



A DISPLACEMENT BASED SEISMIC DESIGN METHOD CONSIDERING DAMAGE CONTROL AND PASSIVE DISSIPATION OF ENERGY FOR BUILDINGS

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Abstract

Supplemental damping is being used for seismic damage control purposes in the retrofit of existing and the design of new buildings. In this paper this goal is achieved by introducing within a displacement-based design procedure, devices that dissipate part of the input earthquake energy, and thereby reduce the displacements of the structural systems to meet those considered as targets. This paper briefly describes a displacement-based seismic design procedure for frame structures considering the use of linear viscous dampers and damage control. In order to consider the non-proportional damping produced by the viscous dampers, in a conventional modal spectral analysis the effect of non-proportional damping matrix is approximated by equivalent modal damping ratios. The framework of this method is constructed around the concept of a reference system, defined as the bilinear single degree of freedom system associated to the properties of the fundamental mode of the inelastic multi degree of freedom system, from which its performance under seismic design conditions may be approximated. As illustrative example, the displacement-based design of a 12 storey reinforced concrete irregular frame with added linear viscous dampers is presented. The considered seismic demand is the E-W component of the Mexico, 1985 Michoacan earthquake recorded at the SCT site. In this example, it is observed that the performance calculated with a non-linear step by step analysis, for a structure designed with the procedure proposed, represents, with a good approximation, the target design performance.

Keywords: *Displacement-based seismic evaluation and design, damage control, linear viscous dampers, non-proportional damping approximation, modal spectral analysis.*

1. Introduction

The main objective of seismic design is to guarantee adequate behaviour of a building by accomplishing given design performance levels (PL) when subjected to earthquake scenarios which may occur during its service life. However, there are many situations where it is not possible to guarantee the PLs considered due to architectural restrictions and/or code related issues. For this reason, it is necessary to consider other strategies to satisfy the PL, such as the use of passive energy dissipation devices *e.g.*, viscous dampers, which dissipate part of the input earthquake energy and as a consequence reduce the displacements of the structure and the corresponding earthquake induced damage. Moreover, in cases when it is required to add a large amount of supplementary damping to satisfy the PLs (*i.e.*, $\xi_{\text{dampers}} > 15\%$), and this amount is neither practical nor reasonable [1], the alternative is to accept damage in the structure, by using performance-based design procedures adapted to include passive energy dissipation devices.

The most appropriate procedure to analyse structures with viscous dampers is the dynamic step by step analysis; nevertheless, current design practices and most building codes recommend for the seismic evaluation and design of structures the use of modal spectral analysis with seismic demands given by design spectra. Due to this situation, in the majority of cases in which viscous dampers are used as passive energy dissipation devices, the associated damping is non proportional leading to non-classical eigen -values and -vectors, making the conventional modal spectral analysis used in the practice of the seismic design of structures, impossible to apply. The assumption of non-proportional damping as proportional is not always appropriate because it can produce significant errors in the response of structures. As a consequence, diverse studies focusing on the development of simplified procedures to use the modal spectral analysis in structures with viscous dampers have been carried out.

Soong et al. [2] propose a modal analysis procedure that considers non-proportional damping; however, it involves complex mode shapes and frequencies, something that is not practical in the seismic design of structures. On the other hand, Constantinou et al. [3] proposes an approximate simplified procedure, in which the effect of the viscous dampers is considered as supplemental modal viscous damping. In this procedure, the approximate amount of damping, associated to the viscous dampers, is easily determined in terms of the dynamic properties of the structure such as the fundamental mode shapes and periods. Using this approximation, Garcia et al. [4,5] propose a simplified sequential search algorithm to calculate the distribution of dampers and damping coefficients according to interstorey velocities. Along the same lines, Hwang et al. [1] propose a damper distribution based on storey shear strain energy. However, none of these procedures consider explicitly the damage in the structure *i.e.*, the devices dissipates the energy that otherwise would produce damage in the structure.

This paper presents a simplified displacement-based seismic evaluation and design procedure which considers explicitly a combination of damage control and added viscous damping to comply with target design displacements. To illustrate the application of this procedure and evaluate the performance of structures designed with the method proposed, a 12-storey reinforced concrete irregular frame is designed. The additional damping required by the structure under design conditions with the accepted damage distribution is calculated using the approximation proposed. To distribute the nominal damping coefficients of the devices that produce the design amount of damping, assuming that they are located at the central spans of each floor, a procedure using as relative weights of these damping coefficients the drifts in the structure. As design seismic demand, the EW component of the SCT record of the 1985 Michoacan earthquake in Mexico is considered. The design obtained, is evaluated by comparing the interstorey drifts and overall structural performance. The accuracy of the results is assessed by comparing the performances extracted from the results of the step by step non-linear dynamic analysis of the structure designed with the target design performance. Finally, some conclusions about the design method and the results obtained are presented stressing the most relevant advantages of the displacement-based design of structures with viscous dampers and damage control.

