



## PROBABILISTIC METHODS IN OPTIMIZATION OF STRUCTURES UNDER EARTHQUAKE EXCITATION

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### **Abstract**

The process of analysis and design of structures under earthquake excitations depends on many random variables, reflecting the uncertainties in the mechanical properties, dimensions of structural components and, most important, in the characteristics of the ground motions. It is necessary to take into account all these uncertainties. The process of performance-based design then calls for an optimization, usually for minimum total cost, under minimum reliability constraints for the different performance requirements. This process can be organized in three main blocks:

a) A nonlinear dynamic analysis is used to obtain structural responses for each combination in a deterministic set of the random variables, and for a set of ground motions likely to occur at the site. These sets are specified with the help of experimental design, and the ground motions could be either obtained from historical data or be artificially generated. The analysis is performed for each ground motion and each variable combination, and the results stored in databases. These, in turn, are used to train neural networks which permit the approximate evaluation of the structural responses, avoiding further dynamic analyses.

b) The reliability level achieved for different combinations of the design parameters (usually, in the case of reinforced concrete, the means of structural dimensions or steel reinforcement ratios) are then calculated. These reliability levels are obtained by simulation, using the neural networks for structural responses. The reliability levels achieved for each performance level are also represented by neural networks, with the design parameters as inputs.

c) The optimization involves the minimization of a total cost objective. The total cost, or life-cycle cost, includes the original associated with construction, plus the one associated with repairs due to damage produced by future earthquakes during the structure's economic life. The earthquakes are assumed to arrive randomly during the structural lifetime. Also the human costs of injuries or casualties during an earthquake are included, plus those for temporary rent during repair. The paper describes an effective optimization algorithm based on a random search, avoiding the calculation of gradients. The results provide the set of design parameters which minimize the total cost, while satisfying minimum target reliability levels for each of the performance criteria.

As an example, the paper includes optimization results for a four-story reinforced-concrete office building with different attributes: (i) a basic structure with portal frames; (ii) the same structure but including energy dissipation devices based on steel yielding; (iii) the same basic structure but introducing lead rubber bearing type base isolators. The results compare the total costs for each case, the corresponding optimal design parameters and the achieved reliabilities when minimum targets are not imposed.

*Keywords: Performance-based seismic design, Optimization, Control devices, Reliability, Social costs.*



## 1. Introduction

Performance-based design is an optimization application in order to find optimum values for design parameters, satisfying minimum performance requirements and possibly minimum reliability levels and other constrain objectives. This optimization must take into account the uncertainty in all the intervening variables.

In the context of earthquake engineering [1, 2], the uncertainty in the ground motions is, of course, most important. A set of ground motions (accelerograms) likely to occur at the site must be considered. In the absence of an adequate set of historical records, artificially generated accelerograms must be used. There are many ways of generating these records, but the main objective in this paper is not in discussing these different approaches. Rather, we choose a particular way of generating records and, given this information, we concentrate on describing a methodology for the performance-based design optimization of a structure, taking into account specified minimum reliability constraints for different performance levels. The damage is estimated in reference to deformation limits which are compared with deformations calculated from nonlinear dynamic analyses of a degrading structure subjected to the ground motion. The future damage is weighted by the arrival rate of the different earthquakes.

In this paper, the objective function is taken to be the total cost over the service life (LCC). This total cost includes: (i) the initial construction cost of the resisting structure, of the control devices, of the non-structural elements and the building contents; (ii) repair costs associated with damage due to the occurrence of earthquakes during the building's service life; and (iii), social costs, including those associated with injuries and death in case of severe damage and structural collapse, plus those for temporary rent during repair. The problem considered is the seismic design of a reinforced concrete structure belonging to an office building. Three variants of structures are optimized: (i) a basic structure with bared portal frames; (ii) a structure with portal frames including energy dissipation devices at each floor level based on steel yielding; and (iii) a structure with portal frames with lead rubber bearing type base isolators.

The initial step in the solution is the determination of the structural response parameters, maxima during the earthquake (maxima displacements, inter-story drifts, damage index, floor accelerations, demand in control devices). These values result from nonlinear dynamic analyses, for each specific combination of the variables in the problem and each ground motion. These responses are discrete values and, for estimation of probabilities of failure, it is convenient to represent them with continuous functions. These approximations, called response surfaces, can then be used as substitutes for the true structural responses for combinations not considered [3]. Different forms of response surfaces have been studied [4], with neural networks offering advantages of flexibility and adaptability, and have been adopted in this work. Estimating responses via a response surface, whichever form is adopted, facilitates the evaluation of the performance functions and makes probability estimation via Monte Carlo simulation very efficient [5].

