



SPECTRAL DECAY CHARACTERISTICS f_{max} AND κ FOR STRONG GROUND MOTION PREDICTION

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Abstract

Spectral decay characteristics of ground motions for inland crustal earthquakes in Japan are examined. It is very important to clear spectral decay characteristics in high frequency range for strong ground motion prediction. Total number of analyzed target earthquakes is 65 and those moment magnitudes (M_w) are from 3.6 to 6.9. Four large earthquakes ($M_w > 5.9$) are included in the target earthquakes. In this study, spectral decay characteristics in high frequency range are evaluated by two approaches. One is f_{max} filter, the other is spectral decay parameter, κ (Kappa). In result, f_{max} 's of four large earthquakes are estimated to be 6.5Hz to 9.4Hz and f_{max} filter shapes of four large earthquakes are almost same. Larger the earthquakes are, smaller f_{max} 's tend to be. Correction filter, $P_C(f)$, which is a filter to correct difference of spectral decay characteristics between large and small earthquakes are suggested. κ 's of four large earthquakes are estimated to be 0.0142 to 0.0277. There are positive correlation between s as power coefficient of high-frequency decay of f_{max} filter and κ as spectral decay parameter and between f_{max} for the f_{max} filter and f_E for parameter κ . f_E is a frequency at which spectrum starts to decrease on log-linear scale. f_E is a very important parameter for strong ground motion prediction, however f_E has not been examined carefully enough in previous studies. It is confirmed that corrected high frequency characteristics using f_{max} filter are almost same as that using spectral decay parameter, κ agree well each other. Both methods, f_{max} filter and spectral decay parameter, κ , provided us almost the same high frequency spectral characteristics to predict strong ground motions, as long as the parameters, f_{max} and s for the f_{max} filter and κ and f_E for the spectral decay parameter are empirically given.

Keywords: Strong ground motion prediction; Crustal earthquakes; spectral decay characteristics; Cut-off frequency; κ

1. Introduction

It is well known that seismic motions are composed of source, path and site characteristics. Recently, these characteristics have been accurately evaluated using observed records. Recipe for predicting strong ground motions from future large earthquakes based on fault rupture propagation model is proposed by Irikura *et al.* [1] as a Japanese standard for strong ground motion prediction. Procedure of source modeling and Green's function estimating considering path and site characteristics is summarized in the recipe. One of the problems for predicting strong ground motion based on fault rupture propagation model is characteristics of seismic motion in high frequency range. If Fourier amplitude spectrum of the seismic motion follows the omega squared model (Aki [2]), a shape of the acceleration Fourier amplitude spectrum is flat in high frequency range. Actually, observed acceleration Fourier amplitude spectrum shows decaying with increasing frequency above a certain frequency called cut-off frequency, f_{max} (Hanks [3]). On the other hand, Anderson and Hough [4] point out a trend of exponential decay, $e^{-\pi f \kappa}$ at high frequencies range of the acceleration amplitude spectrum by analyzing observed records. Many researchers have calculated the spectral decay parameter, κ , using observed records. For example, Houtte *et al.* [5] examined source, path, and site contributions to κ , separately, using inversion scheme.

It is very important to make clear seismic moment dependency of spectral decay characteristics in high frequency range for strong ground motion prediction using empirical Green's function method (Hartzel [6], Irikura [7]), stochastic Green's function method (Kamae *et al.* [8]) and hybrid method (Irikura and Kamae [9]).



In this study, spectral decay characteristics in high frequency range of inland crustal earthquakes in Japan are evaluated by two approaches. One is f_{max} filter, the other is spectral decay parameter, κ . Seismic moment dependency of parameters of f_{max} filter and κ are evaluated based on obtained results. Moreover, we discuss the relationships between parameters of the f_{max} filter and those of the κ evaluation.

2. Target earthquakes and observed average Fourier spectrum at hard rock sites

2.1 Target earthquakes

Target earthquakes are inland crustal earthquakes which occurred in and around Japan. Total number of target earthquakes is 65 and those moment magnitudes (M_w) range from 3.6 to 6.9. Four large earthquakes, the 2003 Miyagi-Ken Hokubu earthquake (M_w 6.1), the 2005 Fukuoka-Ken Seiho oki earthquake (M_w 6.1), the 2008 Iwate-Miyagi Nairiku earthquake (M_w 6.9), and the 2011 Fukushima-Ken Hamadori earthquake (M_w 6.6) are included in the target earthquakes. Small earthquakes used in this study are aftershocks of the four large earthquakes, except a several events. Epicenters of the target earthquakes and hard rock sites used in this analysis are shown in Fig.1.

