16th World Conference on Earthquake, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N° 1670

Registration Code: S-M1463077088

TIME-VARYING SEISMIC FRAGILITY ASSESSMENT OF A CORRODING RC BRIDGE PIER CONSIDERING NON-UNIFORM CORROSION

S.A. Faroz(1), N.N. Pujari(2), S. Ghosh (3), J. Ghosh(4), S. Shekhar(5)

(1) *Research Scholar, Department of Civil Engineering IIT Bombay, Mumbai, India, iamsharvil.faroz@gmail.com*
(2) *Research Scholar, Department of Civil Engineering IIT Bombay, Mumbai, India, pujarinikil@gmail.com*
(3) *Professor, Department of Civil Engineering IIT Bombay, Mumbai, India, sghosh@civil.iitb.ac.in*
(4) *Assistant Professor, Department of Civil Engineering IIT Bombay, Mumbai, India, jghosh@civil.iitb.ac.in*
(5) *Research Scholar, Department of Civil Engineering IIT Bombay, Mumbai, India, shivang12287@gmail.com*

Abstract

Highway bridges form an integral and important part of a nation's infrastructure. Estimation of the seismic vulnerability of these bridges is a necessary step to identify the associated risk and ensure the safety of lifeline systems. Reinforced concrete (RC) dominates bridge construction around the world (along with prestressed concrete). Corrosion has been identified as a major degrading factor in concrete bridges, which makes these structures more vulnerable to extreme event loads, such as earthquake or wind forces, over their design life. Seismic vulnerability assessment of bridge structures is typically performed through a seismic fragility analysis in the probabilistic domain. For about two decades, researchers have been using this tool to effectively estimate the seismic failure probabilities of bridge structures. More recently, researchers have focused on the seismic fragility estimation for concrete bridges deteriorating primarily due to corrosion. However, most of these studies have not properly dealt with (i) the significant uncertainties in the corrosion process and (ii) the effects of pitting corrosion. The current paper presents a unique framework of seismic fragility estimation of corroded concrete bridges, incorporating a probabilistic treatment of the non-uniform corrosion process and integrating it with the Bayesian updating of the corrosion model based on monitored data (measurements through non-destructive testing). The time-varying seismic fragility of a RC bridge pier is estimated in an illustrative numerical example. Conclusions are drawn based on how the seismic fragility for this bridge column vary over time and on the effectiveness of the framework proposed.

Keywords: pitting corrosion; seismic fragility; bridge; adaptive Metropolis; curvature ductility

1. Background

Highway and railway bridges, which form an integral part of the transportation infrastructure, gradually degrade, primarily due to environmental effects and possible overloading. This results in degeneration of the performance of a structure over its lifetime. Ignoring this inevitable fact can lead to misinformed decisions regarding various issues, such as, inspection, maintenance, repair/rehabilitation and replacement of the structure. In reinforced concrete bridges, corrosion of rebars in a saline environment is the most common factor causing its degradation [1]. This degradation typically renders the structure more vulnerable to extreme natural events such as earthquakes. The vulnerability of a structural component (or, a system) to possible earthquakes at a particular site, is typically quantified using fragility curves/functions. The fragility function of a structure expresses the likelihood of its failure for any given seismic intensity, in a probabilistic framework. When properly modelled, a fragility curve/function can capture the "important" variabilities that exist in estimating the possible earthquake and in structural parameters, and also the uncertainties induced due to the use of a particular analysis model. Degradation of a structure over its design life obviously brings in a change in its seismic fragility function. However for most situations, only the "pristine" or "as-built" fragility of a structure is used for all decision-making on and the management of a degrading structure [2].

The integration of corrosion growth modelling and seismic assessment for a structure is not trivial, as the existing corrosion prediction models themselves are deficient in terms of modelling capability [3]. However, this can be alleviated by supplanting the corrosion model with routine or planned corrosion inspections/monitoring.

Such inspections take place at discrete time instances, and these measurements do not provide a clear understanding of the degradation process in a continuous sense. These limited/sparse information, however, can be effectively used in conjunction with the existing corrosion growth model using Bayesian inference. Bayesian inference can be used to obtain an updated model, which is expected to provide a better picture of the degradation as a continuous time function over the structure's life.

Considering these reasons, the main objective of this study is to estimate the time-varying seismic fragility of a bridge due to the corrosion of its rebars, using a Bayesian framework to integrate the data obtained through corrosion inspection/measurements. Furthermore, the effect of pitting (non-uniform corrosion) is added to the updated uniform corrosion model using a pitting factor, which represents the heterogeneity (spatial variation) of steel loss in rebars. Our goal is to develop an overall framework to analyse the effect of time-varying pitting corrosion on the seismic fragility of a structure and illustrate it through the example of a typical two-span integral bridge. It should be noted that although corrosion in concrete is reported to affect the ductility and bond strength of rebars, this work considers only the loss of rebar cross-section (for both main and shear reinforcements) to affect the capacity of the section.

2. Overall framework

Previous studies on the seismic risk assessment of bridge structures have shown that the seismic fragility of a simple bridge, such as the one considered in this work, is determined by the fragility of its columns [4]. Therefore, the estimation of time-varying fragility in this paper focuses entirely on the pristine fragility and the gradual degradation of the bridge column due to corrosion. Choe et al. [5] have also shown that corrosion in columns reduces the capacity of a bridge and increases its seismic fragility. The overall framework adopted here has three major aspects as presented in Fig. 2: (i) Bayesian updating of the corrosion growth model using inspection data, (ii) Sampling of corroded bridge columns incorporating the effect of pitting over the an updated corrosion growth model, and (iii) Seismic fragility estimation of a pristine or corroded column using multi-IDA.