Here we consider the optimization of structural parameters like element dimensions, reinforcement ratios and control devices properties, while maintaining fixed the layout of the building. The optimization strategy must consider minimum reliability constraints for each mode of failure. Optimization algorithms can implement different strategies [6, 7], some requiring the calculation of gradients within schemes of steepest descent or of conjugate gradients [8, 9]. Other strategies are not gradient-dependent, and utilize heuristic schemes like random search or genetic algorithms [10, 11, 12]. Here, an algorithm for a random search is utilized that has been developed in previous work [13, 14].

Using a different approach to that from previous works, this paper does not use fragility data and integration over earthquake intensity demands [15]. Fragilities represent the structural global response to a specified demand, and include the effect of underlying design variables like structural dimensions, steel reinforcements, etc. Each change in structural parameters will demand a recalculation of the fragilities and this, in turn, will not be efficient for structural optimization.

The major contribution of this paper is the comprehensive discussion of all the topics that must be present in a performance-based design: (i) a nonlinear dynamic analysis for the degrading structural responses; (ii) the approximation and characterization of the possible ground motions at the site; (iii) the definition of the design



variables and the uncertainty in all other intervening random variables; (iv) the use of response surfaces (neural networks) to estimate structural responses in lieu of further dynamic analyses; (v) the efficient use of those response surfaces for the estimation of achieved reliabilities at each performance level; and for an efficient optimization algorithm to minimize total cost under reliability constraints.

## 2. The three variants of structures to be optimized

The structure to be optimized is a four-story office building in the city of Mendoza, Argentina, with spans and columns as shown in Fig.1, both in plan and elevation for a portal frame with its three variants along the x-direction, which is the assumed direction of the earthquake motion.

