



Design spectrum-based evaluation of seismic demand of tall buildings

Y. Liu⁽¹⁾, J.S. Kuang⁽²⁾, N. Zhang⁽³⁾

⁽¹⁾ Ph.D. Student, The Hong Kong University of Science and Technology, Kowloon, Hong Kong. E-mail: yliubp@connect.ust.hk

⁽²⁾ Professor, The Hong Kong University of Science and Technology, Kowloon, Hong Kong. E-mail: cejkuang@ust.hk

⁽³⁾ Ph.D. Student, The Hong Kong University of Science and Technology, Kowloon, Hong Kong. E-mail: nzhangab@connect.ust.hk

Abstract

Design spectrum-based evaluation of the seismic demand of tall buildings using modified consecutive modal pushover analysis (MCMP) method is presented, whereas paper, only the multi-stage consecutive pushover procedure is adopted and the contribution of different modes and target displacements are obtained from response spectrum analysis using design spectrum. The final response is the maximum value of the responses at the end of each pushover stage. Comparison of the results from the nonlinear response time history analysis (NL-RTHA) method, the modal pushover analysis (MPA) method, the modified consecutive modal pushover analysis (MCMP) method and the proposed design spectrum-based evaluation (DSE) procedure is made. It can be seen that by using the MCMP in the DSE procedure, the evaluation procedure can predict the seismic demand of buildings, such as inter-storey drift ratio and hinge plastic rotation efficiently and accurately. The results are very close to those obtained from the NL-RTHA method. Owing to its simplicity, accuracy and efficiency, the proposed DSE procedure is considered to be a very promising tool for fast prediction of the seismic demand of tall buildings.

Keywords: seismic demand; tall buildings; nonlinear time history; design spectrum-based evaluation



1. Introduction

In countries and regions of low or moderate seismicity, the use of non-seismic design methods has resulted in little overall seismic resilience of structures and buildings, in particular tall buildings. Assessing non-seismically designed buildings that may have high seismic vulnerability is of critical importance for setting priority criteria for possible strengthening of those buildings. Hence, quick assessment of the seismic demand of existing tall buildings that were designed and built without seismic consideration is one of the most critical and tough challenges for the engineers and researchers.

By very careful selection and scaling of earthquake motions and proper modelling assumptions, the nonlinear time history analysis (NL-RHA) method is the most rigorous and accurate method to obtain the seismic demand of buildings. Indeed, the analysis using NL-RHA requires very heavy computational effort and is very time consuming, especially when the structure has a large number of degrees of freedom. This makes the NL-RHA method unfavourable for quick evaluation of the seismic demand of huge amount of tall buildings. Alternatively, nonlinear static pushover (NSP) method, with the basic idea that applying a set of equivalent lateral forces to the structures to obtain the strength and ductility demand of the structure, is another powerful method to evaluate the seismic demand of buildings. Owing to its high efficiency and reasonable accuracy, the NSP method has gained great popularity and is a standard tool for seismic assessment in many codes of practice. Nevertheless, for tall buildings, where the contribution of high modes is significant, the conventional NSP is not capable of estimating the seismic demand of tall buildings with high accuracy [1-3].

To obtain more accurate estimation of seismic demand of tall buildings, while keeping the efficiency of the NPS method, numbers of modified pushover methods were proposed. The modal pushover analysis (MPA) method and another more efficient method, named modified modal pushover analysis (MMPA) method, developed by Chopra [4,5], are two of the most powerful and prevalent pushover methods considering the contribution of higher modes. In the MPA and MMPA method, the seismic demand of each modal pushover procedure is determined separately and combined to get the final estimation of the seismic demand with appropriate combination law, such as SRSS method or CQC method. Similarly, enveloping methods, like the N2 method, the extended N2 method and the envelope-based pushover method [6-8], which consider several modal and failure modes, and envelope the results of several pushover procedures to estimate the seismic demand of buildings, are also modified NSP that calculate the contributions of different modes separately. The MPA method and enveloping methods can provide much more accurate estimation of seismic demand than conventional NSP procedure. However, as the seismic demand of different modes is considered separately, it is not the case when considering the nonlinear performance of a building; hence these methods are not capable of estimating the local seismic demand of tall buildings, especially for the accurate estimate of hinge rotation.

