16th World Conference on Earthquake, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N°1282

Registration Code: S-U1462419371

Application of inertial mass for a 2-DOF mass-varying structure

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Abstract

This paper presents a new method of reinforcement of two-story mass-varying structure using inertial mass. In a mass-varying structure like an elevated station, the mass of the second story (the upper story) is much smaller than that of the first story (the lower story), and seismic force of the second story tends to increase significantly. If the mass of the second story is simply increased so as to adjust the mass ratio in the vertical direction, it would be necessary to redesign the entire structure. To adjust the mass ratio of the structure, application of inertial mass is presented in this paper. Inertial mass is the apparent mass in vibration phenomenon. Using this, it would be able to adjust the mass ratio of the structure during an earthquake without large increase in substantial mass. However the method of controlling seismic response of building using inertial mass has developed recently, there is no case of the application to mass-varying structure such as railway building. In view of this, the method of adjusting the mass ratio is presented. Through shaking table test and analysis, this study shows that the large seismic response of the second story is reduced by application of inertial mass installed in the second story only.

Keywords: Inertial Mass; Mass-varying Structure; Mass Ratio; Shaking Table Test; Response Control

1. Introduction

Elevated station has small structures on viaduct, for example, shed, waiting room and shop. The mass of the small structures is extremely smaller than that of viaduct. For this reason, the seismic behavior of the elevated station is different from the ordinal buildings. Specifically, the seismic force of the small structures tends to increase significantly. To evaluate the seismic force of the buildings, Japanese building codes regulates “Ai distribution” and its simplified calculation method. However, the method is suitable for the ordinal buildings and is difficult to be applied to mass-varying structure like the elevated station. Regarding this matter, some researches have been reported. Ozeki et al. (2013) studied the seismic force of small structures on the viaduct and Shimizu et al. (2008) and Yamada et al. (2013) revealed the seismic behavior of the elevated station by microtremor measurement of a real station and analysis of skeleton model. In addition, by the analysis of the mass system model, Yamada et al. (2008) revealed that response acceleration of shed on the viaduct is very large when the natural frequency of the viaduct and that of the shed are close to each other and proposed a design method of a shed on the viaduct. Furthermore, Shimizu et al. (2009) examined a reinforcement method of a shed on the viaduct, using oil damper and steel damper, by analysis of frame model. In the analysis, considering installation conditions of real stations, the dampers are of knee brace type and are installed at the corner of the shed.

As one way to reduce seismic response of the small structure on the viaduct, it is conceivable to adjust the mass ratio in the vertical direction. However, in the case of simply increasing mass of the small structure, it is necessary to redesign the entire structure because its own weight increases. Therefore, as a method of reducing the seismic response, it is considered to apply inertial mass, which generates mass only when earthquake occurs, to small structure. In particular, a device capable of ensuring mass effect of thousands times of substantial mass has been developed (for example, Nakaminami et al., 2005). Using this device, it would be able to adjust the mass ratio of the structure during an earthquake without large increase of substantial mass. The apparent mass in the vibration phenomenon, which is realized by such devices, is also called "Dynamic mass". As a study of application of Dynamic mass, Furuhashi and Isimaru (2004) proposed a method to manipulate vibration mode by using Dynamic mass, to control seismic response of building. In addition, Sugimura et al. (2010) and Kida et al. (2011) proposed a method of controlling seismic response of building using "Tuned viscous mass damper". Tuned viscous mass damper is a device in which Dynamic mass and viscous damping elements are arranged in parallel and connected in series to a support member element having a suitable rigidity. This method is expected to be a high-performance improvement technique of seismic performance. However, there is no case of application study to mass-varying structure such as railway building.

In view of the above, this study examined whether it is possible to reduce seismic response acceleration of the upper layer of the two-story structure by application of inertial mass. In this study, a device for generating inertial mass (inertial mass damper) is applied only to the upper layer. By shaking table test, using mass-varying two-layer test body that is assumed as an elevated station, we examined the seismic behavior of mass-varying structure and effect of inertial mass. Based on the result of test and analysis, we examined whether it is possible to reduce seismic response acceleration of the upper layer by adjusting mass ratio. In the case of applying inertial mass damper to real elevated station, the knee brace type (Fig. 1) will be installed so as not to interfere with the passenger and railway operation. However, for the purpose of purely considering effect of inertial mass on vibration properties, this study was conducted in a state that inertial mass damper is installed at the interlayer.