The existing analytical models for corrosion growth over time do not provide a good estimate of the loss of steel in rebars [3]. Monitoring/inspection at regular intervals is expected to help in getting a better judgement of the existing conditions. Integration of these inspection data with an existing steel loss model using Bayesian inference [3] can be a suitable alternative to relying entirely either on analytical models or on discrete time measurements. For this study, we adopt a Bayesian MCMC scheme as presented in Fig. 1 and explained in detail in Section 7. Monitoring is done at discrete time instances which is assumed to represent the true picture of the actual degradation. The randomness present in the parameters involved in any steel loss model (for example see the one later in Eq. (3) results in a probabilistic distribution of steel loss at any given instant ($f(W|t)$). Assuming that the measurement techniques are not precise enough to give the steel loss due to pitting corrosion, we adopt a corrosion model for uniform or average corrosion in the Bayesian updating process.

The Bayesian updating of the corrosion model results in a probability distribution of average steel loss (W) or the average (corroded) cross-section (A_{avg}). In order to propagate this variation of steel loss in rebars along with the randomness in the input ground motion, a stratified simulation format using Latin Hypercube sampling (LHS) is adopted in the seismic fragility analysis. In this, an equal number ($= g$) of LHS samples are generated along both the dimensions. Each of these samples represents a single bridge column model and incremental dynamic analysis (IDA) for a single earthquake sample. To incorporate the effect of pitting corrosion (elaborated in Section 6.1) in each bridge model, we discretize the column into 'e' elements of equal length along its height and obtain LHS samples of the pitting factor (R) for each element. This results in different minimum residual cross-section (A_{min}) for each element. Assuming that the seismic demands do not change noticeably over the length of each element, A_{min} is assigned to the whole length of an element. This process is repeated for each bridge model-ground motion combination obtained earlier.

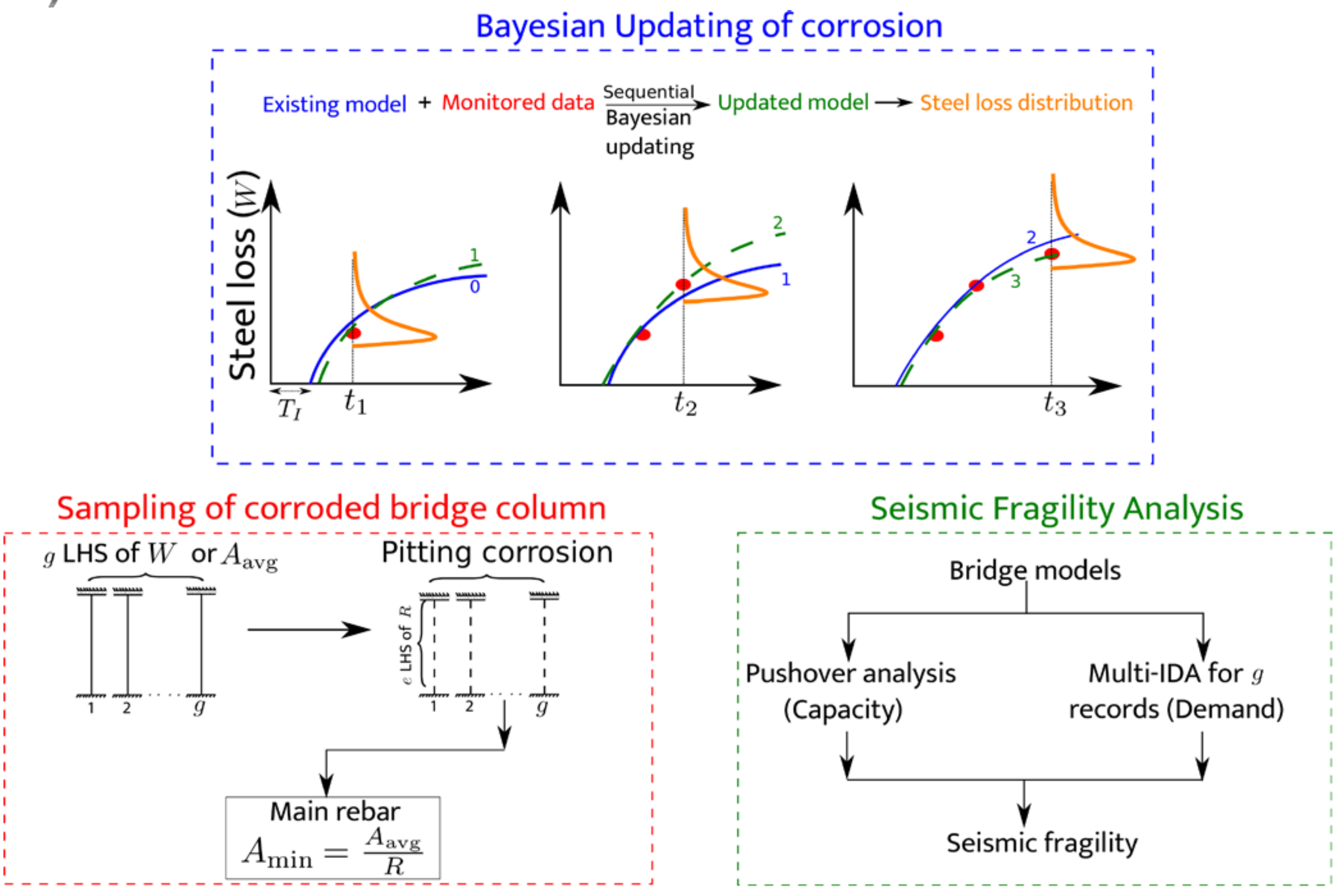

Fig. 1 – A schematic showing the three major aspects of the framework proposed here