2. DISPLACEMENT – BASED SEISMIC DESIGN PROCEDURE WITH VISCOUS DAMPERS AND DAMAGE CONTROL

2.1 Fundamentals of the procedure proposed

The procedure proposed is based on the assumption that an approximation to the performance of a non-linear multi-degree of freedom system (MDOF) structure may be obtained from the performance of a reference simplified non-linear single-degree of freedom system (SDOF), normally associated to the fundamental mode of the structure [6]. The principle of this evaluation/design method is that the non-linear capacity curve of a MDOF structure can be approximated by a bilinear curve, using the equivalence of deformation energies corresponding to the real capacity curve and its bilinear approximation, and that, in accordance with basic principles of structural dynamics, the bilinear capacity curve of the reference SDOF system, also referred to as the behaviour curve of the reference system may be directly extracted from this capacity curve. The behaviour curve of the reference system is obtained from the results of two conventional modal spectral analyses, one for the elastic phase of behaviour, structure without damage, and other for the inelastic phase, structure with the assumed distribution of damage. The slope of the first branch of the behaviour curve represents the elastic stiffness of the reference SDOF system whereas the slope of the second branch, the stiffness corresponding to the inelastic range. The slope of this second branch is defined by the assumed damage distribution associated to the proposed maximum displacement of the target performance level. The spectrum used in this procedure is modified to consider the damping due at the effect of the viscous dampers added to the structure, with the purpose of satisfying the PL considered.

2.2 Correction proposed

As mentioned above, most seismic analysis procedures approximately consider the effect of viscous dampers in the response of the structure by assuming a corresponding proportional damping matrix. However, this assumption is not strictly valid, as the response of the structure may show significant errors when compared against the obtained using step by step dynamic analysis. For this reason, a correction to the procedure is proposed as follows:

The equation that describes the dynamic equilibrium of a MDOF structure can be written as:

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = -[M]\{1\}\{\ddot{u}_g\} \quad (1)$$

where:

$[M]$ = Mass matrix

$[C]$ = Damping matrix

$[K]$ = Stiffness matrix

$\{\ddot{u}_g\}$ = Acceleration at the foundation of the building vector

$\{u(t)\}, \{\dot{u}(t)\}, \{\ddot{u}(t)\}$ = Displacement, velocity and acceleration vectors

For a structure with viscous dampers, the damping matrix can be defined as:

$$[C] = [C_0] + [C_D] \quad (2)$$

where:

$[C_0]$ = Inherent damping matrix (proportional)

$[C_D]$ = Tridiagonal viscous dampers matrix (non-proportional)

Since the damping matrix associated to the viscous dampers is not proportional neither to the mass nor the stiffness matrices, it is reasonable to approximate a part of this matrix as proportional ($[C_{DP}]$) and the other part as a residual matrix ($[C_{DR}]$), this is:

$$[C_D] = [C_{DP}] + [C_{DR}] \quad (3)$$

The proportional matrix can be defined according to Rayleigh damping:

$$[C_{DP}] = a_{0D}[M] + a_{1D}[K] \quad (4)$$

To solve the Eq. 1 uncouple is required, whereby a change of coordinates is performed, as follows: $\{u(t)\} = [\Phi]\{x(t)\}$, $\{\dot{u}(t)\} = [\Phi]\{\dot{x}(t)\}$, $\{\ddot{u}(t)\} = [\Phi]\{\ddot{x}(t)\}$, then the Eq. can be write as:

$$[M][\Phi]\{\ddot{x}(t)\} + [C][\Phi]\{\dot{x}(t)\} + [K][\Phi]\{x(t)\} = -[M]\{1\}\{\ddot{u}_g\} \quad (5)$$

where:

$[\Phi]$ = Modal matrix normalized

Premultiplying each term in this equation by $\{\phi\}_i^T$ gives

$$\{\phi\}_i^T [M] \{\phi\}_i \{\ddot{x}(t)\} + \{\phi\}_i^T [C] \{\phi\}_i \{\dot{x}(t)\} + \{\phi\}_i^T [K] \{\phi\}_i \{x(t)\} = -\{\phi\}_i^T [M] \{1\} \{\ddot{u}_g\} \quad (6)$$

