



NONLINEAR RESPONSE ANALYSIS MODEL OF RC MEMBERS BASED ON FORCE-BASED FIBER MODEL WITH SHEAR EFFECT

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Abstract

In nonlinear Response analysis of reinforce concrete (RC) structures and/or members, the shear effect is usually neglected for the members with large shear span ratio, such simplification is acceptable and does not results in much error. While the inelastic shear deformation cannot be ignored for those with small shear span. These members include deep beams, short columns, shear walls, etc., which are often subjected to damage in previous earthquakes. Current beam-column elements with fiber section offer an accurate and efficient nonlinear analysis model for RC structures and/or members, which are only suitable for the members with large shear span ratio. In order to consider the shear effect on the members with small shear span ratio and make full use of the advantages of the flexibility method, in this paper the shear effect is added into the force-based fiber model, which could separately consider shear yielding and/or flexural yielding. Firstly, the force interpolation function is modified to establish shear force field in the integration section. Secondly, the famous Park three-parameter hysteretic model is employed to simulate the relationship of the shear force-shear deformation. Thirdly, the parameter value of the feature points in Park three-parameter hysteretic model for two kinds of members (short column and shear wall) are suggested based on plenty of literature investigation, therefore the model is more practical. Fourthly, the modified fiber model is embedded into the user subroutine UEL in ABAQUS by second development. Finally, two numerical examples (short column and shear wall) are carried out based on the modified force based fiber model. The results show that the force-based fiber model with shear effect is reliable and effective, and that it is appropriate for the members with small shear ratio to consider the hysteresis of shear effect.

Keywords: nonlinear analysis; flexibility method; fiber model; shear effect



1. Introduction

In nonlinear analysis of reinforced concrete (RC) structures or members, shear effect is usually neglected for the members with large shear span ratio, such simplification is acceptable and does not result in much error. While the inelastic shear deformation cannot be ignored for those with small shear span. These members include deep beams, short columns and shear walls, which are often subjected to serious damage in the previous earthquakes. So, it is an important subject in the field of seismic research how to precisely simulate nonlinear response of this kind of members.

Fiber model is an effective tool for nonlinear analysis of structures and/or members. There are the stiffness method and the flexibility method for beam column element respectively according to different virtual work principle element formulation [1]. For stiffness method, displacement interpolation function is employed, and such element is called displacement-based element (DBE) [2]. In DBE, the displacement field became different from the real one. Often very fine subdivision of the structural member is necessary, which will result in double increase of the calculation work and the instability of numerical analysis. For flexibility method, force interpolation function is adopted, and such element is called force-based element (FBE) [3]. In FBE, for the given internal force distribution and on the condition of not considering the element load distribution, no matter what state the structural element is, even if the member is in softening stages, the equilibrium conditions always can be strictly met. Besides, one FBE can well simulate one member, thus the degrees of freedom of the whole structure is reduced, and time consumption is saved.

Currently, fiber model is mainly applied to the members dominated by bending deformation. To add shear effect to fiber model, a series of researches have been carried out. There are mainly two groups for considering shear effect. One is at the material level by means of choosing a proper material constitutive model, the other is at the section level by building the shear force-deformation relationship of the section [4].

Although fiber models with shear effect have been rapidly developed, two problems still exist. One is the calculation accuracy and computational efficiency. Most modified models to account for shear effect are established based on the stiffness method. How to overcome the deficiency of the stiffness method itself is a problem that needs to be considered in the shear model. The other one is the selection of the shear hysteresis model. Currently such models are too few, and their ability is limited. To solve these problems, the element stiffness matrix considering shear effect is established based on the flexibility method. The famous Park three-parameter hysteretic model is employed to simulate the shear force-shear deformation relationship of the section, and the parameter values of the feature points in the Park three-parameter hysteretic model for two kinds of members (short column and shear wall) are suggested. Numerical examples (short column and shear wall) are carried out to verify the effectiveness and reliability of the modified force-based fiber model.