Seismic fragility is typically defined as the conditional probability of a damage measure (DM) exceeding its threshold of limiting value (DM_l), given an intensity measure (IM). Adopting a two-parameter lognormal model, we can express the fragility of the column (or, the bridge) as

$$F_r = P(\mathrm{DM} > \mathrm{DM}_l \,/\, \mathrm{IM}) = \Phi\left[\frac{\ln\left(x/m_a\right)}{\beta_r}\right] \tag{1}$$

where, m_a is the median ground acceleration capacity and β_r is the aleatory uncertainty due to the variability in ground motion. Seismic fragility of a given structure can be evaluated using various approaches, such as, judgement based, empirical, experimental, analytical and hybrid. A detailed description of these different approaches for bridges has been presented recently in a review paper [6]. In this work, the seismic fragilities are derived analytically based on results obtained by performing multi-IDA (that is for multiple ground acceleration records). The basic idea behind IDA [7] is that the influence of ground motion variability in structural response can be captured by performing multiple nonlinear response-history analyses (NLRHA) at scaled intensity levels. A multi-IDA can become very computation-intensive de-pending on the number of intensity levels at which NRLHA need to be performed and the number of different ground motion records used. In order to reduce this computation, we make use of Bayesian inference as used by researchers in the past [8]. Traditionally adopted fragility models for bridges [2] require a prior estimate of the probabilistic distribution of seismic demand, whereas the multi-IDA based approach followed here does not require this information. Instead, the NLRHA results are directly compared against the capacity of the structure defined by (deterministic) performance criteria. The performance limit states for the bridge column are determined by performing a pushover analysis (presented in detail in Section 4) at the mean material properties with the original cross section area of rebars. The same limit states are considered for the degraded column as well, for the lack of any standard guideline on defining such limit states for corroded structures. Discrete seismic fragility data obtained from the multi-IDA analysis are used in a Bayesian MCMC framework to obtain a continuous fragility curve for the bridge column at every time instant.

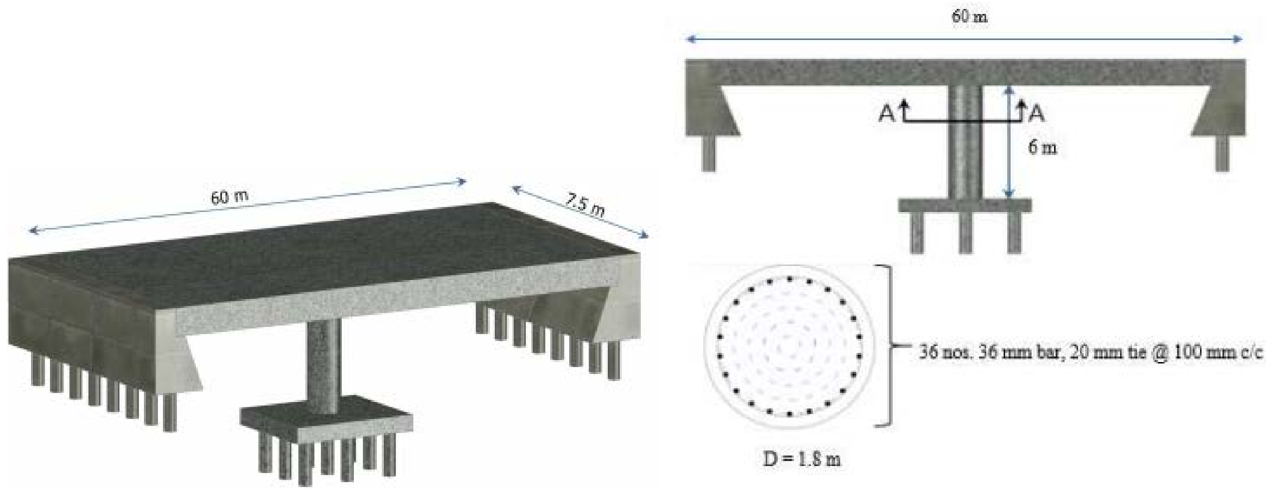

Fig. 2 – Details of the bridge considered for this study

3. Study structure and its modelling

The bridge considered in this study is a two-span single column box girder integral bridge having span length of 30.0 m (Fig. 2). The design concrete strength of the substructure and superstructure is 35 MPa and 40 MPa, respectively. The dead weight of the superstructure is 166.67 kN/m which produces a gravity load of 5000.0 kN on the column. The lateral seismic forces are determined as per the current version of Indian Road Congress (IRC) codes [9, 10] with an assumption that the bridge lies in seismic Zone V (very severe earthquake risk) [11] and belongs to the seismic class of “important” bridges.