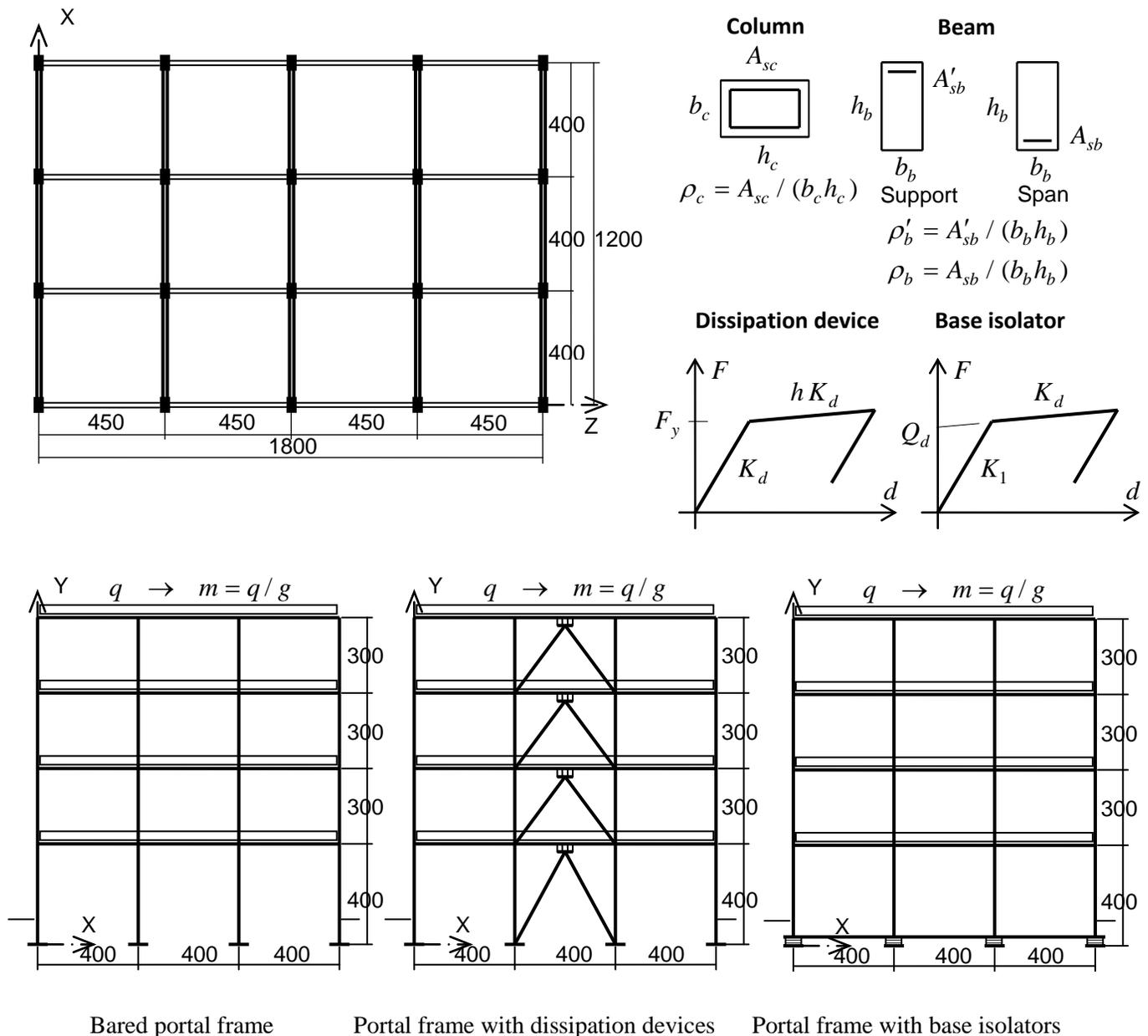


Fig. 1 – Structures to be optimized



### 3. The general process of optimization

The Fig.2 shows the general optimization process, which is divided into blocks that can be executed separately but in sequence.

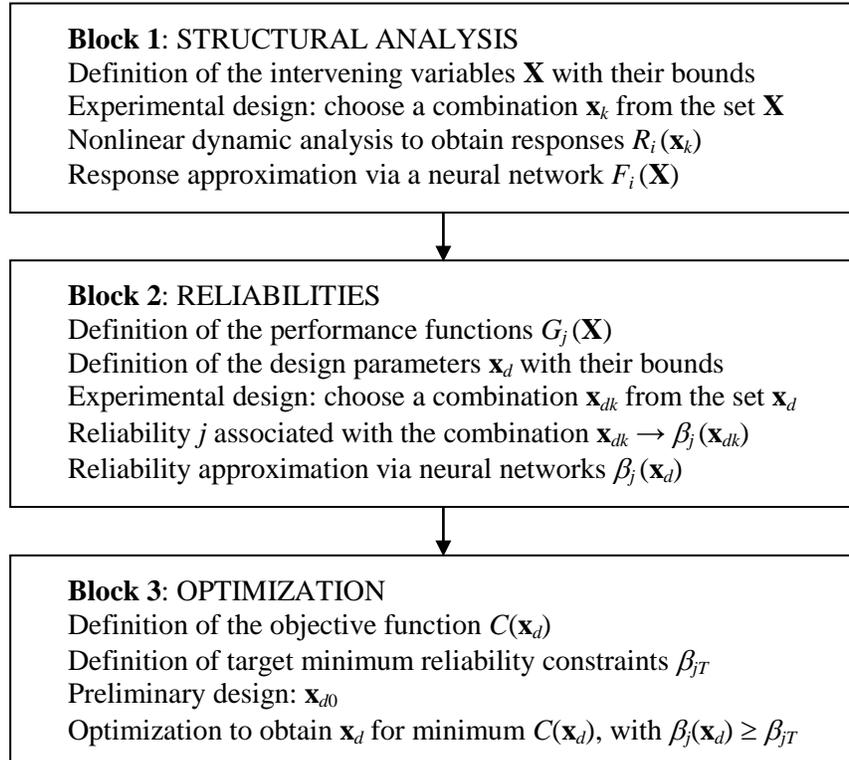


Fig. 2 – Block organization for the optimization process

#### 3.1 Block 1: Structural analysis

Table 1 shows the variables considered in the calculation of the discrete responses, along with the lower and upper bounds. The reinforcement ratios have bounds corresponding to requirements from building Codes, in order to provide minimum strength against gravitational loads. The design procedure [16] is based on the load combination of permanent ( $D$ ), live ( $L$ ) and earthquake ( $E$ ) loads:  $1.0 D + 0.25 L \pm E$ , which, for the structure studied here,  $D = 7.66 \text{KN/m}^2$ ,  $L = 3 \text{KN/m}^2$ , giving a total weight per floor of  $W = 1817 \text{KN}$ . Because the slabs are considered rigid in their plane, each portal frame must support 1/5 of the total seismic load, resulting in a mean value for the mass of the beams  $m = 3.09 \cdot 10^{-4} \text{KN s}^2 / \text{cm}^2$ .