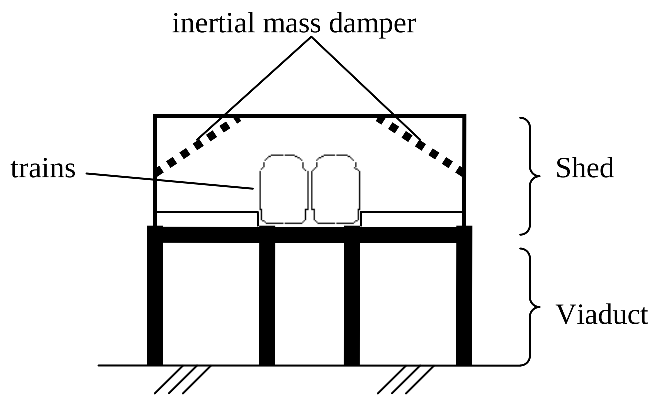


Fig. 1 – Conceptual diagram of application of inertial mass to elevated station

2. Shaking table test

2.1 Outline of the experiment

The test body of the shaking table test is a two-story model (Fig. 2 and Fig. 3). Its mass of the second story (upper story) is much smaller than that of the first story (lower story) in order to reproduce behavior of the elevated station. The first story is assumed as the viaduct, and the second story is assumed as the shed on the viaduct. Table 1 shows the property of the first story of the test body. The first story, supported by four laminated rubber bearings, is placed on the shaking table. The mass is 21.79 ton (measured value). Sum of horizontal stiffness of the laminated rubbers is estimated from experimental results and is approximately 9000 kN/m under the conditions of this experiment (amount of deformation of the laminated rubber is about 15 mm). Sum of equivalent damping factor of the laminated rubbers is 9.2% on average. The natural frequency of the first story alone is 3.23 Hz (calculated from its mass and stiffness). Table 2 shows the property of the second story of the test body. The second story is placed on the first story with roller support device inserted in between. To study influence of different values of the second story properties, 8 cases of the test body were prepared in total. In this paper, "Mass ratio" is defined as shown below:

$$\alpha_m = m_2 / m_1 \qquad (1)$$

where α_m is the mass ratio, m_1 is the mass of the first story, and m_2 is the mass of the second story.

"Natural frequency ratio" is also defined as shown below:

$$\alpha_f = f_2 / f_1 \qquad (2)$$

where α_f is the natural frequency ratio, f_1 is the natural frequency of the first story alone, and f_2 is the natural frequency of the second story alone.

The mass of the second story, considering the case of the real shed, was set so that the mass ratio becomes approximately 0.05, 0.10, 0.20. The stiffness of the second story, was set so that the natural frequency ratio becomes approximately 0.8, 1.0, 1.5. The stiffness of the second story was adjusted by exchangeable steel spring, and the natural frequency of the second story alone would be changed. The case of mass ratio 0.20 at natural frequency ratio of 1.5 is not implemented by the circumstances of the test body.

Inertial mass damper was arranged in parallel with steel spring only in the second story (Fig.3, Fig.4). Inertial mass is 20.0 ton, 5 to 20 times the mass of the second story, comparable to the mass of the first story. Because this study aims to purely verify inertial mass effects, viscous body is not sealed. In addition, static friction force is 1.7 kN (single test result by the manufacturer).

The input earthquake motions were as follows: Hachinohe-EW wave, Wave in notification (The earthquake motion stipulated by the Building Standard Law) and JMA Kobe-NS wave. Time axis of Hachinohe-EW wave and Kobe-NS wave were compressed to half. This is arranged so that the predominant frequency of the input waves and the natural frequency of the first story are not so much deviated from each other. The size of each ground motion is set as large as possible within performance acceptable range of the test body. Measurement items of each story are as follows: displacement, acceleration, shear force.

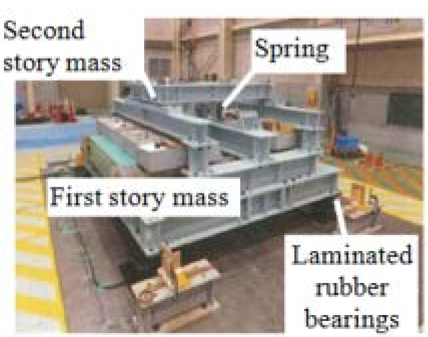


Fig. 2 – Overall view of test body

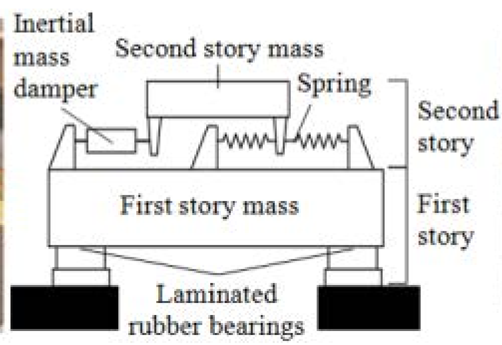


Fig. 3 – System of test body

Fig. 4 – Inertial mass damper

Table 1 – Specifications of the test body (first story)

Specifications of the test body (first story)	Mass[ton] (m_1) *1	Stiffness[kN/m] (k_1) *1	Natural freqency[Hz] (f_1) *1
	21.79	9000	3.23

*1 m_1 and k_1 are measurement values. f_1 is calculated by the following formula : $f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}}$

Table 2 – Specifications of the test body (second story)

Case			Specifications of test body (second story)		
No.	Natural freqency ratio	Mass ratio	Mass[ton] (m_2) *2	Stiffness[kN/m] (k_2) *2	Natural freqency[Hz] (f_2) *2
1	$\alpha_f = 0.8$	$\alpha_m = 0.05$	0.97	266	2.63
2		$\alpha_m = 0.10$	1.88	489	2.57
3		$\alpha_m = 0.20$	3.71	923	2.51
4	$\alpha_f = 1.0$	$\alpha_m = 0.05$	0.97	489	3.57
5		$\alpha_m = 0.10$	1.88	923	3.53
6		$\alpha_m = 0.20$	3.71	1800	3.51
7	$\alpha_f = 1.5$	$\alpha_m = 0.05$	0.97	923	4.91
8		$\alpha_m = 0.10$	1.88	1800	4.92

