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DAMPING MODELS FOR INELASTIC TIME-HISTORY ANALYSES-A PROPOSED MODELLING APPROACH.

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Abstract

Although a lot of issues have been identified with the classical Rayleigh damping model, to date this model remains the most popular choice for nonlinear time-history analysis. So, in order to have a model devoid of the untoward effects of those issues, a new paradigm was proposed by the authors in Puthanpurayil et al (2016) which formulates damping at an elemental level. Two new damping models, Elemental Rayleigh and Elemental Wilson-Penzien, were proposed which are the elemental adaptations of their respective global damping models. In this paper a further extension of the elemental Wilson-Penzien damping model implemented in the form of a tangent damping matrix is proposed. Implementation of the tangent model enables elemental nonlinearity, and thus the overall system nonlinearity, to be reflected in the damping matrix. The performance of this proposed model in comparison to the existing models (both elemental and global) is illustrated by reporting Incremental Dynamic Analysis (IDA) of a four-storey reinforced-concrete frame designed to Eurocodes. The IDA study shows that the tangent form of the elemental Wilson-Penzien model provides a more reliable estimate of the structural responses in comparison to all other models.

Keywords: Viscous damping; Nonlinear seismic analysis; Nonlinear dynamic analysis; inherent damping; in-structure damping

1. Introduction

Conventional nonlinear time-history analysis requires the explicit description of a damping matrix. To date the most popular model used in such analyses is Rayleigh damping using the initial stiffness of the structure. One of the main advantages of the Rayleigh damping model has been that, in linear dynamic analysis, it fits into the computationally-efficient modal analysis framework by preserving the damping proportionality in a similar manner to the mass and stiffness proportionality, but in fact only providing the correct level of damping at two frequencies. This mathematical simplicity, the preconceived notion that damping forces are small [1], and any inaccuracy in the modelling of these forces has negligible effects on the overall structural responses, made Rayleigh damping very popular among analysts and software developers. For these same reasons use of the model was extended to seismic nonlinear time-history analysis.

This paper highlights the issues associated with the use of the Rayleigh damping model in nonlinear time-history analysis. It also gives an overview of recent developments in inherent damping modelling, and proposes and recommends an elemental viscous damping model which the authors believe reflects the damping phenomenon expected (with no untoward effects) in nonlinear dynamic analysis.

2.0 Issues with Rayleigh damping

Although the Rayleigh damping model performs reasonably well in linear time-history analysis, its extension to the nonlinear realm was accompanied by certain significant shortcomings - especially owing to the fact that the relative contribution of the damping forces increased considerably [2-4]. Considerable research effort had been expended in the past on studying the issues associated with the adaptation of the Rayleigh model to inelastic time-history analysis. Crisp [2] was the first known to highlight the appearance of un-realistic damping forces in the nonlinear time-history analysis of a six-storey frame - especially at the point of yielding of the structure when an initial-stiffness-based Rayleigh damping model was used. Figure 1 represents the appearance of such un-realistic damping actions in the nonlinear time-history analysis of a four-storey frame the details of which are given in Appendix A of this paper. The plot in Figure 1 is of the middle beam-column junction in the first storey. The yielding moment of the girder meeting the joint was computed as 100 kNm. It can be clearly seen from Figure 1 that damping moments are like instantaneous, velocity-based, force impulses which are almost like adding instantaneous viscous dampers at the onset of nonlinearity.

