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MULTI-OBJECTIVE LOSS OPTIMIZATION OF SEISMIC RETROFITTING OF MOMENT RESISTING FRAMES USING VISCOUS DAMPERS

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Abstract

The effectiveness of control strategies in achieving the objectives of a performance-based-design is well accepted in the earthquake engineering community. Consequently, various methods have been proposed for optimal design of dampers and their distribution along the building height. Most of the formulated methods concentrate mainly on reducing the responses with no explicit consideration of their long-term economic impact. In this study, a novel multi-objective optimization problem is formulated for optimally distributing viscous dampers by simultaneously minimizing the initial cost and the total expected seismic loss. An intensity based assessment is used for the computation of the total expected loss. The Pareto front between the initial cost and the total expected loss is generated by assuming the parent frame to remain linear. An adaptive aggregate gradient based multi-objective framework is employed to solve the formulated optimization problem. Implementation scheme of the optimization framework is outlined in detail. The efficacy of the proposed procedure is illustrated by applying it on a four storey reinforced concrete frame.

Keywords: Viscous damping; Nonlinear seismic analysis; Nonlinear dynamic analysis; inherent damping; in-structure damping

1. Introduction

The introduction of control techniques in structural engineering was mainly due to an increasing demand to have lesser damage during a major seismic event. Among the different control techniques implemented in structures to reduce seismic responses, from a retrofitting perspective, application of viscous dampers seems to be more common. Main reason for this could be attributed to the fact that the damper force is either linearly or nonlinearly proportional to velocity and is mostly out of phase from the column displacements. As a result, the columns or foundations are not subjected to significant additional demand, and may not need to be strengthened [1-3]. This paper is mainly concerned with the seismic performance enhancement of existing frames using viscous dampers.

The main task needed to be addressed by the engineer in the retrofitting design using viscous dampers is to efficiently quantify and position the dampers. This should take into account both the initial cost that needs to be invested and the achievement of the performance objective. Various optimal design methodologies for retrofitting are available in the literature. Some of them primarily address the problem of distributing a given total added damping to achieve the best performance (minimize damage measures) whereas some other methods minimized the total added damping subject to a constraint on the performance of the structure (allowable inter-story drifts). The relevant references in the direction of optimally positioning the dampers for a given total quantity are [4-12]. Relevant references in the direction of minimizing the total added damping subject to a constraint on the performance are [13-17]. While the latter optimization formulation lends itself to the performance-based-design framework, the allowable inter-story drifts, or performance measures, are determined based on code requirements and are not determined explicitly based on the economic consequences.

All the above cited works involved optimization of a single objective with the other objectives adopted as constraints with predetermined values. For an ideal retrofitting scheme to be economically viable, the initial cost should be minimized without compromising the anticipated performance in terms of response reduction. A reduction in response directly translates to a reduction in loss. But it could clearly be seen that both of these criteria are mutually competing against each other; i.e. minimization of initial cost would result in lesser damping and hence an increase in the seismic lose due to increase in the system response and a minimization of seismic loss would result in an increase in initial cost due to the requirement of more damping. So how would one decide which of these two objectives is more important for a specific decision making scenario?

1.1 Multi-objective optimization

In order to decide on which of the objectives is more important for a chosen decision making scenario, one needs to know the effect of minimization of both of the objectives. A very well-established way of accomplishing this is by the notion of Pareto optimality [18]. Philosophy of Pareto optimality is that rather than attempting to identify a single optimal design, one seeks to determine the entire family of designs that lie on a Pareto front. So basically a Pareto front design represents a feasible solution for which an improvement in any of the objectives can only be realized by a degraded performance in the other. So in the context of this paper, the Pareto front presents a solution of decreased expected total loss only by accepting an increase in the initial cost; but the Pareto front as a whole is very attractive to the decision makers because it presents with a complete picture of the potential design solutions and the implication of a specific decision in terms of its effect on both the expected loss and initial cost

A focus to minimize both initial cost and expected total seismic loss together presents itself with the need for a multi-objective framework. One of the earlier works in this related to seismic control was by Lavan and Dargush [19]. Though a Pareto optimality criterion was adopted, there was no explicit consideration of seismic loss in the study. Also Genetic Algorithm (GA), a zero order optimization scheme was used for generating the Pareto front. The main issue with a GA scheme is that a very large number of function evaluations are needed for the solution which poses a very high computational demand. The Other relevant reference which combined both the initial cost and seismic loss and assessed the performance in a framework of life cycle cost reduction was given in [20].