*2 m_2 and k_2 are measurement values. f_2 is calculated by the following formula : $f_2 = \dfrac{1}{2\pi}\sqrt{\dfrac{k_2}{m_2}}$

2.2 Test results (Time history of response)

Fig 5 shows the time history of response to Hachinohe-EW wave input (the test body is No.6 case: $\alpha_f = 1.0$; $\alpha_m = 0.20$). Without the damper, the displacement and the absolute acceleration of the second story are large compared with those of the first story. On the other hand, the case with the damper, the displacement of the second story is greatly suppressed as compared with the case without the damper. Regarding the waveform, it is observed that the wave cycle is extended by the effect of the damper. The absolute acceleration and the shear force of the second story were also largely suppressed. From the historical loop (Damper force - Relative story acceleration (the second story) relationship), it was confirmed that the damper took inertial mass of the design value. For all of the response of the first story, there was no significant difference between the case with damper and the case without damper.

2.3 Test results (Maximum response)

Fig 6 shows the maximum displacement of the test results. In the case of without damper, the displacement of the second story tends to increase as the mass ratio decrease and to increase as the natural frequency ratio decreases. As the mass ratio increases, the displacement of the first story increases slightly. The maximum displacement of the second story is much larger than that of the first story. On the other hand, in the case with the damper, the displacement of the second story is largely suppressed even in all the cases with any mass ratio, and any natural frequency ratio. The displacement of the first story is almost unchanged in the case of without damper except for some. The above trends are common to the three input earthquake motion cases.

Fig 7 shows the maximum acceleration of the test results. In the case of without damper, the acceleration of the second story is significantly larger than that of the first story. The acceleration of the second story tends to increase as the mass ratio decreases and is large as the natural frequency ratio is close to 1.0. These trends are similar to the trends of the analysis results of Yamada et al. (2008). Therefore, it was confirmed that the test body simulated the response properties of the actual elevated station. For the first story, the maximum acceleration is almost unchanged. On the other hand, in the case with the damper, the acceleration of the second story is largely suppressed even in all the cases with any mass ratio, and any natural frequency ratio. The maximum acceleration of the first story has a trend similar to the displacement. Fig 8 shows the maximum shear force coefficient of the test results. They have a trend similar to the acceleration.

3. Analysis study

3.1 Analytical verification of the test results

Analytical verification was performed on the case with the damper shown in Fig. 5 (α_f = 1.0, α_m = 0.20, Hachinohe-EW wave inputted). Fig.9 and Table 3 show the analysis model and the analysis condition. With respect to the specifications of the second story, the mass and stiffness are the measured values of the test body, and the frictional force of the roller supporting device is the value estimated from the test results. In addition, damping factor is assumed to be 2.0%. With respect to the specifications of the first story, the mass is the measured value of the test body, and the stiffness is the value estimated from the test results, the damping factor is the value estimated by the manufacturer. The specifications of the inertial mass damper are also the values estimated by the manufacturer.

Fig.10 shows analytical verification of the test results. The response of the second story and the inertial mass damper of the test results are correctly reproduced by the analysis. On the other hand, the difference between the test results and the analytical results is seen in the historical loop (Shear force (the first story) - Displacement (the first story) relationship). This is because the stiffness of the first story (laminated rubber) is actually non-linear but is modeled as linear for the convenience of analysis. However, it was confirmed that the response properties of the damper and the whole system of the test body were reproduced by the analysis.