Adaptive modal pushover analysis (AMPA) method, developed by Gupta [9], Kalkan [10] and Antoniou [11], combined the response of individual modal pushover analyses to account for the influence of higher modes and incorporated the effects of changing modal properties during inelastic response through its adaptive feature. This method is capable of reproducing the essential response features and providing a reasonable measure of the likely contribution of higher modes in all phases of the response. However, to achieve high accuracy, the modal properties should be updated so frequently that makes the computation effort of the AMPA method even comparable with the NL-RHA method. This means that the AMPA method is much less efficient than other pushover methods. The modified consecutive modal pushover analysis (MCMP) method, proposed by Khoshnoudian et al. [12], is another method where the seismic demand of different modes is not considered separately. For the MCMP method, the force vectors for different modes are applied to the structures consecutively and the coupling effect on the structural performance of the buildings is also consecutively taken into account. Although the coupling effect of different modes is simplified, it is found that the MCMP has a good estimation of the local seismic demand.

The main objective of this paper is to present a design spectrum-based evaluation (DSE) procedure for quick estimation of the seismic demand of tall buildings. The MCMP is modified and used in the DSE procedure, where only the multi-stage consecutive pushover procedure is adopted and the contribution of different modes and target displacements are obtained from response spectrum analysis using design spectrum. The final response is the maximum value of the responses at the end of each pushover stage. To verify the



applicability and effectiveness of the propose procedure, two special steel moment-resisting frames (SSMRF) with different heights are used for analysis. Comparison of the results from the nonlinear response time history analysis (NL-RTHA) method, the modal pushover analysis (MPA) method, the MCMP method and the DSE procedure is made. It is found from the comparison that the DSE procedure can predict the seismic demand effectively and accurately and the results of the DSE procedure match those obtained from the NL-RTHA method well.

2. Response spectrum analysis

In the design spectrum-based evaluation procedure, the roof displacement is the key parameter revealing the overall performance of the structure under the action of earthquake ground motion. The elastic response of an N-degree of freedom system under the ground motion is governed by

$$m\ddot{u} + c\dot{u} + ku = -mi\ddot{u}_g(t) \quad (1)$$

where u is the floor displacements vector, m , c and k are the mass, classical damping and stiffness of the system respectively; i is the unit vector. The right-hand side of equation (1) represents the effective earthquake force and can be written as

$$p_{eff}(t) = -mi\ddot{u}_g(t) = -s\ddot{u}_g(t) \quad (2)$$

where s represents the spatial distribution of the effective forces over the height of the building, and can be expanded as a summation of the modal inertia force distributions s_n , given by

$$s = \sum_{n=1}^N s_n = \sum_{n=1}^N \Gamma_n m \phi_n \quad (3)$$

in which ϕ_n is the n th natural vibration mode of the structure, and

$$\Gamma_n = \frac{L_n}{M_n}, \quad L_n = \phi_n^T m i, \quad M_n = \phi_n^T m \phi_n \quad (4)$$

By the summation of the model response, the displacement of an N-degree of freedom system can be expressed as

$$u(t) = \sum_{n=1}^N \phi_n q_n(t) \quad (5)$$

where the modal co-ordinate $q_n(t)$ is governed by

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = -\Gamma_n \ddot{u}_g(t) \quad (6)$$

The solution of Equation (6) is given by

$$q_n(t) = \Gamma_n D_n(t) \quad (7)$$

where $D_n(t)$ is governed by the equation of motion for a single degree-of-freedom (SDOF) system subjected to $\ddot{u}_g(t)$:

$$\ddot{D}_n + 2\zeta_n \omega_n \dot{D}_n + \omega_n^2 D_n = -\ddot{u}_g(t) \quad (8)$$