2. Force-based fiber model with shear effect

2.1 Element stiffness matrix considering shear effect

The element stiffness matrix of the force-based element is derived based on the coordinate system in which the rigid body modes are removed (without rigid body mode). Fig.1 shows the degrees of freedom and deformations of the beam element in the basic coordinate system. There are six degrees of freedom, in which q_1 denotes the element axial displacement, q_2 and q_3 denote the rotation of node i and j with respect to the z axis, respectively, q_4 and q_5 denote the rotation of node i and j with respect to the y axis, respectively, q_6 is the relative rotation of node i and j with respect to the x axis.

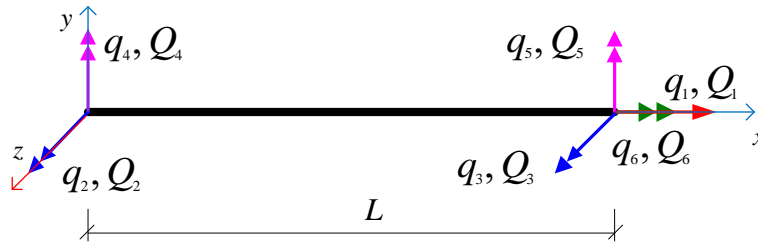


Fig. 1 – Degrees of freedom and deformation of beam element without rigid body mode

So the deformation vector of beam element in the basic coordinate system is

$$\mathbf{q} = \{q_1 \ q_2 \ q_3 \ q_4 \ q_5 \ q_6\}^T \quad (1)$$

and the corresponding element force vector is

$$\mathbf{Q} = \{Q_1 \ Q_2 \ Q_3 \ Q_4 \ Q_5 \ Q_6\}^T \quad (2)$$

The relationship between section force $\mathbf{D}(x)$ and element force \mathbf{Q} in FBE can be written as:

$$\mathbf{D}(x) = \mathbf{b}(x)\mathbf{Q} \quad (3)$$

where $\mathbf{b}(x)$ is force interpolatin function, and it is expressed as

$$\mathbf{b}(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ x-1 & x & 0 & 0 & 0 & 0 \\ 0 & 0 & x-1 & x & 0 & 0 \\ -\frac{1}{L} & -\frac{1}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{L} & -\frac{1}{L} & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \mathbf{x} = \frac{x}{L} \quad (4)$$

and the section force and deformation vectors can be expressed as follows

$$\mathbf{D}(x) = \begin{bmatrix} N(x) \\ M_z(x) \\ M_y(x) \\ M_x(x) \\ V_y(x) \\ V_z(x) \end{bmatrix}, \quad \mathbf{d}(x) = \begin{bmatrix} \alpha(x) \\ c_z(x) \\ c_y(x) \\ c_x(x) \\ g_y(x) \\ g_z(x) \end{bmatrix} \quad (5)$$

where $N(x)$ is section axial force, $M_x(x)$, $M_y(x)$ and $M_z(x)$ is section bending moment with respect to x, y and z axis, respectively, $V_y(x)$ and $V_z(x)$ is the section shear force along y and z axis.

The relationship between section deformation vector $\mathbf{d}(x)$ and section force vector $\mathbf{D}(x)$ is written as

$$\mathbf{d}(x) = \mathbf{f}(x)\mathbf{D}(x) \quad (6)$$



where $f(x)$ is section flexibility matrix, which is obtained by the inverse of the section stiffness matrix as follows

$$f(x) = k^{-1}(x) \quad (7)$$

In order to considering shear effect in force-based fiber model, the method used in this paper is that the shear stiffness is added to the section stiffness matrix of Bernoulli-Euler beam directly. Therefore, the calculation of shear effect is directly based on section level rather than superposition of single fibers. And the axial force, bending moment and shear force can be coupled through force interpolation function in element level, while the coupling effect between torsion and above forces is neglected. So the section stiffness matrix with shear stiffness is expressed as Eq.(8) :