But as the adopted optimization scheme was formulated as a single objective and again zero order, it required a very large number of function evaluations.

In this present paper a novel multi-objective gradient based first order optimization framework is proposed which attempts to generate the Pareto front by minimizing the initial cost and seismic loss simultaneously. Computationally the optimization framework is very efficient mainly because it adopts a gradient based approach and the amount of function evaluations required are very less. Also the Pareto front generated gives a full sphere of the possible solutions helping a decision maker place his decision on informed quantified facts.

2.0 Problem formulation

For an existing building to be retrofitted using viscous dampers, the mass and the stiffness of the structure are already known. So the initial cost in the case of enhancing seismic performance with viscous dampers can be assumed to be directly proportional to the cost of the added dampers and their installation. Total expected loss is computed as the aggregation of the losses of the components.

2.1 Equations of motion

The equations of motion of the linear frame with added dampers are given as,

$$\begin{aligned} &\mathbf{M}\ddot{\mathbf{u}}(t) + \left[\mathbf{C} + \mathbf{C}_{damper}(\mathbf{c}_d)\right]\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M}\mathbf{I}\ddot{u}_g(t) \\ &\mathbf{u}(0) = 0; \dot{\mathbf{u}}(0) = 0 \end{aligned} \tag{1}$$

In eq. (1), **M** represents the mass matrix and **K** represents the stiffness matrix; in the case of a linear frame both the matrices remain constant. Similarly, **C** represents the inherent damping matrix (in this study represented by initial stiffness Rayleigh damping), $\mathbf{C}_{damper}(\mathbf{c}_d)$ is the added supplemental damping matrix, $\mathbf{c}_d$ is the added damping vector, **I** represents the ground motion directional vector, $\ddot{\mathbf{u}}(t), \dot{\mathbf{u}}(t)$ and $\mathbf{u}(t)$ represent the relative acceleration, relative velocity and relative displacement, and $\ddot{u}_g(t)$ represents the acceleration due to gravity.

2.2 Loss computation

One of the ways of measuring the economic consequences can be in terms of expected annual loss which actually corresponds to the economic loss that, on an average, occurs every year in the building [21-23]. In the classical detailed loss assessment framework, the expected annual loss or the loss expected over a period of time is computed as [18],

$$E[L_T] = \frac{1 - e^{-\lambda t}}{\lambda} \int_0^\infty E[L_T \mid IM] dv(IM) \tag{2}$$

where $E[L_T]$ is the expected annual loss, λ is the discount rate (to convert the future loss to net present value), t is the period for which the rate is applied, $E[L_T \mid IM]$ is the expected loss conditioned on the intensity measure *IM*, and *v(IM)* is the mean annual rate of exceedance of the intensity measure. As this study aims to present the optimization methodology, the expected loss is computed only at a single value of intensity measure. Hence, the computed loss is independent of period *t*; thereby making eq. (2) not readily usable. Hence, in the present study, the total expected loss in no-collapse scenario conditioned on the mean engineering demand parameter ($\overline{EDP}$) which in turn is conditioned on the selected intensity measure (IM_1) is used and is assumed to be [21],

$$E\left[L_T \mid NC, \overline{EDP}_{IM_1}\right] = \sum_{j=1}^{N} a_j \left(E\left[L_j \mid NC, \overline{EDP}_{IM_1}\right]\right) \tag{3}$$

Here, a_j is the cost of the j^{th} component and NC refers to no-collapse state, N refers to the number of components. Eq. (3) is period independent and $\overline{EDP}$ is computed for the specific intensity measure (IM_1). As dampers are added into the structure and the structure is assumed to behave linearly, collapse probability can be argued to be zero. Hence, only the no-collapse state is used for estimating loss in this paper.