2.2 Observed average Fourier amplitude spectrum at hard rock sites

Fourier amplitude spectrum averaged for amplitude spectra at an average hypocentral distance converted from observed records at several hard rock sites is calculated. Borehole data in the digital strong-motion seismograph network (KiK-net) deployed by National Research Institute for Earth Science and Disaster Prevention (NIED) are used as observed records at hard rock sites. Effects of rupture directivity and radiation pattern can be removed from the observed spectra by calculating from several rock sites. Vectorial summation of two horizontal components after correcting amplitude characteristics of seismometers is used as each observed spectrum. The multiple taper (Thomsom [10], Lees and Park [11]) is used to improve precision of spectral calculation. Q-factor in propagation path expressed in Eq. (1), (2), and (3) are used to correct path characteristics from the observed spectrum. Q-factors are obtained by the spectral inversion analysis using observed records at target sites during target earthquakes except earthquakes occurred in Fukuoka region. Q-factor for earthquakes occurred in Fukuoka region was already obtained by Kawase and Matsuo [12].

$$\text{For earthquakes in Iwate and Miyagi region} : Q(f) = 60.0 \times f^{0.95} \quad (1)$$

$$\text{For earthquakes in Fukuoka region} : Q(f) = 112.0 \times f^{0.70} \quad (2)$$

$$\text{For earthquakes in Fukushima region} : Q(f) = 70.8 \times f^{0.97} \quad (3)$$

3. Approach 1 [Spectral decay characteristics using f_{max} filter]

3.1 Method

Four parameters are estimated by comparing observed average Fourier spectrum at hard rock sites mentioned above with theoretical spectrum. The theoretical spectrum, $A_f(f)$ are calculated, based on the omega squared source characteristics convolved with propagation-path effects and f_{max} filter shapes, $P(f)$ [Eq. (4)]. $P(f)$ is the Butterworth type high-cut filters with cut-off frequency, f_{max} and its power coefficient of high-frequency decay, s shown in Eq. (6) (Boore [13]) in this study. Four parameters, seismic moment, M_o , corner frequency, f_c , cut-off frequency, f_{max} , and its power coefficient, s , are estimated. M_o and f_c are evaluated by the automated objective method (Andrews [14]) and f_{max} and its power coefficient of high-frequency decay, s , are done by the reannealing method (Ingber and Rosen [15]).

$$A_f(f) = CM_o S(f) \frac{1}{X} \exp \frac{-\pi f X}{Q(f) \beta} P(f), \quad (4)$$

where $S(f)$ is acceleration source spectrum according to the omega squared model (Eq (5), Aki (1967) [2]), X is average hypocentral distance for several target rock sites, C is constant, as follows.

$$S(f) = \frac{(2\pi f)^2}{1 + \left(\frac{f}{f_c}\right)^2} \quad (5)$$

$$P(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{\max}}\right)^{2s}}} \quad (6)$$