This bridge column is modelled as a 2D beam/stick, with a lumped mass for the super-structure at the top. This idealisation is realistic owing to the fact that in case of integral bridges, the overall bridge fragility is governed by the nonlinear response of bridge columns. The superstructure is assumed to remain elastic during earthquakes and also the contribution of abutment to the overall bridge fragility is assumed to be minimal. The bridge column, which is expected to display inelastic responses under design earthquake intensities, is modelled with the *NonLinearBeamColumn* element in OpenSees [12] using the *FiberSection* approach. It models the cross-section with circular concrete patches and circular layers of reinforcement. The concrete, both confined and unconfined, is modelled using the *Concrete01* properties in OpenSees. The parameters for confined concrete are calculated based on the model proposed by Mander et al. [13], which depends on the amount of transverse reinforcement. For unconfined concrete, the peak compressive strength is assumed to occur at a strain of 0.002 with its strength reducing to zero at a strain value of 0.005. The longitudinal reinforcement is modelled using *uniaxialMaterial Hysteretic* material capable of capturing pinching of force and deformation, damage due to ductility and energy, and degraded unloading stiffness based on ductility. The fibre based modelling of the RC section can only consider interaction between biaxial bending and axial force. The *SectionAggregator* command is used to “add” the shear deformation behaviour to the existing fibre section. An elastic-perfectly plastic shear force-deformation behaviour is assumed with an elastic slope equal to GA_s, where G is the shear modulus and A_s is the effective shear area. The calculated shear strength of a section depends on the grades of concrete and steel, the amount of transverse reinforcement, and the axial load. The aggregated section can accommodate axial, bending and shear behaviours.

4. Performance limit states

The performance of bridge components are typically expressed in the form of some damage states (DS) or limit states (LS). These limit states are described in terms of different engineering demand parameters (EDPs), that have been discussed in detail in a recent paper [6]. In our work, the structural capacity for each limit state is obtained using the mean values of structural parameters by performing a static pushover analysis of the bridge column. Limit states for this bridge column are defined in terms of the maximum curvature ductility. Four damage states are defined: ‘slight’, ‘moderate’, ‘extensive’ and ‘collapse’. These damage states are characterised with respect to concrete compression and steel tension limits taken from literature. Based on the work by Hwang et al. [14], the LS of slight damage is defined as the curvature (ϕ_{y1}) at which the longitudinal steel yields (the initial yield point). The idealised yield point (ϕ_y) is obtained using an elasto-plastic idealisation of the actual pushover curve such that the area under the two curves are the same. The LS of extensive damage (ϕ_e) is defined as the point at which the extreme fibre of unconfined concrete reaches a strain of 0.003, which is assumed to be the strain limit for the spalling of concrete cover. The derived limit states in terms of curvature are converted into curvature ductility (μ_φ) by dividing the curvature limits by the initial yield curvature. The LS of collapse is adopted from the work of Hwang et al. [14], which defined collapse as the extensive damage state value plus 3.0. This value is in terms of displacement ductility (μ_Δ), which is converted into curvature ductility [15]:

$$\mu_\phi = 1 + \frac{\mu_\Delta - 1}{\frac{3L_p}{L}\left(1 - 0.5\frac{L_p}{L}\right)} \tag{2}$$

LS capacities for the pristine condition of bridge are shown in Table 1 for all four damage states. For the lack of any specific guideline on what limiting curvature ductilities should be considered for a corroded bridge column, we use the pristine LS values for all time instances over the life of the structure.

Table 1 – Curvature ductility (μ_φ) limit states (LS) of bridge column

LS	Slight	Moderate	Extreme	Collapse
μ_φ	1	1.35	4.66	8.73

5. Selection of a suite of ground motions

The selection of ground motions plays an important role in the development of fragility curves for a structure. Several studies in the past showed that the record-to-record variability of ground motions is the primary contributor to the (aleatory) randomness. Since the number of recorded ground motion suitable for fragility analysis for Northeast India (Zone V) is insufficient, recorded acceleration time-histories from other parts of the world having similar fault mechanism, site condition and seismic potential are selected, in addition. The initial selection criteria for ground motion are decided based on the (i) maximum earthquake magnitude potential (5.0 - 7.5 M_w), (ii) distance from the nearest fault (10-30 km), (iii) fault mechanism (normal, strike-slip, reverse) and (iv) site conditions (shear wave velocity V_s = 180-1500 m/s). These criteria are selected based on past literatures in order to represent Northeast India. The next criterion is based on the minimum seismic intensity level, and ground motions having PGA < 0.05g and PGV < 0.15 m/s are not considered. The last criterion applied is limiting the number of records per each earthquake event to two to avoid event based bias. A total of 32 ground motions are thus selected from the PEER NGA database. Further, eight records are randomly selected from past earthquakes records of Northeast India, after applying the criteria of PGA > 0.05g with not more than two records selected from a single event.

The selected 40 records are normalised based on FEMA (2009) guidelines. As per this method individual records of a given set are normalised by their peak ground velocities (PGV). A normalisation by PGV is an easy and simple way to remove the unwarranted variability among records (those due to inherent difference in event magnitude, source type, site condition, and source distance), while still maintaining the inherent record-to-record variability necessary for accurately estimating the fragility. After normalisation, the records are scaled to different hazard levels (0.1g to 1.5g) to perform multi-IDA.