The floor displacement can then be represented as

$$u(t) = \sum_{n=1}^N u_n(t) = \sum_{n=1}^N \Gamma_n \phi_n D_n(t) \quad (9)$$

The peak value of D_n can be determined from deign spectrum. According to the square-root-of-squares (SRSS) combination rules, an estimation of the peak value to roof displacement can be expressed as

$$u_{r0} \approx (\sum_{n=1}^N (u_{rn0})^2)^{0.5} = (\sum_{n=1}^N (\Gamma_n \phi_{rn} D_n)^2)^{0.5} \quad (10)$$

3. Design spectrum-based evaluation procedure

The MCMP method is modified and adopted. In the MCMP method, both multi-stage and single-stage pushover analysis procedure are included. The multi-stage consecutive pushover technique adopted is that when one pushover analysis procedure completed, the next pushover procedure started with initial structural state (like internal stress, strain and displacement) the same as the structural state at the end of previous pushover analysis procedure. The force vectors used for the multi-stage pushover procedure are mode-shape distributions that are obtained from eigenvalue-analysis of linear structures and a uniformly distributed force vector is used for single-stage pushover analysis procedure. The changes of modal properties and mode shape at the nonlinear stage are ignored. The order of the modal pushover analysis should follow the order of modes, from the first mode to higher modes. The number of modes and modal pushover analysis included depend on the height and the configuration of buildings. The final response is taken as the maximum response of that obtained at the end of multi-stage consecutive pushover procedure and single-stage pushover procedure.

Fig. 1 presents typical force vectors used in the multi-stage consecutive pushover procedure in MCMP method for a 9-storey building. It can be seen that the forces at first 4 storeys and the 7th and 8th storey are not in the same direction for different mode cases. This means that during the multi-stage consecutive pushover procedure, these floors will experience push and pull forces in turn, while only push forces act on the roof storey. As a result, the response at the end of the multi-stage consecutive pushover procedure, where final target roof displacement has reached, the displacement, drift ratio and other responses of other stories may not be the peak values. In other words, the peak responses of other stories might have been reached before the roof displacement reaches the target displacement value. Thus, taking the responses at the end of the multi-stage consecutive pushover procedure is not reasonable and not enough. The solution for this in the MCMP is to have a single-stage pushover analysis, where a uniform distributed force vector is used, and the final response is taken as the maximum response of that obtained at the end of multi-stage consecutive pushover procedure and single-stage pushover procedure.

In the DSE procedure, another solution for the irrationality of taking the responses at the end of the multi-stage consecutive pushover procedure is proposed and used, in which only the multi-stage consecutive pushover procedure is adopted, and the final response is the maximum value of the responses at the end of each pushover stage. This is based on that the force vector keeps invariant in each stage, and the response at the end of each stage represents the maximum response that the force vector can induce in the corresponding stage.

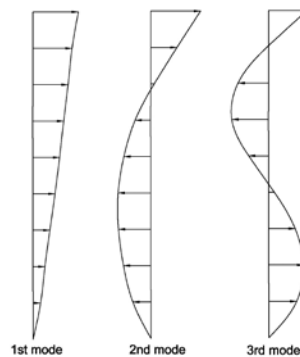


Fig. 1 – Typical force vector used for MCMP method

Follow this way, the MCMP method is modified in the DSE procedure. Furthermore, in the DSE, the contribution of different modes and target displacements are obtained from response spectrum analysis using design spectrum, and the details of the proposed DSE procedure is summarised as:

- (1) Eigenvalue analysis of structure. Calculate the natural frequencies, ω_n , and the mode-shapes, ϕ_n . The mode-shapes are normalized so that the roof component of ϕ_{rn} equals unity ($\phi_{rn} = 1$).
- (2) Compute $s_n^* = m\phi_n$, where s_n^* shows the distribution of incremental lateral forces over the



height of the structure for the n th stage of multi-stage pushover analysis.

(3) Obtain the design spectrum of building.