$$C = \frac{R_{\theta\phi} \times FS \times PRTITN}{4\pi\rho\beta^3} \quad (7)$$

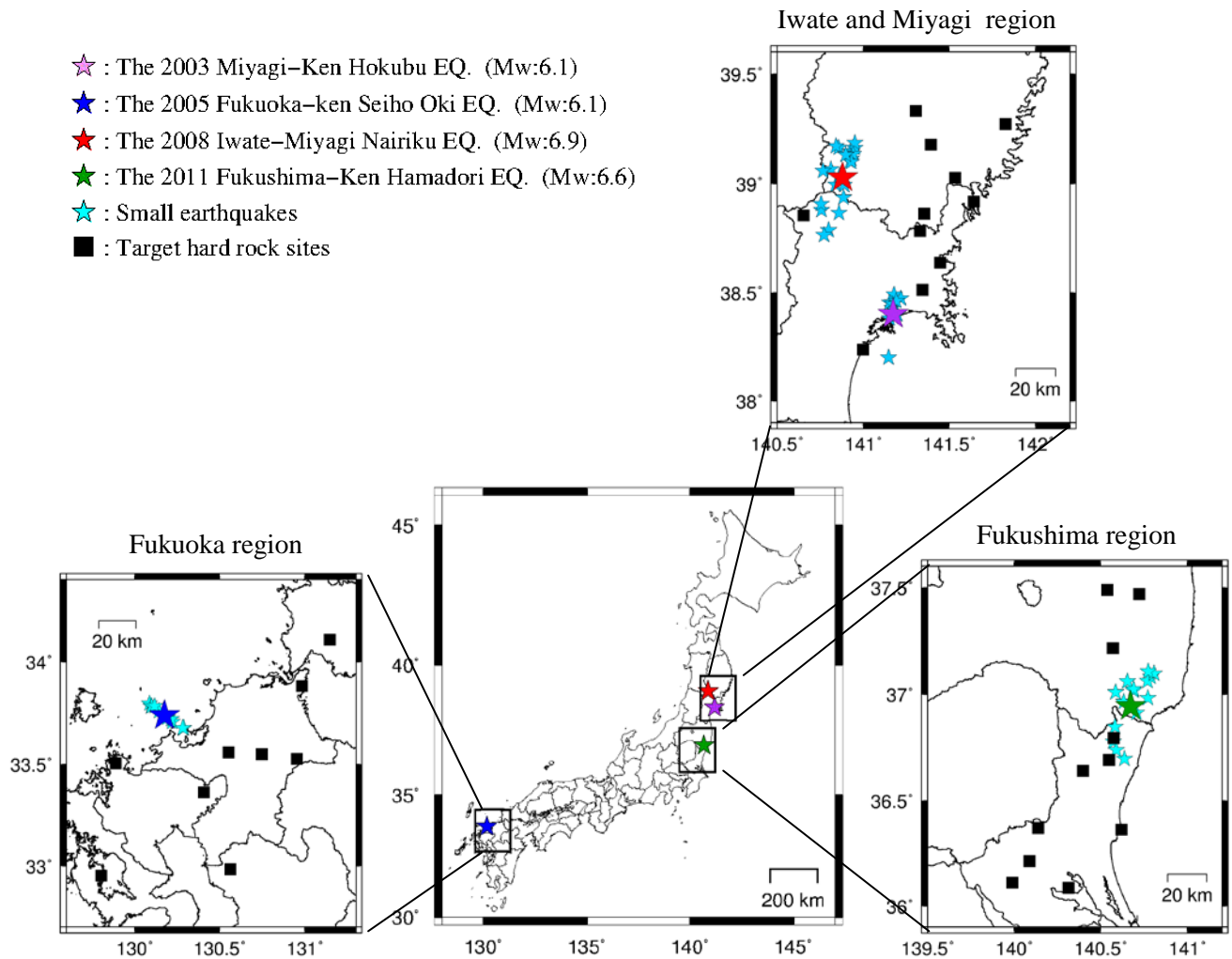


Fig.1 Epicenters of target earthquakes and hard rock sites used in this study



where $R_{\theta\phi}$ is coefficient of radiation pattern (0.63 is assumed for average value after Boore and Boatwright [16]), FS is the amplification due to the free surface (taken as 2.0), $PRTITN$ is the reduction factor that accounts for the partitioning of energy into horizontal components (taken as 1.0, because the vectorial summation of two-components spectra is used in this study), ρ is density (assumed as 2.7g/cm³), β is S-wave velocity (assumed as 3.6km/sec).

3.2 Results

3.2.1 f_{max} filters

Comparisons of the observed average amplitude spectrum at hard rock sites and the theoretical spectrum, $A_f(f)$ for the large four earthquakes are shown in Fig.2. Validity of obtained four parameters (M_o , f_c , f_{max} , and s) is confirmed from good agreement between the observed average spectra and the theoretical ones.

Equations of the obtained f_{max} filter, $P(f)$ of four large earthquakes and their average one are expressed in Eqs. (8) to (12) and shown in Fig.3.

$$\text{The 2003 Miyagi-Ken Hokubu earthquake } (M_w \ 6.1) \quad : \quad P(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{8.0}\right)^{2 \times 0.93}}} \quad (8)$$

$$\text{The 2005 Fukuoka-Ken Seiho oki earthquake } (M_w \ 6.1) \quad : \quad P(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{6.5}\right)^{2 \times 0.90}}} \quad (9)$$

$$\text{The 2008 Iwate-Miyagi Nairiku earthquake } (M_w \ 6.9) \quad : \quad P(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{9.4}\right)^{2 \times 0.84}}} \quad (10)$$

$$\text{The 2011 Fukushima-Ken Hamadori earthquake } (M_w \ 6.6) \quad : \quad P(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{8.0}\right)^{2 \times 0.78}}} \quad (11)$$

$$\text{Average characteristics} \quad : \quad P(f) = \frac{1}{\sqrt{1 + \left(\frac{f}{7.8}\right)^{2 \times 0.85}}} \quad (12)$$

For four large earthquakes with $M_w > 5.9$, f_{max} 's and power coefficients of high-frequency decay of f_{max} filter, s , are estimated to be 6.5Hz to 9.5Hz and estimated to be 0.78 to 0.93. It is clear that f_{max} filter shapes, $P(f)$ of the four large earthquakes are almost same.

For small earthquakes, f_{max} 's and s are estimated to be 8.8Hz to 20Hz and 0.62 to 2.35, respectively. Seismic dependencies of f_{max} and the power coefficients, s , are shown in Fig.4. It is clear that f_{max} 's of large earthquakes are smaller than those of small earthquakes. On the other hand, seismic moment dependency of the power coefficients, s , is not confirmed.

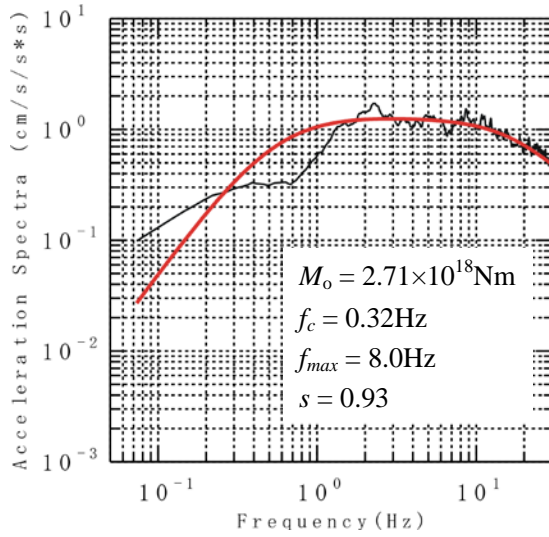


3.2.2 Method to correct difference of spectral decay characteristics between large and small earthquakes

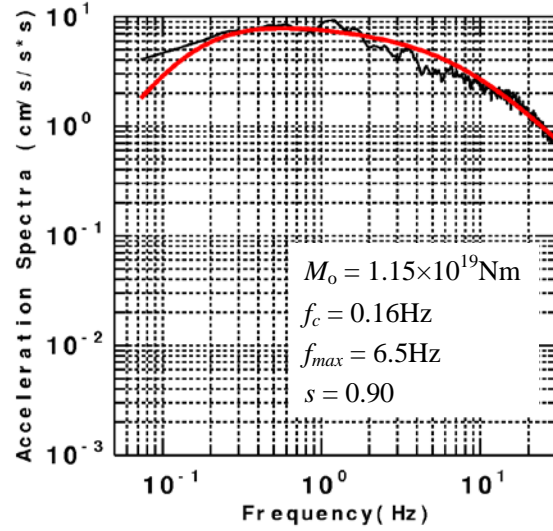
The relationship between f_{max} filter of large earthquake, $P_L(f)$, and that of small earthquake, $P_s(f)$ is expressed as follows.