$$k^i(x) = \begin{pmatrix} \sum_{i=1}^{n(x)} E_i A_i & - \sum_{i=1}^{n(x)} E_i A_i x_{y_i} & \sum_{i=1}^{n(x)} E_i A_i x_{z_i} & 0 & 0 & 0 \\ \sum_{i=1}^{n(x)} \bar{a}_i E_i A_i x_{y_i} & \sum_{i=1}^{n(x)} \bar{a}_i E_i A_i x_{y_i}^2 & \sum_{i=1}^{n(x)} \bar{a}_i E_i A_i x_{y_i} x_{z_i} & 0 & 0 & 0 \\ \sum_{i=1}^{n(x)} E_i A_i x_{z_i} & \sum_{i=1}^{n(x)} \bar{a}_i E_i A_i x_{y_i} x_{z_i} & \sum_{i=1}^{n(x)} \bar{a}_i E_i A_i x_{z_i}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sum_{i=1}^n G_i A_i [y_i^2 + z_i^2] & 0 & 0 \\ 0 & 0 & 0 & 0 & G(g_y) A_w & 0 \\ 0 & 0 & 0 & 0 & 0 & G(g_y) A_w \end{pmatrix} \quad (8)$$

2.2 The process of element state determination

Generally, iterative computations are needed in both structure and element levels for force-based beam-column element. And in each iteration calculation, the loading and unloading history information of all the integral sections and all the fiber material of element should be saved so that the computation task is very huge. Neuenhofer and Fillippou [5] proposed an effective method of element state determination, in which iteration is not needed in element level, thus computation task is reduced obviously. The method is employed for element state determination in this paper. The main process is as follows:

Assuming that the element deformation increments at i -th iterative step DQ^i in structure level are known, the corresponding element force increments DD^i can be obtained through the element displacement increments as

$$DD^i = (F^{i-1})^{-1} DQ^i \quad (9)$$

where F^{i-1} is the element flexibility matrix in the previous iterative step. The section force increments are expressed as

$$DD^i(x) = b(x)DD^i + \bar{D}^{i-1}(x) \quad (10)$$

where $\bar{D}^{i-1}(x)$ are the unbalance section forces in the previous iterative step.

The increment of section deformation field can be obtained by section force increment

$$Dd^i(x) = f^{i-1}(x)DD^i(x) \quad (11)$$

where $f^{i-1}(x)$ is the section flexibility matrix in the previous iterative step. And the current section deformation can be updated by Eq. (12)



$$\mathbf{d}^i(x) = \mathbf{d}^{i-1}(x) + \mathbf{D}\mathbf{d}^i(x) \quad (12)$$

For section state determination, the cross section is divided into fibers, and the geometric matrix $\mathbf{I}(x)$ is first used to obtain the strain of fibers \mathbf{e}^i . Then the stress \mathbf{s}^i and tangent modulus \mathbf{E}^i of fibers are evaluated based on material constitutive relation. The section resisting forces $\mathbf{D}_R^i(x)$ can be obtained by summation of axial force and biaxial bending contribution of all fibers through Eq. (13)

$$\mathbf{D}_R^i(x) = \mathbf{I}^T(x) \mathbf{A} \mathbf{s}^i \quad (13)$$

and the new section tangent stiffness matrix $\mathbf{k}^i(x)$ can be obtained through Eq. (14)

$$\mathbf{k}^i(x) = \mathbf{I}^T(x) \mathbf{A} \mathbf{E}_{\tan}^i \mathbf{A} \mathbf{I}(x) \quad (14)$$

where \mathbf{A} is a diagonal matrix with entries of areas of all fibers. The section flexibility matrix can be obtained by inversion of the section stiffness matrix as follows

$$\mathbf{f}^i(x) = [\mathbf{k}^i(x)]^{-1} \quad (15)$$

According to the principle of virtual displacement, integration of the residual section deformations

$$\mathbf{r}^i(x) = \mathbf{f}^i(x) \left(\mathbf{D}^{i-1}(x) + \mathbf{D}\mathbf{D}^i(x) - \mathbf{D}_R^i(x) \right) \quad (16)$$

yield the residual element displacements as Eq. (17)

$$\mathbf{r}^i = \int_0^L \mathbf{b}^T(x) \mathbf{r}^i(x) dx \quad (17)$$

These residuals displacements are transformed to residual forces such that the element resisting forces become

$$\mathbf{Q}^i = \mathbf{Q}^{i-1} + \mathbf{D}\mathbf{Q}^i - (\mathbf{F}^i)^{-1} \mathbf{r}^i \quad (18)$$

where \mathbf{F}^i is the current element flexibility matrix and expressed as

$$\mathbf{F} = \frac{\mathbf{I} \mathbf{q}}{\mathbf{I} \mathbf{Q}} = \int_0^L \mathbf{b}^T(x) \mathbf{f}(x) \mathbf{b}(x) dx \quad (19)$$