(4) Compute the target roof displacements of the structure for different mode $\mathbf{u}_{r_{i0}}$ and the total roof displacement \mathbf{u}_{r0} based on the response spectrum analysis method using design spectrum.

(5) Carry out the modal consecutive pushover analysis procedure. This step consists of a few pushover analyses. First, apply the gravity loads and then perform the displacement-control pushover analyses according to the following sub-steps:

- a. In the first stage, perform the nonlinear static pushover analysis, using the incremental lateral forces $s_1^* = m\phi_1$, until the displacement increment at the roof reaches $u_{r1} = \alpha_1 u_{r0}$, where $\alpha_i = \frac{u_{ri0}}{\sum_{i=1}^N u_{ri0}} = \frac{\Gamma_i \phi_{ri} D_i}{\sum_{n=1}^N \Gamma_n \phi_{rn} D_n}$, which represents the contribution of different modes. Obtain the peak value of the response such as displacements, storey drifts, and hinge plastic rotations for this stage, the response is noted by r_1 .
- b. Implement the second stage of analysis using the incremental lateral forces $s_2^* = m\phi_2$ until the displacement increment at the roof equals $u_{r2} = \alpha_2 u_{r0}$. It is noted that the initial condition in this stage of pushover analysis is the same as the state at the last step of analysis in the previous stage. Obtain the peak value to the response r_2 for this stage;
- c. Repeat step 5.2 (b) for the rest of the modes considered, with the target displacement of i th step being $u_{ri} = \alpha_i u_{r0}$ and the initial condition in this stage pushover analysis being the same as the state at the last step of analysis in the previous stage. Obtain the peak value to the response r_i for this stage;

(6) Enveloping the peak response for each step:

$$r = \max\{r_1, r_2, \dots, r_n\} \quad (14)$$

The value of r shows the seismic response, such as drift ratio, displacement for each level, hinge plastic rotation, et al. computed by the DSE procedure.

4. Case study

4.1 Ground motion characteristics

In the case study, a typical ASCE-10 code design spectrum [13] was selected as the target response spectrum. The key parameters for the target response spectrum were: $S_{ds} = 1.2g$, $S_{d1} = 0.85g$, $T_L = 10s$. Overall 15 ground motion records from the strong ground motion database of PEER (<http://peer.berkeley.edu/>) were selected and scaled by minimising the computed weighted mean squared error of record, and suite average, with regards to target spectrum, or simply, minimizing MSE method. The weight value was 1.0 for periods 0.1s, 1s, 4s and 10s to ensure that the response spectrums of the scaled selected records fit the target spectrum well in a wide range of the period. The selected strong ground motions had Ms magnitudes range between 6.5-8. The motion records were far field records, and the corresponding distance between the recording stations and the epicentre were at least 12 km. The soil type for the site is NEHRP site class C, and no pulse like records was considered. Details of the selected records are listed in Table 1. The pseudo-acceleration spectrum and displacement spectrum of the target ASCE spectrum and the selected ground motions are shown in Fig. 2.

4.2 Description of the models

Two special steel moment-resisting frames (SSMRFs) with different heights of 9 and 20 stories, respectively, were used for the case study. The SSMRFs, representing typical medium and high rise buildings design for the Los Angeles, California, were designed for the SAC Phase II Steel project and meet seismic code requirement of



the 1994 UBC. In this paper, only the perimeter of frames in the north–south direction was modelled and used for case study. The characteristics and modal information are summarized in Table 2. Details about the member size, seismic mass and materials have been given by Ohtori et al. [14].