Assuming that the inclusion of the viscous dampers in the structure do not modify the mode shapes, a set of uncoupled dynamic equilibrium equations expressed in terms of modal coordinates $x_i(t)$ is obtained:

$$\ddot{x}_i(t) + 2(\xi_{0i} + \xi_{DPi})\omega_i \dot{x}_i(t) + \omega_i^2 x_i(t) = \{\Gamma_i\} \{\ddot{u}_g\} - \{\phi\}_i^T [C_{DR}] \{\phi\}_i \dot{x}_i(t) \quad (7)$$

where:

ω_i = Frequency for mode i

ξ_{0i} = Inherent viscous damping ratio for mode i

ξ_{DPi} = Proportional viscous damping ratio of devices for mode i

Γ_i = Modal participation factor

The term $\{\phi\}_i^T [C_{DR}] \{\phi\}_i$ is not a diagonal matrix. Nevertheless experimental evidence has shown that if the damping ratio of a structure is increased, the responses of the higher modes of the structure may be ignored [7,8]. As a consequence, for practical applications in the simplified procedure only the first mode of the MDOF system is usually considered. With the previous information can be define as $\{\phi\}_i^T [C_{DR}] \{\phi\}_i = 2 \xi_{DRi} \omega_i$, therefore the Eq. 7 can be rewritten as:

$$\ddot{x}_i(t) + 2 (\xi_{0i} + \xi_{DPi} + \xi_{DRi}) \omega_i \dot{x}_i(t) + \omega_i^2 x_i(t) = \{\Gamma_i\} \{\ddot{u}_g\} \quad (8)$$

where:

ξ_{DRi} = Residual viscous damping ratio of devices for mode i

The proportional viscous damping ratio of devices can be calculated using the energy based approximation propose by Constantinou et al. [3]:

$$\xi_{DPi} = \frac{T_i \sum_{j=1}^{nd} (C_j) (\cos \theta_j) (\phi_i - \phi_{i-1})^2}{4 \pi (\sum_{i=1}^n m_i \phi_i^2)} \quad (9)$$

where:

T_i = Natural period of vibration of the mode i

θ_j = Angle of the viscous damper j

ϕ_i = Horizontal modal displacements of the i mode

C_j = Damping coefficient of the damper at the j storey

The residual viscous damping ratio of devices of mode i can be defined as:

$$\xi_{DRi} = \frac{\{\phi_i^T\} [C_{DR}] \{\phi_i\}}{2 \omega_i} = \frac{\{\phi_i^T\} [C_D - C_{DP}] \{\phi_i\}}{2 \omega_i} \quad (10)$$

Thus, ignoring the contribution of the higher modes, the total damping ratio for the first mode is defined as:

$$\xi_{T1} = (\xi_{01} + \xi_{DP1} + \xi_{DR1}) \quad (11)$$

3. Design procedure

In accordance with the aforementioned concepts the application of the procedure proposed to design framed structures with viscous dampers intended to satisfy the life safety limit state, LSLS, can be summarized in the following steps:

1. Preliminary configuration and dimensioning of the elements of the structure according to engineering judgement and/or designer experience. The objective of this first step is to define a realistic stiffness distribution of the structural elements throughout the height of the structure, allowing the definition of design displacement shapes that are consistent with actual structures.
2. Modal analysis of the elastic bare structure designed in the previous step. From this analysis, the participation factor, the fundamental period of the structure, T_E , and the displacement shape of the first mode are obtained. From this displacement shape, the spectral yield displacement of the reference SDOF, S_{dy} , is calculated using the following equation:

$$Sd_y = \frac{IDT_y H_k}{PF_1^E (\phi_{k,1}^E - \phi_{k-1,1}^E)} \quad (12)$$

where:

IDT_y = Yield interstorey drift

H_k = Height of the critical storey k (where maximum drift occurs)

PF_1^E = Modal participation factor of the fundamental mode (elastic structure)

$\phi_{k,1}^E$ = Modal shape ordinate of the critical interstorey, k

The yield interstorey drift for a reinforced concrete structure can be defined by the proposal of Lopez [9]:

$$IDT_y = \frac{0.3 \varepsilon_y L_1}{h_{v1}} \left(\frac{\frac{I_{v1}}{L_1} + \frac{I_{v2}}{L_2} + \frac{I_{ck}}{H_k} + \frac{I_{ck+1}}{H_{k+1}}}{\left[\frac{I_{ck}}{H_k^2} + \frac{\phi_{k+1,1}^E I_{ck+1}}{\phi_{k,1}^E H_{k+1}^2} \right]} \right) \quad (13)$$

where:

ε_y = Yield strain of the reinforcing steel

L_1 = Length of the span to the left of the node nearest to the centre of the storey where maximum drift occurs

L_2 = Length of the span to the right of the node nearest to the centre of the storey where maximum drift occurs

H_k = Height of the storey where maximum drift occurs

H_{k+1} = Height of the storey above the storey where maximum drift occurs

I_{v1}, I_{v2} = Moments of inertia of the beams in the spans 1 and 2

I_{ck}, I_{ck+1} = Moments of inertia of the columns of the storeys k and $k+1$

h_{v1} = Beam depth at span 1 to the left of the node nearest to the centre of the storey where maximum drift occurs

3. Definition of a design damage distribution for PL in accordance with the characteristics of the structure and, the design demands, using a strong-column weak-beam strategy and considering the inclusion of the viscous dampers in the structure. Structural damage is introduced at the ends of the elements, where damage is accepted to occur under design conditions adding hinges with zero or residual rotational stiffness, equal to a reduced bending stiffness of the damaged element section.
4. Modal analysis of the damaged bare structure to obtain the fundamental modal shape, participation factor and period, T_D , and, from this period, the slope of the second branch of the idealized bilinear behaviour curve of the reference SDOF system. From this modal shape, the target spectral displacement of the reference SDOF, Sd_{PL} , is obtained using the following equation:

$$S_{d_{PL}} = \frac{IDT_{PL} H_k}{PF_1^D (\phi_{k,1}^D - \phi_{k-1,1}^D)} \quad (14)$$

where:

IDT_{PL} = Interstorey drift of PL

5. Calculation of the target yield and ultimate spectral displacements of the reference SDOF system, $S_{d_{PL}}$ and S_{d_y} respectively, corresponding to the fundamental mode using the results of modal analysis. Its ductility, μ , and post-yielding to initial stiffness ratio, α , are obtained using Eqs. (15) and (16)

$$\mu = \frac{S_{d_{PL}}}{S_{d_y}} \quad (15)$$

$$\alpha = \left(\frac{T_E}{T_D} \right)^2 \quad (16)$$

6. Modification of the effective viscous damping ratio from the displacement spectrum for the given μ and α , until the spectral displacement associated to the fundamental period is equal to the target spectral displacement of the frame (step 4). See Fig. 1:

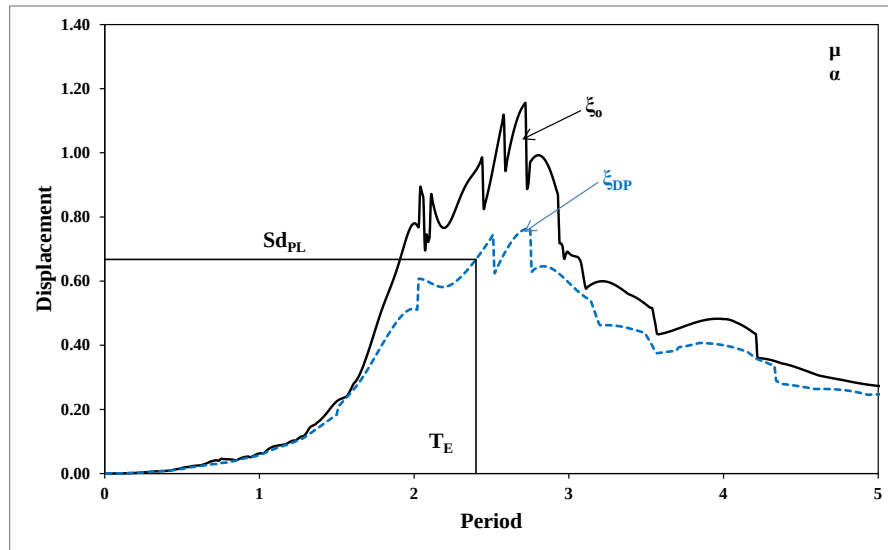


Fig. 1. Inelastic displacement spectrum

7. Application of the correction proposed, *i.e.*, residual viscous damping for the first mode calculated using Eq. (10).
8. Calculation of the damping coefficients of the viscous dampers, starting with the total damping coefficient of the formulation proposed by Constantinou et al. [3]. The damping coefficient for each interstorey is calculated in proportion to the relative modal displacement drifts normalized with the interstorey height of the building.