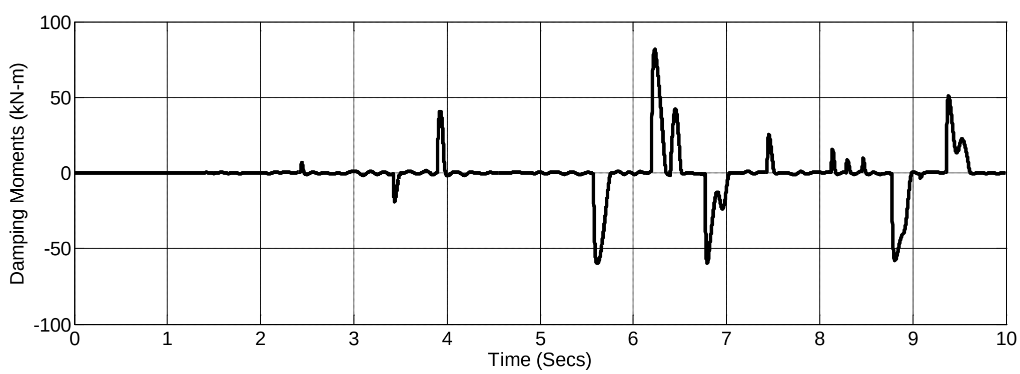

Figure 1.0 Damping moment plot for the four-storey frame for the first storey middle node

Previous and recent studies highlight the fact that the presence of such damping moments affects the member moments and, as a result, the structural displacement response.

For decades researchers have investigated the issue and suggested possible improvement to the damping modelling in nonlinear time-history analysis. Some of the relevant suggestions in this direction include, [5-12]. A majority of these works identify the appearance of unrealistic damping forces as one of the main impediments to the extension of the Rayleigh damping model into the nonlinear domain, and they also suggest different ways of rectifying this. Except for Chopra and McKenna [11] and Puthanpurayil et al [12], a majority suggest different adaptive modifications of the basic Rayleigh damping model to correct the unrealistic damping forces. Since their underlying basic models are still Rayleigh based, from a commercial software point of view some of their modified models have serious limitations with regard to their ease of implementation and the associated computational demand. In Chopra and McKenna [11], a reduced Wilson-Penzien damping form, which includes a lesser number of modes compared to the complete number required by the classical Wilson-Penzien format, is suggested as the recommended model. Though the idea of using Wilson-Penzien as the recommended model is not new [2-4], the suggestion to use a reduced number of modes in the damping matrix computation is an improvement; but, unfortunately, how many modes should be used in a relatively unsymmetrical complex structure to represent the *un-modelled* dissipation in a reasonable manner is still a question which needs further research. The Perform3D [13], an analysis software, offers the option of Wilson-Penzien damping for up to the first 50 modes and suggests adding some Rayleigh damping to cover the higher modes.

3.0 Proposed approach to modelling inherent damping [12]

Puthanpurayil et al [12] proposed an innovative way of formulating dissipation phenomenon by defining the damping model at the element level and assembling the elemental damping in a form similar to a stiffness or mass matrix to obtain the system damping matrix. It has been demonstrated that the damping matrix formulated at element level is devoid of the inherent issues present in the current inherent damping modelling approaches. Two new damping models which are the elemental level adaptations of their global counterparts were introduced in Puthanpurayil et al [12]. In this paper we investigate further the option of 'updated' elemental Wilson-Penzien damping not included in Puthanpurayil et al [12].

3.1 Overview of the elemental damping models

Figure 2 is an overview of the elemental models. A majority of the models except for the updated elemental Wilson-Penzien model further investigated in this paper, were presented in Puthanpurayil et al [12].