Table 1 – List of selected motion records

Result ID	Earthquake	Year	Magnitude (Ms)	Station Name	Component (deg)	Scale factor
1	Cape Mendocino	1992	7.01	Fortuna - Fortuna Blvd	0	4.1683
2	Landers	1992	7.28	Amboy	0	4.0531
3	Chi-Chi_ Taiwan	1999	7.62	HWA033	E	3.8879
4	Chi-Chi_ Taiwan	1999	7.62	HWA051	N	4.1792
5	Chi-Chi_ Taiwan	1999	7.62	TCU015	E	3.6098
6	Chi-Chi_ Taiwan	1999	7.62	TCU042	E	2.548
7	Chi-Chi_ Taiwan	1999	7.62	TCU106	E	2.467
8	Chi-Chi_ Taiwan	1999	7.62	TCU116	E	2.2468
9	Hector Mine	1999	7.13	Amboy	90	2.6191
10	Cape Mendocino	1992	7.01	College of the Redwoods	270	3.4667
11	Cape Mendocino	1992	7.01	Ferndale Fire Station	270	1.4044
12	Cape Mendocino	1992	7.01	Loleta Fire Station	270	2.1273
13	Landers	1992	7.28	North Palm Springs Fire Sta #36	90	4.1362
14	Chuetsu-oki_Japan	2007	6.8	Sawa Mizuguti Tokamachi	NS	4.1456
15	Iwate_Japan	2008	6.9	Yuzawa Town	NS	2.8624

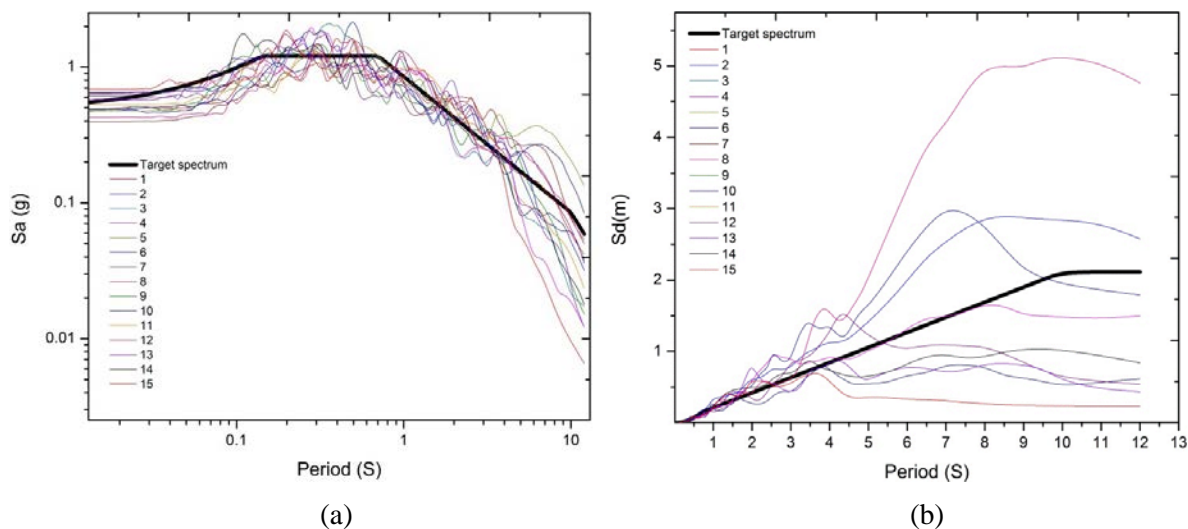


Fig. 2 – Analysis of spectrums. (a) Pseudo-acceleration spectrums; (b) displacement spectrums with damping ration=5%



The nonlinear behaviour of the structures is assumed to be concentrated on the discretized hinges that located at the ends of the frame members. For columns, the hinges were modelled based on the interaction of axial forces and bending moment, while for beams, the hinges just under the action of bending moment. The properties of the hinges and modelling parameters were calculated according to FEMA 365, and Fig. 3 shows a typical force-deformation relationship for the hinges.