$$C_t = \frac{4 \pi \xi_{DPI} \left(\sum_{j=1}^{nd} \frac{\phi_i - \phi_{i-1}}{H_j} \right) \left(\sum_{i=1}^n m_i \phi_i^2 \right)}{T \sum_{j=1}^{nd} (\cos \theta_j) \left(\frac{(\phi_i - \phi_{i-1})^3}{H_j} \right)} \quad (17)$$

$$C_j = \left(\frac{\left(\frac{\phi_i - \phi_{i-1}}{H_j} \right)}{\sum_{i=1}^{nd} \left(\frac{\phi_i - \phi_{i-1}}{H_j} \right)} \right) C_t \quad (18)$$

where:

H_j = Height of the storey j

9. Determination of the yield strength, R_y , for the period from elastic model from the inelastic strength spectrum, PL, corresponding to the values of μ and α previously calculated. See Fig. 2:
10. Calculation of the ultimate strength, R_u , of the reference system using the follow equation:

$$R_u = R_y [1 + \alpha (\mu - 1)] \quad (19)$$

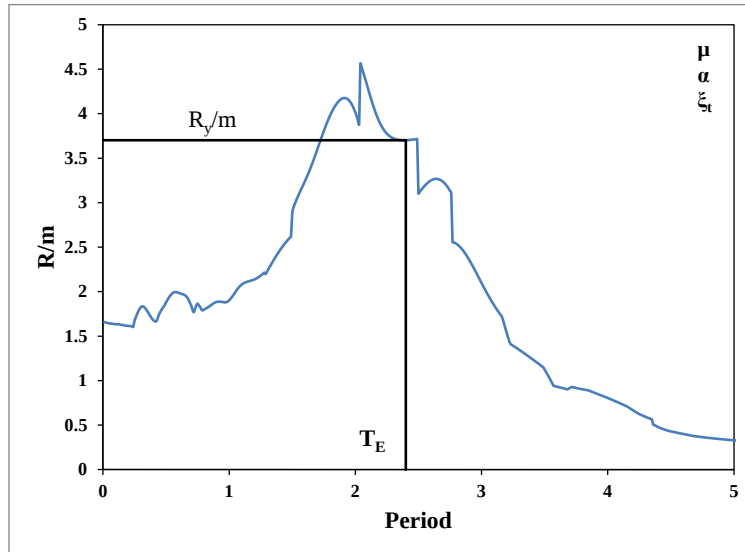


Fig. 2. Strength per unit mass spectrum for μ and α , associated to the PL

11. Determination of the design forces of the elements using the results of three different analyses: a gravity load analysis of the undamaged structure, a modal spectral analysis of the undamaged structure, using the elastic design spectrum scaled by the ratio of the strength per unit mass at the yield point of the behaviour curve and the elastic pseudo-acceleration for the initial period, λ_E , and a modal spectral analysis of the damaged structure, using the elastic spectrum scaled by the ratio of the difference of ultimate and yield strengths per unit mass and the pseudo-acceleration for the period of the damaged structure, λ_D . Each modal spectral analysis considers the total viscous damping ratio (ξ_T). The design

forces are obtained by adding the forces due to gravity loads and the forces of the modal spectral analyses of the undamaged and damaged structure.

12. Determination of the design of every structural element in accordance with the forces obtained from the analysis of the simplified models and using the applicable design rules. The design process must be carried out in such a way that the design criteria of the code do not alter significantly the expected performance.

3. APPLICATION EXAMPLE

To illustrate the application of the design method developed in this paper, a 12 storey irregular reinforced concrete frame (see Fig. 3 (a)), was designed. The nominal properties of the materials used in the design are: for the concrete, a compressive strength $f'_c = 3.00 \times 10^4$ kN/m², a modulus of elasticity $E_c = 27.00 \times 10^6$ kN/m², and a weight density $\gamma = 23.53$ kN/m³, and for the steel reinforcement a yield stress $f_y = 4.50 \times 10^5$ kN/m² and a modulus of elasticity $E_s = 2.00 \times 10^8$ kN/m². Based on the results of the preliminary design of the frame, the sections of the structural elements were defined; *i.e.*, for all columns 0.55 x 0.55 m, and 0.45x0.30m for all beams.

The seismic demand considered was the response spectra of the EW component of the record of the 1985 Michoacán earthquake, obtained at the SCT site. To validate the seismic performance of the designed structure, the drifts, obtained from this non-linear step by step analysis of the frame, were compared with the maximum drift considered as design target. The non-linear step by step analysis was carried out with the PERFORM 3D [10] program with the following considerations: 1) nearly elasto - plastic bilinear stable hysteretic behaviour for all beams and columns, 2) Non classical damping matrix due to the incorporation the viscous dampers, 3) nominal yield moments for beam and columns obtained from the design method proposed and 4) the drift considered as design target was 0.03, as specified by the current GDF 2004 [11] to satisfy the LSLS.