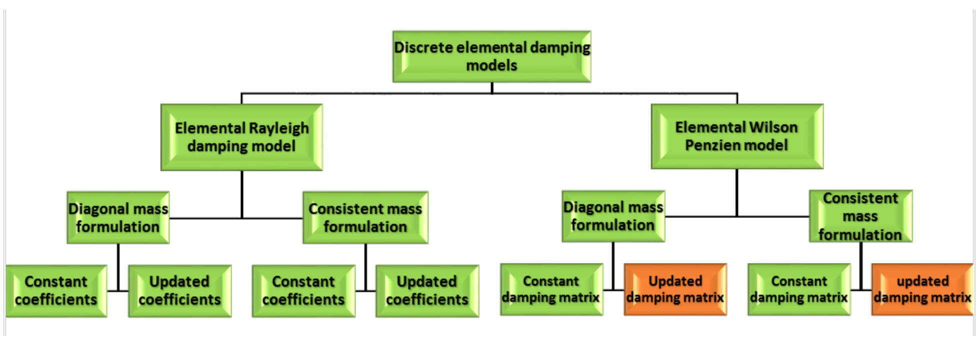

Figure 2 :Overview of the elemental damping models

3.2 Why we need elemental damping models? A philosophical justification

In order to understand the need for *elemental damping models*, one needs to understand what is the main function of a mathematical damping model in nonlinear time history analysis. Its main function is to model the *un-modelled dissipation*. *Un-modelled dissipation* includes all those tiny mechanisms such as cracking of non-structural components, cracking of gravity supporting structures, cracking of plaster, etc., that are not included in the hysteretic response of the elements in a structure. To model these mechanisms explicitly is almost impossible. The model representing damping should mimic this *un-modelled dissipation* mechanism as closely as possible with minimum computational penalties and, preferably, with no untoward unexpected effects when the system becomes nonlinear. From a realistic physical point of view, there is no doubt that this *un-modelled dissipation* phenomenon emanates from the elements. This is because all the gravity and non-structural components contributing to this phenomenon are either attached directly or indirectly to the elements contributing to the hysteretic response. Damping occurs when these attached primary members deform to cause cracks and failure in the non-structural components, along with the damping of the elastic portion of the inelastic members. In effect it is essential that the damping model should be defined at the element level.

3.3 Elemental Rayleigh damping model

A brief overview of the Elemental Rayleigh damping model is given in this section. Its formulation was mainly motivated by the simplicity of the Rayleigh damping model normally applied at system global level in classical structural dynamics and its familiarity. This is not a totally new approach as it was implemented in Ruaumoko, a nonlinear analysis software [14] using different global multipliers for different parts of the structure. Elemental Rayleigh damping is a direct adaptation of the global Rayleigh damping at element level. The form of the element damping matrix is given as,

$$\mathbf{C}_e = \alpha_{eRD}\mathbf{M}_e + \beta_{eRD}\mathbf{K}_e \tag{1}$$

where α_{eRD} and β_{eRD} are the elemental damping coefficients. In Elemental Rayleigh damping the coefficients are computed as:

$$\left.\begin{aligned} \alpha_{eRD} &= 2\xi_{eR1}\frac{\omega_e^i\omega_e^j}{\omega_e^i+\omega_e^j} \\ \beta_{eRD} &= 2\xi_{eR1}\frac{1}{\omega_e^i+\omega_e^j} \end{aligned}\right\} \tag{2}$$

where ω_e^i and ω_e^j are the i^{th} and the j^{th} elemental frequencies. ξ_{eR1} and ξ_{eR2} are elemental damping ratios which need to be parameterised as outlined in Puthanpurayil et al [12]. At the present time the requirement of this parametrization is one of the main shortcomings of the Elemental Rayleigh damping model. The main difference between Elemental Rayleigh damping and classical Rayleigh damping predominantly implemented at a global level exists in the computation of these damping coefficients as the Elemental Rayleigh damping uses the elemental frequencies rather than the global frequencies. The main justification for the use of elemental frequencies is the fact that inherent damping in reality is a function of the elemental deformations and before the system matrix assembly these deformations depend on the elemental frequencies and elemental mode shapes. For more details on the implementation, refer to Puthanpurayil et al [12].

3.4 Elemental Wilson-Penzien model

For easier reference, a brief overview of the mathematical background of the Elemental Wilson-Penzien model as adapted from Puthanpurayil et al [12] is presented in this section:

Let Φ_e represent the full $N_e \times N_e$ elemental modal matrix and ω_e^2 represent the $N_e \times N_e$ diagonal frequency matrix. By classical modal analysis, elemental displacements $\mathbf{d}_e(t)$ can be expressed as:

$$\mathbf{d}_e(t) = \Phi_e\mathbf{q}_e(t) \tag{3}$$

where $\mathbf{q}_e(t)$ is the generalised coordinate in modal analysis. In the derivation of the global Wilson-Penzien model, the damping matrix assumes the same orthogonality property as the mass and stiffness matrices [15]. At the elemental level, the same assumption is valid. So, by pre-multiplying the equation of motion by Φ_e^T and using Equation (3):

