



## A Response Spectrum Method for Surface Strata Site Amplification

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### Abstract

Many seismic codes have suggested that the response spectrum should be specified at bedrock level, and then, the calculated free field response spectrum from bedrock will incorporate local site effects. This paper proposes a simple and accurate procedure to obtain the free field response spectrum of a layered soil profile from a specified bedrock response spectrum, taking into account the nonlinear behavior of soil. In this procedure, the soil profile is modeled as a multi-degree-of-freedom system with a rigid base, by replacing radiation damping by equivalent material damping. Using the developed model, the nonlinear behavior of soil is evaluated by classical model analysis with bedrock response spectrum directly. Finally, the free field response spectrum is obtained directly from the input bedrock response spectrum by using a method developed for calculating the floor response spectrum.

To demonstrate the validity and usefulness of this approach, eight examples are presented. The free field response spectra of these eight soil profiles are obtained using our proposed method, and are then compared with the results obtained using the SHAKE program. The results of our proposed procedure are found to be accurate.

*Keywords: site effect, response spectrum, response spectrum method.*

### 1. Introduction

It has long been recognized that the effects of a local site on ground motion should be considered in the seismic design of structures. Many seismic codes [1-3] have suggested that the response spectrum should be specified at bedrock level, and then, the calculated free field response spectrum from bedrock will incorporate the local site effects. For this purpose, many methods have been developed to determine the free field response spectrum from the bedrock response spectrum [1-7]. Among these, the most conventional one is to transform the bedrock response spectrum into several accelerograms, and then, obtain the average free field response spectrum by site response analysis. However, conducting several site response analyses using the generated accelerograms is very cumbersome. Additionally, in many methods [4-5], the bedrock response spectrum is converted to power spectrum density (PSD), and using the transfer function of the soil profile, the free field PSD can be obtained, which can then be used to generate the free field response spectrum. Since an iterative calculation is needed to transform the response spectrum into PSD, this method is not straightforward. An approximate method has been developed by Miura [6], and a new version of this method has been developed in the Japanese seismic code [2]. In this method, the amplification ratio of the response spectrum is approximately evaluated using the envelope of the transfer function of the soil profile, which is constructed using the fundamental period and corresponding peak of the transfer function, this is called the first resonance peak (FRP). The FRP is evaluated by replacing multi-layer soil by an equivalent single-layer soil by weighted averaging of the soil shear-wave velocity and density. However, this method might underestimate the FRP when the impedance contrast among the soil layers is large [7-9]. This can lead to underestimation of the free field response spectrum and is not conservative for structure design.

This paper proposes a simple procedure to obtain the free field response spectrum of a layered soil profile from a specified bedrock response spectrum, taking into account the nonlinear behavior of soil. The rest of the paper is organized as follows. First, the soil profile is modeled as a multi-degree-of-freedom (MDOF) system with a rigid base by replacing the radiation damping by equivalent material damping. Then, using the developed



model, the nonlinear behavior of soil is evaluated by classical model analysis using the bedrock response spectrum directly. Finally, the free field response spectrum is obtained directly from the input bedrock response spectrum using a method developed for calculating the floor response spectrum. Since (1) the bedrock response spectrum is used as input directly without transforming to time history or power spectrum density (PSD) and (2) each soil layer is considered without being replaced by an equivalent single layer when evaluating the free field response, the proposed model is considered a simple and accurate method applicable to earthquake engineering.