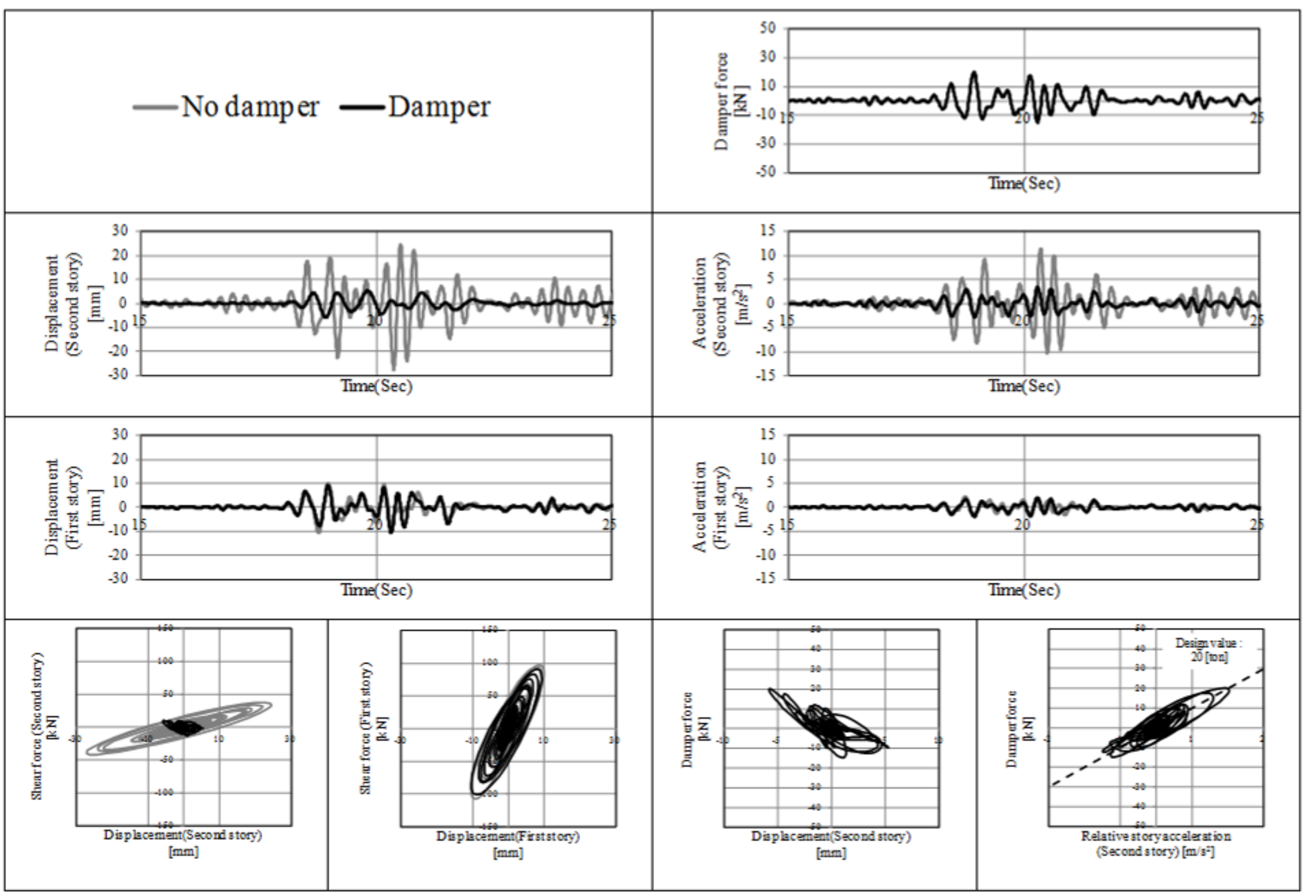
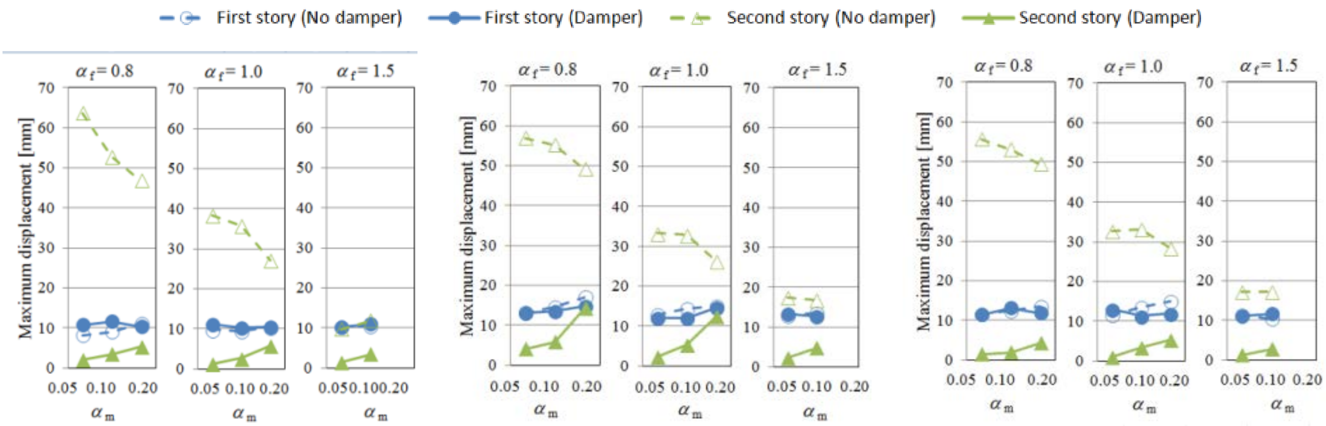
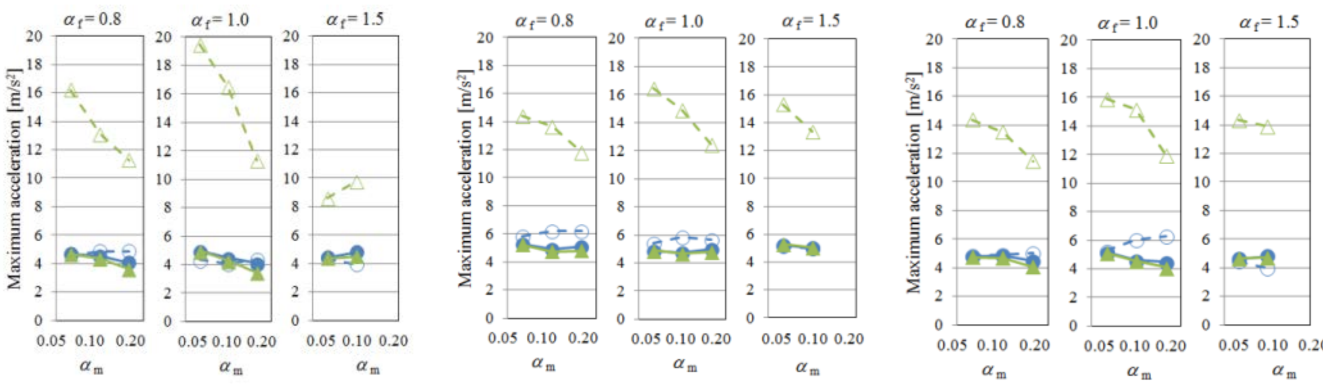