$$\mathbf{M}_{di}\ddot{\mathbf{q}}_e(t) + \mathbf{C}_{di}\dot{\mathbf{q}}_e(t) + \mathbf{K}_{di}\mathbf{q}_e(t) = \mathbf{F}_e(t) \tag{4}$$

where, $\mathbf{M}_{di}$, $\mathbf{C}_{di}$ and $\mathbf{K}_{di}$ represent the diagonalised mass, damping and stiffness matrices, respectively. The individual terms in this are given as follows:

$$\left.\begin{aligned} M_{di}^{i} &= \left(\phi_e^i\right)^T \mathbf{M}_e \left(\phi_e^i\right) \\ C_{di}^{i} &= \left(\phi_e^i\right)^T \mathbf{C}_e \left(\phi_e^i\right) \\ K_{di}^{i} &= \left(\phi_e^i\right)^T \mathbf{K}_e \left(\phi_e^i\right) \end{aligned}\right\rangle i = 1,2........N_e \tag{5}$$

An elemental-level constant, denoted as ξ_{eWP}^{i} corresponding to the i^{th} elemental mode, is introduced and the elemental modal damping coefficient C_{di}^{i} can be then written as:

$$C_{di}^{i} = 2M_{di}^{i}\xi_{eWP}^{i}\omega_e^i \tag{6}$$

Now, from Equation (5) we get:

$$\mathbf{C}_e = \left(\Phi_e^{-1}\right)^T \mathbf{C}_{di}\left(\Phi_e^{-1}\right) \tag{7}$$

where:

$$\left(\Phi_e^{-1}\right)^T = \mathbf{M}_e\Phi_e\mathbf{M}_{di}^{-1} \tag{8}$$

$$\left(\Phi_e^{-1}\right) = \mathbf{M}_{di}^{-1}\Phi_e^T\mathbf{M}_e \tag{9}$$

By substituting eq. (6), eq. (8) and eq. (9) in eq. (7), we get:

$$\mathbf{C}_e = \Theta_e\Psi_e\Theta_e^T \tag{10}$$

where Θ_e is the mass-normalized elemental mode shape matrix, and Ψ_e is a diagonal matrix with diagonal elements given by:

$$\psi_e^i = \frac{2\xi_{eWP}^{i}\omega_e^i}{M_{di}^{i}} \tag{11}$$

In the present study for application convenience, ξ_{eWP} (named here as the Elemental Wilson-Penzien damping ratio) is assumed to be equal for all elements and *assumed to be the same as the global damping ratio (i.e., the damping ratio used for the global Wilson-Penzien model)*. However, it has to be noted here that there is no restriction requiring ξ_{eWP} to be a constant; if better parameterization methodologies are available, then ξ_{eWP} can be treated as a variable for each mode of each of the elements comprising the whole system. This shows the generality of the Elemental Wilson-Penzien formulation. This approach also allows for different damping ratios in different parts of the structural system easily, allowing for different damping ratios such as in the foundation members or for differentiating steel and concrete frame members.

4.0 Implementation of the proposed models in incremental Newmark frameworks

A majority of the commercial software implements an incremental version of Newmark Constant Average Acceleration method for solving the equations of motion. Figure 3 presents the flow chart for implementing the Elemental Wilson-Penzien model in tangent stiffness form. The main modification in the existing nonlinear framework is the addition of a sub-step (in the second block) to compute the assembled tangent-damping matrix which involves computation of elemental eigen parameters at every time step. However, as the matrices involved are very small, the computational penalty imposed by such a sub-step is close to negligible.