6. Corrosion model

The mass loss of steel due to uniform corrosion is expressed as [16]:

$$W = \sqrt{2\pi D i_{\mathrm{corr}} A_w \alpha_{sr}^2 t} \tag{3}$$

where, D is the pristine diameter of the rebar and i_{corr} is the mean annual rate of corrosion. In this equation, A_w and α_{sr} are related to the corrosion process and the amount of rust generated, respectively. Performing a regression analysis on available data, for D measured in mm, i_{corr} in μA/cm^2, W in mg/mm and t in years, Bhargava et al. [16] found: A_{w} = 2.486 and α_{sr} = 0.6131. To take into account the corrosion initiation period (t_{in}), the equation can be modified as

$$W = \sqrt{2\pi D i_{\mathrm{corr}} A_w \alpha_{sr}^2 \left(t - t_{\mathrm{in}}\right)} \qquad t > t_{\mathrm{in}} \tag{4}$$

Deterministic relationships, such as Eq. (4), are inadequate in capturing the corrosion phenomenon properly, because of the parameter uncertainty and scatter in actual steel loss data [3]. However, in a probabilistic setting, such deterministic models can be “completed” by introducing “correcting” random variables [17] into the model, as in

$$\ln W = \ln \sqrt{2\pi D i_{\mathrm{corr}} A_w \alpha_{sr}^2 \left(t - t_{\mathrm{in}}\right)} + E \tag{5}$$

Where, $E \sim N(0,\sigma)$ is the correcting random variable. The reason for using a logarithmic transform of the original form of Eq. (4) is to ensure that the distribution of W has a positive support. Eq. (5) can be written in an alternate form as $W = e^E \sqrt{2\pi D i_{\mathrm{corr}} A_w \alpha_{sr}^2 \left(t - t_{\mathrm{in}}\right)}$. This implies that the mass loss at an instant t follows a lognormal distribution: $W \sim LN\left(\sqrt{2\pi D i_{\mathrm{corr}} A_w \alpha_{sr}^2 \left(t - t_{\mathrm{in}}\right)}, \sigma\right)$.

6.1 Pitting model

The uniform corrosion model of Eq. (4) is a simplistic assumption, whereas a realistic failure assessment of a structure due to corrosion needs to account for pitting effects, as well. Here we use the pitting factor R [18], which relates the minimum rebar area ($A_{\min}$) to the rebar area based on uniform corrosion (A_{avg}):

$$R = \frac{A_{\mathrm{avg}}}{A_{\min}} \tag{6}$$

R expresses the area spatial heterogeneity. Zhang et al. [18] created a time varying probabilistic (Gumbel) model for R, where the parameters of its distribution are expressed as functions of the mass loss ratio (η_t) at time t. This ratio is defined as

$$\eta_t = \frac{W}{W_0} \tag{7}$$

where, W_0 is the mass per unit length of an uncorroded bar and W is obtained from Eq. (4). The average residual area of a rebar can be now expressed in terms of η_t:

$$A_{avg} = A_0(1-\eta_t) \tag{8}$$

where, A_0 is the original cross-sectional area of a rebar. Zhang et al. [18] derived the statistics of R for rebars of diameter of 14 mm and 20 mm. In order to use these statistics for a 36 mm diameter rebar, we use linear extrapolation for the distribution parameters for an element length of 150 mm. It should be noted here that such linear extrapolation of the parameter of an Extreme-type distribution is usually cautioned against. However, for the lack of necessary data we follow linear extrapolation of parameters here. For any given location, all rebars are assumed to be fully correlated in terms of minimum corroded area. Also, corrosion in the tie and the main reinforcements, at any specific location, are assumed to be in the ratio of their original cross-sectional areas, accounting for the difference in cover concrete.

7. Bayesian inference

We assume that a routine NDT scheme is used for monitoring corrosion in the column and only steel loss (W) data are available at discrete time instances. Generally, it is not possible to measure the model parameters: $\boldsymbol{\theta} = [A_w \quad \alpha_{sr} \quad t_{\text{in}} \quad \sigma]^T$. These parameters remain unobservable, and need to be "updated" through an inverse analysis by Bayesian inference, using the observations on W. Using Bayes' theorem, the prior information – described by the joint probability density function (PDF), $f(\boldsymbol{\theta})$, when combined with the monitored data – quantified by a likelihood function, $f(D_n|\boldsymbol{\theta})$, results in an updated distribution, $f(\boldsymbol{\theta}|D_n)$:

$$f\left(\boldsymbol{\theta}\middle|D_n\right) \propto f\left(D_n\middle|\boldsymbol{\theta}\right) f\left(\boldsymbol{\theta}\right) \tag{9}$$

In the present scenario, the model parameters, $\boldsymbol{\theta}$, are updated using observations $D_n = [d_1 \quad d_2, \quad \ldots, \quad d_i, \quad \ldots, \quad d_n]^T$. Note that each d_i represents the pair $[W_i, t_i]$. The likelihood for such a single pair of data can be obtained using Eq. (5).

$$f\left(W_i\middle|t_i,\boldsymbol{\theta}\right) = \frac{1}{W_i\sigma}\left[\frac{\ln W_i - \ln\sqrt{2\pi D i_{\text{corr}} A_w \alpha_{sr}^2 \left(t-t_{\text{in}}\right)}}{\sigma}\right] \tag{10}$$

For n observations, with the assumption that they are statistically independent, we have

$$f\left(D_n\middle|\boldsymbol{\theta}\right) = \prod_{i=1}^{n} f\left(W_i\middle|t_i,\boldsymbol{\theta}\right) \tag{11}$$