## 2. Development of MDOF system

One-dimensional site response analysis methods can be divided into frequency domain analysis and time domain analysis. In frequency domain analysis, the soil column is modeled as a layered system, as shown in Fig. 1(a), and the input for soil response analysis is required in the form of a Fourier spectrum or PSD. For the case where the bedrock response spectrum is given as input, the response spectrum can be converted into a Fourier spectrum or PSD, and then, the free field Fourier spectrum or PSD can be obtained using the transfer function of a layered system, which can in turn be used to generate the response spectrum. However, this procedure is costly and cumbersome. In time-domain nonlinear analysis, the soil column is always discretized into individual layers using an MDOF lumped parameter model [10]. Each individual layer is represented by a corresponding mass, spring, and a dashpot. The base of the soil column is always modeled as an elastic half-space. For an input specified at outcrop bedrock, the energy radiated back into the underlying medium, called radiation damping, is represented by a viscous damping equal to  $\rho_B \cdot V_B$  (where  $\rho_B$  and  $V_B$  are the density and shear-wave velocity of bedrock) [10], as shown in Fig. 1(b). For this MDOF model, not only the time-history analysis method but also a widely used method for seismic analysis of structures (the response spectrum method) can be used for response analysis. Naturally, for the response spectrum input, the response spectrum method is desirable. However, because (1) both the damping magnitude and damping characteristics of the soil and boundary are dramatically different and (2) there is only a dashpot and no spring at the boundary, the classical modal analysis cannot be used to analyze this model.

Zhao [11] developed a method to replace single-layer soft soil on elastic bedrock by single-layer soil on rigid bedrock, by converting the radiation damping to an equivalent material damping,  $\xi_e$ , and using a response enhancement factor,  $R_e$ , expressed as

$$\xi_{eq} = \frac{aV_s}{\omega H} \quad (1)$$

$$R_e = \cosh(a) \quad (2)$$

where  $\omega$  is the circular frequency of excitation,  $V_s$  is the shear-wave velocity,  $H$  is the thickness of the single layer, and  $a$  is the impedance ratio of the surface single layer to bedrock. For a multi-layer soil profile, the multi-layer soil can be approximately replaced by the equivalent single-layer soil, and the equivalent shear-wave velocity can be calculated by the weighted averaging of the soil shear-wave velocity, as follows:

$$V_s = \frac{\sum_1^n V_i H_i}{\sum_1^n H_i} \quad (3)$$

where  $i$  is the soil layer number, each having finite thickness  $H_i$  and shear-wave velocity  $V_i$ . Using this substitution, radiation damping represented by viscous damping can be converted into equivalent material damping using eqs. (1 to 3), and then, an MDOF system with a rigid base, shown in Fig. 1(c), can be constructed. In addition, the damping ratio of soil due to soil's nonlinearity is hysteretic damping; according to reference [12], the equivalent damping ratio of each model can be calculated by the weighted-averaged damping ratio of each layer:

$$h_{eq} = \frac{\sum h_i E R_i}{\sum E R_i} \quad (4)$$

$$ER_i = \frac{G_i}{2H_i} (\Delta u_i)^2 \quad (5)$$

where  $h_i$  is the hysteretic damping ratio,  $G_i$  is the shear modulus, and  $\Delta u_i$  is the relative displacement of the  $i$ -th soil layer.

Thus, using this MDOF model, classical model analysis can be conducted and the response of the soil can be directly estimated using the bedrock response spectrum by the response spectrum method.