Optimization problem

The optimization problem is formulated as,

$$\left.\begin{array}{l}\min\left(\Gamma(\mathbf{c}_d),\Theta\right)\\ 0\leq \mathbf{c}_d\end{array}\right\} \tag{4}$$

where Γ represents the initial cost which in the retrofitting case is assumed proportional to the quantity of damping required; $\mathbf{c}_d$ represents the vector of damper coefficients; Θ is the total expected loss and is given as,

$$\Theta=\sum_{i=1}^{N_d}(\theta_i) \tag{5}$$

Here θ_i refers to the expected total loss at the i^{th} degree of freedom computed based on the maximum peak response. In normal structures, non-structural components especially drift-sensitive components are attached to the primary structural members. Any damage to the non-structural component thus is a result of the response of the structural component to which it is attached. So in the present study the non-structural component is assumed to be discretely lumped at the nodes of the structure to which it is attached. An effective distribution of the component is assumed for each structural node. Only the translational degree of freedom of the lumped node is considered in the loss computation. The idea is very similar to a lumped mass concept used in structural dynamics.

Mathematically θ_i is given as,

$$\left.\begin{array}{l}\theta_i=\chi\left(\max\left(abs\left(\mathbf{r}_i(t)\right)\right)\right)\\ \text{where } \chi \text{ represents a function form as given in [24]}\\ \text{and } \mathbf{r}_i(t) \text{ is the drift response vector which satisfies the following equation,}\\ \mathbf{M}\ddot{\mathbf{u}}(t)+\left[\mathbf{C}+\mathbf{C}_{damper}(\mathbf{c}_d)\right]\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=-\mathbf{M}\mathbf{I}\ddot{u}_g(t)\\ \mathbf{u}(0)=0;\dot{\mathbf{u}}(0)=0\end{array}\right\}\forall\ddot{u}_g(t) \tag{6}$$

3.0 Optimization Algorithm

An aggregate gradient based multi-objective framework is used for the study [25]. As already explained in the introduction section in contrast to the first order optimization schemes used by the earlier works, a gradient based approach is highly efficient as it requires very less function evaluations. This section gives stepwise implementation scheme of the optimization procedure. Though the algorithm only employs intensity based assessment for loss computation, it is equally valid for time based assessments. An ensemble of ground motions is selected to match the target mean spectrum corresponding to the specific intensity level of interest. Following steps are involved in the proposed optimization scheme.

Step 1: Identification of critical ground motion

A methodology to identify the active ground motions is given in [16]. For the whole ground motion ensemble matching the target mean spectrum, spectral response curves of a single degree of freedom having the same fundamental natural frequency as that of the parent structure vs. the damping coefficient is generated. In the present study the maximum displacement is taken as the response quantity. The ground motion which produces the largest spectral response curve for a reasonable range of damping is taken as the critical active ground motion. This significantly reduces the analysis effort and makes the scheme more appealing for practical application.

3.1 Start of Multi-Objective Framework [25]:

The aggregate gradient based methodology adopted for the multi-objective optimization framework is presented in detail in this section.