Through a compilation of data reported in literature, Jamali et al. [19] reported the values of α_{sr} to be in the range of 0.348 to 0.778, corresponding to various possible corrosion products. In the absence of any further information on the composition of rust products, we conservatively adopt a uniform distribution for the prior probability model of α_{sr}:

$$\alpha_{sr} \sim U\left(0.348, 0.778\right) \tag{12}$$

Based on experimental results sourced form literature, Bhargava et al. [16] estimated the parameters of the steel loss model. The prior distribution of A_w, is based on the 28 samples obtained from the same test data. The prior probability model for A_w is obtained by fitting this statistic using the Kolmogorov-Smirnov goodness-of-fit test. A lognormal distribution is found suitable to model this parameter: $\ln A_w \sim N$ (0.9073, 0.6752). The use of a lognormal distribution is also justified based on Eq. (4), which requires A_w to be non-negative. To ensure that even after updating, the values of A_w remain positive, we adopt the transformation $A_w = e^a$, where

$$a \sim N(0.9073, 0.6752) \tag{13}$$

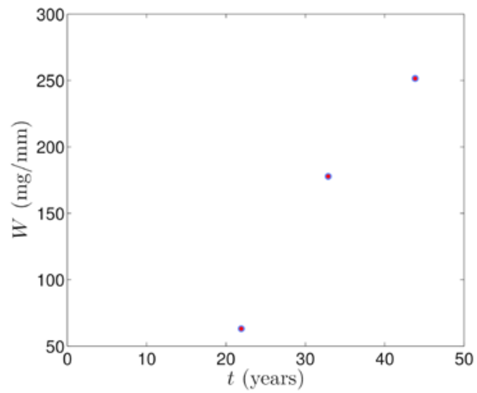

Fig. 3 - Rebar mass loss data, normalised to i_{corr} = 1.0 µA/cm^2

The corrosion initiation time t_{in} is the third random variable in our treatment. t_{in} has to be in the range between zero to the time of first non-zero measurement of steel loss due to corrosion (t_1, here). We adopt a uniform prior for t_{in}

$$t_{\mathrm{in}} \sim U(0, t_1) \tag{14}$$

The standard deviation of the correction term, E (Eq. 5), must always be positive. We adopt a transformation, $\sigma = e^s$, and assume a standard normal prior distribution:

$$s \sim N(0,1) \tag{15}$$

The joint prior distribution is taken as the product of the densities of individual parameters, by assuming each parameter to be independent of the other:

$$f(\boldsymbol{\theta}) = f(a)\, f(\alpha_{sr})\, f(t_{\mathrm{in}})\, f(s) \tag{16}$$

The data used for the present study is sourced form Azad et al.'s work [20]. From their experiment on accelerated corrosion, we select three test specimens, BT1-2-4, BT1-2-6 and BT1-2-8, which were subjected to a corrosion rate of 2000 µA/cm^2 for 4, 6 and 8 days, respectively. However, we normalise these data to scale down the corrosion rate to 1.0 µA/cm^2, which is classified as 'moderate' by Dhir et al. [21]. Keeping mass loss observations the same, the time instants of observation are changed to 21.92, 32.88 and 43.84 years, as presented in Fig. 3.

7.1 Delayed rejection adaptive Metropolis algorithm

One of the popular methods for drawing random samples from an unknown posterior distribution us-ing the Markov chain is the Metropolis algorithm [22]. As the posterior is unknown, in this algorithm samples are drawn from a Gaussian proposal distribution $q\,(\cdot|\cdot)$, which can be a multivariate function depending on the number of parameters of a model. Further details on this commonly adopted algorithm can be found in the original paper by Metropolis et al. [22]. A significant challenge which arises while adopting this algorithm, typically in high dimensions, is the tuning of the proposal distribution for an efficient sampling. To alleviate this challenge, in the present study we adopt the delayed rejection adaptive Metropolis (DRAM) technique [23], which is an application of the adaptive Metropolis (AM) algorithm within the delayed rejection (DR) algorithm.

In AM, the proposal automatically adapts based on the history of the chain of samples. The proposal at the mth state can be chosen as a Gaussian distribution centered at $\boldsymbol{\theta}_m$ with the covariance matrix $\boldsymbol{C}_{m+1}$, such that

$$\begin{aligned} \boldsymbol{C}_{m+1} &= \boldsymbol{C}_0 \qquad m \le m_o \\ &= s_p\left(\tilde{\boldsymbol{C}}_m + \varepsilon \mathbf{I}_p\right) \quad m > m_o \end{aligned} \tag{17}$$

where we choose a strictly positive definite covariance matrix $\mathbf{C}_0$, for the initial "non-adaptive" states upto m_0. $s_p = 2.38^2/p$, where p is the number of parameters. $\varepsilon > 0$ is a very small number which ensures that $\boldsymbol{C}_{m+1}$ does not become singular (in the case all samples are rejected) and $\mathbf{I}_p$ is a p-dimensional identity matrix. $\tilde{\boldsymbol{C}}_m$ is the empirical covariance matrix computed as

$$\tilde{\boldsymbol{C}}_m = \frac{1}{m-1}\sum_{i=1}^{m}\left(\boldsymbol{\theta}_i - \bar{\boldsymbol{\theta}}_m\right)\left(\boldsymbol{\theta}_i - \bar{\boldsymbol{\theta}}_m\right)^T \tag{18}$$

where, $\bar{\boldsymbol{\theta}} = \frac{1}{m}\sum_{i=1}^{m}\boldsymbol{\theta}_i$ and $\boldsymbol{\theta}_i \in \mathbb{R}^p$ is a column vector. At any mth state of the chain positioned at $\mathbf{x} = \boldsymbol{\theta}_m$, if a candidate $\mathbf{y}_1$ generated form the proposal ("first stage"; obtained using AM so far) is rejected, another proposal distribution is used to generate the sample ("second stage"). If the candidate is rejected even at this stage, the process can be repeated l times (upto lth stage) until a candidate is accepted. In the present study, a Gaussian proposal depending only on the last rejected sample with a scaling of the covariance matrix at the lth stage as $\mathbf{C}_m/2^l$, is adopted [24]. The acceptance probability at the lth stage DR algorithm becomes [24]

$$A_l\left(\mathbf{x}, \mathbf{y}_1, \ldots.., \mathbf{y}_l\right) = \min\left\{1, \frac{\max\left\{0, \left[f\left(\boldsymbol{y}_l \middle| D_n\right) - f\left(\boldsymbol{y}^* \middle| D_n\right)\right]\right\}}{f\left(\mathbf{x} \middle| D_n\right) - f\left(\boldsymbol{y}^* \middle| D_n\right)}\right\} \tag{19}$$

where, $\boldsymbol{y}^* = \arg\max_{j<l} f\left(\boldsymbol{y}_j \middle| D_n\right)$. Once the sample is accepted, we set $\boldsymbol{\theta}_{m+1} = \boldsymbol{y}_l$.