3.1 Results of the design procedure proposed

According to the structural configuration and the dimensioning of structural elements of the proposed preliminary design, the model of the bare structure was built and a modal analysis over it was carried out. From these results the fundamental period of the undamaged structure was obtained ($T_E = 2.40$ s). Subsequently, a damage distribution, was proposed. For this case, the beams of levels 1 to 10 were considered damaged at both ends and all columns were assumed undamaged. The modal analysis for this model was carried out to obtain the fundamental period of the damaged structure ($T_D = 9.75$ s). Fig. 3 (c) shows the distribution of damage used in this design.

From the results obtained from the bare undamaged model, the yield spectral displacement ($S_{d_y} = 0.6480$ m) was calculated with Eqs. 10 and 11. On the other hand from the results obtained from the damaged model, the target roof displacement ($S_{d_{LSLS}} = 0.633$ m) was determined.

With the previous information and Eqs. 13 and 14, the ductility ($\mu = 1.20$) and the post-yielding to initial stiffness ratio ($\alpha = 0.06$) were calculated. Using the calculated values of μ and α , a displacement spectrum was constructed and, from it, the displacement associated to the period T_E , obtained. The additional viscous damping ($\xi_{DP} = 5\%$) was modified until the spectral displacement matched the target displacement for the LSLS considered.

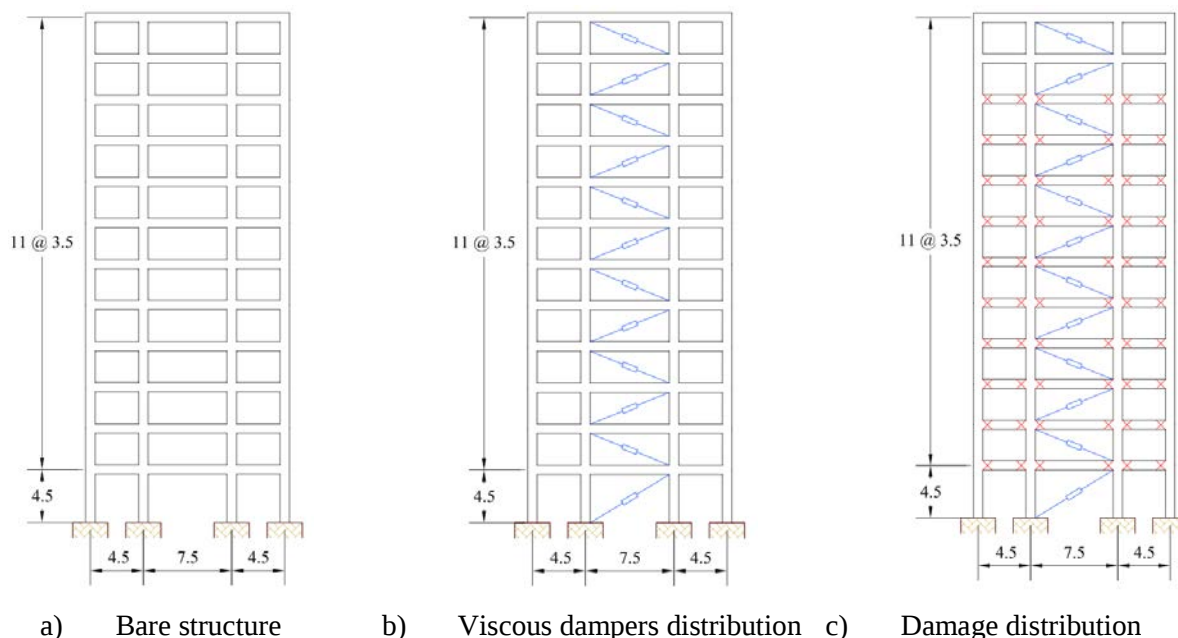


Fig. 3 Example frame

The arrangement of viscous dampers proposed in this example is shown in Fig. 3(b), *i.e.*, one damper per storey. Using Eqs. 15 and 16, the damping coefficients of these devices were calculated (See Table 1).

Once the residual damping ratio was calculated ($\xi_{DP} = 1.00\%$), the strength spectrum associated to the final values of α , μ and ($\xi_T = 11.00\%$) was constructed and from it, the yield strength ($R_y = 3.65 \text{ m/s}^2$), for the period that satisfies the target displacement read. The ultimate strength ($R_u = 3.694 \text{ m/s}^2$) was obtained using Eq. 17. Finally, the factors $\lambda_E = 0.52$ and $\lambda_D = 0.26$ were determined, the modal spectral analyses were carried out the design forces of each structural element were obtained.