The internal force distribution $\mathbf{D}_R^i(x)$ according to Eq. (13) generally violates equilibrium and therefore is not compatible with the force interpolation functions $\mathbf{b}(x)$, which enforce equilibrium along the element in a strict sense. Consequently, unbalanced section forces are defined as

$$\bar{\mathbf{D}}^i(x) = \mathbf{b}(x) \mathbf{Q}^i - \mathbf{D}_R^i(x) \quad (20)$$

The stiffness matrix of fiber model considering shear effect is compiled with Fortran language. It is embedded into ABAQUS[6] finite element software through user's defined element subroutine. Thus, nonlinear response analysis method of RC members based on forced-based fiber model with shear effect is realized.

3. Hysteresis model of section shear force- shear deformation

The hysteresis curve of shear damage dominated structures and/or members is characterized with strength degradation, stiffness degradation and pinch effect. These factors should be considered so as to correctly describe the actual mechanical characteristics of such members. Park et al. (1985)[7] proposed a restoring force model with three parameters, and pointed that the strength degradation, stiffness degradation and pinch effect are not only correlate with the maximum of inelastic deformation of the member, but also associate with the cycle

number of inelastic deformation. The Park three parameters model is regarded as the one with complete consideration, so it is employed to simulate the relationship of the shear force-shear deformation.

As shown in Fig.2, the skeleton curve of Park model is piecewise. The three parameters including stiffness degradation coefficient α , strength degradation coefficient β and pinch effect coefficient γ are used to reflect the hysteresis rules. Where, Stiffness degradation coefficient α denotes the ratio of unloading target strength to yield strength, it is introduced by a common point to which all the unload line in the skeleton point. The strength degradation is defined in the model as

$$F_{new} = F_{max} \left(1.0 - \beta_e \frac{\int dE}{M_y \phi_u} - \beta_d \frac{\phi_{max}}{\phi_y} \right) \quad (21)$$

where β_e and β_d are used as energy dissipation and ductility function to determine the strength degradation coefficient β , usually $\beta_e = \beta_d = \beta$ is assumed; M_y and ϕ_y are yield moment and curvature, respectively; ϕ_{max} and ϕ_u are maximal curvature and ultimate curvature, respectively; γ is the ratio of the strength to the deformation corresponding to the values when first unloading to $M=0$, and then reverse loading equal to the initial cracking deformation, which reflects the phenomenon of pinch and slip. The influence of the three parameters on the hysteresis characteristics can be refer to [8].

