *balanced orthogonal*
#3 (in 3 out of 5 test cases): *unbalanced orthogonal*
#4 (in 4 out of 5 test cases): *pinwheel orthogonal*
#5 (in 5 out of 5 test cases): *stewart*

Conclusions

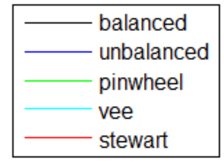
All actuator configurations have dynamic cross-coupling to some degree, with the *vee* configuration having the least amount and the *stewart* configuration having the most. The issue is to what degree it degrades test fidelity. Most seismic tests are centered at the home position, and only occasionally and briefly does the table move to a large nonzero position where cross-coupling due to position offset may be significant; in such cases the impact on test fidelity may be minimal. Cross-coupling due to CG offset is more of a concern, since it exists at the home position where the table operates most of the time. Whether or not significant cross-coupling is a problem depends on the extent to which the frequency content of the test acceleration record overlaps the frequency response of the cross-coupling (which in the test cases presented in this paper becomes significant near 10 Hz and above). The earthquake acceleration records used in full-scale model testing typically have frequency content below 10 Hz, where cross-coupling is generally low. However, the acceleration records used in scaled model testing or equipment qualification testing often have frequency content exceeding 10 Hz. In such cases cross-coupling may be quite noticeable. Advance control techniques exist that mitigate cross-coupling effects, but at a cost of complexity or requiring multiple test runs. Owners of existing tables have no choice but to rely on such methods, but buyers of new tables may find it preferable to purchase a table whose geometry has inherently low cross-coupling.

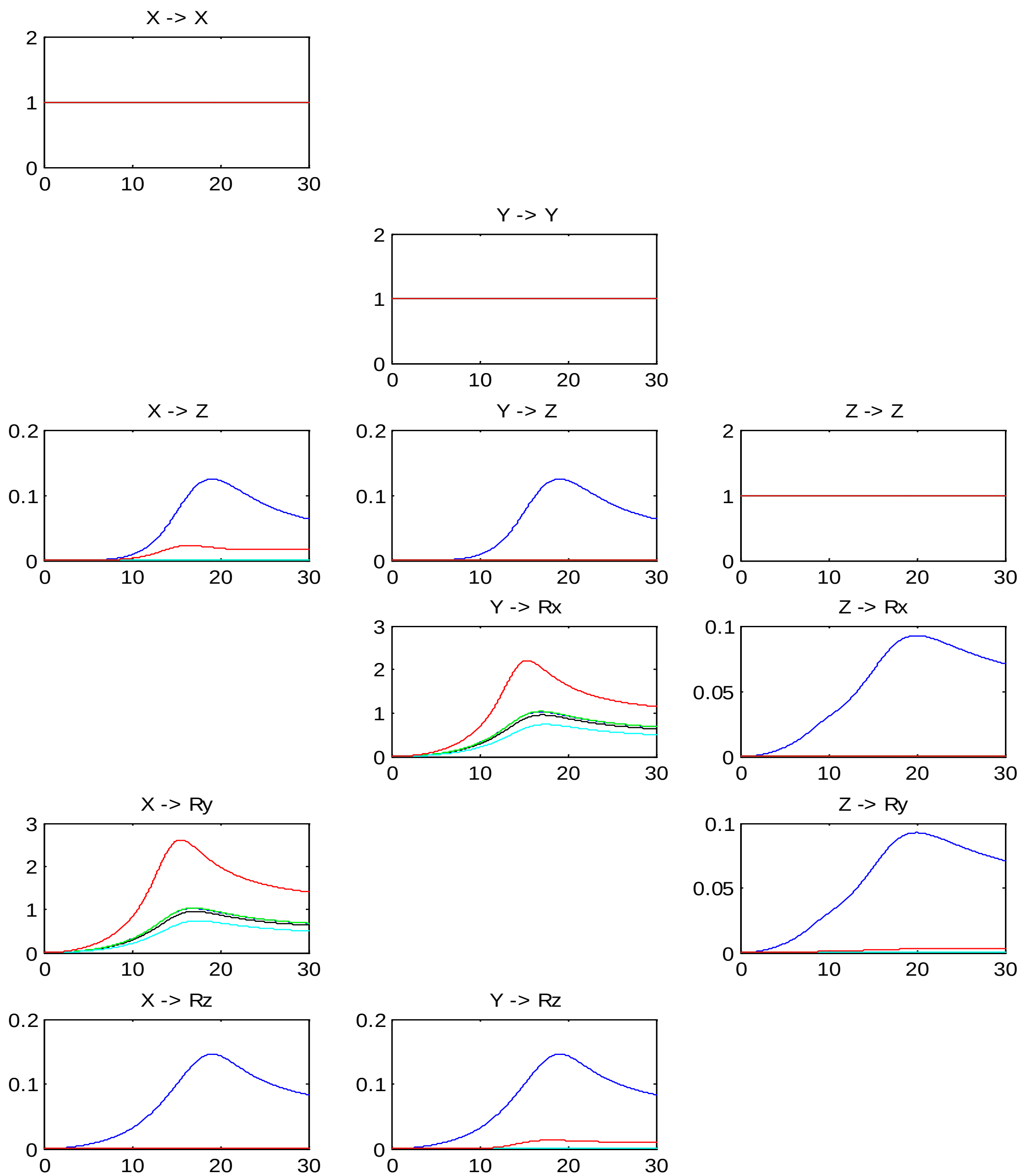
Case 1: Position offset = {0 0 0}, CG offset = {0 0 0}



