



Evaluation of inhomogeneous structures in seismic propagation path in Japan based on the fractal characteristic of observed earthquake motion phase

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Abstract

The amplitude of seismic ground motion attenuates because of the wave scattering phenomenon due to the inhomogeneity of the wave propagation path. In the framework of strong ground motion estimations, this attenuation effect is modeled by Q-value. In the past study, Q-value in and around Japanese Archipelago has been estimated by the tomography using observed ground motions and Spectral Inversion Technique proposed by Iwata and Irikura (1986). Sato (2013) has developed a new methodology to evaluate inhomogeneity of wave propagating medium using phase characteristics of observed strong ground motions. It is assumed that the phase characteristics of ground motion have the self-affine similarity because the inhomogeneous crustal medium of the wave propagation path shows the self-affine similarity nature. The self-affine similarity of phase is evaluated by Hurst index. In this paper, the inhomogeneity of the wave propagation medium in the whole of Japan is evaluated based on Hurst index which is calculated by the Sato's method. It is found that distribution of Hurst index has a regional locality and its characteristics are consistent with scattering Q-value distribution evaluated in the past study. From these results, we can simulate earthquake motion phase considering the regional inhomogeneity of medium.

Keywords: phase characteristics, Hurst index, Fractal, Scattering, Q-value

1. Introduction

Earthquake motion generated from an epicenter is scattered due to the inhomogeneity of wave propagation medium. Therefore, the observed ground motion is composed of not only simple P-wave and S-wave but also the wave group called S-wave coda. It is the clearest evidence that wave propagation medium has the inhomogeneity [1]. The study about the scattering of earthquake motions has been energetically carried out. As beginning from the stochastic approach by Aki [2] and Aki and Chouet [3], it has developed to the theoretical clarification of the scattering phenomena [4] and the modeling of the entire envelope shape of earthquake motion [5]. In recent years, the simulation of scattering phenomena has also been analyzed in not only random medium but also the cracks and voids. The state-of-arts of the scattering phenomenon is detailed in [6].

For keeping high accuracy of strong ground motion estimations, evaluation of the attenuation structure of wave propagation medium around Japan has been promoted. There are a geometric and inertial damping, scattering in the inhomogeneous medium as the factors of ground motion attenuation. The latter two of these factors are generally quantified by Q-value.

Evaluation of Q-value has been performed by two methods. The first method is the tomographic inversion of observed ground motion data. Hashida and Shimazaki [7] have identified the attenuation structure using seismic intensity data. Although the physical relationship between seismic intensity and Q-value is ambiguous, it is possible to identify a wide 3-D structure because the great amount of observation records can be used. Thereafter, large-scale problems can be solved by the development of ARTB method [8], so it implemented in the Q-value inversion method for the whole Japan by Nakamura et al [9]. In Nakamura [10], The 3-D attenuation structure is evaluated by using the ground motion data observed in the high density observation network which has been established after the Southern Hyogo prefecture earthquake in 1995.

The second method for the Q-value estimation is based on the spectrum of strong ground motion. Q-value in and around Japanese Archipelago has been estimated using observed ground motions and Spectral Inversion Technique proposed by Iwata and Irikura [11].

Sato [12] has developed a new methodology to evaluate inhomogeneity of wave propagating medium using phase characteristics of observed strong ground motions. It is assumed that the phase characteristics of ground motion have the self-affine similarity because the inhomogeneous crustal medium of the wave propagation path shows the self-affine similarity nature. The self-affine similarity of phase is evaluated by Hurst index. In this paper, we purpose a method to estimate inhomogeneous structures in the seismic propagation path using Hurst index. The distribution characteristics of Hurst index in the whole of Japan is compared with the attenuation structure modeled by Q-value distribution obtained in the past studies.

2. The methodology to evaluate inhomogeneous structures

Earthquake motion generated from the epicenter propagates in inhomogeneous medium until it reaches at the observation site. The earthquake motion from the epicenter is repeatedly reflected and refracted in propagation medium, so the ground motion observed at the site contains the phase delay caused by the scattering phenomenon in the inhomogeneous medium.