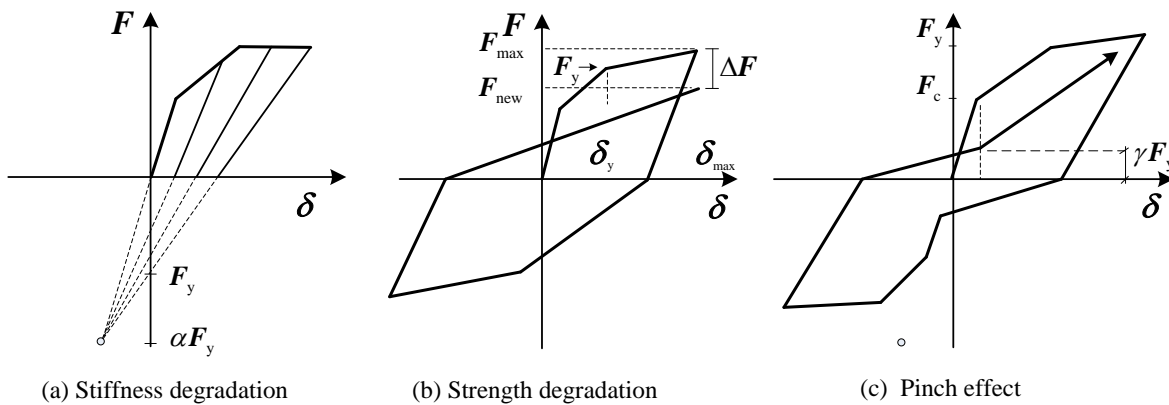


Fig. 2 – Park three-parameter hysteresis model

Whether the characteristic parameter values of restoring force model are accurate or not will directly affect the precision of the calculation results. On the basis of references, the parameter values of the feature points in shear force-shear deformation curves of short columns and shear walls are suggested, They are introduced as follows, respectively.

3.1 Parameter value of feature points in shear force-deformation curve of short columns

For reinforce concrete shor column, its shear span ratio is small, stiffness is large, ductility and energy dissipation performance is poor. the value of feature points in shear force-shear deformation curve of short columns are given by referring to Reference[9].

(1) Crack shear force-shear deformation

Before cracking, the shear strength of the members are completely undertaken by concrete. The stress of the longitudinal reinforcement and transverse reinforcement is so low that their contribution is ignored. Thus, cracking shear force is expressed as

$$V_{cr} = k\sqrt{f'_c}A_e \quad (22)$$



where f'_c is compressive strength of concrete, A_e is effective area of the cross section, and k is a parameter which reduces with the increase of the curvature ductility.

(2) Yield shear force and the secant stiffness at yield point

The yield shear force of reinforce concrete short column is composed of three parts

$$V_y = k\sqrt{f'_c}A_e + V_p + V_s \quad (23)$$

where the first part is the shear strength offered by concrete, and it mainly due to aggregate interlocking and dowel action of bending longitudinal steel, which is the same as Eq. (22). The second part is shear strength increase resulted by axial force through arch effect. The third part is the shear strength offered by transverse stirrups through truss mechanism.

For the second part offered by axial force, the corresponding shear force is expressed as:

$$V_p = \frac{D-a}{2D(M/VD)}P_u \quad (24)$$

where P_u is axial compressive force, D is the height of cross section, a is the height of compressive zone in stress diagram, and M/VD is the shear span ratio.

For the third part offered by transverse stirrups, the corresponding shear force is expressed as:

$$V_s = \begin{cases} \frac{A_v f_{yh} D'}{s} \cot 30^\circ, & \text{for rectangle cross section} \\ \frac{\pi A_{sp} f_{yh} D'}{2s} \cot 30^\circ, & \text{for circle cross section} \end{cases} \quad (25)$$

where A_v and A_{sp} are the cross section area of transverse stirrups, f_{yh} is the yield strength of transverse stirrup, and D' is the height of core concrete measured from the external edge of stirrup.

The yield deformation can be obtained by transformation of the secant stiffness at yield point. It can be seen Reference[10] in detail.

3.2 Parameter value of feature points in shear force-deformation curve of shear walls

Reference[11] gave the parameter evaluation method of feature point in shear force-shear deformation curve of the horizontal spring for the multiple-vertical-line-element model. The method is borrowed to determine related parameters of the Park three parameters model in this paper.

(1) Crack shear force and crack stiffness

Crack shear force V_{cr} and crack stiffness K_{cr} are expressed as follows:

$$V_{cr} = 0.438\sqrt{f'_c}A_w \quad (26)$$

$$K_{cr} = GA_w / \chi \quad (27)$$

where G is shear modulus, A_w is cross section area of shear wall, and χ is shape coefficient of shear deformation, and it can be determined as follows

$$\chi = 3(1+u)[1-u^2(1-v)] / 4[1-u^3(1-v)] \quad (28)$$

where u and v is the geometrical coefficient of the shear wall cross section.