Table 1 – Damping Coefficients of viscous dampers

Level	1	2	3	4	5	6	7	8	9	10	11	12
Damping Coefficient (MN-s/m)	0.843	1.353	1.443	1.435	1.380	1.294	1.184	1.053	0.904	0.742	0.573	0.416

Fig. 4 shows the maximum interstorey drifts calculated using step by step analyses for four different cases: (a) bare structure, (b) structure with dampers (non-proportional damping matrix), (c) structure without the correction proposed and (d) structure with correction proposed in this paper. It may be observed that when the bare structure is subjected to the design seismic demand corresponding to the LSLS, some interstoreys exceed lightly the permissible drifts for this limit state (IDT_{LSLS}). However when the viscous dampers are added to the structure, the interstorey drift prescribed by the code is not exceeded at in any interstoreys. In addition, the procedure proposed with the correction proposed gives a better approximation when is compared with the results of the non-linear step by step analysis (see Fig. 5). With regarding to the damage distributions, the results of the design procedure proposed show the same damage distribution than that obtained from the results of the non-linear step by step analysis.

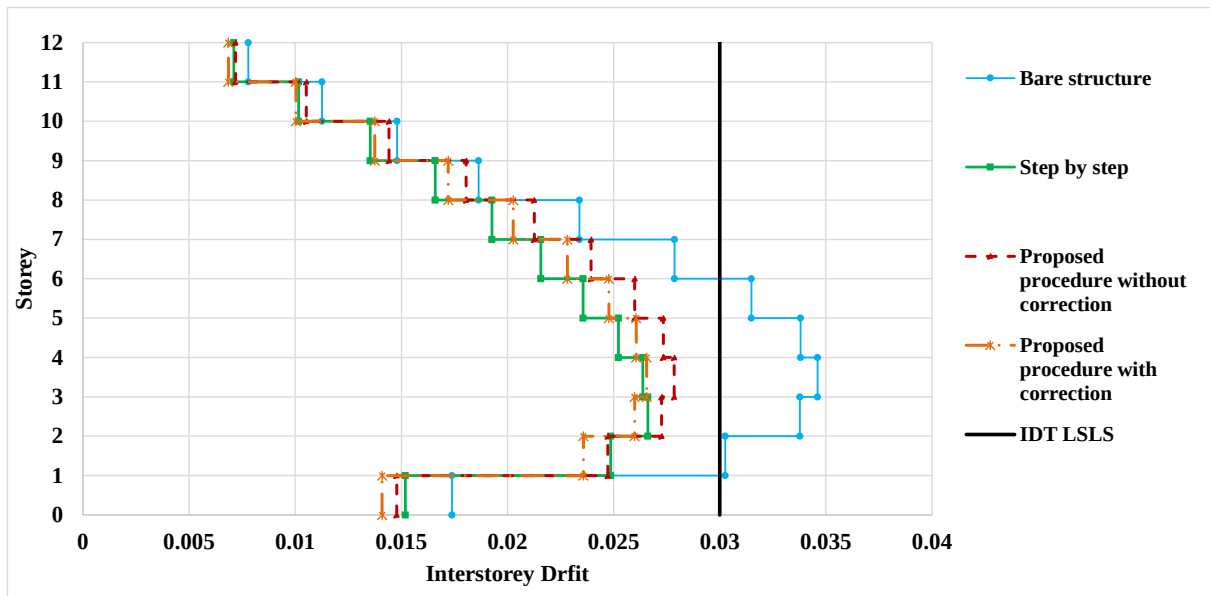


Fig. 4. Interstorey drifts in the example frame

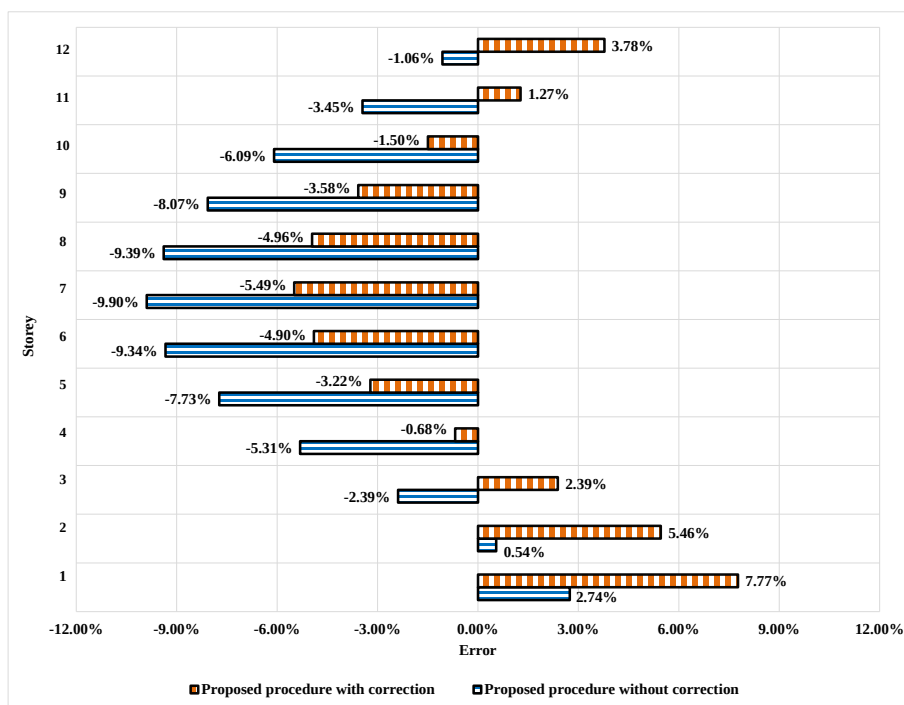


Fig. 5. Errors in interstorey drifts

4. CONCLUSIONS

This paper illustrated the application of a displacement-based seismic evaluation/design and also a retrofit procedure for frame was presented. To validate the design procedure proposed, the performance of the designed structure was obtained using non-linear step by step dynamic analyses using as seismic demand the same demand used for its design. From the analysis of the results obtained the following conclusions may be extracted:

1. For the frame designed with the procedure proposed, the maximum interstorey drifts, produced by the design demand applying the correction, are closer to those obtained from the non-linear step by step analysis of the structure, considering a non-proportional damping matrix, than to the procedure without the correction. Nevertheless, considering that the design interstorey drift is recommended to limit the damage of the LSLS, both results can be considered satisfactory. However, to guarantee this conclusion it is necessary to carry out additional evaluation/design examples, considering structures of different configurations and subjected to different seismic demands.
