

STOCHASTIC MODELING OF GROUND MOTIONS MATCHING SPECTRAL ACCELERATION, CUMULATIVE ARIAS INTENSITY AND DURATION

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Abstract

In performance-based earthquake engineering, a large number of ground-motion time histories are needed for analyzing the distribution of dynamic response of nonlinear systems. In current design practice, ground motion effects on structures are represented by an elastic response spectrum, yet the response spectrum doesn't contain other important ground-motion features such as energy build-up, duration as well as ground-motion nonstationarity, which are found to be important in the analysis of certain types of structures. In this study, a stochastic ground-motion simulation and modification technique is developed to generate energy-compatible and spectrum-compatible (ECSC) synthetic motions through wavelet-packet characterization and modification. The ECSC method significantly advances traditional ground-motion modification approaches, because it generates ground motions that not only match target spectral accelerations, but also match other important parameters such the Arias intensity, its temporal accumulation, as well as measures of ground-motion duration. A salient feature of the ECSC method is that ground motions are modified in both frequency and time domain iteratively. The great similarity between the ECSC simulated motions and the actual recorded motions is demonstrated through one-to-one comparison of a variety of intensity measures. Furthermore, nonlinear structural responses subjected to the ECSC ground motions and their recorded counterparts are compared using a 12-story reinforced concrete perimeter frame model. The numerical analyses demonstrated that the NGA and ECSC ground motions result in very consistent structural responses. The ECSC method can be easily implemented in the generalized conditional intensity measure (GCIM) framework to directly simulate a set of motions following a targeted distribution of multiple intensity measures. The simulation can be repeated for many realizations such that a collection of the simulated motions will follow the targeted distribution of spectral accelerations, duration and Arias intensity for a specific earthquake scenario. Therefore, the ECSC method has great potential to be used in performance-based earthquake design and analysis.

Keywords: synthetic ground motions, spectrum matching, wavelet packet analysis, ground-motion duration



1. Introduction

In performance-based earthquake engineering, a large number of ground-motion time histories are needed for analyzing the distribution of dynamic response of nonlinear systems. However, actual recorded ground motions are often limited. Especially, recorded motions at design levels are very rare, and may not be sufficient for characterizing structural responses. In current engineering practice, ground-motion time histories need to be selected and modified from recorded strong-motion database [1-4], or numerically generated using either deterministic or stochastic approaches. In this paper, stochastic simulation refers to a class of methods that directly generate ground-motion time histories using a few empirically calibrated parameters [5, 6]. Compared with physics-based approach, stochastic simulation doesn't rely on seismological principles to describe the seismic source mechanism and wave travel path that vary significantly from region to region. Instead, model inputs of the stochastic method are seismological parameters that directly related to a specific earthquake scenario and site condition, which can be readily used by practicing engineers.

In current design practice, ground motion effects on structures are represented by an elastic response spectrum, which is defined as the peak acceleration of a damped elastic single-degree-of-freedom system under the input motion. For design purposes, ground motions need to be simulated or modified to match a target response spectrum. For example, most existing ground motion selection and modification (GMSM) methods develop time-history datasets in aggregate, whose response spectra can resemble a specified target spectrum. Many studies have highlighted the importance of spectral shape of ground motions at the design level in the GMSM process [7]. On the other hand, ground motions can be stochastically simulated to be compatible with a prescribed response spectrum. A notable example is SIMQKE, which uses Gaussian random process and a time-varying modulating function to generate ground motions matching a target spectral shape [8]. However, one of the limitations of the above procedure is that the simulation process doesn't link to any seismological environment. Also, frequency content of simulated accelerograms cannot vary with time.

Given a response spectrum, ground motions that reasonably match the target response spectrum are not unique, because the response spectrum provides only a partial picture of real ground-motion characteristics and doesn't contain other important features such as energy build-up, duration as well as ground-motion nonstationarity. These parameters have been found to be important in the analysis of certain types of structures. For example, Chandramohan *et al.* [9] demonstrated that ground-motion duration is important in risk assessments of structural collapse; Wang [10] showed that the Arias intensity (Ia), in addition to spectral acceleration, is critical for estimating seismic slope displacements.

Therefore, rigorous ground-motion selection and modification process requires consideration of multiple intensity measures (IMs) collectively. It has been demonstrated that considering multiple IMs are important in seismic hazard analyses [11]. Recently, a generalized conditional intensity measure (GCIM) approach was developed that allows the construction of conditional distribution of multiple ground-motion IMs. Following this line, ground motion can be selected based on joint distribution of specific IMs derived from the GCIM framework [12]. However, none of the existing method can directly simulate ground-motion time histories targeting at multiple IMs.