Table 2 – Modal properties of the structures

No.	Storey	Period (s)		
		T_1	T_2	T_3
F1	9	2.268	0.856	0.496
F2	20	3.832	1.325	0.767

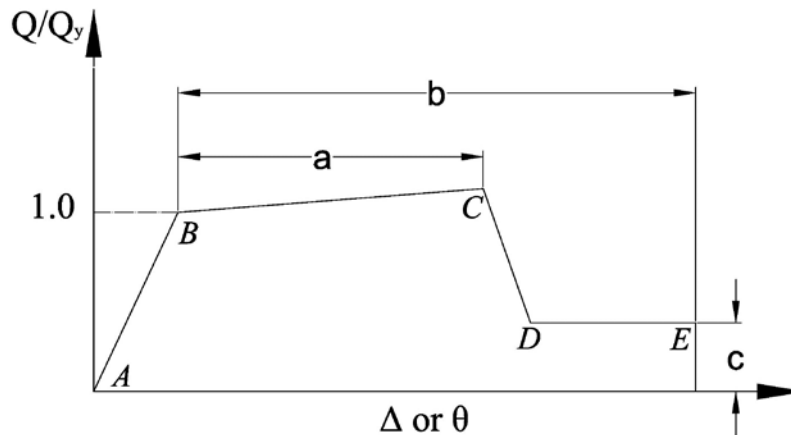


Fig. 3 – Force-deformation relationship for hinges

4.3 Details of analytical models

To verify the accuracy and applicability of the DSE procedure, a comparison of the results from the DSE procedure, the MCMP method, the MPA method and the NL-RHA method is made. In this study, the NL-RHA method and the MPA method were carried out for all the selected ground motions, the median value of the responses was used for comparison. The numerical implicit Wilson- θ time integration method, in which the parameter θ was assumed to have a value of 1.4 was adopted to perform the NL-RHA method. A 5% damping ratio was assumed for the first and third mode of builds for analysis. For the MPA method, 3 modes were considered for the 9-storey structures, while 5 modes were included for the 20-storey structures. In MCMP method, three-stage analysis procedure is used for both structure, and the target roof displacements for the MCMP method were obtained from median roof displacement of NL-RHA method, which were 67.53 cm and 98.50 cm for F1 and F2 respectively. While for the proposed DSE procedure, 3 modes were considered for both structures. The target displacements as well as the contribution of different modes were calculated based on the response spectrum analysis using the selected ASCE design spectrum. The P- δ effect was included in all the methods. The nonlinear version of SAP 2000 was adopted for all the analysis procedures.

5. Results and discussions

The height-wise variation of the inter-storey drift ratio and hinge plastic rotation of F1 and F2 are shown in Fig. 4 and Fig. 5 respectively. It should be noted that the results of the MPA and NL-RHA method in the figures are



the median values. The maximum as well as the minimum response calculated from the NL-RHA method are also plotted in all the figures, and were denoted as MAX and MIN respectively.

From Fig. 4, it can be seen that all the simplified analysis procedures provide satisfactory estimation of the inter-storey drift ratio, as almost all the results are close to the median inter-storey drift ratio and vary between the maximum and minimum inter-storey drift ratios from NL-RHA. As for the inter-storey drift ratio of the 9-storey building, the MPA method tends to overestimate the drift ratio at the lower part of the building, but underestimate the drift ratio at upper part of the building. While for the MCMP method, the drift ratio at the upper and lower parts of the building is overestimated, while the drift ratio at the middle part is lower than that of the NL-RHA method. Among the three pushover methods, the DSE procedure provides the best estimation of the storey drift ratio, as height-wise variation of the inter-storey drift ratio of the DSE procedure matches that of the NL-RHA method very well. The inter-storey drift ratio of the 20-storey building calculated from the MPA method shares the same trend with that of 9-storey building from the MPA method, since the drift ratio by the MPA method is smaller at lower part and larger at upper part, compared with that from NL-RHA method. While for the MCMP method and the MCPA method, the inter-storey drift ratio at upper and lower part is overestimated, and the inter-storey drift ratio is underestimated at the middle part of the building. It can also be seen from Fig. 4(b) that it is still the DSE procedure that provides the closest estimation of the inter-storey drift ratio, since the drift ratio calculated by the DSE procedure has the least bias compared with that of the NL-RHA method.