(2) Yield shear force and yield deformation

The empirical formula offered by Hiroswawa[11] is adopted to determine the yield shear force, which can be written as:

$$V_y = [0.0679\rho_t^{0.23}(f'_c + 17.6) / (M / VL + 0.12)^{1/2} + 0.845(f_{wh}\rho_{wh})^{1/2} + 0.1\sigma_0]b_e j \quad (29)$$

where ρ_t and ρ_{wh} are reinforcement ratio of tensile longitudinal steels and transverse stirrups, respectively, b_e is the average width of the shear wall cross section, S is the spacing of transverse stirrup, M / VL is shear span ratio of the calculation section; $j=(7/8)(L-a/2)$, and f'_c is compressive strength of concrete.

Yield shear deformation is expressed as the function of yield strains of longitudinal reinforcement and transverse stirrup according to reference [12]

$$\gamma_y = \frac{f_y}{E_s} + \frac{\tau_y - n}{\rho_v E_s} + \frac{4\tau_y}{E_c} \quad (30)$$

where τ_y is yield shear stress, n is vertical (axial) compression stress, ρ_v is the reinforcement ratio of longitudinal steel, and E_s, E_c are the elastic modulus of steel and concrete, respectively.

4. Verification examples

In order to verify the effectiveness of the modified force-based fiber model, short column and shear wall are taken as examples, respectively. The material parameters and value of feature points in shear force-deformation restoring force model are determined according to section 3, as listed in Table 1.

Table 1 –Material parameters and value of feature points of Park three parameters model

Examples	Initial stiffness	Crack shear force	Crack shear strain	Yield shear force	Yield shear strain
Short column	3074094105	56840	0.00001849	96628	0.0032
Shear wall	1688558370	616166	0.0000364	918000	0.0033

Note: the unit of stiffness is (N/mm), and the unit of shear force is (N).

4.1 Example 1- Short column

The cycle loading test of RC short columns carried out by Yoshida (1989) [13] is chosen as the first verification example. The effect of different loading direction on the performance of short column is studied in the test. The test device, loading conditions and the specimen are shown in Fig. 3. The cross section dimension of the specimen is 18cm×18cm. The shear span ratio of the short column is 1.25.

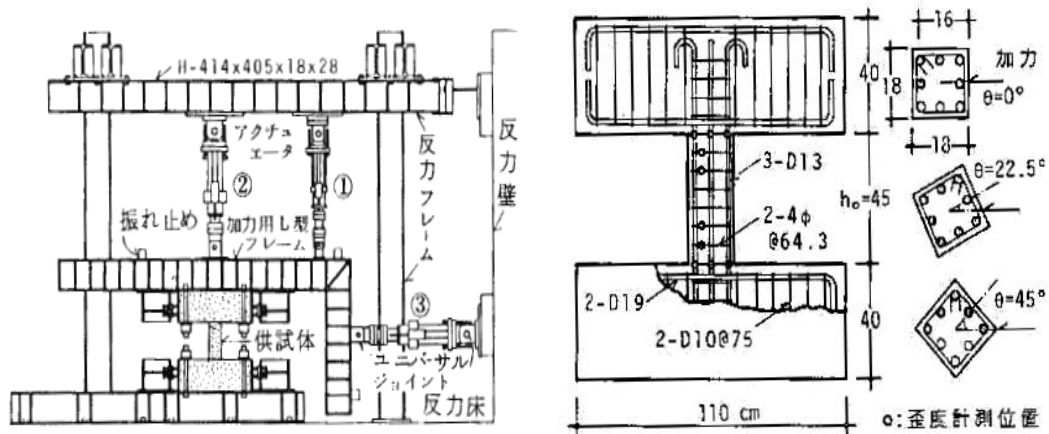


Fig. 3 – Loading conditions and the cross section dimension of the specimen

The modified force-based fiber model given by this paper is adopted to conduct the nonlinear analysis of No. OAO short column specimen under the same cycle loading path. The cross section of the short column is divided into 216 fibers. The analytical result and testing result are compared as listed in Table 2. From Table 2, it can be seen that the maximum relative error of the shear force between analytical value with test value is only 3.3%, which demonstrates that the model given by this paper is reliable.