Because there exists the self-affine similarity between the macroscopic and microscopic structure in the wave propagation medium, it is possible to assume that the phase characteristics of ground motion has the self-affine similarity. We propose a method to evaluate the inhomogeneity of the propagation medium by focusing on the phase characteristics of the ground motion in this paper. Based on the discrete Fourier transformation, a general time function $f(t)$ is expressed by Eq. (1).

$$f(t) = \sum_{i=1}^N A(\omega_i) \cos(\omega_i t + \phi_i(\omega_i)) \quad (1)$$

If the cosine time function $\cos(\omega_i t)$ is assumed to be generated at the epicenter, its phase $\phi_i(\omega_i)$ changes through the propagation path (Fig.1). In Fig.1, L denotes the length of the shortest path from the seismic source

to the site. We assume that there are many wave propagation paths. M is the number of total propagation paths. L_j denotes the length of j^{th} path and ΔL_j denotes the difference between the shortest path, L and L_j . When the phase velocity of the propagation wave is v , the travel time t_j from seismic source to site at the j^{th} path is given by Eq. (2).

$$t_j = \frac{L_j}{v} = \frac{L}{v} + \frac{\Delta L_j}{v} = \frac{L}{v} + \Delta t_j \quad (2)$$

Δt_j represents the difference of the travel time between the shortest path and the j^{th} -path. In case of existing many propagation paths from seismic source to site as shown in Fig.1, the phase delay $\phi_i(\omega_i)$ should rewrite as Eq. (3).

$$\phi_{i,j}(\omega_i) = -\omega_i t_j = -\omega_i \left(\frac{L}{v} + \Delta t_j \right) = -\omega_i \frac{L}{v} - \omega_i \Delta t_j \quad (3)$$

where the first term in Eq. (3) is the linear part of the phase delay, the second term is the fluctuation part of phase, which is defined as $\Psi_j(\omega_i)$ and given by Eq. (4).

$$\Psi_j(\omega_i) = -\omega_i \Delta t_j \quad (4)$$

Therefore, the observed motion is expressed by combining the all arriving waves through M paths as Eq. (5).

$$f(t) = \sum_{i=1}^N \sum_{j=1}^M A_j(\omega_i) \cos \left(\omega_i t - \omega_i \frac{L}{v} - \Psi_j(\omega_i) \right) \quad (5)$$

where $A_j(\omega_i)$ is the Fourier amplitude to j^{th} path. The fluctuation of ground motion phase generated due to the inhomogeneity of the propagation medium is expressed by the third term of the phase as $\Psi_j(\omega_i)$ in Eq. (5). In this paper, this is referred as the fluctuation part of phase. The second term of the phase part ($\omega_i L/v$) in Eq. (5) is referred as the linear part of the phase delay. This delay is generated due to the global velocity structures between seismic source and site, so it is not a subject of discussion in this paper.

There is a similarity between microscopic and macroscopic inhomogeneity in the wave propagating medium. Such similarity is called as self-similarity. The frequency range of ground motion phase affected by the inhomogeneous medium is changed according to the scale of the inhomogeneity. So it is thought that the self-similarity can be seen in the fluctuation part of ground motion phase generated by the scattering in the inhomogeneous medium. If the fluctuation part of phase Ψ has such a self-similarity, the following expression was obtained in [12].

$$\Psi(\omega_i) = \gamma \Psi(c^{-1} \omega_i) \quad (6)$$

If Eq. (6) is satisfied for any arbitrary ω_i , $\Psi(\omega_i)$ should be represented by a power function of ω_i , then we obtain Eq. (7).

$$\Psi(\omega_i) \sim \omega_i^H \quad (7)$$

Substituting Eq. (7) into Eq. (6), the coefficient γ in Eq. (6) is expressed by Eq. (8).

$$\gamma = c^H \quad (8)$$

Here, H is the parameter which represents the degree of the self-similarity and it called Hurst index. The fluctuation part of phase is expressed by Eq. (9).

$$\Psi(\omega_i) = \omega_i^H \frac{\Psi(\omega_1)}{\omega_1^H} \quad (9)$$

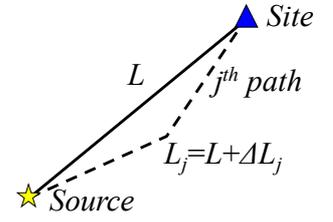


Fig.1 – Propagating paths from seismic source to site

where ω_1 is an arbitrary circular frequency. From Eq. (9), the variance of fluctuation part of phase is represented by Eq. (10).

$$E[\Psi(\omega_i)^2] = \omega_i^{2H} \frac{E[\Psi(\omega_1)^2]}{\omega_1^{2H}} \quad (10)$$

If the first-order difference of fluctuation part of phase, Ψ , is stationary, Ψ is satisfied the following equation for any d .

$$\{\Psi(\omega_{i+d}) - \Psi(\omega_{i+d-1})\} = \{\Psi(\omega_i) - \Psi(\omega_{i-1})\} \quad (11)$$

Then, we have

$$\{\Psi(c^{-1}\omega_{i+d}) - \Psi(c^{-1}\omega_{i+d-1})\} = c^{-H} \{\Psi(\omega_i) - \Psi(\omega_{i-1})\} \quad (12)$$

Therefore, the difference of Ψ has the self-similarity and the parameter represents the degree of self-similarity is Hurst index, H . The variance of phase difference is expressed by Eq. (13).