In this work, a stochastic ground-motion simulation and modification technique is developed to generate energy-compatible and spectrum-compatible (ECSC) synthetic motions through wavelet-packet characterization and modification. The stochastic ground-motion simulation model used in this study was firstly developed by Yamamoto and Baker [13] based on wavelet-packet transform (WPT). Compared with many of its kind, WPT has basis functions that are orthogonal and localized in time and frequency domains. The salient feature allows for flexible control of the temporal and frequency resolution of ground-motion nonstationarity and easy reconstruction of waveforms, as the wavelet-packet coefficients can be independently modified on time axis and on frequency axis. Therefore, one can simulate ground motions with target time and frequency characteristics including response spectra, cumulative Arias intensity and duration. The current study can be regarded as stochastic modeling of ground motions conditional on given seismological environment, as well as response spectrum, duration and cumulative energy. In this paper, an example is illustrated to directly simulate suite of ground motions based on conditional distribution of IMs including spectral acceleration, $D_{5.75}$, $D_{5.95}$ and Arias



intensity build-up. This example demonstrates the great potential for the ECSC method to be implemented in the GCIM framework.

2. Stochastic model for generating energy compatible and spectrum compatible ground motions

2.1 Stochastic modelling of ground motions using wavelet packets

Artificial ground motions can be generated using many numerical methods. As one special form of spectral presentation, the wavelet packet transform was recently used to simulate nonstationary ground motions by Yamamoto and Baker [13]. The wavelet packet transform is an advanced time-frequency analysis tool that decomposes a ground-motion time series x(t) into a set of wavelet packets localized in the time (t) and frequency (f) domain. For a time series x(t), the wavelet packet coefficients are defined as:

$$c_{j,k}^{i} = \int_{0}^{\infty} x(t) \psi_{j,k}^{i}(t) dt$$
 (1)

where $\psi_{j,k}^{i}(t)$ represents the wavelet packet basis function, which is chosen as the Meyer wavelet function throughout this study, due to its good orthogonal property for time/frequency domain decomposition. The method has several salient features: (1) Model parameters have been empirically calibrated as functions of seismological variables such as earthquake magnitude, source-to-site distance and site conditions etc. Therefore, synthetic ground motions can be generated based on hazard-consistent scenarios etc. Ground motions can be synthesized based on hazard-consistent scenarios (2) Simulated motions from the stochastic model have been validated with similar median and variability for a variety of IMs, which are consistent with empirical groundmotion prediction models (GMPM). Therefore, the model can be used to extrapolate for simulating future earthquakes, and (3) the stochastic model can effectively characterize both amplitude and frequency nonstationarity of ground motion. The nonstationarity of ground motions can greatly affect nonlinear structural responses. It is worth mentioning that the stochastic model has also been extended for simulating spatially correlated ground motions by Huang and Wang [14, 15].

Fig. 1 illustrated an example of decomposition of a ground-motion time history using wavelet-packet analysis. Given an acceleration time history as shown in Fig. 1(a), the time and frequency distribution of squared wavelet-packet coefficients is the wavelet-packet spectrum (WPS), as presented in Fig. 1(b). In this example, the time series contains 4096 data points with a time interval 0.01 s. The resolution of the wavelet-packet spectrum is 0.1953 hz in the frequency domain and 2.56 s in the time domain. Therefore, the WPS contains 16 column in the time domain and 256 rows in the frequency domain. Using inverse transformation, ground-motion time history x(t) can be precisely reconstructed through summation of wavelet packets over all columns and rows:

$$x(t) = \sum_{i=1}^{2^{j}} \sum_{k=1}^{2^{N-j}} c_{j,k}^{i} \psi_{j,k}^{i}(t)$$
(2)

Additionally, the build-up of Arias intensity with time is known as the Husid function, which can be normalized by Arias intensity, such that the normalized function asymptotically approaches one as time increases. As illustrated in Fig. 2, the normalized Husid plot shows the build-up of energy of an accelerogram with time. Intuitively, the shape of H(t) can be determined by summing up the squared coefficients in each column. It is important to emphasize that the temporal distribution of energy content can be fully described and modulated using accumulation of column summation of coefficients over time, that is, the Husid function can be adjusted via modifying the wavelet-packet coefficients in the corresponding column, as shown in Fig. 1 (d). On the other hand, the response spectrum can be modified by adjusting the wavelet-packet coefficients in the corresponding frequency, as shown in Fig. 1 (c). The method takes full advantage of orthogonality of the wavelet packets, so time/frequency characteristics can be varied rather independently. As is well known, the time interval for 5 to 95 percent energy build-up is defined as significant duration, $D_{5.95}$, and the time interval for 5 to 75



percent energy build-up is defined as significant duration, D_{5-75} . Therefore, matching the Husid function is more restrictive requirement than just matching a target Arias intensity and duration.