$$P_L(f) = P_s(f) \times P_C(f), \quad (13)$$

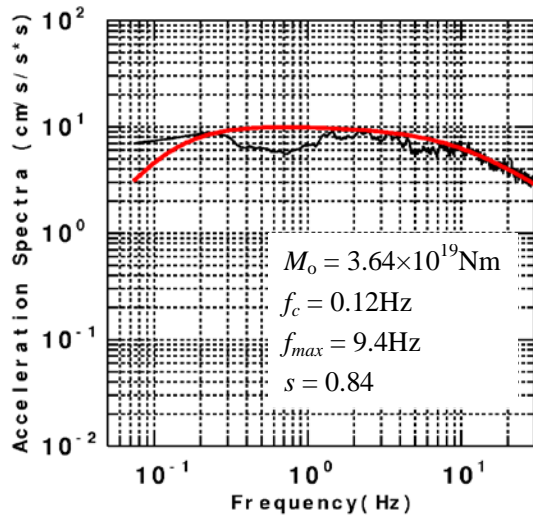
where $P_C(f)$ is a filter to correct the difference of the spectral decay characteristics between large and small earthquakes. It is necessary to correct the spectral contents of predicted strong motions with $P_C(f)$, when small earthquake records are used in a process of prediction such as empirical Green's function method. Fig.5 shows



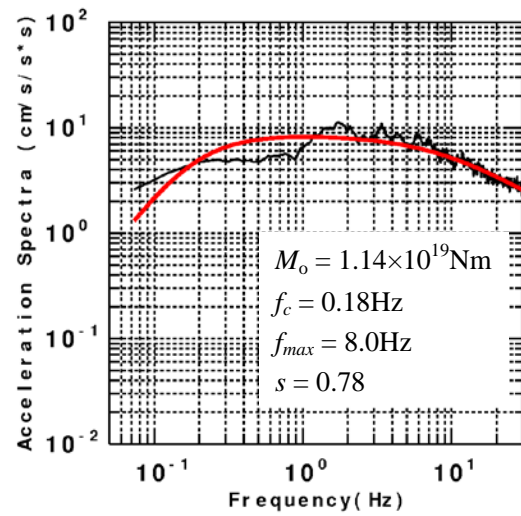
(1) The 2003 Miyagi-Ken Hokubu EQ



(2) The 2005 Fukuoka-Ken Seiho oki EQ



(3) The 2008 Iwate-Miyagi Nairiku EQ



(4) The 2011 Fukushima-Ken Hamadori EQ

Fig.2 Comparisons of observed average Fourier amplitude spectra at hard rock sites (--) and theoretical spectra, $A_f(f)$, calculated by Eq.(4) (--) for large earthquakes



the correction filters, $P_c(f)$. The average characteristics of f_{max} filter from four large earthquakes are expressed as $P_L(f)$, while those from 61 small earthquakes are used as $P_s(f)$ in Eq.(13). The correction filter, $P_c(f)$, does not show decaying with increasing frequency unlike $P_L(f)$ and $P_s(f)$, because the power coefficients of high-frequency decay, s , of large and small earthquake are almost same.

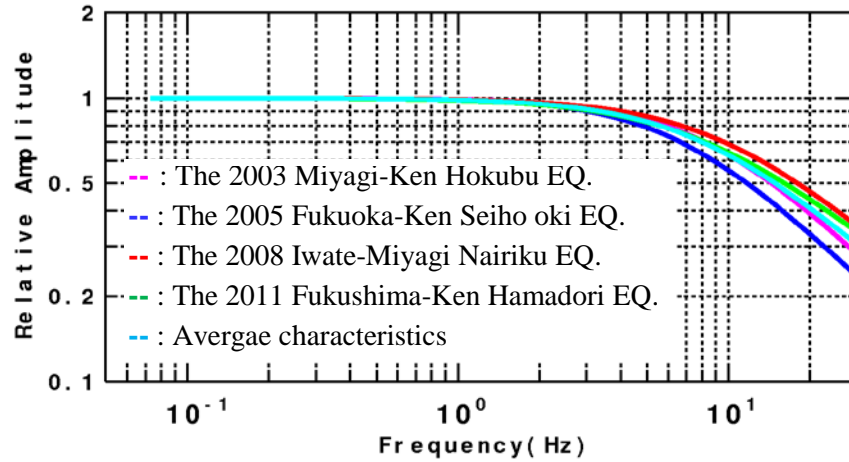


Fig.3 The f_{max} filter shapes, $P(f)$ of four large earthquakes and their average characteristics