Fig. 5 – Time history of response (α_f = 1.0, α_m = 0.20, Hachinohe-EW wave inputted)



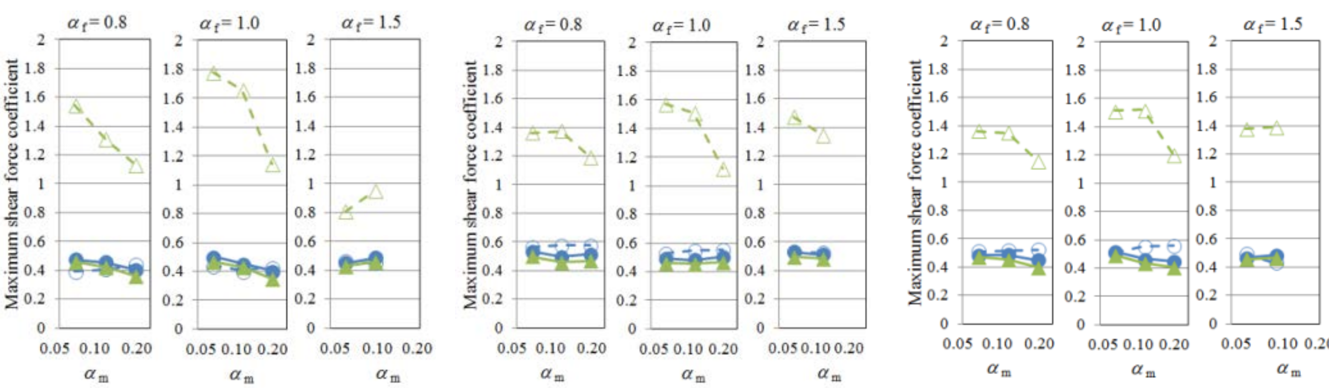
(a) Hachinohe-EW wave (b) Wave in notification (c) JMA Kobe-NS wave

Fig. 6 – Maximum displacement



(a) Hachinohe-EW wave (b) Wave in notification (c) JMA Kobe-NS wave

Fig. 7 – Maximum acceleration



(a) Hachinohe-EW wave (b) Wave in notification (c) JMA Kobe-NS wave

Fig. 8 – Maximum shear force coefficient

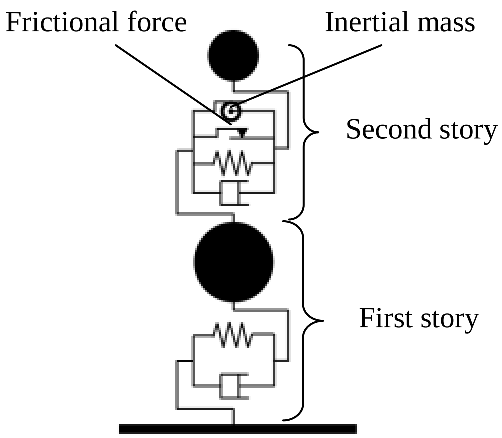


Fig. 9 – Analysis model

Table 3 – Analysis condition

Second story	Mass [ton]	3.71
	Stiffness [kN/m]	1800
	Damping factor	2.0%
	Frictional force [kN]	0.5
First story	Mass [ton]	21.79
	Stiffness [kN/m]	9000
	Damping factor	9.2%
Inertial mass damper	Inertial mass [ton]	20.00
	Frictional force [kN]	1.7