Fig. 1- Wavelet-packet spectrum, showing the distribution of squared wavelet-packet coefficients of the recorded acceleration time history at the San Valley – Roscoe Blvd station in the 1994 Northridge earthquake.



Fig. 2 – Normalized Husid function and fitted lognormal CDF



To quantify the distribution of WPS, 13 wavelet parameters are proposed by Yamamoto and Baker to characterize the wave energy and time-domain/frequency-domain distribution of the wavelet coefficients. The mean of each wavelet parameter can be predicted using seismological variables, such as the moment magnitude, site-to-source distance and site conditions, through regression analysis of strong-motion data:

$$\overline{Y}(M, R, V_{s30}) = \alpha + \beta_1 M_w + \beta_2 \ln(M_w) + \beta_3 \exp(M_w) + \beta_4 (R_{hyp} - R_{rup}) + \beta_5 \ln \sqrt{R_{rup}^2 + h^2} + \beta_6 \ln(V_{s30})$$
(3)

where \overline{Y} denotes median of natural logarithm of a wavelet-packet parameter, M_w is moment magnitude, R_{rup} is the rupture distance, R_{hyp} is the hypocenter distance and V_{s30} is the average shear-wave velocity in top 30 m. The empirical model can be used to generate synthetic motions based on hazard-consistent scenarios.

2.2 Algorithm for modifying ground-motion time series

In this section, ground motions are initially simulated and iteratively modified to match the spectral acceleration and Husid function from a recorded target motion. Wavelet-packet coefficients $c_{j,k}^i$ of the initial motion x(t) are adjusted in the time and frequency domains iteratively to match a target response spectrum and target accumulative wave energy. The step-by-step illustration of the algorithm is presented as follows.

1. Generate a seed motion given a specific earthquake scenario

An initial seed motion x(t) is first generated following Eq. (3), given a hazard-consistent earthquake scenario (M, R, V_{s30} etc.)

2. Adjustment of wavelet-packet coefficients $c_{i,k}^{i}$

The seed motion is modified in time and frequency domain iteratively via applying a multiplier w(i,k) to the wavelet packet coefficients, or applying an enveloping function. The adjustment can be realized using a simple procedure to minimize the mismatch of spectrum and energy build-up iteratively.

2(a) Frequency-domain modification

Using superscript (*n*) to denote quantities of the *n*th iteration, the simulated ground motion can be written $y^{(n)}(t) = \sum_{i=1}^{2^j} \sum_{k=1}^{2^{N-j}} c_{j,k}^{i(n)} \psi_{j,k}^i(t)$. The weight of the WPS coefficients for the $(n+1)^{th}$ iteration $w^{(n+1)}(i,k)$ can be determined exception of the structure of the str

be determined according to the ratio of the target to the simulated spectral acceleration at frequency f_i .

2(b) Time-domain modification

Accumulation of Arias intensity corresponds to the summation of acceleration over time. To match the energy build-up process, the increment of Husid function of the target and simulated motions should match within each WPS time interval. Accordingly, a time-domain adjustment factor is applied to the acceleration time history in each time interval interactively to match the target Husid function. It is equivalent to apply a step-wise enveloping function to the time histories.

3. Evaluate compatibility of H(t) and Sa(f)

To measure the compatibility of spectral acceleration and Arias intensity build-up, the mean squared errors (MSE) is adopted to quantify the relative difference in Sa and H(t) between the simulation and the target. Steps 2(a) and 2(b) will be iterated until the MSE in Sa and H(t) reach a desired level. In the last, baseline correction on the modified accelerogram will be performed to avoid drift in velocity and displacement time



It is worth mentioning that the proposed approach is different from many existing spectrum-matching approaches that adjust ground-motion time histories either in the frequency domain or in the time domain [16, 17]. Although the time-domain approach can result in a modified motion that preserves the normalized Arias intensity build-up process of the seed motion, it has been reported that the procedure cannot control the value of Arias intensity itself [18]. The method is only suitable for slight modification of the spectral shape and relies heavily on the sensible choice of seed motions.