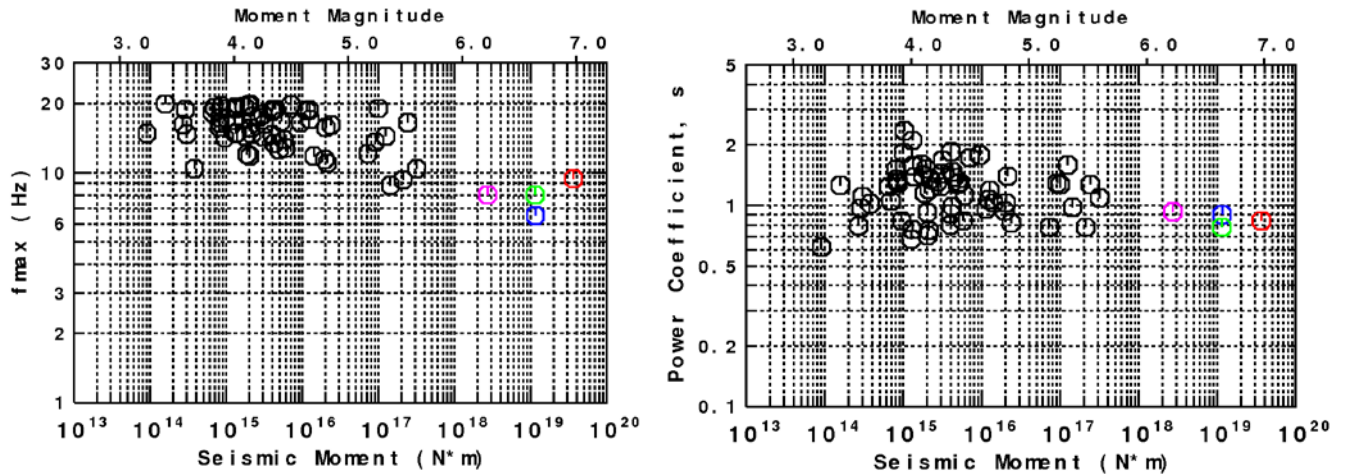


Fig.4 Seismic moment dependency of f_{max} (Left) and power coefficient, s .(Right)

- : The 2003 Miyagi-Ken Hokubu EQ.
- : The 2005 Fukuoka-Ken Seiho oki EQ.
- : The 2008 Iwate-Miyagi Nairiku EQ.
- : The 2011 Fukushima-Ken Hamadori EQ.
- : Small earthquakes

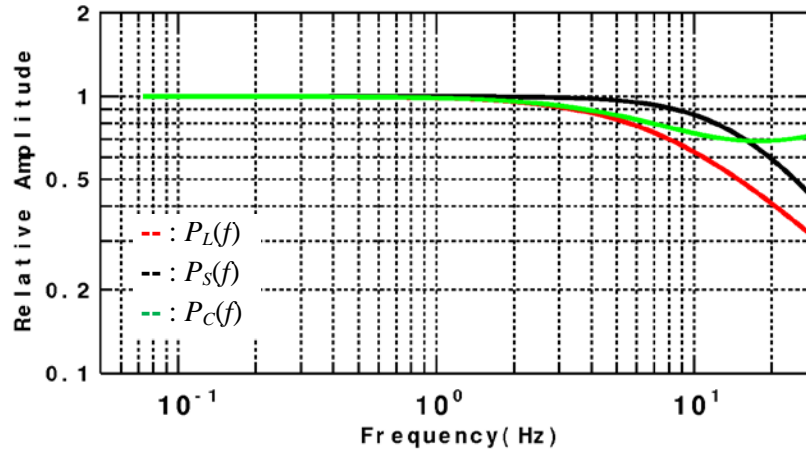


Fig.5 Average characteristics of f_{max} filter for large and small earthquakes [$P_L(f)$, $P_S(f)$] and correction filter, $P_C(f)$

4. Approach 2 [Spectral decay characteristics using parameter κ (Kappa)]

4.1 Method

The spectral decay characteristics in the high frequency range are expressed in Eq. (14) using a spectral decay parameter κ (Anderson and Hough [4]). The parameter κ of observed average Fourier amplitude spectrum at hard rock sites mentioned above is estimated by the least-squares method in the frequency range from f_E to 30Hz.

$$A_\kappa(f) = A_0 e^{-\pi f \kappa} \quad (f > f_E), \quad (14)$$

where A_0 is a source and propagation path-dependent amplitude, f_E is a specific frequency above which the spectral shape is linear on log-linear scale. f_E is visually estimated as the frequency below which the corner frequency exists and at which the spectrum starts to decrease linearly with frequency. f_E is a very important parameter for strong ground motion prediction, however f_E has not been examined carefully enough in the previous studies (for example, Anderson and Hough [4], Houtte *et al.* [5]).

4.2 Results

The comparisons of observed average Fourier amplitude spectrum at hard rock sites and theoretical spectrum, $A_\kappa(f)$ calculated by Eq.(14) are shown in Fig.6. Theoretical spectrum, $A_f(f)$ calculated by Eq.(4) and obtained parameters, κ and f_E , are also shown in Fig.6. Validity of the obtained two parameters (κ and f_E) is confirmed from the comparison between observed and theoretical ones in Fig.6. κ 's of the four large earthquakes are estimated to be 0.0142 to 0.0277 and f_E 's are estimated to be 2Hz to 5Hz.

κ 's of small earthquakes are estimated to be 0.0036 to 0.0408 and f_E 's are estimated to be 2.6Hz to 15Hz. Seismic moment dependencies of f_E and κ are shown in Fig.7. The seismic moment dependency of f_E is confirmed, but that of κ is not confirmed. These results correspond with the seismic moment dependency of f_{max} and non-dependency of power coefficient of high-frequency decay of f_{max} filter, s .