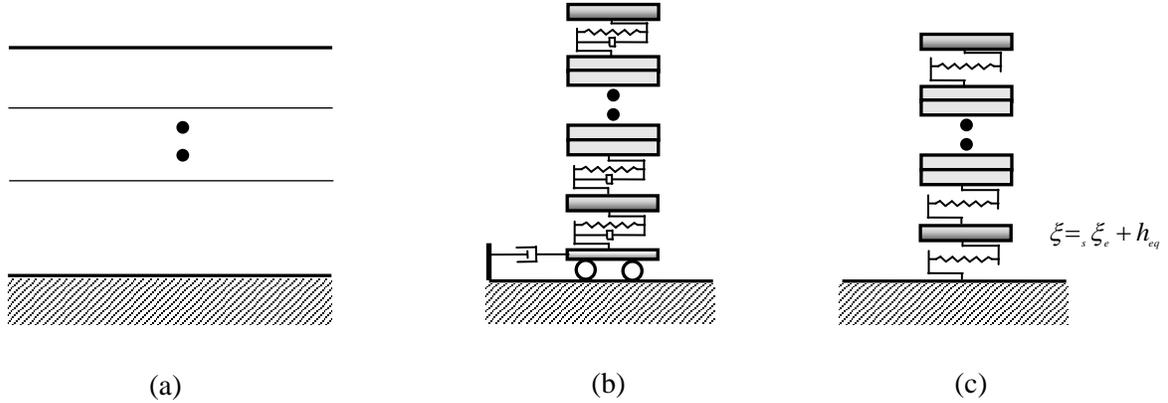


Fig. 1 Diagram of method used to approximate multiple layers with a multi-lumped mass model

### 3. Calculation of soil nonlinearity

This section presents a response spectrum method to evaluate soil nonlinearity according to the response spectrum at bedrock directly. As soil nonlinearity (including the degradation of shear modulus and increase in damping ratio) depends on the shear strain of the soil layer, and according to the equivalent linear method [13], soil nonlinearity can be evaluated according to the maximum shear strain of the soil layer, the maximum strain of each layer should be evaluated first. According to the widely used complete quadratic combination rule, the maximum relative displacement  $|\delta_i|_{\max}$  of the  $i$ -th layer can be evaluated by the using the following equations from the MDOF model:

$$|\delta_i|_{\max} \approx \sqrt{\sum_{s=1}^n \sum_{r=1}^n (\beta_s \beta_r \Delta u_i) S_d(T_s, h) \rho_{\omega} (\beta_r \Delta u_i) S_d(T_r, h) R_e^2} \quad (6)$$

$$\Delta u_i = u_i - u_{i+1} \quad (7)$$

$$\rho_{\omega} = \frac{8\sqrt{h_r h_s (h_r h_s \chi)^{3/2}}}{(1 - \chi^2)^2 + 4h_r h_s \chi (1 + \chi^2) + 4(h_r^2 + h_s^2) \chi^2} \quad (8)$$

where  $\beta_s$  is the participation factor for the  $s$ -th model,  $u_i$  is the model shape of the  $s$ -th model,  $\chi_r = \omega_r / \omega_s$ ,  $\omega_s$  is the frequency of the  $s$ -th model,  $S_d(T_s, h)$  is the pseudo-displacement response spectrum corresponding to the  $s$ -th model's natural period  $T_s$  and the  $s$ -th model's damping ratio  $h_s$ , which is calculated from the acceleration response spectrum  $S_a(T_s, \xi)$  as

$$S_d(T_s, \xi) = \left( \frac{T_s}{2\pi} \right)^2 S_a(T_s, \xi) \quad (9)$$

The damping ratio for the  $s$ -th model is considered as the sum of the equivalent material damping  $h_{eq}$  using Eq. (4) and the equivalent damping  $\xi_e$  from Eq. (1), where the circular frequency is used as the frequency of the  $s$ -th modal,  $\omega_s$ ,



$$\xi = \xi_s + h_{eq} \quad (10)$$

The effect of the damping ratio is taken into account in the form of a response reduction factor  $F_h(\xi)$  [14] expressed as

$$S_a(T, \xi) = S_a(T, 0.05) \times F_h(\xi) \quad (11)$$

where

$$F_h(\xi) = \frac{1.5}{1 + 10 \times \xi} \quad (12)$$

Then, using the equivalent linear method proposed in reference [13], the effective shearing strain,  $\gamma_i$  of the  $i$ -th layer can be calculated by

$$\gamma_i = R_\gamma |\delta_i|_{\max} / H_i \quad (13)$$

where  $R_\gamma$  is the ratio of effective shear strain to maximum shear strain. For estimating  $R_\gamma$ , a magnitude-dependent formula [15] and a frequency-dependent formula [16] have been developed. For convenience,  $R_\gamma$  is taken to be 0.65, as it is frequently used in engineering practice.