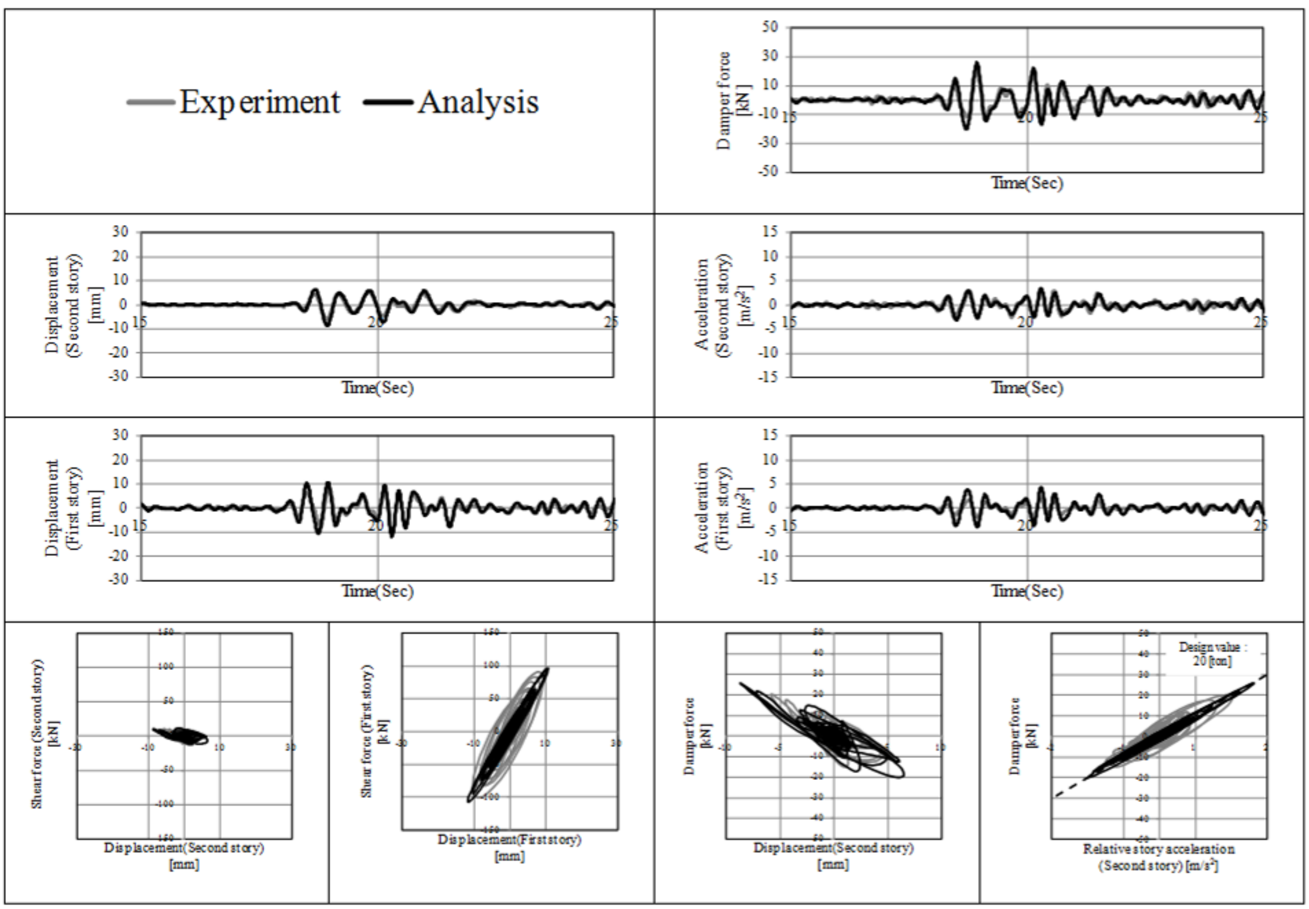


Fig. 10 – Analytical verification of the test results (α_f = 1.0, α_m = 0.20, Hachinohe-EW wave inputted)

3.2 Parametric study

Parametric study was performed with the case of the analysis model shown in Fig. 9. The parameters of the first story is constant at the values shown in Table 3. Regarding the parameters of the second story, the mass and the stiffness are changed so that the mass ratio is between 0.03 and 0.20 (with increments of 0.025) and the natural frequency ratio is between 0.5 and 2.0 (with increments of 0.0125). Regarding the parameters of the inertial mass damper, the inertial mass was changed in the ratio of the mass of the second story. In this paper, “Added mass ratio” is defined as below:

$$\alpha_d = m_d / m_2 \tag{3}$$

where α_d is the added mass ratio, m_d is the inertial mass of the damper, and m_2 is the mass of the second story.

Added mass ratio is set at 0.00 (without damper), 0.50, 1.00 and 2.00. In addition, the friction forces of the second story and the damper are not taken into consideration in order to purely consider the effect of the inertial mass. The input earthquake motion was white noise (The maximum acceleration is 200 cm/s^2).

Fig 11 shows the maximum displacement of the analysis result. In the case of without damper, the displacement of the second story is much larger than that of the first story. The displacement of the second story tends to increase as the natural frequency ratio decreases. This is similar to the trends of the experimental results shown in Section 2.3.