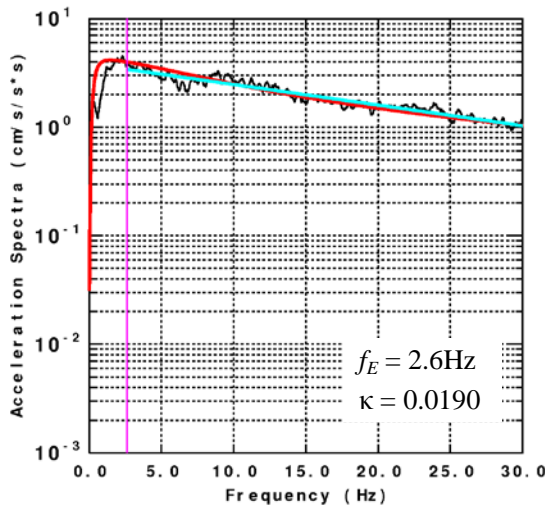
5. Relationship between parameters of f_{max} filter and κ

The relationship between s as power coefficient of high-frequency decay of f_{max} filter and κ as spectral decay parameter and between f_{max} for the f_{max} filter and f_E for parameter κ are shown Fig.8. A positive correlation is confirmed between the power coefficient of high-frequency decay of f_{max} filter, s , and κ and between f_{max} and f_E .

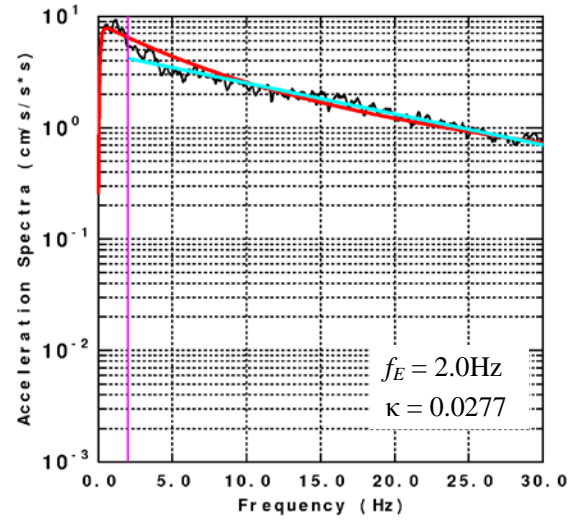
These are reasonable results, because the spectral decay characteristics for the same observed spectrum are evaluated by different approaches.

6. Effect of spectral decay characteristics on strong motion prediction

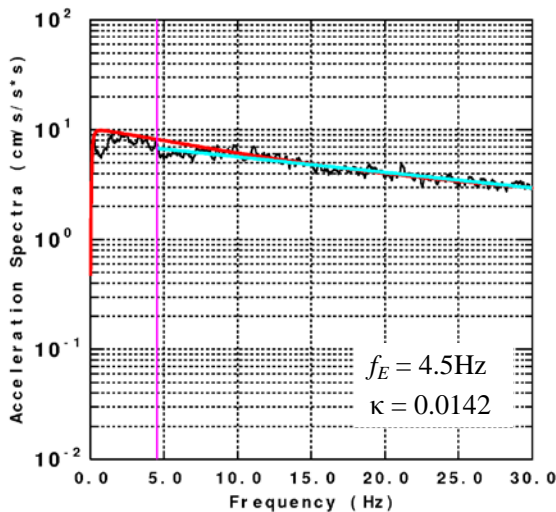
Finally, the effects of the spectral decay characteristics on strong ground motions are examined. The stochastic Green's function method is adopted to predict strong ground motion. Target in this study is to estimate strong ground motion at hard rock site in near-source region from source fault. Source parameters are given by the



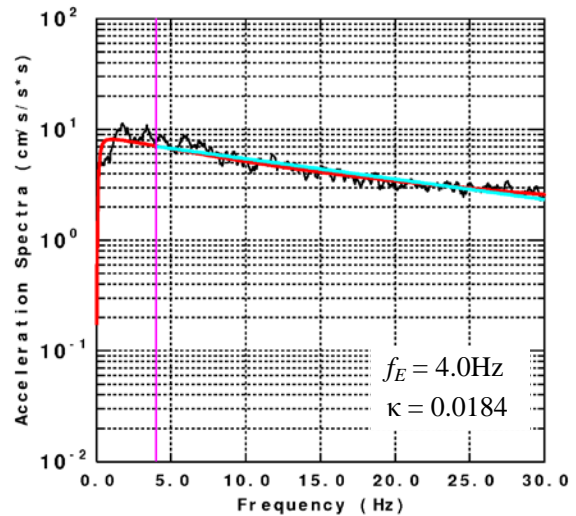
(1) The 2003 Miyagi-Ken Hokubu EQ



(2) The 2005 Fukuoka-Ken Seiho oki EQ



(3) The 2008 Iwate-Miyagi Nairiku EQ



(4) The 2011 Fukushima-Ken Hamadori EQ

Fig.6 Comparisons of observed average Fourier amplitude spectra at hard rock sites (--) and theoretical spectra, $A_{\kappa}(f)$ (--) calculated by Eq.(14) for large earthquakes. Theoretical spectra $A_f(f)$ (--) calculated by Eq.(4) are also shown. (---: f_E)