Fig. 5 shows the hinge plastic rotation results of the 9-storey and 20-storey buildings respectively. It is found from the results that all the methods provide reasonable estimation of the hinge plastic rotation, since all the estimations are close to the NL-RHA median value and vary between the maximum and minimum results. When it comes to the comparison of results from pushover methods and the NL-RHA method, it can be seen from Fig. 5 that the trend of bias does not change with the change of storey height. The MPA method overestimates the hinge plastic rotation at lower part, but underestimates the hinge plastic rotation at upper part. The hinge plastic rotation of the MCMP and DSE procedure is overestimated at the upper part and lower part, but at the middle part, the hinge plastic rotation from the MCMP and DSE procedure is underestimated. Similarly, it still can be found from Fig. 5 that the hinge plastic rotation of the DSE procedure matches the best with that of the NL-RHA method.

The comparison of the results shows the great applicability of the proposed DSE procedure. With the modifications in the proposed procedure, the DSE procedure can even provide better estimation of the seismic demand from the MCMP method. Except for the high accuracy, the design spectrum-based computation procedure is another attractive aspect of the DSE procedure. Since on site ground motion data is not available for most of the site, design spectrum is one of the best representations of the intensity of the possible earthquake at a particular site. What is more, for most of structural engineers, design spectrum is the most recognised tool for the seismic design. As a consequence, the DSE procedure would be very easy for practical engineers to acquire and use. Thus owing to its simplicity, accuracy, efficiency and spectrum-based calculation procedure, the proposed DSE procedure is considered to be a very promising tool for predicting the seismic demand of tall buildings.

6. Conclusions

In this paper, a design spectrum-based evaluation (DSE) procedure for quick evaluation of the seismic demand of tall buildings is presented. A comparative case study was conducted to verify the accuracy and applicability of the DSE procedure. On the basis of the research findings, the following conclusions can be drawn.

1. All the simplified methods, including the MPA method, the MCMP method and the DSE procedure, can provide satisfactory estimation of the seismic demand of the buildings, since the results of the simplified methods are close to the median response of the NL-RHA method, and vary between the maximum and minimum responses of the NL-RHA method.
2. Among all the simplified methods, the DSE procedure has the best prediction of both inter-storey drift ratio and hinge plastic rotation of buildings with different heights.



3. Owing to its high efficiency, accuracy and the spectrum-based calculation procedure, the proposed DSE procedure is considered as the one of most promising tools for fast seismic prediction of tall buildings.