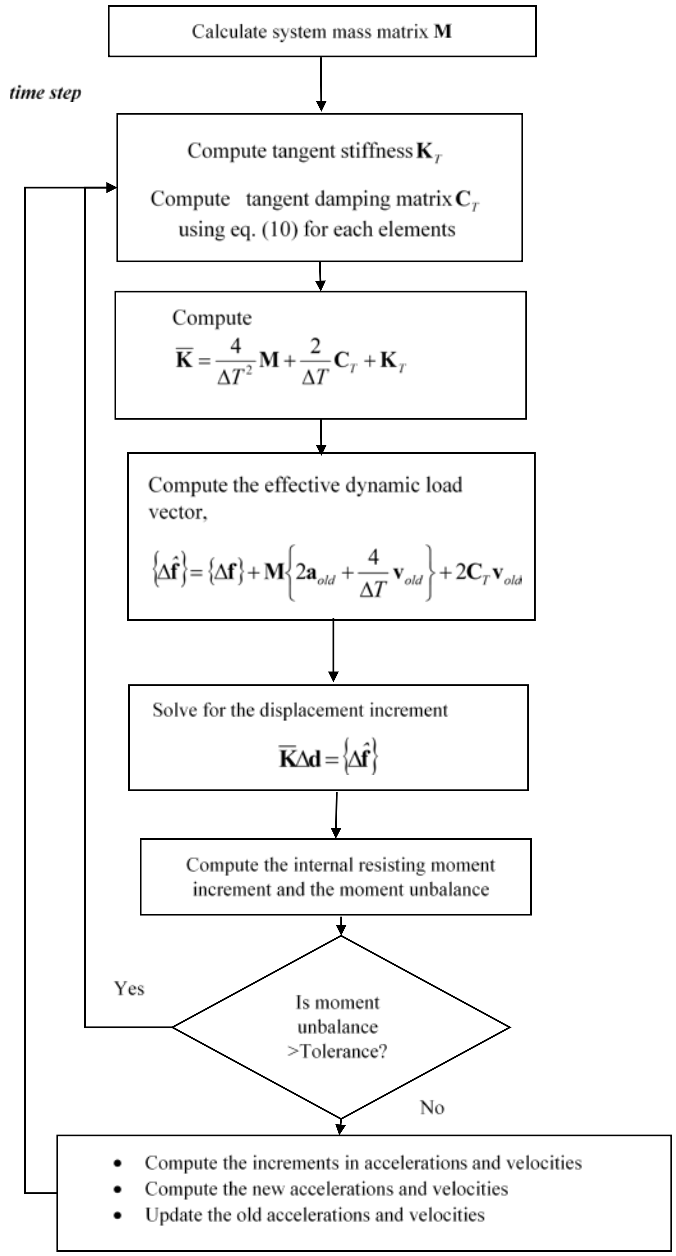

Figure 3 Flow chart for the implementation of elemental Wilson-Penzien in tangent form

Furthermore, the elemental formulation for this viscous damping model is such that a structure damping matrix is not required. In normal time integration processes the damping contributions to the dynamic stiffness matrix must be obtained by multiplying the global damping matrix by the global current velocity; but in the case of the elemental Wilson-Penzien model, these can be obtained together with the element stiffness matrices. At each time step the element damping actions can be computed at the element level, transformed to the global coordinates and summed to obtain the structures damping actions. This will reduce the memory requirements in the analysis as there is no need for the storing of a system damping matrix; also the requirement for large multiplication of the global damping matrix and current global velocities are eliminated producing a much better computational platform. The damping actions at each time step are computed from the product of the damping matrix (computed using the tangent stiffness) and the current velocities. The damping matrix is used as a secant matrix which avoids the significant unrealistic damping hysteretic effects.

A reason for updating the damping at each time-step in a non-linear structure is that, if the damping coefficients remain constant, as the stiffness reduces it is implied that the fractions of critical damping increase which is not regarded as realistic as the energy dissipation effects of the non–linearity is measured as plastic work.