The shear modulus  $G$  and damping ratio  $h$  of soil is dependent on the soil shear strain. The degradation of the shear modulus, defined by the ratio of  $G$  to the initial value  $G_{\max}$ , depends on shear strain. This relationship is proposed by Hardin and Drnevich [17] and is expressed as

$$G/G_{\max} = \frac{1}{1 + \gamma/\gamma_r} \quad (14)$$

where  $\gamma_r$  is the reference shear strain, determined according to plasticity indices. The damping ratio depending on soil shear strain is expressed as

$$h = h_{\max} (1 - G/G_{\max}) \quad (15)$$

where  $h_{\max}$  is the maximum damping ratio.

For the evaluation of soil nonlinearity using this procedure, iterative calculation is needed until the estimated shear strain in consecutive iterations becomes stable.

#### 4. Free field response spectrum

To obtain the free field response spectrum according to the bedrock response spectrum directly, a method to obtain the floor response spectrum is used. For a classical MDOF system, many methods have been developed to obtain the floor response spectrum directly from the input response spectrum. Among these methods, the one developed in [18] is used in this study. In this method, the floor response spectrum  $S_{Fa}(A\omega, Ah)$  is calculated by

$$S_{Ei}(\omega_A, h_A) = \sqrt{\frac{1 + 4(h_A + h)^2 (\omega_A / \omega)^2}{[1 - (\omega_A / \omega)^2]^2 + 4(h_A + h)^2 (\omega_A / \omega)^2}} \cdot \sqrt{\left[ \left( \frac{\omega_A}{\omega} \right)^2 S_a(\omega, h) \right]^2 + S_a(\omega_A, h_A)^2} \quad (16)$$

$$S_{Fa}(\omega_A, h_A) = \sqrt{\sum_i (\beta_s u \cdot S_{Ei})^2} \quad (17)$$

where  $S_{Fa}(A\omega, Ah)$  is the floor acceleration response spectrum corresponding to the natural frequency  $A\omega$  and damping ratio  $Ah$ . Actually, the top floor response spectrum of the MDOF model is the free field response spectrum, so the free field response spectrum,  $S_{Fa}(A\omega, Ah)$ , can be directly obtained using this equation.