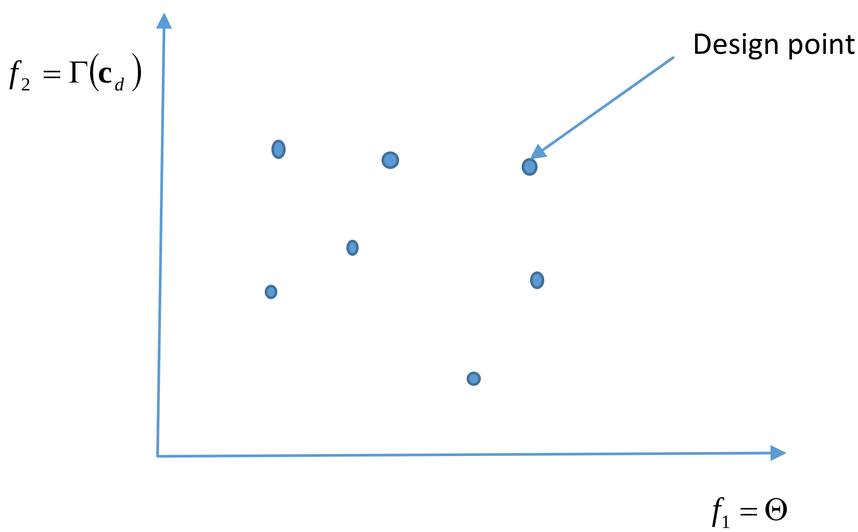

Fig. 1 Objective function space

Step 2 Initialization of design variables and generation of initial design points

Design point is basically obtained by computing the two objective functions given in eq. (4) assuming a specific random distribution for the design variable which are the damper coefficients. Mathematically this means, for q design variables (damper coefficients), generate K design points using random values for the design variables. For e.g. if we assume q=2 and K=7, then there are seven random distributions of the two dampers and each design point in the objective function space as shown in fig.1 corresponds to evaluation of the objective functions in the eq. (4) subject to the constraint on the damper coefficients. The objective function $\Gamma(\mathbf{c}_d)$ is simply the addition of the capacities of the two dampers which reflect the initial cost that is needed and the evaluation of the expected total loss Θ corresponds to the evaluation of eq. (5) through eq. (6).

Step 3 Compute weighting coefficients as per Data Envelopment Analysis (DEA)

In order to generate the Pareto front, the design points shown in fig. 1.0 needs to move towards the Pareto frontier point that is closest to its current position in the objective function space. But as the Pareto frontier is not known prior to optimization calculation, the points in the objective function space needs to be updated using an adaptive weighting method. Only a very brief detail is given in this step and for details interested readers should refer to [25].

DEA computes the efficiency of the M^{th} point by solving a linear programming problem as,

$$\left.\begin{aligned}&\min\left(\Lambda^{M}\right)=\sum_{i=1}^{m} w_i^M\, f_i^{\,M}\\&\text{Subject to,}\\&\sum_{i=1}^{m} w_i^M\, f_i^{\,k}\geq 1\\&w_i^M\geq 0\end{aligned}\right\}\ (\text{for k} = 1,2,\ldots\ldots\ldots K) \qquad (7)$$

Here $f_i^{\,k}$ is the k^{th} point's i^{th} objective function value and w_i^M represents the weighting coefficients.

Step 4 Compute the sensitivities of the objective functions for the M^{th} point

Gradient for the objective function Γ^M is trivial as it is a direct function of the damping vector $\mathbf{c}_d$ and the sensitivity will return a vector **1.** But the gradient of the objective function Θ^M is not trivial. One way to determine the gradient is by finite difference approach; but this has serious limitations in terms of computational demand as it requires *n+1* analysis for *n* design variables. So in the present study gradients are computed analytically using the adjoint Variable method as outlined in [26]

Step 5 Update the design variables of the M^{th} point

A minimization of the weighted sum of the objective functions is done using sequential linear programming (SLP) and the design variables are updated. SLP uses a suitable move limit in order to arrive at the updated value of the design variable. For the M^{th} point we get,

$$\left.\min\ f^{\,M} = \sum_{i=1}^{m} w_i^M \sum_{j=1}^{n} \frac{\partial f_i\left(\mathbf{c}_d^M\right)}{\partial c_{dj}} c_{dj}\right\} \qquad (8)$$

Subject to

$$\mathbf{c}_d^L \leq \mathbf{c}_d \leq \mathbf{c}_d^U \qquad (9)$$

Here $f^{\,M}$ is the weighted sum of the objective functions, $\mathbf{c}_d^M$ is the design variable vector of the M^{th} point before updating, $\mathbf{c}_d^L$ and $\mathbf{c}_d^U$ are the lower and upper move limits of the design variable. If $M=n$ then proceed to step 6, else adopt $M=M+1$ and proceed to step 5. For more details on this step interested readers should refer [25].