The seismic action  $E$  corresponds to the specified ground motions. These have to be selected to constitute a set of records likely to occur at the site. Of course, this is not a simple task, and several techniques are normally used to produce such a set, including the use of historical records (when available). Here, ground accelerograms are artificially generated following a Shinozuka procedure for which two basic variables are required: the peak ground acceleration  $a_G$  at the site and the central frequency for the ground filter  $f_g$ . A Normal random phase angle is introduced with each of the frequencies and a modulation function is applied to introduce nonstationarity. Other similar methods can be used for the specific characterization of the ground motions, as the approach chosen does not affect the optimization procedure described in this work.

Design of experiments was applied to generate, randomly, combinations of the intervening variables. The range of  $X(12)$  between its corresponding bounds was divided into three sectors, while only one sector was used between the bounds of each of the remaining variables. Each combination was then obtained by choosing a random value for a variable within each corresponding sector. The process was repeated 150 times, resulting in a



total of NP = 450 variable combinations. This number permits (i) an adequate coverage of the variable ranges, and (ii) that the neural network which will be used to approximate the corresponding dynamic responses could have an architecture with a maximum of 25 neurons in one hidden layer, in order to achieve satisfactory precision in the predictions.

For each of the 450 combinations of the variables in Table 1, a set of NS = 10 sub-combinations were obtained. Each of these correspond to: (i) a different random choice for the phase angle associated with each frequency in the generation of an artificial accelerogram, that is, a different ground motion, and (ii) a different concrete strength and steel yield point, parameters that directly affect the moment-curvature relation for the beams and columns.

Table 1 – Variables and their bounds

Variable	Lower bound	Upper bound	Definition
$X(1) = m \text{ (KN s}^2 / \text{cm}^2)$	$2.00 \times 10^{-4}$	$4.00 \times 10^{-4}$	Beam mass per unit length
$X(2) = b_b \text{ (cm)}$	15	30	Beam section width
$X(3) = h_b \text{ (cm)}$	30	70	Beam section depth
$X(4) = b_c \text{ (cm)}$	20	40	Column section width
$X(5) = h_c \text{ (cm)}$	30	100	Column section depth
$X(6) = \rho_b$	0.00298	0.01389	Beam reinforcement ratio (midspan)
$X(7) = \rho'_b$	0.00298	0.01389	Beam reinforcement ratio (supports)
$X(8) = \rho_c$	0.008	0.04286	Column reinforcement ratio
$X(9) = f_r / f'_{c0}$	0	0.15	Confinement pressure (normalized)
$X(10) = F_y \text{ or } Q_d \text{ (KN)}$	25 / 20	2300 / 450	Yield force of dissipator / isolator device
$X(11) = K_d \text{ (KN / cm)}$	45 / 250	37000 / 1700	Stiffness of dissipator / isolator device
$X(12) = a_G \text{ (cm / s}^2)$	10	1200	Peak ground acceleration
$X(13) = f_g \text{ (Hz)}$	1.00	4.00	Central ground filter frequency

A nonlinear dynamic analysis is performed for each combination of the variables and for each of the corresponding sub-combinations. The background for the structural model used in this analysis is described elsewhere [17] and is similar to that proposed by Filippou et al. [18]. The maximum responses, which enter into the different performance functions, are then obtained: *UMAX*: maximum horizontal displacement at the top of the structure; *AMAX* (*i*), *i*=1,4: maximum acceleration for the *i*<sup>th</sup> story; *DISTM*: maximum inter-story drift; *DIES*: global damage index; *DILOM*: maximum local damage index. This work uses damage indices as defined by Park and Ang. For the structure with dissipation devices, the maximum ductility of the devices are obtained: *DUCDM* (*i*), *i*=1,4. For the structure with base isolators, the maximum shear strain of the devices are obtained: *DISAM* (*i*), *i*=1,4.

If *R* is a generic response, the corresponding calculated responses  $R_{k,j}$  are obtained for each combination  $j = 1, \text{NP}$  and each sub-combination  $k = 1, \text{NS}$ . For each of the NP combinations, the results are then used to calculate the mean response and the standard deviation over the set of NS sub-combinations:

$$\bar{R}_j = \frac{1}{NS} \sum_{k=1}^{NS} R_{k,j} \quad \sigma_{R_j} = \sqrt{\frac{1}{NS-1} \sum_{k=1}^{NS} (R_{k,j} - \bar{R}_j)^2} \quad (1)$$