$$\begin{aligned} E[(\Psi(\omega_t) - \Psi(\omega_s))^2] &= E[(\Psi(\omega_t - \omega_s) - \Psi(\omega_0))^2] \\ &= |\omega_t - \omega_s|^{2H} \frac{E[\Psi(\omega_1)^2]}{\omega_1^2} \\ &= |\omega_t - \omega_s|^{2H} \sigma_0^2 \end{aligned} \quad (13)$$

When the variance of phase difference of Ψ in each $\Delta\omega$ ($= \omega_t - \omega_s$) is evaluated from the phase of observed motions, the parameters, H and σ_0 , can be obtained by the regression analysis using the expression of Eq. (13). These parameters express the inhomogeneity of the propagation medium around the site.

3. The evaluation of Hurst index in Kyushu region

3.1 The database of ground motions

In this chapter, we evaluate the inhomogeneity of the wave propagating medium based on Hurst index and σ_0 in Kyushu region located in the southwest of Japan. Table 1 shows source parameters of two earthquakes. These earthquakes occurred at the almost same location and $M_w=5.5$ earthquake is the aftershock of $M_w=6.1$ main shock. The observed ground motions of these earthquakes are propagating through the same medium, so it is expected that two sets of Hurst index evaluated from these earthquake motions have the almost same values.

The recorded ground motions at K-NET and KiK-net observation stations are used for analyses. The epicenter distance of these data is smaller than 100km. Hurst index is independently evaluated to NS and EW component of the observed ground motions. If the difference of Hurst index between NS and EW components is greater than 0.1, these datasets are discarded.

Table 1 – Source parameters of two earthquakes

	time	M_w	Source Lat.	Source Long.	Source depth(km)
Main shock	1997 3/26 17:31	6.1	31.97	130.36	12
Aftershock	1997 4/3 4:33	5.5	31.97	130.32	15

3.2 The result at KGS006 station

The analyzed result of using the record at KGS006 station observed during the 1997 northwestern Kagoshima earthquake is shown in Fig.2. Hurst index is evaluated through the following 4 steps.

Step1: The phase of the S-wave in the observed motion is calculated.

- Step2: The fluctuation part of phase is extracted from the phase calculated in Step1 by subtracting the linear delay part. Concretely, the linear part of phase delay ($\omega_i L/v$) in Eq. (3) is removed from the phase in Step1.
- Step3: The variance of stochastic process of $\Delta \Psi$ for different $\Delta \omega (= \omega_r - \omega_s)$ is calculated from the fluctuation part of phase calculated in Step2
- Step4: The parameters, H and σ_0 , can be identified by the regression analysis based on Eq. (14).

$$\text{var}(\Delta \Psi) = E[(\Psi(\omega + \Delta \omega) - \Psi(\omega))^2] = \Delta \omega^{2H} \sigma_0^2 \quad (14)$$

Fig.2(a) shows the time history of the ground motion used for the analysis. The group delay time is calculated from this ground motion as shown in Fig.2(b). We obtain the phase by integrating the group delay time with respect to the circular frequency. Fig.2(c) shows the fluctuation part of phase. Finally, the variance of phase difference, $\Delta \phi = \Psi(\omega_r) - \Psi(\omega_s)$, for different $\Delta \omega$ is evaluated as shown black line in Fig.2(d). And the parameters H and σ_0 are obtained by the regression analysis using Eq. (11) and drawn in the black line. If the stochastic process has a self-similarity, the value of Hurst index exists within the range from 0.5 to 1.0. In the example shown in Fig.2, it is $H=0.65$.