Step 6 Check for termination condition

If termination condition is satisfied (maximum number of iteration), the procedure ends else returns to step 2.

4.0 Numerical Study

A four story reinforced concrete frame described in [27], designed in accordance with Eurocode 8 (EC8) and Eurocode 2(EC2) is used to illustrate the proposed optimization procedure. The frame is designed for high seismicity assuming a PGA of 0.3g. The geometric details of the frame with the location of the partition walls and the arrangement of the dampers are given in Fig.2. It has to be noted that in the analysis, partition wall was not modelled and only the bare frame with the dampers are analyzed. Modelling details for the frame is given in Appendix A.

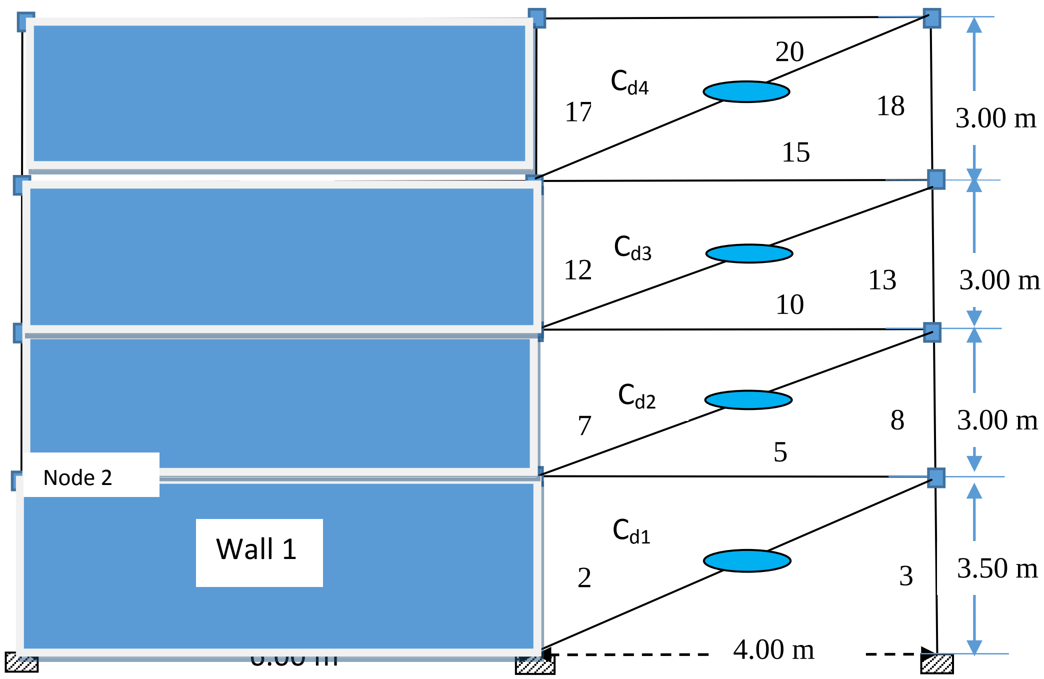

Fig. 2.0 C_{di} refers to added dampers and $i=1\dots4$

As the whole purpose of this paper is to demonstrate the multi-objective optimization framework, only loss to partition walls is presented in this study. Normalized loss is computed as described in [24]. Losses in the partition walls are assumed to be lumped to the nodes of the bay to which the wall is attached by assuming a suitable tributary area. So for e.g. for wall 1, 50% of the loss is lumped to node 2 shown in fig. 2. The interstorey drift associated with node 2 which is the same for the other nodes at the same level (in the case of node 2, the first floor leve) causes the loss in the partition wall.