The NP discrete results  $R(\mathbf{X})$  from (1) are then represented by two response surfaces, neural network in this work: one for the mean values  $Y_m(\mathbf{X})$  and another for the standard deviations  $\sigma_Y(\mathbf{X})$ . The approximation of neural network is never perfect, and the scattering of differences can be quantified using the standard deviation of the relative error, which is an average coefficient of variation, as follows

$$\sigma_{\varepsilon_r} = \sqrt{\frac{1}{NP-1} \sum_{k=1}^{NP} \left(\frac{Y_k - T_k}{Y_k}\right)^2} \quad (2)$$

in which  $Y_k$  is the value calculated with the network,  $T_k$  is the value obtained with the dynamic analysis and NP is the number of combinations used in the database. The mean value and the standard deviation of the responses over the sub-combinations can then be written taking into account the dispersion error

$$\bar{F}(\mathbf{X}) = Y_m(\mathbf{X}) (1. + \sigma_{\varepsilon_m} X_{N_1}) \quad \sigma_F(\mathbf{X}) = \sigma_Y(\mathbf{X}) (1. + \sigma_{\varepsilon\sigma} X_{N_2}) \quad (3)$$

in which  $\sigma_{\varepsilon_m}$ ,  $\sigma_{\varepsilon\sigma}$  are the deviations obtained from Eq. (2) and  $X_{N_1}$ ,  $X_{N_2}$  are Standard Normal random variables. Finally, the mean values and standard deviations from Eq. (3) can be used to predict the response  $R(\mathbf{X})$ , using a Standard Normal random variable  $R_{N_1}$ , and assuming a Lognormal distribution to represent the variability over the sub-combinations:

$$R(\mathbf{X}) \cong F(\mathbf{X}) = \frac{\bar{F}(\mathbf{X})}{\sqrt{1. + \left(\frac{\sigma_F(\mathbf{X})}{\bar{F}(\mathbf{X})}\right)^2}} \exp\left[R_{N_1} \sqrt{\ln\left(1. + \left(\frac{\sigma_F(\mathbf{X})}{\bar{F}(\mathbf{X})}\right)^2\right)}\right] \quad (4)$$

Equation (4) allows a quick estimation of the responses, using the neural networks to estimate the effect of the basic variables from Table 1 and with the Standard Normal variable  $R_{N_1}$  providing the variability over the sub-combinations (which include the effect of different ground motions).

### 3.2 Block 2: Reliabilities

Table 2 shows all the random variables considered in the problem, including the basic ones shown in Table 1, and their assumed probability distribution and statistics. Each symbol “?” appearing in this table indicates a design parameter, that is, one of the outputs from the optimization.

Table 2 – Random variables

Variable	$\bar{X}$	$\sigma_X$	Type	Variable	$\bar{X}$	$\sigma_X$	Type
$X(1) = m$	$3.09 \cdot 10^{-4}$	$3.09 \cdot 10^{-5}$	Normal	$X(11) = K_d$	?	$0.10 \bar{X}$	Lognormal
$X(2) = b_b$	20 cm	1 cm	Normal	$X(12) = \bar{a}_G$	$48 \text{ cm/s}^2$	$78 \text{ cm/s}^2$	Lognormal
$X(3) = h_b$	? cm	$0.05 \bar{X}$	Normal	$X(13) = f_g$	2.50 Hz	0.375 Hz	Normal
$X(4) = b_c$	30 cm	1.5 cm	Normal	$X(14) = \sigma_{\bar{a}_G}$	0	0.25	Normal
$X(5) = h_c$	? cm	$0.05 \bar{X}$	Normal	$X(15) = a_G$	$X(15) = X(12) [1.0+X(14)]$		
$X(6) = \rho_b$	?	$0.10 \bar{X}$	Lognormal	$X(16) = R_{N_1}$	0	1	Normal
$X(7) = \rho'_b$	?	$0.10 \bar{X}$	Lognormal	$X(17) = X_{N_1}$	0	1	Normal
$X(8) = \rho_c$	?	$0.10 \bar{X}$	Lognormal	$X(18) = X_{N_2}$	0	1	Normal
$X(9) = f_r / f'_{c0}$	0.10	0.01	Normal	$X(19) = X_{N_3}$	0	1	Normal
$X(10) = F_y - Q_d$	?	$0.10 \bar{X}$	Lognormal				



It is assumed that the ordinate of the INPRES-CIRSOC 103 Parte I [16] design spectrum has an exceedance probability of 10% in 50 years and, for the zone that includes the city of Mendoza, the mean peak ground acceleration is  $a_G = 0.35 g$ . Following the procedure from FEMA 356 [2], accelerations corresponding to other return periods can be obtained. Thus, other levels of earthquake can be represented as frequent, occasional, rare and very rare, and this set can then be represented by a Lognormal distribution for  $a_G$ :  $X(12)$  in Table 2.