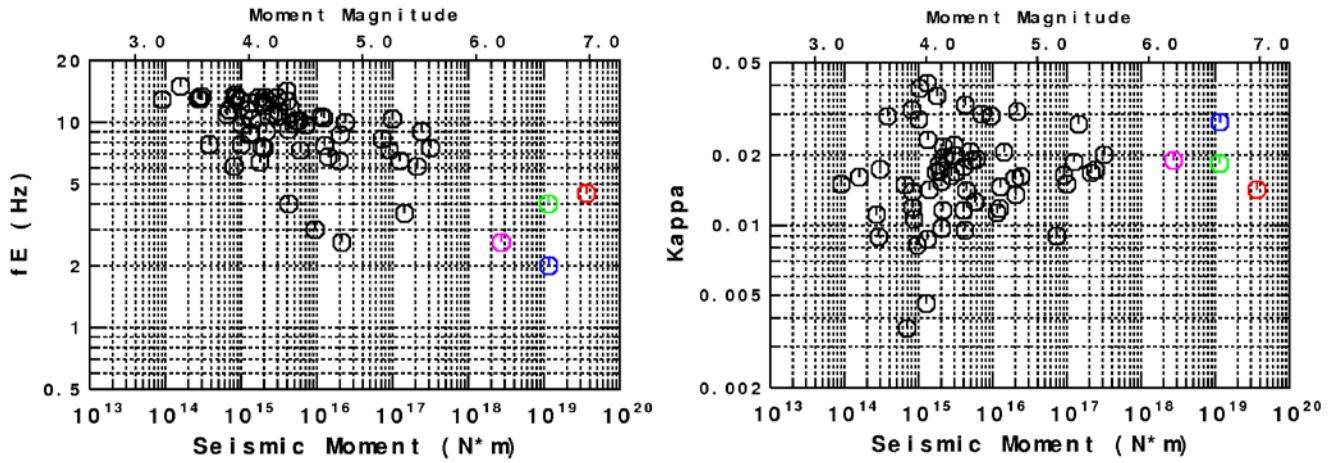
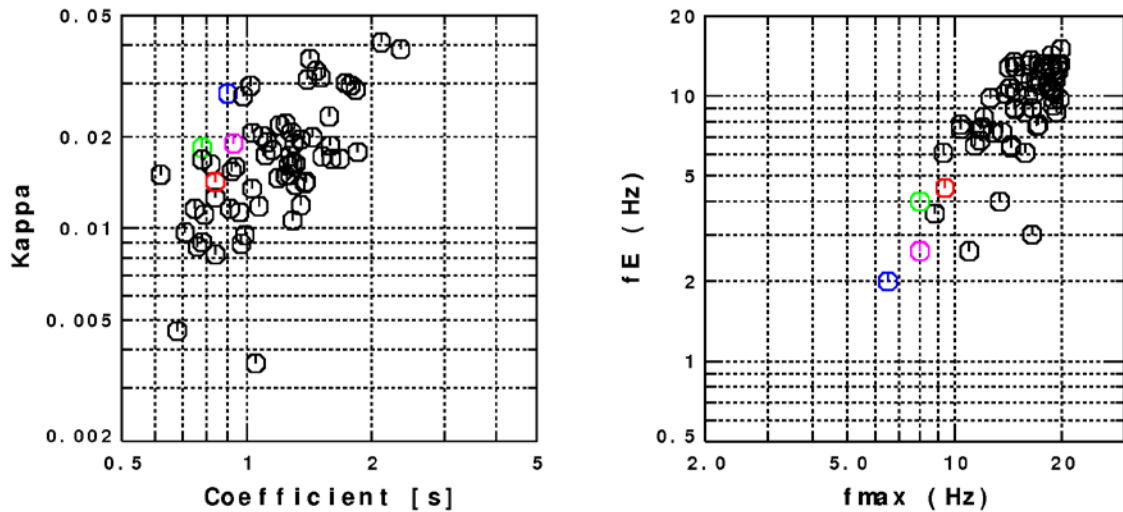


Fig.7 Seismic moment dependency of f_E (Left) and κ (Right)

- : The 2003 Miyagi-Ken Hokubu EQ.
- : The 2005 Fukuoka-Ken Seiho oki EQ.
- : The 2008 Iwate-Miyagi Nairiku EQ.
- : The 2011 Fukushima-Ken Hamadori EQ.
- : Small earthquakes



(1) Relationship between s and κ

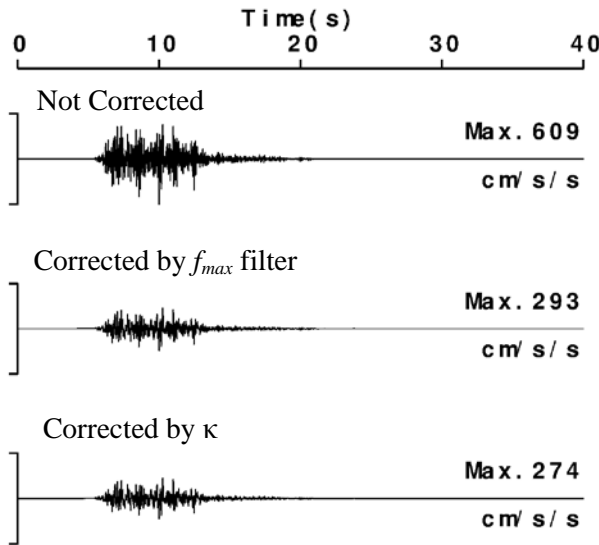
(2) Relationship between f_{max} and f_E

Fig.8 Relationship between s as power coefficient of high-frequency decay of f_{max} filter and κ as spectral decay parameter and between f_{max} for the f_{max} filter and f_E for parameter κ

- : The 2003 Miyagi-Ken Hokubu EQ.
- : The 2005 Fukuoka-Ken Seiho oki EQ.
- : The 2008 Iwate-Miyagi Nairiku EQ.
- : The 2011 Fukushima-Ken Hamadori EQ.
- : Small earthquakes

Table 1 Outer fault parameters and spectral decay characteristics used.