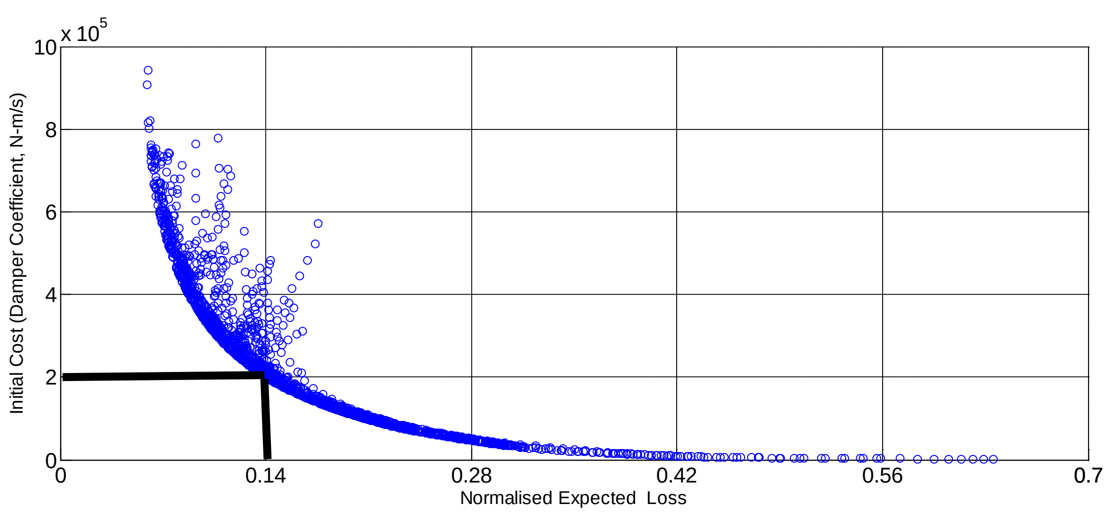

Fig. 3 Migration of generated solutions to the pareto-front.

A suite of 7 artificial ground motions scaled to match a EC8 design spectra with PGA 0.45g is used for the present study. Un-controlled frame analysis has revealed that this level of ground motion intensity can incur inelastic excursions in the parent frame due to drifts greater than the order of about 1.5% [27]. It has been observed in the public response to the Christchurch sequence of earthquakes that when the building tends to yield the owners tend to demolish the buildings as they could claim insurance to build new ones. So an effective retrofitting scheme should incur minimum yielding state in the parent structure. To achieve this objective in the present study the parent frame is assumed to be linear. Only drift sensitive non-structural loss is accounted in the present study. Multi-objective optimization is performed as per the methodology described in section 3.0. Sequential Linear Programming method is used for solving eq. (8). For the present study only 40 design points are generated in the objective space, i.e. K=40 in step 2 and $q=4$ as there are only 4 dampers. Constraint *move limit* as required by eq. (9) is adopted as 5% of the design damping vector.

Fig. 3 shows all the generated solutions and its migration of the solutions to the Pareto frontier. Each of this point in the objective function space corresponds to a specific quantity of dampers and its distribution. In order to illustrate this, a specific point is adopted on the pareto front as shown in fig.3. The selection of the point on the Pareto front is shown in fig. 3. This point corresponds to a total expected normalized loss of ~14% loss and a total quantity of dampers is $200\ kNms^{-1}$. The distribution of the dampers corresponding to this quantity across the height of the buiding is shown in fig. 4.