Fig. 3 - Illustration of the ECSC simulation and modification algorithms

2.3. An illustrative example

Fig. 4 further illustrates the convergence of simulated response spectrum and Arias intensity build-up, measured by mean squared errors in Eq. (8) and (9). In general, the mean squared errors keep reducing with increasing number of iterations, and reaches a stable state after 20 iterations.



Fig.4 – Comparison of time histories, response spectra and Arias intensity build-up for the ground motion recorded at the Foster City during the Loma Prieta earthquake from the ECSC dataset and NGA dataset

3. ECSC Simulated ground-motion dataset

In this study, a large number of ground-motion records from three large earthquakes in California are selected in the validation test. They are the 1979 Imperial Valley earthquake, the 1989 Loma Prieta earthquake and the 1994 Northridge earthquake. Ground motions are recorded in both strike-normal and strike-parallel directions and are retrieved from the PEER-NGA strong-motion database. Using the ECSC method, each actual recorded motion is stochastically simulated using their spectral accelerations and accumulative Arias intensity, as well as seismological variables (M, R, V_{s30}) associated with that record as targets. In this way, 562 waveforms are "reproduced" and referred to as the ECSC simulated dataset.

We select a group of 50 simulated ground motions and their counterparts from the ECSC dataset and the NGA database, as shown in Fig. 5. Ground motions in the ECSC dataset and their recorded counterparts in the NGA dataset are compared in terms of seven important ground-motion IMs, including PGA, PGV, PGD, Ia, CAV and D_{5-95} . The residual term $r_{\ln IM}$ is defined to quantify the relative difference between IMs (in the natural log scale), as follows:

$$r_{\ln IM} = \ln \left(IM_{NGA} \right) - \ln \left(IM_{ECSC} \right) \tag{4}$$

From Fig. 5, it is clear that the mean residuals between these pairs of motions are generally limited to 5% error bound for a variety of different IMs. The standard deviations of residuals are also limited to 10%, except for PGV and PGD. Note that PGV and PGD are parameters hard to control in the ECSC simulation. The ECSC ground motions are generally similar to the recorded ones in the first place, before comparisons can be made to study their effects on the structural responses.



Fig. 5 – Selected 50 simulated and recorded ground-motion pairs for oscillator analyses. (a) Comparison of spectral accelerations, and (b) mean and standard deviation of IM residuals.

4. Time history analyses of structural responses using ECSC and NGA datasets

4.1 The structural model

Although the ECSC simulated motions resemble recorded motions in terms of response spectrum and accumulative energy, their capability to produce similar structural responses should be tested. The efficiency of the ECSC scheme is demonstrated in this section to predict the nonlinear response of buildings using the NGA and ECSC dataset. The structural model utilized in this study is a modern 12-story reinforced concrete perimeter frame building designed according to the 2003 International Building Code and ASCE7-02. The finite element model, as shown in Fig. 6, was developed in OpenSees by Haselton (known as Building ID 1013 in [19]). The building represents typical ductile frame system with a first-mode period of 2.01 s. The same model has been utilized by the PEER GMSM Working Group to conduct benchmark tests on various ground motion selection and modification methods.

4.2 Predicted structural response

Nonlinear numerical analyses were performed to investigate the seismic response of the 12-story building using NGA and ECSC dataset. The maximum inter-story drift ratio of all stories (MIDR) is selected to evaluate the seismic response. Fig. 7 (a) and (b) show the distribution of the MIDRs for each story using the two datasets, with their mean and standard deviation highlighted in both figures. The story-by-story distribution of MIDRs provides important details for structural optimization. It can be seen that the MIDR distribution of the ECSC dataset is very consistent with that of the NGA dataset. Fig. 8 (a) summarizes the predicted median value and median \pm one standard deviation curves for the two datasets. Consistent results can be observed for all building



stories, indicating the plausibility of the ECSC method in capturing the variability of detained structural response. To further examine the nonlinear performance of structures, ground motions from ECSC and NGA dataset are scaled with a factor of 2. Fig. 8 (b) shows the predicted median value and median \pm one standard deviation of MIDR for the two datasets. It can be seen that excellent agreement between structural responses for all stories from recorded and simulated ground motions is achieved.





Fig. 7 – Distribution of the story-by-story MIDRs using (a) NGA dataset and (b) ECSC dataset



Fig. 8 – Summary of the story-by-story MIDRs using NGA and ECSC dataset with (a) scale factor = 1, and (b) scale factor = 2.