5.0 Numerical Study

A four-storey reinforced concrete frame described in [16], designed in accordance with Eurocode 8 (EC8) and Eurocode 2 (EC2), has been used for the study. The frame was designed for high seismicity assuming a PGA of 0.3 g. The geometric and material properties of the frame are given in Appendix A. A simple lumped-point-plasticity model based on Giberson's one-component element was used for the modelling of the frame. A strength and stiffness non-degrading, bilinear hysteresis with 1 % strain hardening was used as an hysteretic model for the plastic hinge. Theoretically, the choice of this hysteresis over-simplifies the hysteretic performance of the concrete frame. However, as the main focus is to illustrate the performance of the updated Element Wilson-Penzien model, the choice of this simple hysteresis was deemed to be sufficient. All existing elemental and global damping models, including the proposed updated Elemental Wilson-Penzien one, are investigated in this section. The following abbreviations are used to identify different damping models included in the plots:

Global:

ISRD	Initial stiffness based global Rayleigh damping [17]
TSRD1	Tangent stiffness based global Rayleigh damping with constant coefficients [18]
TSRD2	Tangent stiffness based global Rayleigh damping with updated coefficients
GWP	Global Wilson-Penzien model

Elemental:

ELRD1:	Elemental Rayleigh damping with constant elemental proportionality coefficients
ELRD2:	Elemental Rayleigh damping with updated proportionality coefficients
EWP:	Elemental Wilson-Penzien model implemented as a constant damping matrix
UEWP:	Elemental Wilson-Penzien model implemented as a tangent matrix using secant formulation to avoid damping hysteresis.

The EWP model is implemented as a constant matrix outside the nonlinear loop whereas the UEWP model was implemented as per Section 5. In all the plots presented in this section, global models are presented with continuous lines and elemental models are presented with broken lines.

5.1 Consistent mass formulation

In this approach, the mass matrix used for damping matrix computation adopts the classical consistent mass formulation of Euler Bernoulli beams. As the whole purpose of the IDA study is to illustrate qualitatively the effect the choice of the damping models might have on the performance assessment studies, a set of seven artificial far-field ground motions scaled to five intensity measures (represented by peak ground acceleration, PGA) were used. The intensity measures to which the ground motions were scaled are 0.3 g, 0.4 g, 0.5 g, 0.6 g and 0.7 g. In order to have similarity when compared, the global models also adopt consistent mass for the damping matrix computation instead of the more usual lumped mass formulation.

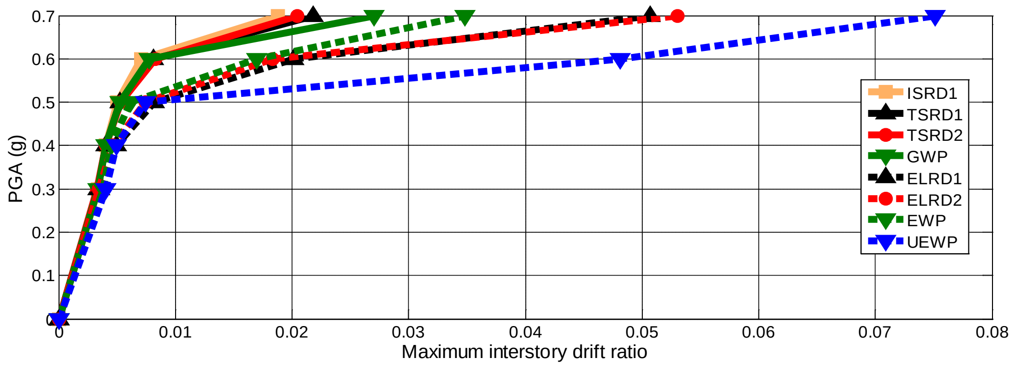

Figure 4 : IDA results for the inter-storey drift ratio

Figure 4 illustrates the the mean IDA curves for location-independent peak inter-storey drift ratio as the engineering demand parameter (EDP). It can be clearly seen that the Updated Elemental Wilson-Penzien model (UEWP) gives a very conservative result from an engineering perspective. All the updated models show greater displacements. This is to be expected as if the damping matrix remains a constant greater damping is implied when the structure becomes non-linear greatly reducing the deflections. The use of the tangent stiffness to compute the damping also increases the displacements. These effects were also observed by Crisp [2]. Although not shown here, damping moments obtained were also very small for this model.