Eqs. (5) to (16) describe the failure functions adopted to describe the structural performance at three different levels: operational, life safety and collapse. A random variable  $X_{N_3}$ , Standard Normal, is introduced in order to take into account modelling error in the calculation of the demand parameters. It is assumed that these quantities show an uncertainty with a coefficient of variation  $COV = 0.10$ .

Operational	$G_{11}(\mathbf{X}) = 0.005 - DISTM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	mode 1	(5)
	$G_{12}(\mathbf{X}) = 0.10 - DILOM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	mode 2	(6)
	$G_{13}(\mathbf{X}) = 1.00 - DUCDM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	only for dissip.devices	mode 3 (7)
	$G_{13}(\mathbf{X}) = 1.00 - DISAM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	only for base isolators	mode 3 (8)
Life safety	$G_{21}(\mathbf{X}) = 0.015 - DISTM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	mode 4	(9)
	$G_{22}(\mathbf{X}) = 0.40 - DIES(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	} mode 5	(10)
	$G_{23}(\mathbf{X}) = 0.60 - DILOM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$		(11)
	$G_{24}(\mathbf{X}) = 1.50 - DISAM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	only for base isolators	mode 6 (12)
Collapse	$G_{31}(\mathbf{X}) = 0.025 - DISTM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	mode 7	(13)
	$G_{32}(\mathbf{X}) = 0.80 - DIES(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	} mode 8	(14)
	$G_{33}(\mathbf{X}) = 1.00 - DILOM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$		(15)
	$G_{34}(\mathbf{X}) = 2.50 - DISAM(\mathbf{X}) \left[ 1 + COV X_{N_3} \right]$	only for base isolators	mode 9 (16)

Within the bounds of the design parameters, and applying again experimental design,  $MC = 285$  combinations are chosen at random for the design parameters  $\mathbf{x}_d$ . For each combination, a Monte Carlo simulation is used to determine the failure probability (or reliability index) for each of the 9 limit states or failure modes in Eqs. (5) to (16). The discrete results  $\beta_j(\mathbf{x}_d)$ ,  $j=1,9$  are then represented by neural networks, which are utilized during the optimization to verify the compliance with the imposed minimum reliability constraints.

### 3.3 Block 3: Optimization

#### 3.3.1 Objective function

The objective function for the optimization is the life cycle cost. This includes the initial cost of construction,  $C_0(\mathbf{x}_d)$ , plus the repair costs  $C_d(\mathbf{x}_d)$  and the social costs  $C_s(\mathbf{x}_d)$  due to the occurrence of earthquakes during the service life of the structure. Accordingly,

$$C(\mathbf{x}_d) = C_0(\mathbf{x}_d) + C_d(\mathbf{x}_d) + C_s(\mathbf{x}_d) \quad (17)$$

(a) Initial cost  $C_0(\mathbf{x}_d)$  that is associated with:

- The costs of beams and columns, both functions of the design parameters, is the volume of concrete multiply by the unit cost  $CUH = 655 \text{ USD/m}^3$  plus the weight of reinforcing steel multiply by the unit cost  $CUA = 2.60 \text{ USD/Kg}$ . These costs reflect both currency and conditions in Argentina.
- The costs of the control devices, nothing for the structure with bared portal frames; the cost of the energy dissipation devices at  $500 \text{ USD/Kg}$  and their attachments at  $2.00 \text{ USD/Kg}$ , for the structure with dissipation



devices; or the cost of the base isolators valued between 30 USD/m<sup>2</sup> and 50 USD/m<sup>2</sup> depending on the isolator size, for the structure with base isolators.

- Costs associated with slabs and foundations do not depend on the design parameters, and are taken here as a constant USD 129600. Similarly, the cost of non-structural elements is taken as a constant USD 472500, and the cost of contents (equipment and furniture) estimated on the basis of 60 workstations as USD 222500.