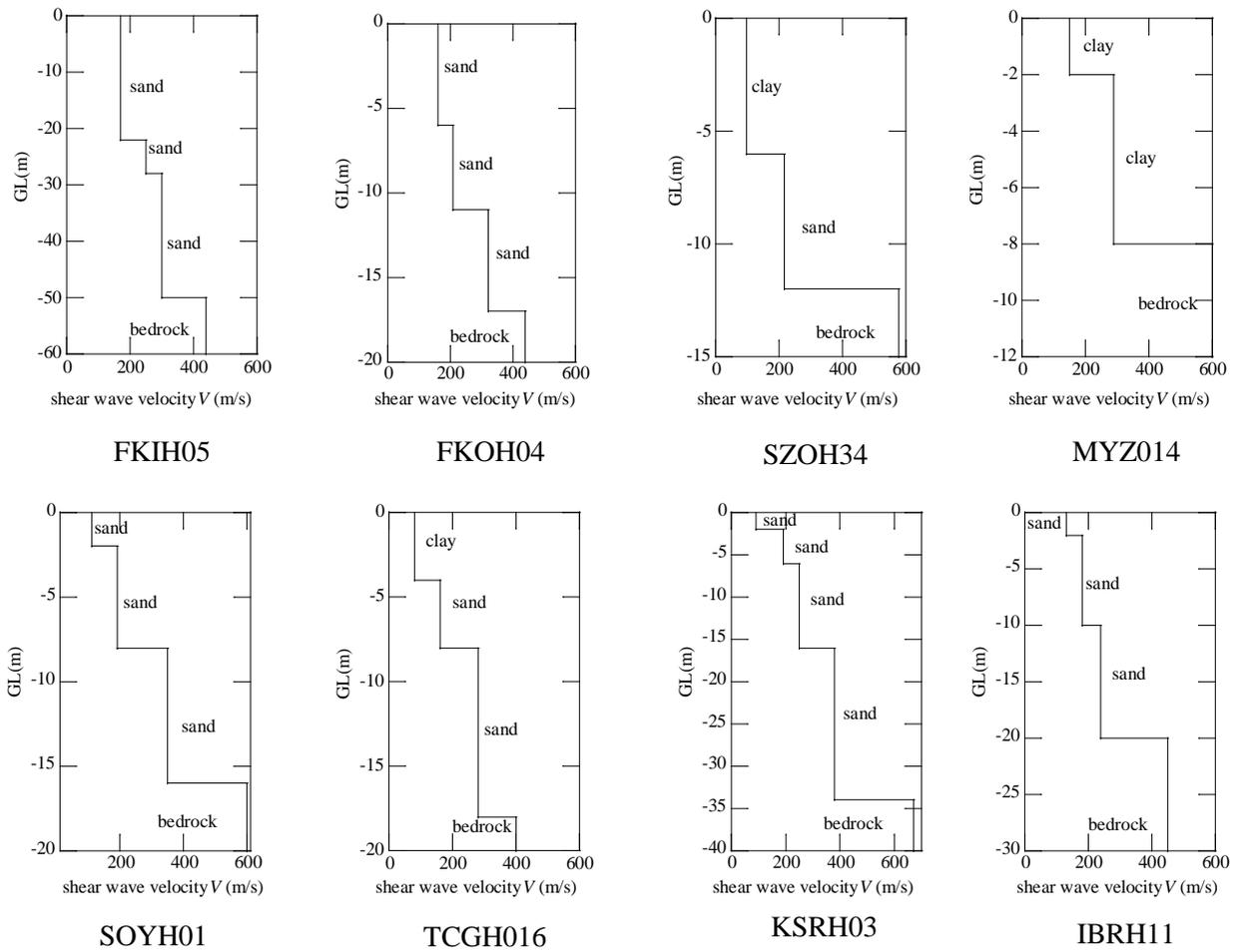
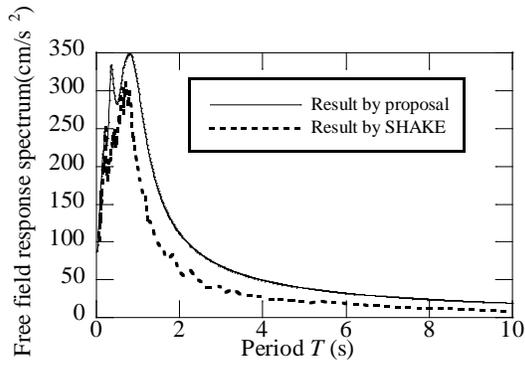