Dip angle (°)	60	Acceleration source spectral level (Nm/s ²)	1.18×10 ¹⁹
Slip angle (°)	135	Average slip(m)	0.95
Fault Length (km)	32	Rupture velocity (km/s)	2.40
Fault width (km)	18	f_{max} filter	f_{max} (Hz)
Fault Area (km ²)	576		s
Sesimic moment (N·m)	1.69×10 ¹⁹		
Moment magnitude	6.8	Spectral decay parameter, κ	f_E (Hz)
Average stress driop (MPa)	3.0		κ



(1) Acceleration waveform

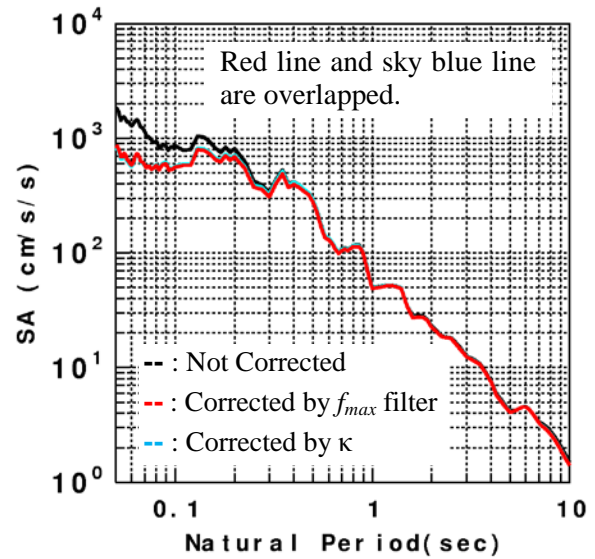

(2) Acceleration response spectrum ($h=5\%$)

Fig.9 Effects of the spectral decay characteristics on strong ground motion

recipe for predicting strong ground motions from future large earthquakes proposed by Irikura *et al.* [1]. The outer fault parameters are shown in Table 1.

Three predicted strong ground motions are compared. First one is simulated motion according to the omega squared model without correcting the high-frequency characteristics. Second one is the omega squared simulated motion corrected by the high frequency characteristics with the f_{max} filter and third one is that corrected by spectral decay parameter, κ . The f_{max} filter and κ obtained from the observed records during the 2003 Miyagi-Ken Hokubu earthquake are used to correct high frequency ground motions. Spectral decay characteristics used in the prediction are shown in Table 1, too.

Predicted strong ground motions are shown in Fig.9. Response spectra of the predicted motions corrected by f_{max} filter (red line) match well those corrected by κ (sky blue line) as shown in Fig.9. The peak ground accelerations and response spectra of the corrected motions at short period range are within about 0.5 times of those of the not-corrected motions. Moreover, it is confirmed that the corrected motion using f_{max} filter and that using spectral decay parameter, κ , are almost same. From this result, both methods using f_{max} filter and using spectral decay parameter, κ , are appropriate to predict strong ground motions.



7. Conclusion

The spectral decay characteristics in high frequency range of inland crustal earthquakes occurring in and around Japan are evaluated by two approaches in this study. One is f_{max} filter, the other is spectral decay parameter, κ . Total number of target earthquakes is 65 and their moment magnitudes (M_w) is from 3.6 to 6.9. Four large earthquakes with M_w 5.9 to 6.9 are included in the target earthquakes. In result, f_{max} 's of the large earthquakes are estimated to be 6.5Hz to 9.4Hz and power coefficients of high-frequency decay of f_{max} filter, s , are estimated to be 0.78 to 0.93. It is clear that the f_{max} filter shapes of the four large earthquakes are almost same. κ 's of the four large earthquakes are estimated to be 0.0142 to 0.0277 and f_E 's are estimated to be 2Hz to 5Hz. f_E is a frequency at which the spectra starts to decrease linearly with frequency. f_E is a very important parameter for strong ground motion prediction, however f_E has not been examined carefully enough in the previous studies.

The f_{max} 's of the large earthquakes tend to be smaller than those of small earthquakes, showing seismic moment dependency of f_E clearly. Positive correlations are confirmed between s as power coefficient of high-frequency decay of f_{max} filter and κ as spectral decay parameter and between f_{max} for the f_{max} filter and f_E for parameter κ . Finally, the effects of the spectral decay characteristics on strong ground motions are examined. It is confirmed that simulated motion corrected by the f_{max} filter and those corrected by the spectral decay parameter, κ , are almost same.

Both methods using f_{max} filter and using spectral decay parameter, κ , are appropriate to evaluate spectral decay characteristics in high frequency range and predict strong ground motions. The results obtained in this paper contribute to strong ground motion prediction in high frequency range.

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