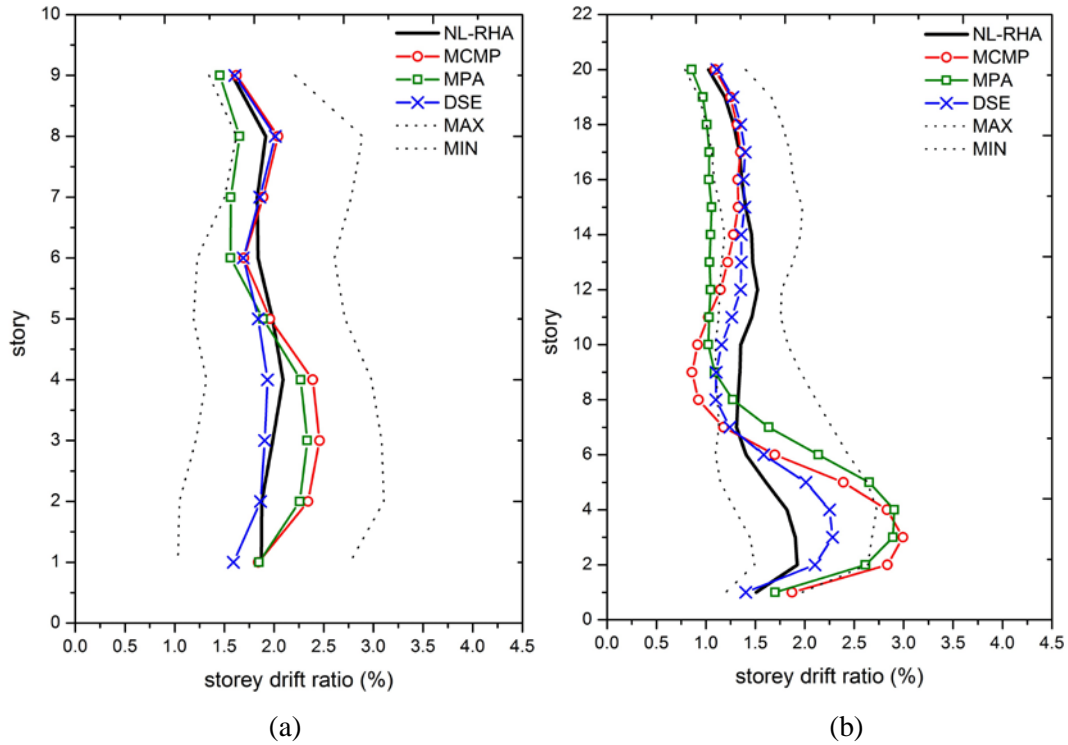


Fig. 4 – Inter-storey drift ratio. (a) 9-storey building; (b) 20-storey building

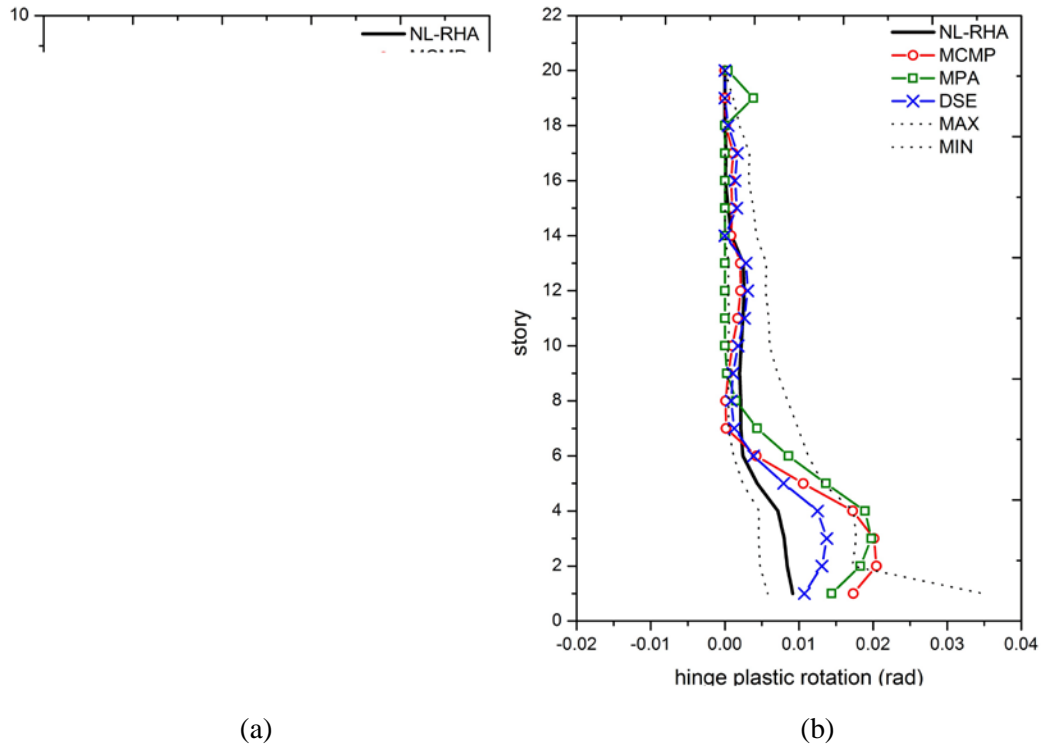


Fig. 5 – Hinge plastic rotation. (a) 9-storey building; (b) 20-storey building



7. Acknowledgements

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