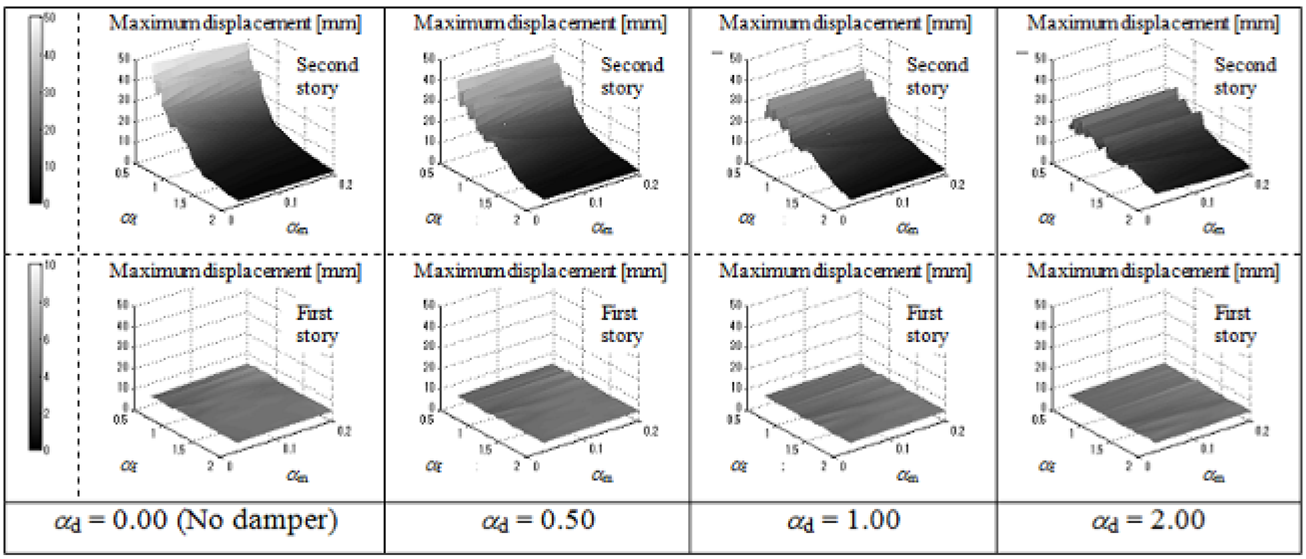


Fig. 11 – Analysis result (Maximum displacement)

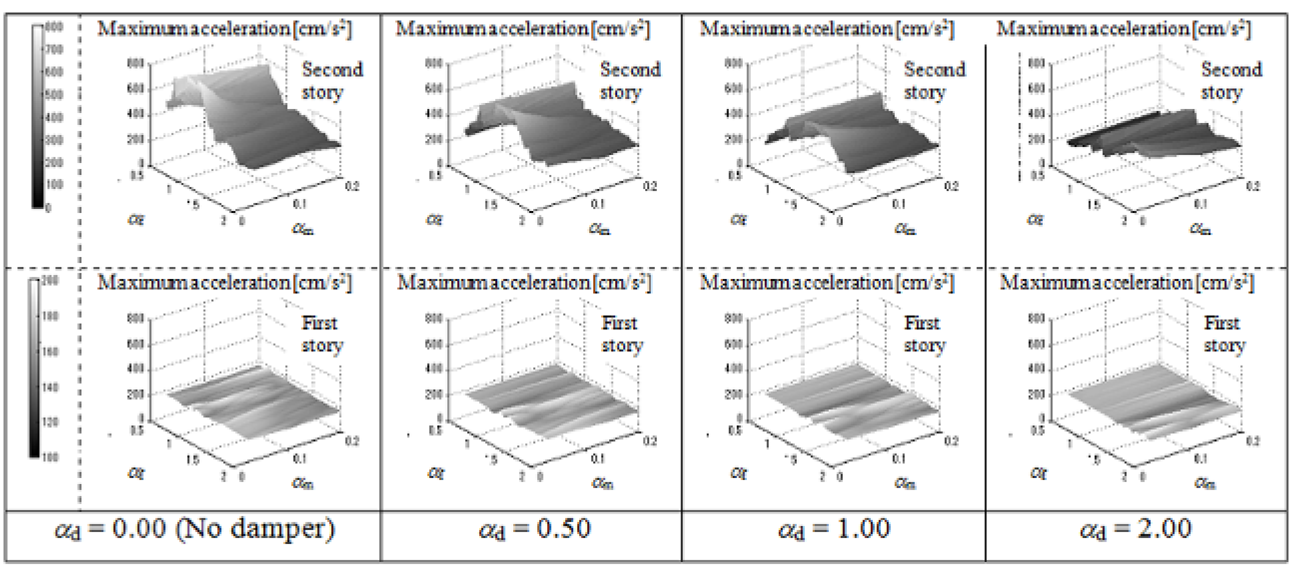


Fig. 12 – Analysis result (Maximum acceleration)

On the other hand, as the added mass ratio increases, the displacement in the low-frequency case decreases but that in the high-frequency case increases in some cases. This is because the second story resonates with the first story as a result of the decrease of the natural frequency by the effect of the inertial mass. However, even in this case, the maximum displacement at resonance is not so large. In the first story, as the added mass ratio increases, the displacement increases slightly.

Fig 12 shows the maximum acceleration of the analysis result. In the case of without damper, the acceleration of the second story is much larger than that of the first story. The acceleration of the second story tends to increase as the mass ratio decreases and as the natural frequency ratio is close to 1.0. These are also similar to the trends of the experimental results shown in Section 2.3. On the other hand, as the added mass ratio increases, the acceleration decreases in the case where the natural frequency ratio is close to 1.0 but increased in the case of the natural frequency ratio is higher than 1.0. However, even in this case, the maximum acceleration is not so large. In the first story, as the added mass ratio increases, the acceleration increases slightly.

4. Modal analysis of two-mass system model with inertial mass

4.1 Formulation of participation vector

For the purpose of examining the effect of inertial mass on participation vector, formulation of participation vector of the two-mass system model with inertial mass was performed. In the following, the damping factors were not considered for the facilitation of the formulation.