Fig. 2 Shear-wave profiles above engineering bedrock used for analyses

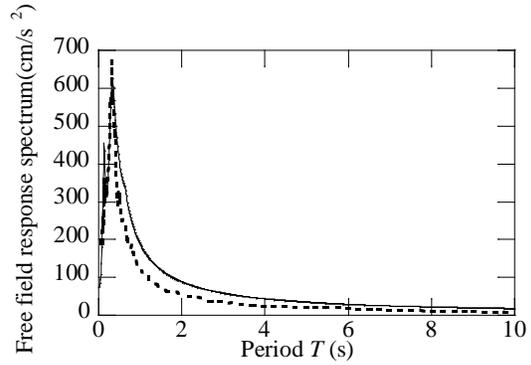
## 5. Verification

Verification analysis has been conducted using eight representative soil profiles selected from Strong-motion Seismograph Networks (K-NET, KIK-net) [19]. An overview of the SWV profiles above engineering bedrock is presented in Fig. 2. Here, engineering bedrock is defined as the layer whose SWV is more than about 400m/s according to the Japanese Seismic Code. As the density of the soil layers is not given in some soil profiles, according to reference [20], the density is empirically given by 1.6 (tf/m<sup>3</sup>) for clay, 1.9 (tf/m<sup>3</sup>) for sand, 2.0 (tf/m<sup>3</sup>) for engineering bedrock whose SWV is in the range 400–800 (m/s), and 2.2 (tf/m<sup>3</sup>) for engineering bedrock whose SWV is over 800 (m/s) [20]. The reference shear strain  $\gamma_r$  and maximum damping ratio  $h_{max}$  used in Eqs. (14) and (15) are determined according to reference [21]:  $\gamma_r = 0.18\%$ ,  $h_{max} = 17\%$  for clay, and  $\gamma_r = 0.10\%$ ,  $h_{max} = 21\%$  for sand. All parameters used in the proposed procedure and the SHAKE program are the same. Two levels of input response spectrum defined on bedrock in the Japanese Seismic Code [21] are considered for this verification.

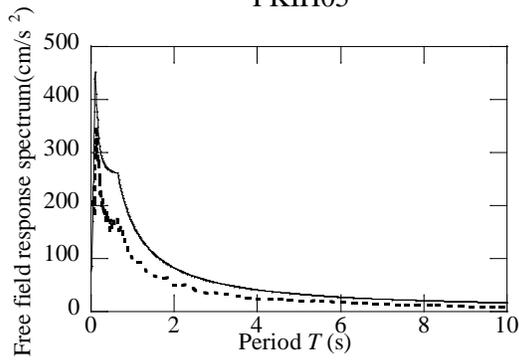
For the response analysis of the soil profile using the SHAKE program, 10 accelerograms are generated corresponding to each level of the response spectrum, and then, the free field response spectrum is calculated by averaging the obtained free field accelerograms. Thereafter, the free field response spectrum of these eight soil profiles are calculated using the proposed procedure and compared with the results obtained using the SHAKE program. The comparison of the results obtained by our proposed method and SHAKE, corresponding to level 1 and level 2 inputs, are shown in Figs. 3 and 4, respectively. The results of our method are represented by solid