Table 2 – Comparison of maximum shear force between test with analytical value

Test No.	Test value V_m (t)	Analytical value V (t)	$(V - V_m) / V_m$
OAO	10.1	9.76	3.3%

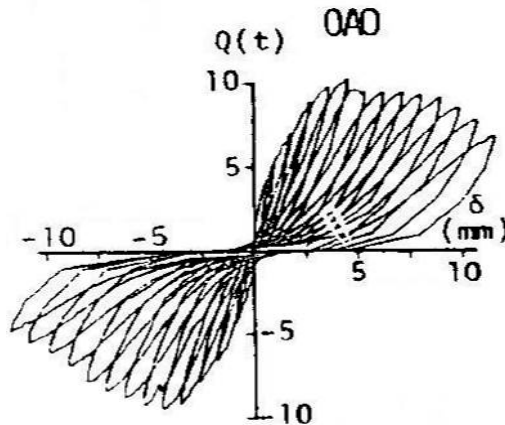


Fig. 4 – Experimental load-displacement curve of the short column

In addition, the influence of considering and non-considering shear effect on the member response is compared. Fig. 4 shows the experimental load-displacement response curve of the short column, while Fig. 5 shows the analytical load-displacement response curves considering and non-considering shear effect. From Fig. 5(a), it can be seen that the stiffness degradation and pinch effect in the experiment can be well simulated by the modified force-based fiber model considering shear effect. As for the strength degradation in analytical results, it is less obvious than the experimental result. The reason may be that the bond slip and buckling of the longitudinal reinforcing steel bars is not considered. From Fig. 5(b), it can be seen that the analytical results of hysteresis model simulated by conventional fiber model without shear effect appear the feature of bending deformation dominated, and that strength degradation and pinch effect in the experimental results are not realized. Therefore, the modified force-based fiber model can well simulate the shear effect and predict the actual shear capacity of the specimen.

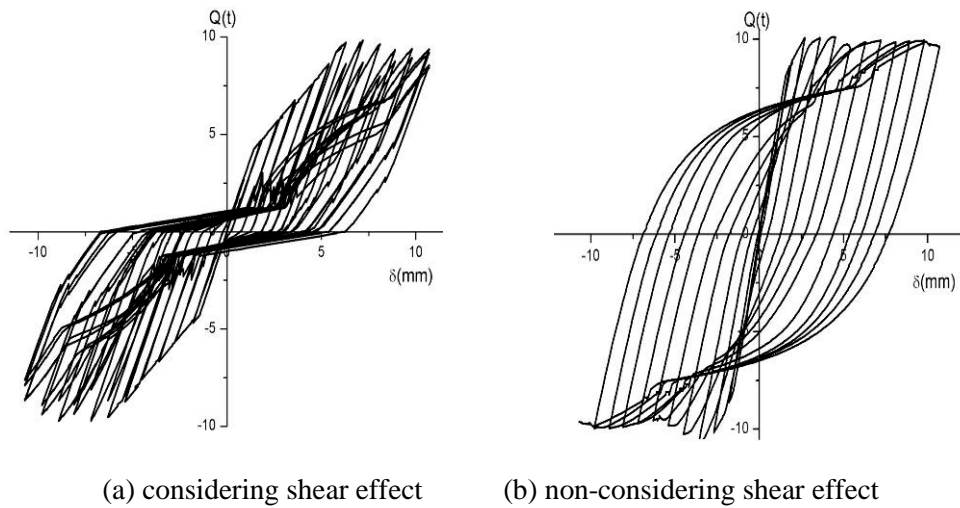


Fig. 5– Analytical load-displacement curves of the short column

4.2 Example 2 – Shear wall

This example is a 1/3 scale testing wall under monotonic load carried out in California University. It is a three-storey shear wall with edge columns, of which the cross section dimension and loading mode are shown in Fig. 6. The details about this experiment can refer to Reference [11]. As for numerical modelling, each storey is treated as a element, which is represented by the fiber model considering shear effect. The cross section is divided into 672 fibers. The coupling beam is simulated by Rigid element in ABAQUS.