(b) Repair costs:  $C_d(\mathbf{x}_d)$

The repair costs, at present values, depend on the level of damage caused by the earthquakes, the uncertainty associated with their arrival, the number of earthquakes during the life  $T_D$  of the structure and the interest rate available for a repair fund from the time of construction until the occurrence of the damages. If  $PR$  is a response parameter used to quantify the damage, and if  $C_f(PR)$  is the cost of repairs required at a time  $t$ , under the assumption that the structure is repaired after each event, returning it to the original conditions, the expected cost  $C_d|_{PR}$  (at present values and conditional on the response  $PR$ ) becomes [13]:

$$C_d|_{PR} = \sum_{n=1}^{\infty} C_f(PR) v \sum_{i=0}^{n-1} \left[ \frac{v^i}{i!} \int_0^{T_D} t^i \exp(-(r+v)t) dt \right] \frac{(vT_D)^n}{n!} \exp(-vT_D) \quad (18)$$

in which  $v = 0.20$  is the mean arrival rate of the earthquakes for Mendoza city,  $r = 0.05$  is an interest rate and  $n$  is the number of earthquake events in  $T_D$ . In general, this cost increases with the number  $n$ , but the occurrence probability of  $n$  events in  $T_D$  diminishes quite rapidly with  $n$ , resulting in  $C_d|_{PR}$  from Eq. (18) approaching a finite value. Then,

$$C_d(\mathbf{x}_d) = \int_0^{\infty} C_d|_{PR} f_{PR}(PR) d(PR) \quad (19)$$

in which  $f_{PR}(PR)$  is the probability density function for the response parameter. For a given set of design parameters  $\mathbf{x}_d$ , and with the help of the response neural network for  $PR$ , a Monte Carlo simulation is used to obtain the mean value and standard deviation of  $PR$ , from which the probability density in Eq. (19) is obtained from a Lognormal distribution assumption. For damage to the structure is  $PR = DIES$ ; for damage to the energy dissipation devices is  $PR = DUCDM$ ; for damage to base isolators is  $PR = DISAM$ ; for damage to non-structural elements is  $PR = DISTM$ ; and, finally, for damage to the contents is  $PR = ACELM$ . It is to be noted that, even if the initial cost of nonstructural elements and of the contents are not functions of the design parameters  $\mathbf{x}_d$ , their corresponding repair costs are, as these depend on the respond parameters which quantify the damage in those elements.

(c) Social costs:  $C_s(\mathbf{x}_d)$

Three scenarios of social costs are considered as a function of damage index: (i) Low damage index  $DIES = 0.10$ ; (ii) Intermediate damage index  $DIES = 0.40$ ; (iii) High damage index  $DIES = 0.80$ . Then, linear interpolation is used. Assuming 60 workers in the building, the cost for each case is:

Low damage index: with the hypothesis of no physical injury to persons, this cost includes the cost of re-insertion into the work routine, the cost of damage assessment and the cost of temporary rent for 3 months: USD 38500.

Intermediate damage index: with the hypothesis that 30% of people have injuries that required 6 months of treatment and 70% have more serious injuries, with 9 months of treatment, this cost includes medical costs and recovery, the cost of damage assessment and temporary rent for 6 months during repairs: USD 351000.

High damage index: with the hypothesis that 50% of people die, 35% had severe injuries requiring 9 months of treatment and 15% have minor injuries with 3 months of medical treatment, this cost includes medical costs and recovery, support for family of people died, compensation payments for death of people, individual life insurance, the cost of damage assessment and temporary rent for 12 months during reconstruction: USD 1293000.



These costs are conditional on the damage parameter DIES, and the total must be obtained by integration using the probability density function for DIES. Thus, the social costs are function of the design parameters  $\mathbf{x}_d$  through the dependence of DIES on those parameters.

### 3.3.2 Optimization algorithm

The algorithm used for optimization is based on a random search, without calculation of gradients. The procedure within the domain of  $\mathbf{x}_d$ , starts from an initial choice for the design parameters, or first “anchor point”,  $\mathbf{x}_{d0}$ . A set of  $n$  combinations of  $\mathbf{x}_d$  are randomly chosen in the vicinity of  $\mathbf{x}_{d0}$ , within a “search zone”. The reliability constraints are verified for each combination, and should any constraint be violated, a new combination is chosen. The total cost is calculated for each combination, and if the minimum cost is lower than that for the anchor point, the corresponding combination becomes the new anchor point and the process is repeated. The process stops when none of the  $n$  combinations within the search zone has a total cost lower than the one for the anchor. This anchor provides, then, an approximation to the optimum solution.

### 3.3.3 Reliability target constraints

The target maximum annual failure probabilities are chosen according to the recommendations by Paulay and Priestley [19]:  $2 \times 10^{-2}$  for operational performance;  $2 \times 10^{-3}$  for life safety and  $2 \times 10^{-4}$  for the limit state of collapse. Considering that earthquakes for the city of Mendoza obey a Poisson arrival process with a mean rate of  $\nu = 0.20$ , those annual probability limits are equivalent to the following reliability indices for the event of earthquake occurrence: 1.276 (operational), 2.326 (life safety), 3.090 (collapse).