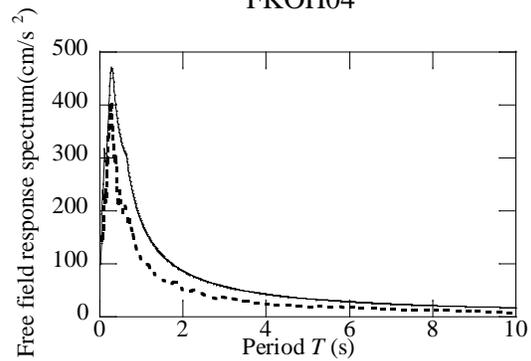
FKIHO5



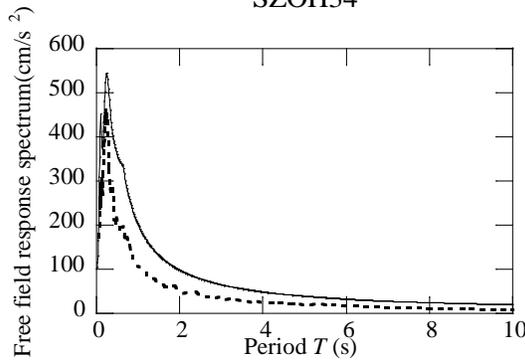
FKOHO4



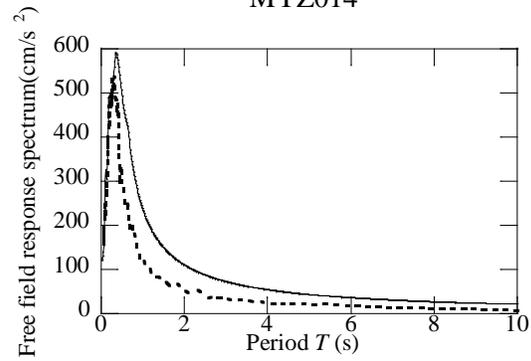
SZOH34



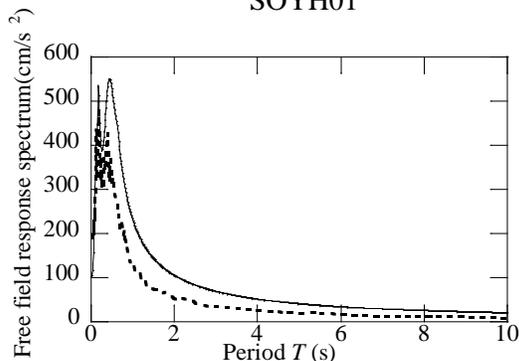
MYZ014



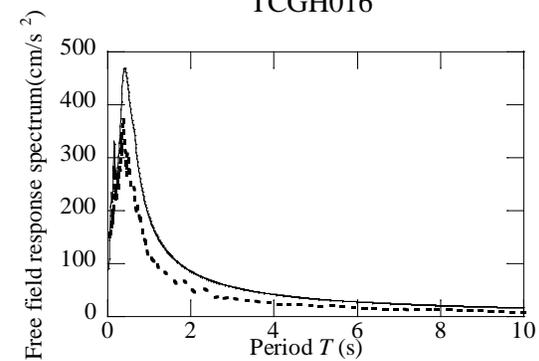
SOYH01



TCGH016



KSRH03



IBRH11

Fig. 3 Comparisons of free field response spectra calculated by our proposed method with results obtained from SHAKE corresponding to level 1 input