The tuning of the proposal covariance matrix is stopped after a predefined burn-in of 2000 samples, after which there is no significant change in $\mathbf{C}_m$. For the initial non-adaptive state upto m_0, $\mathbf{C}_0 = s_p\mathbf{I}_p$ and $m_0 = 5$. In this state, $\mathbf{C}_m$ is updated at every 5th sample and the DR is run until a sample is accepted. After the DRAM algorithm is completed, the generated covariance is used in the conventional Metropolis algorithm to generate 5000 samples of each parameter, which were found adequate as per Geweke's convergence criteria [25]. It should be noted that while generating the samples for t_{in} and α_{sr}, care is taken that any sample falling outside [0, t_1] and [0.348, 0.778], respectively, is discarded.

8. Results and conclusions

As discussed in Section 2, the fragility of the bridge column is obtained at its pristine condition and after each corrosion measurement at t = 21.92, 32.88 and 43.84 years. The seismic fragility of the corroded column is obtained from a sequential Bayesian updating of the mass loss (W) at each of these instances. For all the three time instances, updated samples of W are best fitted with generalised extreme value (GEV) distributions. The fitted distribution of W after the third updating is presented in Fig. 4. For modelling the corroded column, W is resampled using the LHS scheme only in the range (0, W_0). Similarly in the sampling of the pitting factor, R, all samples less than 1.0 are rejected.

At any instant, the seismic fragility parameters m_a (median acceleration capacity) and β_r (lognormal standard deviation of aleatory uncertainty) are obtained by "fitting" the lognormal model of Eq. (1) to the discrete multi-IDA fragility values for curvature ductility.

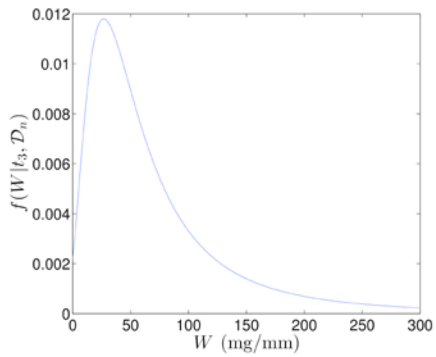

Fig. 4 – Posterior distribution of mass loss W after the third update

m_a and β_r values obtained for the four time instances are presented in Table 2. The m_a values show a gradual deterioration of the median ground acceleration capacity over time for all the four limit states considered. Adopting a service life of 100 years [9] for this bridge, this deterioration in ma is obtained for the whole service life by fitting quadratic curves to this data-set. For example, for LS4 we obtain the following relation, with an R^2 value of 0.9764:

$$m_a(t) = -8.450\times10^{-6}t^2 - 1.602\times10^{-6}t + 1.515 \tag{20}$$

Table 2 – Seismic fragility parameters for four limit states at four different time instances

Limit State	Fragility parameters	Pristine	t_1	t_2	t_3
LS1	m_a	0.6612	0.6552	0.6528	0.6506
	β_r	0.1816	0.1839	0.1792	0.1790
LS2	m_a	0.8912	0.8840	0.8789	0.8744
	β_r	0.1758	0.1793	0.1841	0.1831
LS3	m_a	1.4035	1.395	1.397	1.392
	β_r	0.1504	0.1549	0.1657	0.1680
LS4	m_a	1.514	1.511	1.503	1.498
	β_r	0.1197	0.1287	0.13180	0.1412

Fig. 5 presents the time-varying trend of the median acceleration capacity. There is no such trend in the time-variability of β_r. An average β_r (for example 0.1352 for LS4) is therefore proposed here for estimating seismic fragility throughout the bridge's service life. It should be noted here that there is no significant change in this bridge's seismic fragility due to degradation even after 100 years of service. The primary reason for this could be the fact that the rate of corrosion adopted in this work is moderate, which typically should not pose any challenge for the bridge's life management.