5. ECSC simulation used in GCIM framework

In this section, the ECSC method is implemented in the GCIM framework to generate ground-motion time histories that match multiple target IMs. Different from conventional GCIM practice, ground motions are directly simulated instead of being selected from a database. First, an earthquake scenario is specified: $M_w=7$, $R_{rup}=10$ km, strike-slip faulting. Fig. 9 (a) shows the distribution of 60 ground-motion realizations conditioned on T = 1s with epsilon = 2 for this scenario. The cumulative distribution of significant duration parameters $D_{5.75}$ and $D_{5.95}$, and Ia can be determined through multivariate random realizations using empirical cross-correlation between these IMs [12]. Fig. 9 (a) also highlights a single realization of the spectral acceleration. Its corresponding values of $D_{5.75}$ and $D_{5.95}$, and Ia are computed as 5.1 s, 9.3 s and 0.16 g×s, respectively, which are specified as selected targets for ECSC simulation in this case, as shown in Fig. 9(b) and 9(c). Following this line, a target Husid function can be easily constructed by assuming the Arias intensity build-up process follows a lognormal CDF distribution, given $D_{5.75}$ and $D_{5.95}$, and Ia.

Acceleration, velocity and displacement time histories of the ECSC simulated ground motion following the selected targets are presented in Fig. 10 (a). Fig. 10(b) and 10(c) compare the IMs of the simulated ground motion with the specified targets. Clearly, both response spectrum and Arias intensity build-up of the simulated motion agree well with the targets. It is worth mentioning that $D_{5.75}$, $D_{5.95}$ and Arias intensity of the ECSC simulated motion are calculated as 5.2 s, 9.4 s and 0.16 g×s, respectively, which are in a good agreement of selected targets following the GCIM framework. This ground-motion simulation procedure can be repeated for other selected targets of *Sa*, $D_{5.75}$, $D_{5.95}$ and *Ia* such that a full set of simulated motions can be generated to follow the targeted distribution of these multiple IMs for a specific earthquake scenario. Therefore, the ECSC method can be implemented in the GCIM framework to directly simulate ground-motion time histories given an earthquake scenario.



Fig. 9 –Acceleration, velocity and displacement time histories of the simulated ground motion, (b) Target and simulated spectral acceleration and (c) Arias intensity build-up.







Fig. 10 –Acceleration, velocity and displacement time histories of the simulated ground motion, (b) Target and simulated spectral acceleration and (c) Arias intensity build-up.

6. Conclusions

A new ground-motion simulation and modification procedure was presented that allows the generation of energy-compatible and spectrum-compatible (ECSC) ground motions through wavelet-packet characterization and modification. The wavelet-packet transform has basis functions that are orthogonal and localized in time and frequency domains. The salient feature allows for ground-motion time histories to be flexibly adjusted in frequency domain and time domain simultaneously, thus, modifying their response spectrum and cumulative energy. The procedure is based on three key steps, starting from prediction of a seed motion using seismological constraints (M, R, V_{s30}), followed by iterative adjustments of the wavelet-packet spectrum in both time and frequency domain, and finally, evaluating compatibility of response spectrum and cumulative energy of the simulated motion with targets.

The simulated ECSC ground motions have similar spectral accelerations, cumulative Arias intensity and durations compared with their counterparts recorded in the PEER-NGA database. Furthermore, nonlinear structural responses subjected to the ECSC ground motions and their recorded counterparts are compared using a 12-story reinforced concrete perimeter frame model. The numerical analyses demonstrated that the NGA and ECSC ground motions result in very consistent structural responses. More examples to validate the ECSC ground motions using other structural types or geotechnical systems can be found in Huang [20].

As the ECSC procedure can be regarded as stochastic ground-motion modeling conditioned on a given seismological environment, response spectrum, duration and cumulative energy, the ECSC technique can be easily implemented in the GCIM framework to directly simulate suitable ground motions for performance-based earthquake design and analysis. In this paper, an example is presented to directly simulate ground motions based on conditional distribution of IMs including *Sa*, $D_{5.75}$, $D_{5.95}$ and *Ia*. The simulation can be repeated for many realizations such that a collection of the simulated motions will follow the targeted distribution of spectral accelerations, duration and Arias intensity for a specific earthquake scenario. Different from the previous GCIM studies, the ground motions are directly simulated instead of being selected from a database. The ECSC simulation software can be accessed from the authors' website at http://ihome.ust.hk/~gwang/ ECSC_simulation.html.

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