5.2 Diagonal mass formulation

In this section, the mass matrix used in the damping matrix computation adopts the classical diagonal mass formulation of Euler Bernoulli beams. The same set of seven artificial far-field ground motions were used for this study. All the global models also adopt the diagonal mass formulation for the damping matrix computation.

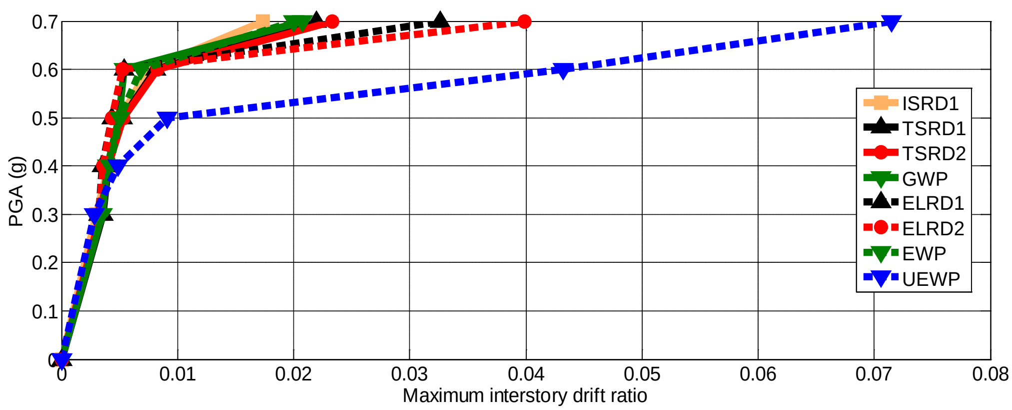

Figure 5 : IDA results for the inter-storey drift ratio

Figure 5 illustrates the the mean IDA curves for location-independent peak inter-storey drift ratio as the engineering demand parameter (EDP). It can be clearly seen that the Updated Elemental Wilson-Penzien method (UEWP) gives a very conservative result from an engineering perspective. Although not shown here, damping moments obtained were also very negligible for this model. One of the problems identified by Puthanpurayil et al [12] was that the EWP model performed very poorly when diagonal mass formulation was used. Figure 5 shows that UEWP performs really well with the diagonal mass formulation reinforcing the fact that an updated Elemental Wilson-Penzien damping model shows itself as as an appropriate model for representing the inherent damping phenomenon. As pointed out the non-updated models give reduced displacements due to excessive damping at time of non-linearity and the elemental models give the ability to match the damping to the expected behaviour of each of the elements in the structure. The elemental Wilson-Penzien damping also avoids the complications of the global Wilson-Penzien model, the large eigenvalue problem require to set up the matrix and computational costs implied by the fully populated damping matrix.

6.0 Conclusions

The tangent Elemental Wilson-Penzien damping model is presented along with the implementation steps in detail. It has been shown that the tangent implementation version of the Elemental Wilson-Penzien damping model performs equally well with either consistent mass formulation or diagonal mass formulation. Through the IDA study it has also been shown that the tangent implementation of Elemental Wilson-Penzien damping produces more conservative results from an engineering perspective in comparison to other global and elemental damping models - making it more conservatively appropriate for performance assessment studies.

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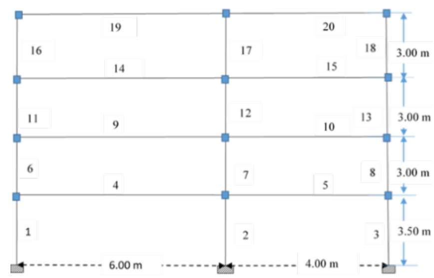
Appendix A: Details of the four storey frame



C_{di} refers to added dampers and $i=1....4$

Material Property

Dynamic Young's modulus $= 3.5 \times 10^{10}\ Nm^{-2}$

Geometric Properties

Member number	Width (mm)	Depth (mm)
1,6,11,16,17,12,7,2,3,8,13,18	450	450
4,5,9,10,14,15,19,20	300	450

Nodal Mass

Floor level	Mass per node (kg)
1st floor	29 800
2nd -4th floors	29 500

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