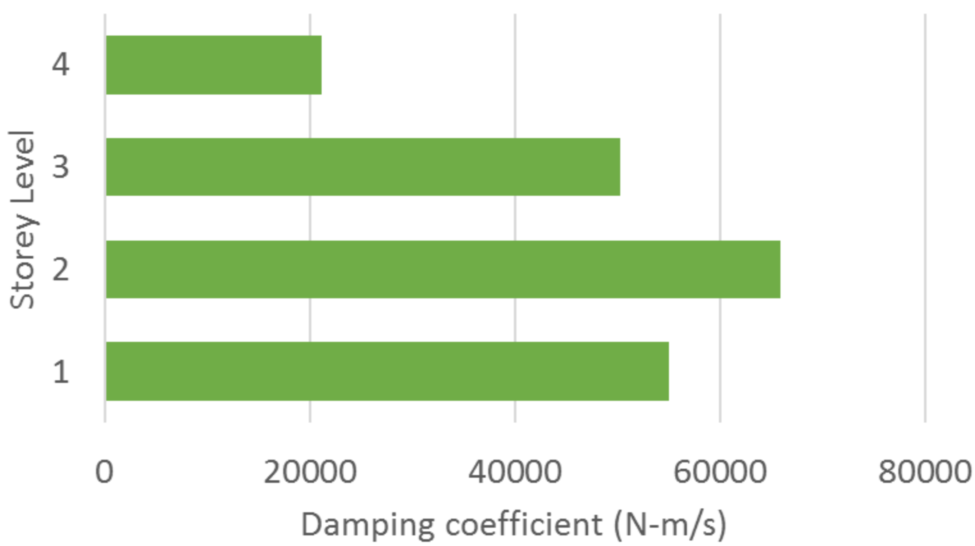

Fig. 4 Optimal distribution of dampers of the selected point in the pareto front

The biggest advantage of this method is that both quantification and distribution of the dampers are achieved simultaneously. It could be clearly seen that as the initial cost increases the loss is minimized considerably.

The Pareto front shown in fig. 3 along with the migrating points illustrates a clear trade-off for the choice of the expected loss and the initial cost. Each point on the pareto front corresponds to a solution which is obtained by the degraded performance of one of the objectives. For e.g., a point on to the extreme right on the x-axis presents a solution where there is a very high loss but minimum initial cost and similarly a point to left we can obtain a minimum loss scenario but with a very high initial cost. So a whole sphere of possible solutions is represented by the Pareto front and enables the designer to quantitatively weigh the trade-offs; i.e. whether to favor one objective in the expense of a degraded performance of the other. Also in the obtained Pareto front it could also be seen that, minimal added damping results in a very large decrease in loss. The point selected to generate fig 4.0 represents such a neighborhood of possible solutions. But for a further reduction in loss, at some point to the left of this point, it could be seen that a very large added damping is required. So it becomes very evident that the generation of the pareto front enables the stake holder to decide on what level of loss is acceptable and gives a more powerful decision making scenario.

5.0 Conclusions

A multi objective optimization problem formulation is presented for the design of viscous dampers in seismic retrofitting. The formulation adopts the initial cost of retrofitting as one objective and the expected loss as the other. Thus, with the obtained Pareto front at hand, the stakeholder, or the decision maker, can choose the best compromise between the two without making any decisions on weighting functions or constraints a-priory. An aggregate gradient based multi-objective framework is adopted for the solution of the problem. Being gradient based, the framework is very efficient for the problem at hand, where the function evaluation is highly computationally demanding. Full detail step by step procedure of the optimization framework is illustrated. Numerical study illustrates that the proposed optimization framework presents itself with a powerful decision making tool for the stakeholder by generating the complete Pareto front. Each point on the Pareto front corresponds to a specific optimal quantity and distribution along with the associated expected total loss.

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Appendix A: Modelling details of the four storey frame

Material Property

Dynamic Young's modulus of concrete $= 3.5 \times 10^{10}\ Nm^{-2}$

Geometric Properties

Member number	Width (mm)	Depth (mm)
1,6,11,16,17,12,7,2,3,8,13,18	450	450
4,5,9,10,14,15,19,20	300	450

Nodal Mass

Floor level	Mass per node (kg)
1st floor	29 800
2nd -4th floors	29 500

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