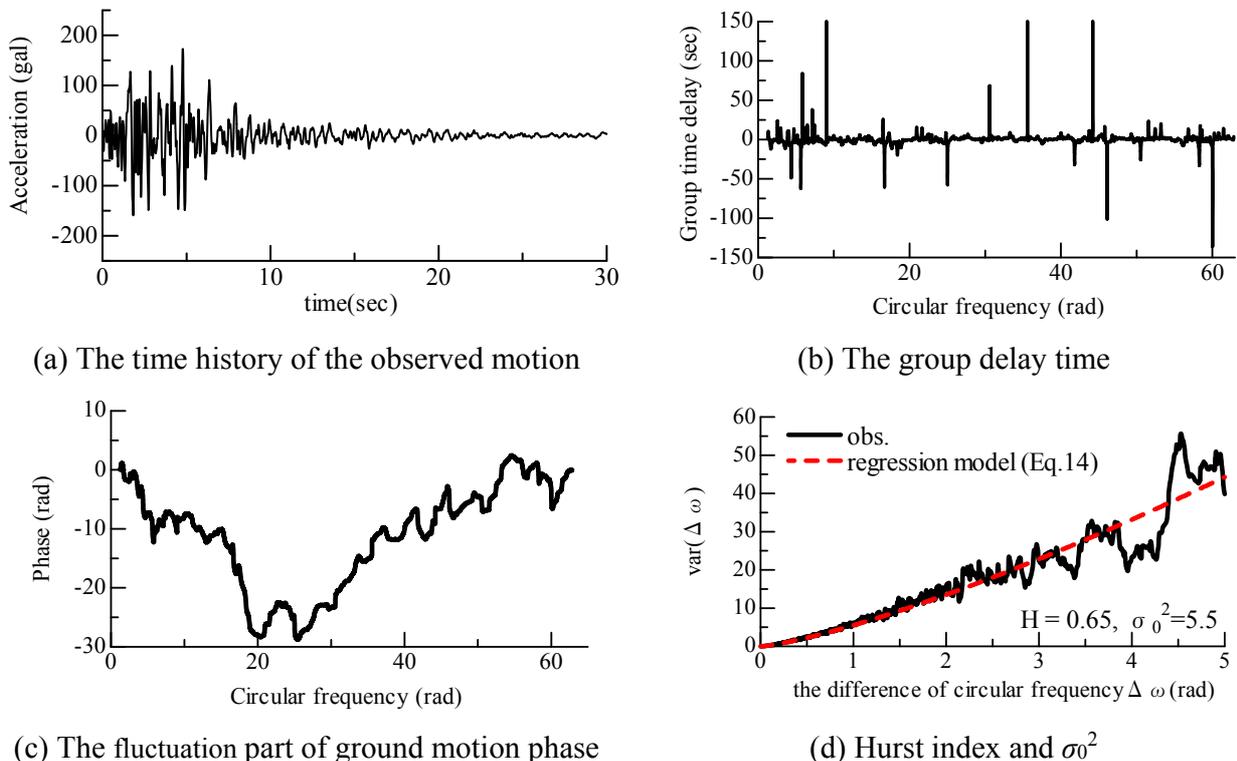


Fig. 2 – The evaluation at KGS006 station during the 1997 northwestern Kagoshima earthquake

3.3 The result in Kyushu region

Fig.3 shows the inhomogeneity of the wave propagation medium based on the distribution of Hurst index (NS component). In Kyushu area, Hurst index has a different value between the coastal and mountain area. The value in the coastal area is about 0.6 and the mountain area is about 0.9. The tendency of the spatial distribution of Hurst index in two earthquakes is generally consistent. In Fig.4, the distribution of $\log_{10} \sigma_0^2$ (NS component) is shown. It is seen that $\log_{10} \sigma_0^2$ concentrically increases from the epicenter. As seen in Fig.3, the distribution of $\log_{10} \sigma_0^2$ at two earthquakes is also consistent.

3.4 Discussion

The epicenter location and source mechanism is almost the same in two earthquakes. Therefore, it is expected that Hurst index or σ_0^2 become a similar value in two earthquakes. Fig.5 shows the result of a comparison of these parameters between main shock and aftershock. It is concluded that both have a strong correlation and the evaluated parameters are reproducible.

In the spatial distribution shown in Fig.3 and 4, the value of σ_0^2 has a clear dependence on hypocentral distance. The relationship between hypocentral distance and Hurst index as well as σ_0^2 is shown in Fig.6. A clear linear relationship can be seen in Fig.6(b). On the other hand, this tendency is not clear in Fig.6(a). The careful consideration should be done on Q-value distributions in the past studies and the location of the volcanic fronts. In this paper we purpose a method to grasp the general tendency of Hurst index, however the more detail investigation of the characteristics of Hurst index should be carried out in the future study.