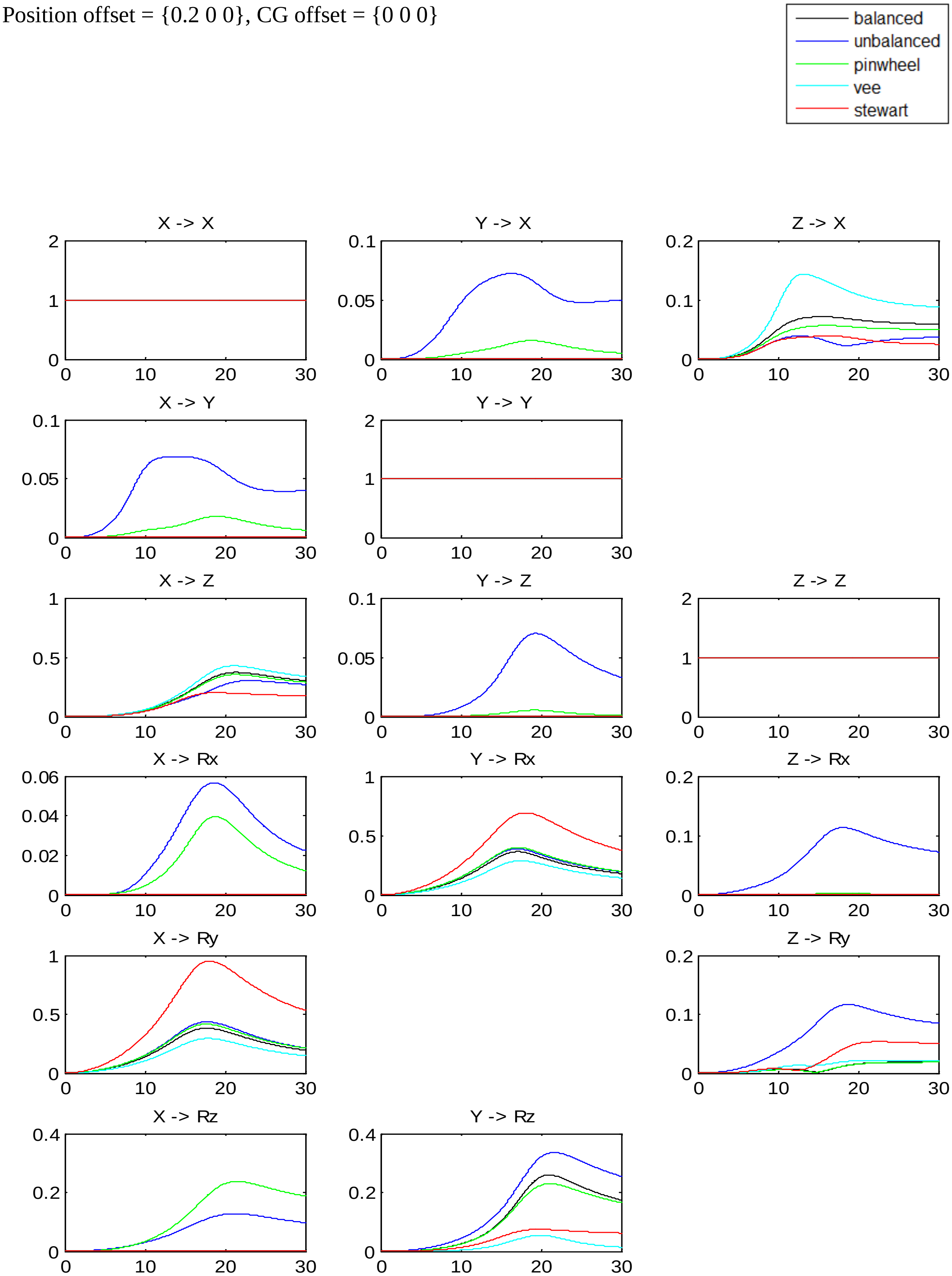


Case 2: Position offset = {0 0 0}, CG offset = {0 0 1}

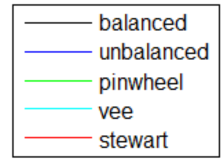


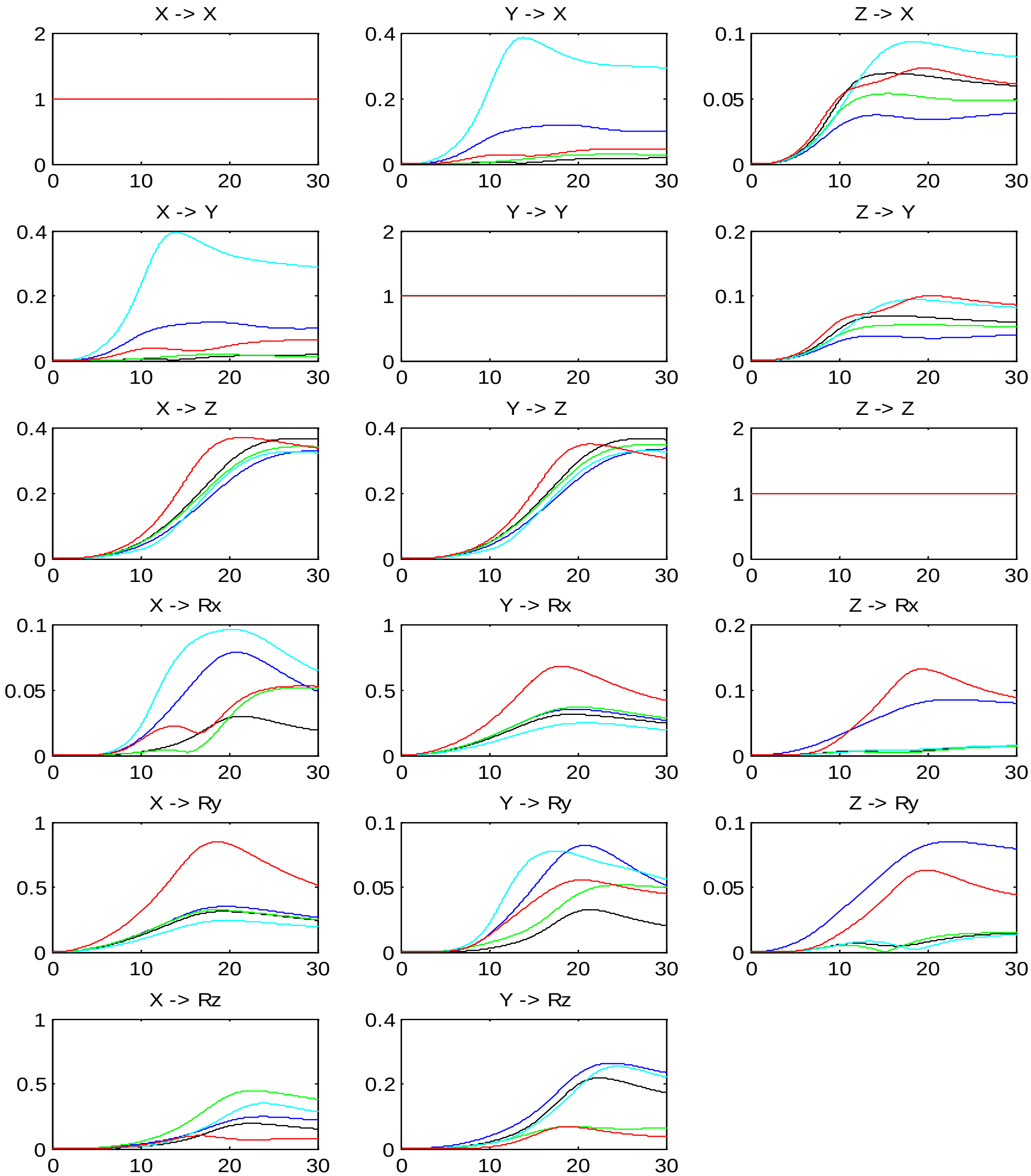


Case 3: Position offset = {0.2 0 0}, CG offset = {0 0 0}



Case 4: Position offset = {0.2 0.2 0.1}, CG offset = {0 0 0}





Case 5: Lightly-damped resonant specimen

(100% of table mass, 5 Hz, 5% damping, 1m CG)