The equation of motion of the two-mass system model to which inertial mass is added in the second story is expressed by Eqn. 4:

$$[M]\{\ddot{\mathbf{y}}\}+[K]\{\mathbf{y}\}=-[M]\{1\}\ddot{\mathbf{y}}_g$$
$$[M]=\begin{bmatrix} m_1+m_d & -m_d \\ -m_d & m_2+m_d \end{bmatrix} \quad , \quad [K]=\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \tag{4}$$

where m_1 is the mass of the first story, k_1 is the stiffness of the first story, m_2 is the mass of the second story, k_2 is the stiffness of the first story, m_d is the added inertial mass, $\{\mathbf{y}\}$ is the displacement vector, and $\ddot{\mathbf{y}}_g$, is the inputted ground acceleration and $\{\mathbf{y}\}$ is assumed to be given as follows:

$$\{y\}=\{u\}e^{i\omega t} \tag{5}$$

where ω is the natural circular frequency, and $\{u\}$ is the eigenvector.

Under free vibration ($\ddot{y}_g=0$), Eqn. 4 changes into Eqn. 6:

$$\left(-\omega^2[M]+[K]\right)\{u\}=0 \tag{6}$$

By solving Eqn.6, ω is obtained and expressed by Eqn. 7:

$$\omega^2=\frac{k_1m_d+k_2m_1+(k_1+k_2)m_2}{2(m_1+m_2)m_d+2m_1m_2} \pm\frac{\sqrt{(k_1m_d+k_2m_1+(k_1+k_2)m_2)^2-4(m_1m_2+(m_1+m_2)m_d)k_1k_2}}{2(m_1+m_2)m_d+2m_1m_2} \tag{7}$$

$\{u\}=\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$ is also obtained by solving Eqn.6 and expressed by Eqn. 8:

$$u_2/u_1=\frac{m_d\omega^2-k_2}{(m_2+m_d)\omega^2-k_2} \tag{8}$$

As a result, the participation vectors are expressed by Eqn. 9 and Eqn. 10:

$$\beta u_1 = \frac{\{u\}^T[M]\{1\}}{\{u\}^T[M]\{u\}} \cdot u_1 = \frac{m_1 + m_2 \cdot \left(u_2/u_1 \right)}{m_1 + m_d - 2m_d \left(u_2/u_1 \right) + (m_2 + m_d) \cdot \left(u_2/u_1 \right)^2} \tag{9}$$

$$\beta u_2 = \frac{\{u\}^T[M]\{1\}}{\{u\}^T[M]\{u\}} \cdot u_2 = \frac{m_1 \cdot \left(u_2/u_1 \right) + m_2 \cdot \left(u_2/u_1 \right)^2}{m_1 + m_d - 2m_d \left(u_2/u_1 \right) + (m_2 + m_d) \cdot \left(u_2/u_1 \right)^2} \tag{10}$$

where βu_1 is the participation vector for the first story, and βu_2 is the participation vector for the second story. Here after, the case where ω becomes smaller is defined as the first mode, and the case where ω becomes larger is defined as the second mode.

4.2 Effects of inertial mass on participation vector

Using the formula formulated in Section 4.1, the effect of the inertial mass on the participation vector was examined. Fig.13 and Fig.14 show the participation vector of some cases where the inertial mass change. They were calculated by Eqn. 9 and Eqn. 10, where the values of the parameters were as follows: m_1, 21.79 ton; k_1, 9000 kN/m; m_2, 1.09 ton; and k_2, 450 kN/m. The value of m_1 and k_1 are the values of the test body, and the value of m_2 and k_2 are set so that the mass ratio is 0.05 and the natural frequency ratio is 1.0. As the inertial mass increases, the interlayer response of the second story on both in the first mode and in the second mode is reduced. Further, the interlayer response of the second story is reduced in the first mode but is increased in the second mode. This suggests that the seismic response of the second story is reduced by the inertial mass installed only in the second story. In addition, it can be said that the lower layer of the response can be increased by the application of the inertial mass.

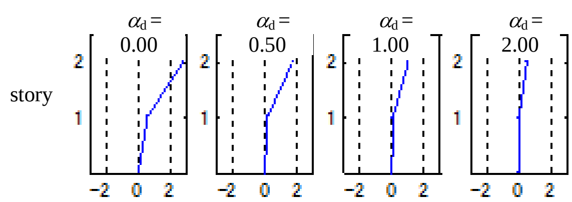


Fig. 13 – Participation vector (First mode)

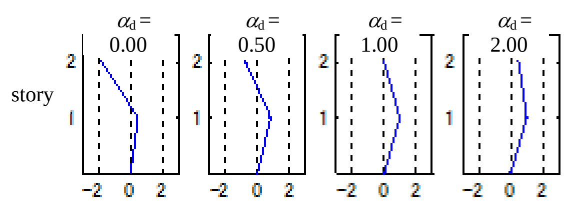


Fig. 14 – Participation vector (Second mode)

5. Conclusion

For the purpose of reducing the response of mass-varying two-story structure like the elevated station, it was examined whether the seismic response of the second story is reduced by application of inertial mass. By the shaking table test, it was confirmed that the inertial mass installed in the second story only to adjust the mass ratio and reduced the seismic response of the second story. Further, by the analysis using the mass system model, where the case of changing the parameters of the second story and the added inertial mass was changed, it was confirmed that the same tendency as that given by the shaking table test is observed.

Part of this study was implemented with the subsidy of the Ministry of Land, Infrastructure, Transport and Tourism for railway technical development.

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