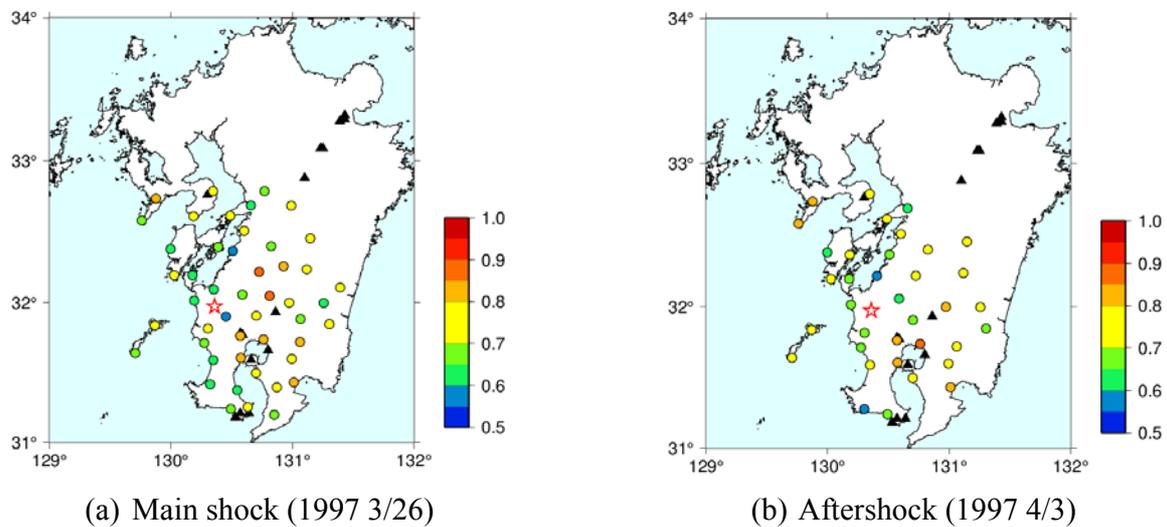


Fig. 3 – The distribution of Hurst index in Kyushu region (NS component)

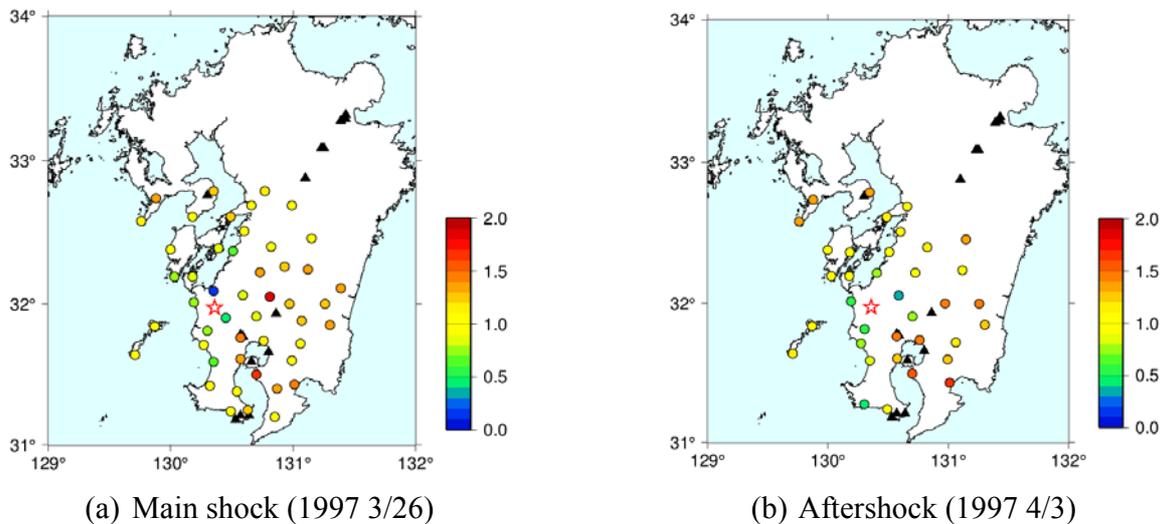


Fig. 4 – The distribution of $\log_{10}\sigma_0^2$ in Kyushu region (NS component)