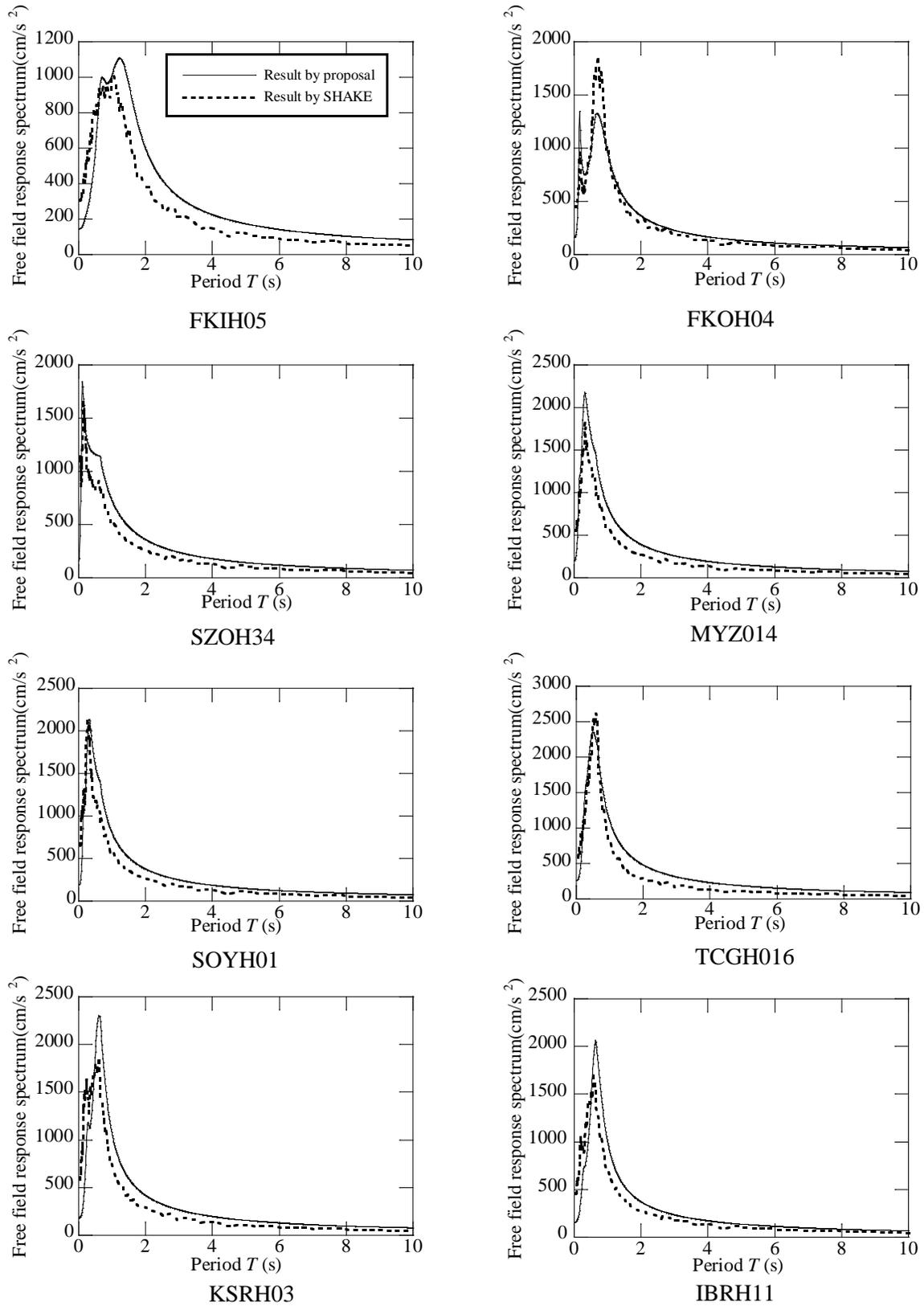


Fig. 4 Comparisons of free field response spectra calculated by our proposed method with results obtained from SHAKE corresponding to level 2 input



lines and those of the SHAKE program are represented by dotted lines. It can be seen that our proposed method agrees well with the SHAKE results at both levels of input. For some cases, such as soil profile FKIHO5, the free field response spectrum can be seen overestimating a little, compared with SHAKE results. As the estimations should be conservative for safety, this is considered acceptable.

## 6. Discussion

In this section, the methodology and results of the proposed method are discussed. The main feature of the proposal is that bed rock response spectrum is used directly for site response analysis, without transforming to time history or PSD. To realize this objective two main approaches are used in this proposal: (1) response spectrum method widely used in structure analysis is adopted to evaluate the nonlinearity of soil, by modeling the layered soil profile as a MDOF system, (2) a method used to estimate floor response spectrum is adopted to obtain free filed response spectrum from response spectrum defined on bed rock. It is clearly that, the proposal is more convenient comparing with current methods without transforming bedrock response spectrum to time history or PSD.

On the other hand, same with current methods, many assumptions introduced above are also used in this proposal to obtain free filed response spectrum. To demonstrate the validity of this development, 8 representative soil profiles are evaluated. The comparison of results by proposal and SHAKE program has been shown in Fig.3 and Fig.4. It can be noted that the results by proposal agree well with those by SHAKE program in the main. But, for nearly all cases, the response spectrum at periods larger than predominate period are overestimated comparing the results by SHAKE. Among these analyzed cases, the error of the soil profile named as FKIHO5 is most notable, and the maximum error is about 30%. However, it is considered that the results by proposal are conservative, and the error are acceptable for engineering design. In this study, only 8 cases are analyzed, it is considered further analyzation of more case is necessary.

## 7. Conclusion

In this study, a simple procedure to obtain the free field response spectrum of a layered soil profile from a specified bedrock response spectrum, taking into the account the nonlinear behavior of soil, is developed. In this procedure, the soil profile is modeled as an MDOF system with a rigid base, by replacing the radiation damping with an equivalent material damping. Using the developed model, the nonlinear behavior of soil is evaluated using the bedrock response spectrum directly in the response spectrum method without transforming to time history or PSD. Finally, the free field response spectrum is obtained directly from the input bedrock response spectrum using a method developed for calculating the floor response spectrum. To demonstrate the validity and usefulness of this approach, eight examples are presented. The free field response spectra obtained using our proposed method are compared with the results obtained using the SHAKE program, and it is found that our results are accurate.

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