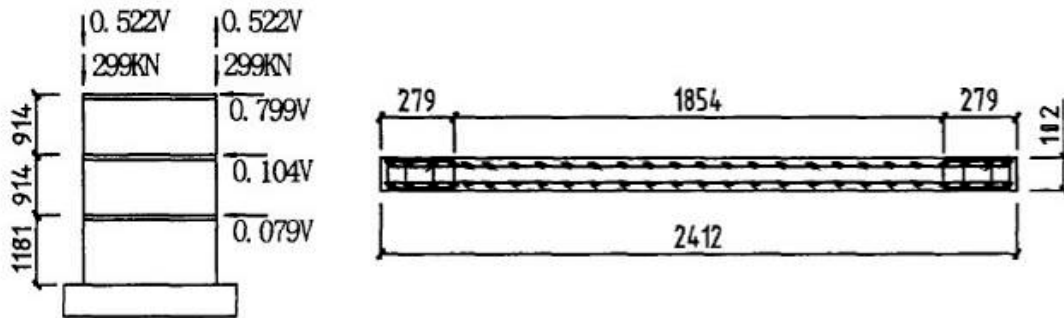


Fig. 6 – Geometric dimension and external loads of the specimen

The relationship of base shear force versus top displacement at the third floor is analysed based on the modified force-based fiber model, and it is compared with the experimental result as shown in Fig. 7. It can be seen that the initial stiffness of analytical result is in good agreement with that of experimental result. However, the post-yield stiffness of the analysis is a little larger than that of testing. The reason may be that the modified force-based fiber model in this paper does not consider the bond slip effect of the steel and concrete. The yield strength and yield deformation of computation is in good agreement with that of testing. Therefore the modified force-based fiber model considering shear effect is reliability to simulate shear wall.

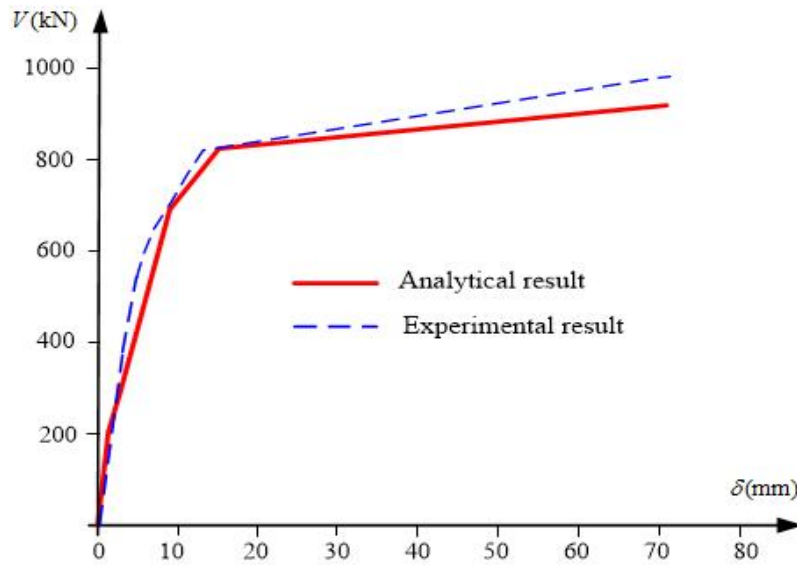


Fig. 7 – Comparison of experimental and analytical results

5. Conclusion

Currently, shear deformation is not very well considered in fiber model. Large errors may be caused if conventional fiber model is used to represent members with large shear deformation. In this paper, the shear effect is added to the force-based fiber model. Park three-parameters hysteretic model is adopted to represent shear force-shear deformation hysteresis relationship, and the parameter value of the feature points in Park three-parameter hysteretic model for two kinds of members (short column and shear wall) are suggested. The modified fiber model is embedded into the user subroutine UEL in ABAQUS by second development. Two numerical examples (short column and shear wall) are carried out based on the modified force based fiber model. The results show that the force-based fiber model with shear effect is reliable and effective, and that it is appropriate for the members with small shear ratio to consider the hysteresis of shear effect.

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