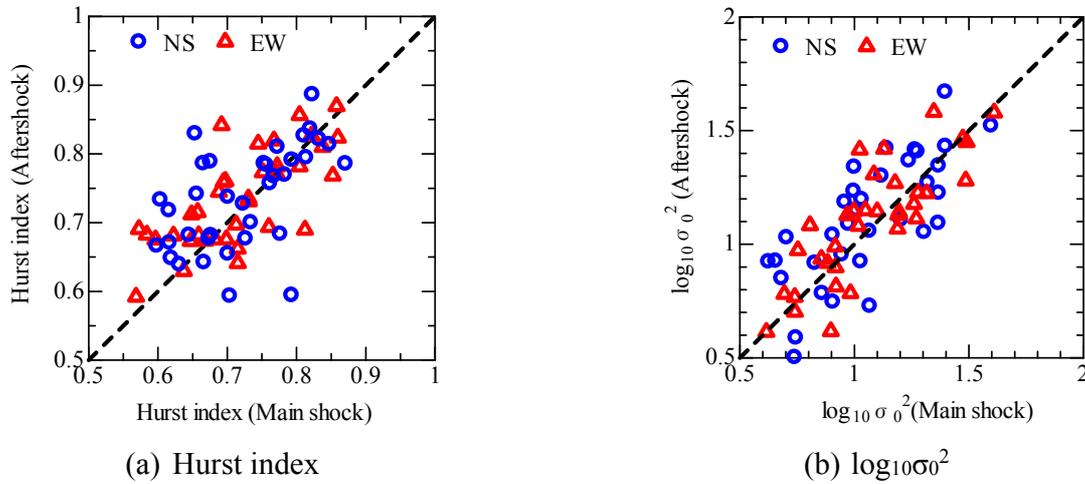


Fig. 5 – The comparison of main shock and aftershock

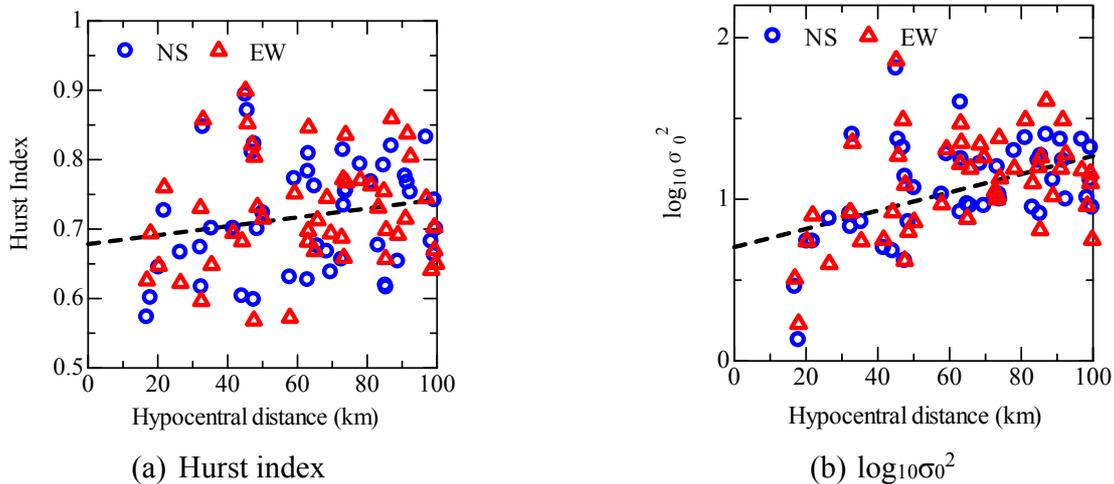


Fig. 6 – The dependency of parameters on hypocentral distance

4. Evaluation of Hurst index in all Japan

4.1 The database of ground motions

In this chapter, the inhomogeneity of the wave propagation medium is evaluated based on Hurst index in the whole of Japan. The ground motions observed at K-NET and KiK-net stations satisfying with the following conditions are used for the analyses.

- Japan Meteorological Agency magnitude (M_j) ≤ 6.5
- Source depth $\leq 40\text{km}$
- The Source located just under the observation stations (Hypocentral distance $< 2 \cdot$ Source depth)

The observation stations where the number of records is smaller than 3 are discarded from the analyses. The duration of ground motions used for analyses is 20.48 seconds from the onset time of S-wave. The transverse component is used, which is converted from NS and EW components taking into account the geometric location of epicenters and stations. The location of observation stations is shown in Fig.7 and the number of these stations is 930. The number of all ground motions used is 15418. The location of seismic sources is shown Fig8. The

value of Hurst index and σ_0 at each observation station is evaluated by calculating the mean value of all the results obtained from records at each station.

4.2 The result in the whole of Japan

The contour map of Hurst index is shown in Fig.9. The value of Hurst index is large in Niigata (enclosed by red circle) and Fukushima Prefecture (green circle), Izu Peninsula (purple circle), the northern part of Tohoku region (blue circle). On the other hand, the value in the western part of Japan is small. The contour map of $\log_{10}\sigma_0^2$ shown in Fig.10 has the same tendency with that of Hurst index. Comparing Figs.9 and 10, some correlation can be seen between Hurst index and σ_0 .

There are the earthquakes of various source depths in the earthquakes chosen in this study as shown in Fig.8. In Fig 11 we investigate the relationship between source depth and Hurst index as well as σ_0^2 . A clear relationship cannot be seen in these parameters with source depth.