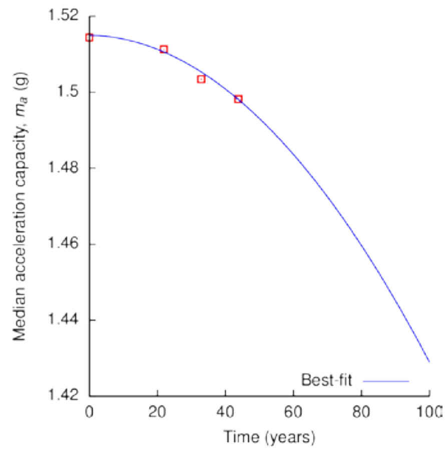

Fig. 5 – Variation of the median acceleration capacity over time

We have presented in this paper a general framework for obtaining the time-varying seismic fragility of an integral bridge, deteriorating due to corrosion. The most important aspect of this framework is that instead of relying upon the available corrosion growth models – that typically result in predictions far off from the reality, it augments these models with monitored data through a scheme of Bayesian updating. This framework is also able to address the issue of pitting or non-uniform corrosion through a spatial heterogeneity factor. The case study of a simple integral bridge in the highest seismic region in India shows that, at a moderate corrosion rate of 1.0 μA/cm^2, corrosion does not have a significant effect on the seismic fragility. However, as one would expect, it shows a gradual deterioration of the structure throughout its service life. This framework can be applied to any deteriorating structure where monitoring is performed on a routine basis. The seismic fragility analysis using multi-IDA makes this process severely computation-heavy. However, adopting less demanding comptutational approches to fragility analysis, such as the 'cloud analysis', may reduce the computational cost. Bayesian inference using the DRAM algorithm is found to be very effective while updating many parameters.

9. References

[1] Enright MP, Frangopol DM (1998): Probabilistic analysis of resistance degradation of reinforced concrete bridge beams under corrosion. Engineering Structures, **20**(11), 960–971.

[2] Nielson B (2005): Analytical Fragility Curves for Highway Bridges in Moderate Seismic Zones. Ph.D. thesis, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, Georgia.

[3] Faroz SA, Pujari NN, Ghosh S (2014): A Bayesian Markov Chain Monte Carlo approach for the estimation of corrosion in reinforced concrete structures. In Topping BHV, Ivanyi´ P, editors, Proceedings of the Twelfth International Conference on Computational Structures Technology. Civil-Comp Press, Stirlingshire, UK. Paper 150.

[4] Choine M, O'Connor A, Padgett J (2013): A seismic reliability assessment of reinforced concrete integral bridges subjected to corrosion. Key Engineering Materials, **569-570**(366-373).

[5] Choe DE, Gardoni P, Rosowsky D, Haukaas T (2009): Seismic fragility estimates for reinforced concrete bridges subject to corrosion. Structural Safety, **31**(4), 275–283.

[6] Billah AHM, Alam MS (2015): Seismic fragility assessment of highway bridges: a state-of-the-art review. Structure and Infrastructure Engineering, **11**(6), 804–832.

[7] Vamvatsikos D, Cornell CA (2002): Incremental dynamic analysis. Earthquake Engineering and Structural Dynamics, **31**(3), 491–514.

[8] Pujari NN, Ghosh S, Lala S (2015): Seismic fragility estimation of a containment shell based on the formation of through-wall cracks. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, **B4015004**.

[9] IRC (2011): IRC: 112 Code of Practice for Concrete Road Bridges. Indian Roads Congress, New Delhi, India.

[10] IRC: 6 (2014): Standard Specifications and Code of Practice for Road Bridges, Section: II, Loads and Stresses. The Indian Road Congress, New Delhi, India.

[11] BIS (2002): IS: 1893 Criteria for Earthquake Resistant Design of Structures, Part 1: General Pro-visions and Buildings. Bureau of Indian Standards, New Delhi, India.

[12] Mazzoni S, McKenna F, Fenves GL (2006): Opensees Command Language Manual. The Regents of the University of California, Berkeley.

[13] Mander JB, Priestley MJ, Park R (1988): Theoretical stress-strain model for confined concrete.

Journal of Structural Engineering (United States), **114**(8), 1804–1826.

[14] Hwang H, Liu J, Chiu Y (2001): Seismic fragility analysis of highway bridges. Technical Report MAEE RR-4, Mid-America Earthquake Center, Illinois, USA.

[15] Priestley M, Seible F, Calvi GM (1996): Seismic Design and Retrofit of Bridges. John Wiley and Sons, Inc., NY, USA.

[16] Bhargava K, Ghosh AK, Mori Y, Ramanujam S (2006): Analytical model for time to cover cracking in RC structures due to rebar corrosion. Nuclear Engineering and Design, **236**(11), 1123–1139.

[17] Ditlevsen O (1982): Model uncertainty in structural reliability. Structural Safety, **1**(1), 73–86.

[18] Zhang W, Zhou B, Gu X, Dai H (2014): Probability distribution model for cross-sectional area of corroded reinforcing steel bars. ASCE Journal of Materials in Civil Engineering, **26**(5), 822–832.

[19] Jamali A, Angst U, Adey B, Elsener B (2013): Modeling of corrosion-induced concrete cover cracking: A critical analysis. Construction and Building Materials, **42**, 225–237.

[20] Azad AK, Ahmad S, Azher SA (2007): Residual strength of corrosion-damaged reinforced concrete beams. ACI Materials Journal, **104**(1), 40–47.

[21] Dhir RK, Jones MR, McCarthy MJ (1994): PFA concrete: chloride-induced reinforcement corrosion. Magazine of Concrete Research, **46**(169), 269–277.

[22] Metropolis N, Rosenbluth A, Rosenbluth M, Teller A, Teller E (1953): Equation of state calculations by fast computing machines. The Journal of Chemical Physics, **21**(6), 1087–1092.

[23] Haario H, Laine M, Mira A, Saksman E (2006): DRAM: Efficient adaptive MCMC. Statistics and Computing, **16**(4), 339–354.

[24] Mira A (2001): On Metropolis-Hastings algorithms with delayed rejection. Metron, **59**(3-4), 231– 241.

[25] Geweke J (1992): Evaluating the Accuracy of Sampling-Based Approaches to the Calculation of Posterior Moments. In Bernardo J, Berger J, Dawid A, Smith A, editors, Bayesian Statistics, pages 169–193. Oxford University Press