2. The distribution of damage, used as target within the design procedure proposed was reproduced in the results of non-linear step by step analysis of the structure with a non-proportional damping matrix. This result was due to the correction to the added damping coefficient in the procedure, something that guaranteed that the response of the structure was reduced, and the accomplishment of the design objective of controlling the distribution of damage in the structural elements.

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REFERENCES

- [1] Hwang JS, Lin WC, Wu NJ (2013): Comparison of distribution methods for viscous damping coefficients to buildings. *Struct Infrastruct Eng*, 9(1):28–41.
- [2] Soong TT; Grigoriu M (1993): Random vibration of mechanical and structural systems. Prentice-Hall. 1st edition.
- [3] Constantinou MC, and Symans MD (1992): Experimental and analytical investigation of seismic response of structure with supplemental fluid viscous dampers. Report No. NCEER-92-0032. New York: *National Center for Earthquake Engineering Research, State University of New York at Buffalo*.
- [4] Garcia DL (2001): A simple method for the design of optimal damper configurations in MDOF structures. *Earthquake Spectra*, 17 (3), 387–398.
- [5] Garcia DL, Soong TT (2002): Efficiency of a simple approach to damper allocation in MDOF structures. *Journal of Structural Control*, 9 (1), 19–30.
- [6] Ayala, AG (2001): Evaluation of seismic performance of structures, a new approach (in Spanish). *Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería*, 17:3. 285-303.
- [7] Hwang JS, Huang YN (2003): Seismic design of structures with viscous dampers, *International training program for seismic design of structures, National Center for Research on Earthquake Engineering*, 217–234.
- [8] Hwang JS (2005): Seismic design of structures with viscous dampers, *International Training Programs for Seismic Design of Building Structures, National Center for Research on Earthquake Engineering*, September. 30, Taipei, pp. 40–53.
- [9] López S (2010): Displacement Based Design of Reinforced Concrete Structures, (in Spanish). Master of Engineering Thesis. Graduate Program in Engineering. UNAM, Mexico.
- [10] CSI (2007): Perform 3D V5. Non-linear Analysis and Performance Assessment for 3D Structures. Computers and Structures, Inc. Berkeley, CA.
- [11] GDF (2004): Construction code for the Federal District, complementary technical standards on criteria and actions for the structural design of buildings and complementary technical standards for seismic design, (in Spanish), Official Gazzette, Mexico.