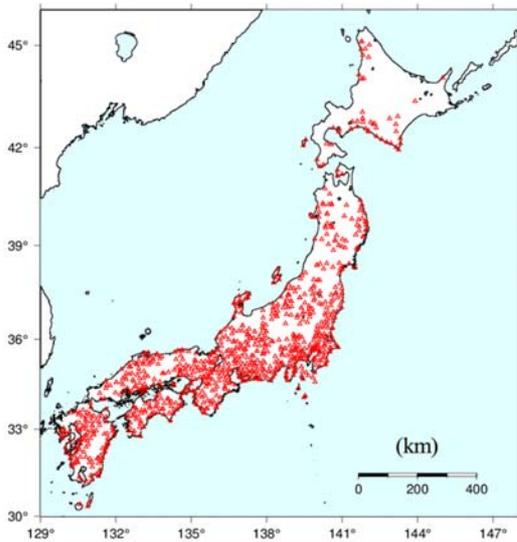


Fig. 7 – The distribution of observation stations

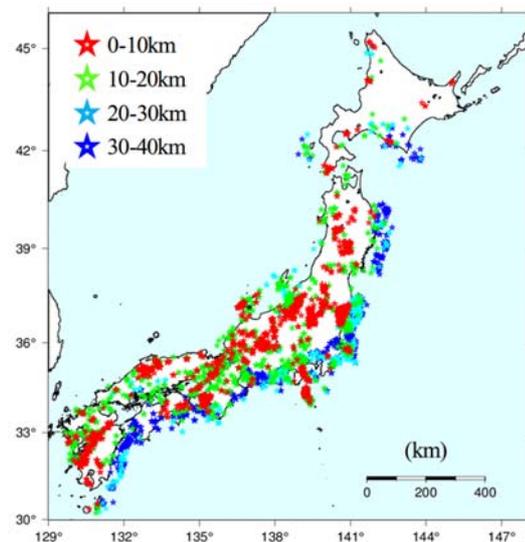


Fig. 8 – The distribution of seismic sources

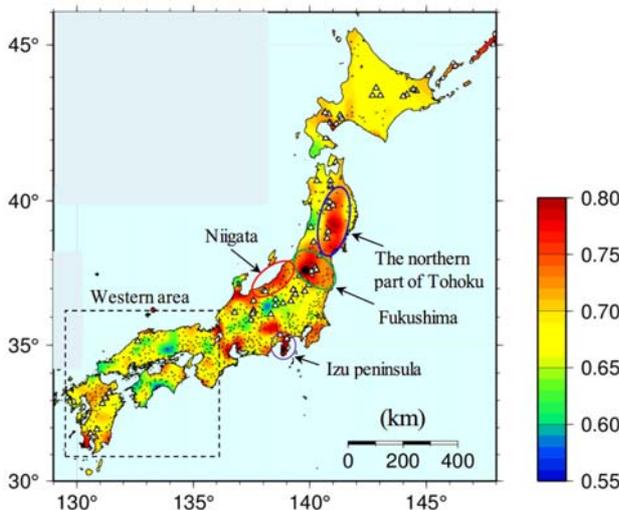


Fig. 9 – The contour map of Hurst index

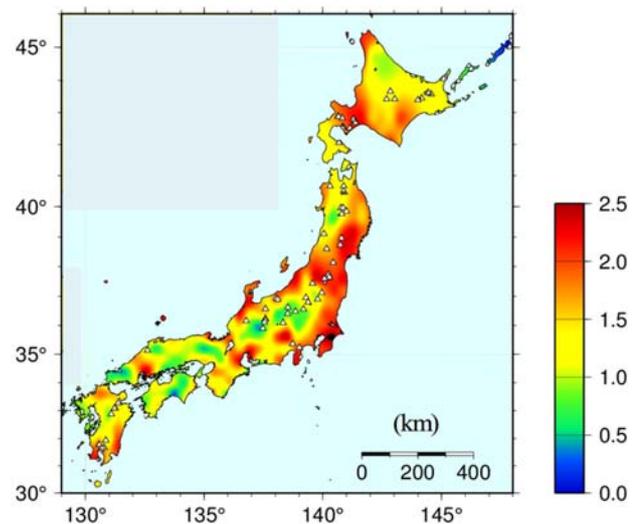


Fig. 10 – The contour map of $\log_{10}\sigma_0^2$

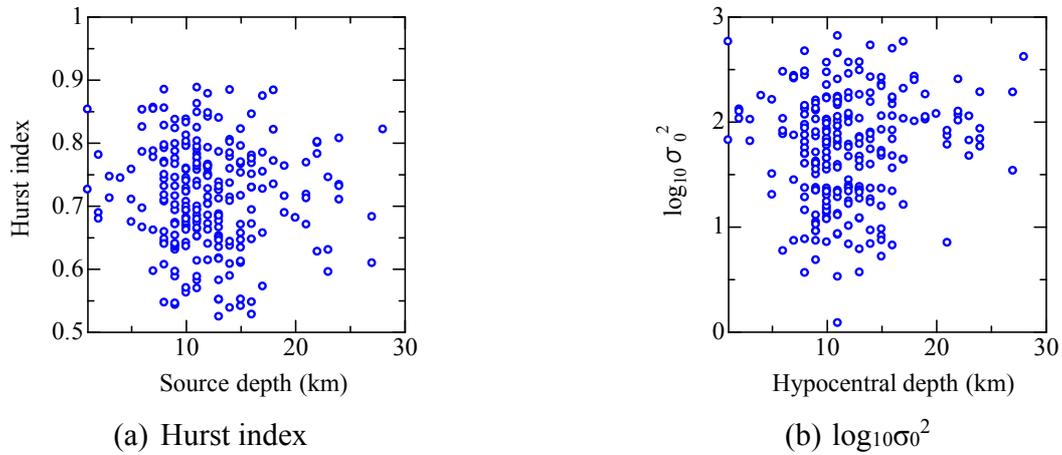


Fig. 11 – The dependency of parameters on source depth (NIG019)

4.3 Discussion

In this section, we compare our result with that of past study. In Carcole and Sato [14], the S-wave attenuation parameters for all Japan are obtained by Multiple Lapse Time Window Analysis (MLTWA). Fig. 12 shows graphical maps of Q-value for all Japan evaluated in [14]. The maximum source depth of target earthquakes is 40km, so it is consistent with our study. The contour of Q-value shows a little different according to the frequency range, so Fig.12 shows the contour map of Q-value for different frequency range (1-2, 2-4, 4-8Hz). The frequency range is consistent with the range used in the calculation for the ground motion phase in our analysis, which is the range from 0.2 to 10Hz.

In this study, Hurst index and σ_0 is calculated from the fluctuation part of ground motion phase caused by the wave scattering in the wave propagation medium. So, these parameters may be related to the scattering Q-value shown in Fig.12. The tendency that the Q-value in western Japan is small but large in Eastern Japan can be seen in Figs.9, 12. Carcole and Sato [13] gave a contour map of intrinsic absorption Q-value, however, we cannot see a clear relationship between intrinsic absorption Q-value and Hurst index. From these results, it is qualitatively suggested that Hurst index expresses the degree of scattering attenuation due to the inhomogeneity of the wave propagation medium.

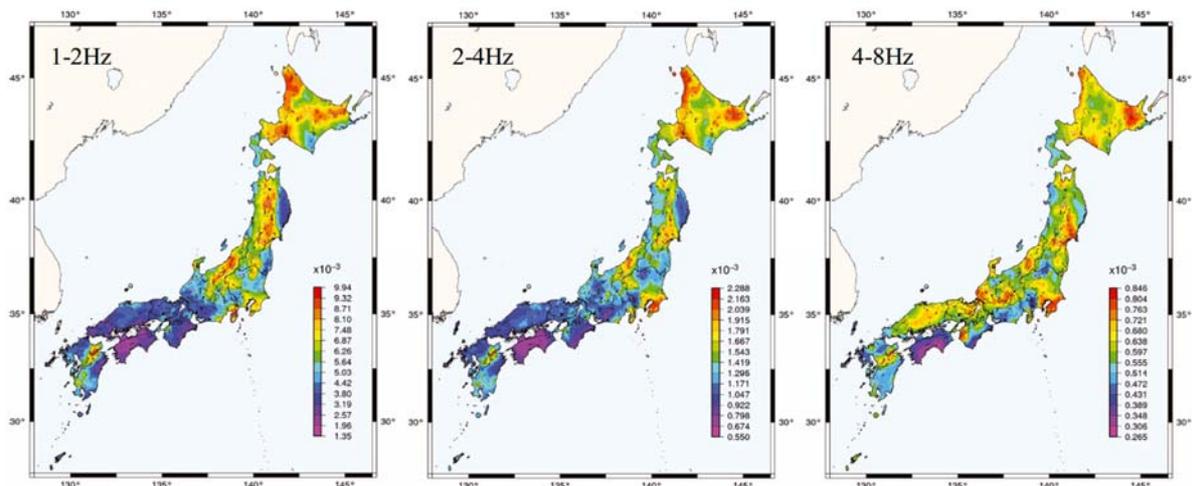


Fig. 12 – Graphical representation of scattering loss Q_s^{-1} for all Japan (Reprinted from [13])

5. Concluding remarks

Sato [12] has developed a new methodology to evaluate the inhomogeneity of the wave propagating medium using phase characteristics of observed strong ground motions. We assume that the phase characteristics of ground motions have the self-affine similarity because the inhomogeneous crustal medium of the wave propagation path shows the self-affine similarity nature. Hurst index calculated from the ground motion phase is used as the parameter which represents the degree of the self-similarity.

At first, the algorithm to calculate Hurst index are described and applied to the observed strong ground motion in Kyushu region to confirm the validation of the proposed method. The Hurst index and σ_0 are calculated from two earthquakes which has the same epicenter location and source mechanism. As a result, it is shown that the values between main shock and aftershock are almost the same.

Then, Hurst index and σ_0 in the whole of Japan are calculated. It is found that Hurst index has a regional locality and its locality is consistent with scattering Q-value distribution proposed in Carcole and Sato [14]. It is suggested that Hurst index represents the degree of scattering attenuation due to the inhomogeneity.

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