## 4. Numerical results and discussion

The summary of the numerical results for the three variants of the building structure is presented in Table 3, considering reliability constraints (WRC) and without these constraints (WORC).

### a) Results about the design parameters:

The dimensions of beams and columns are substantially larger in the structure of bared portal frames than in the structures with control devices. This result is consistent because the ends of beams and the ends of some columns in that structure have to dissipate energy, while in structures with control the greater dissipation is accomplished by the control devices, releasing beams and columns that function. The results indicate that the process implemented capture the operation of the control devices.

When the results between with and without reliability constraints are compared, the results are similar for bared portal frames, changing only larger column dimension for less amount of reinforcement. In the case of portal frames with dissipation devices the beams are weaker but the devices are stiffer without constraints than with constraints. The structure with base isolators presents very similar results in both cases, because the optimal solution is obtained with the smallest isolators that were considered according to the catalog available.

### b) Results about the reliability indices

In the structure of bared portal frames with reliability constraints, the optimal solution is limited by  $\beta_8(\mathbf{x}_d) = 3.101 > \beta_T = 3.09$  which corresponds to limit of damage in the performance level of collapse. For other limit states, the reliability index meets the minimum with some excess. When restrictions are released only seen a small decrease in  $\beta_8(\mathbf{x}_d) = 2.94$ , while other indices of reliability practically hold their value. This result shows that the restrictions are reasonable.

In the structure of portal frames with dissipating energy, the optimal solution is obtained with  $\beta_3(\mathbf{x}_d) = 1.304 > \beta_T = 1.276$  which corresponds to yield limit of the dissipators for operational performance, while other restrictions are satisfied with some excess. The optimal solution without constrained presents a significant decrease in  $\beta_3(\mathbf{x}_d) = -0.432$ , which corresponds to probability of failure 0.667 (steel yielding) when a seismic event occurs. The other indices of reliability have some variation but remain above  $\beta_T$ .



The results of the structure with base isolators with reliability constraints show that all limit states are satisfied with excess, especially those related to the behavior of isolators. The optimal solution is obtained with the smallest isolators available, then, a close-fitting design could be achieved with smaller devices. When restrictions are released, no major changes are observed.

Table 3 – Optimization results

Result	Bared portal frame		Portal frame with dissipation devices		Portal frame with base isolators	
	WRC	WORC	WRC	WORC	WRC	WORC
$x_d(1) = \bar{X}(3) = h_b$ [ cm ]	64.1	64.4	55.4	38.2	44.4	45.6
$x_d(2) = \bar{X}(5) = h_c$ [ cm ]	85.4	90.2	43.3	42.6	43.9	45.3
$x_d(3) = \bar{X}(6) = \rho_b$	0.0119	0.0111	0.0103	0.0076	0.0075	0.0087
$x_d(4) = \bar{X}(7) = \rho'_b$	0.0104	0.0105	0.0087	0.0064	0.0113	0.0091
$x_d(5) = \bar{X}(8) = \rho_c$	0.0184	0.0102	0.0218	0.0165	0.0223	0.0228
$x_d(6) = \bar{X}(10) = F_y - Q_d$			416	525	20.7	20.4
$x_d(7) = \bar{X}(11) = K_d$			4135	8962	281.5	286.2
Operational $\beta_1(\mathbf{x}_d)$	2.097	1.988	2.467	2.607	1.557	1.664
( $\beta_T = 1.276$ ) $\beta_2(\mathbf{x}_d)$	2.087	2.101	2.209	1.803	2.521	2.395
$\beta_3(\mathbf{x}_d)$			1.304	-0.432	1.921	2.069
Life safety $\beta_4(\mathbf{x}_d)$	3.039	2.987	3.749	3.927	2.906	3.070
( $\beta_T = 2.326$ ) $\beta_5(\mathbf{x}_d)$	2.538	2.528	3.071	3.004	2.749	2.702
$\beta_6(\mathbf{x}_d)$					3.461	2.504
Collapse $\beta_7(\mathbf{x}_d)$	3.847	3.764	4.483	6.600	3.452	3.544
( $\beta_T = 3.09$ ) $\beta_8(\mathbf{x}_d)$	3.101	2.940	4.084	3.904	3.229	3.428
$\beta_9(\mathbf{x}_d)$					4.776	2.837
Initial cost, struct. [USD]	144100	137170	98063	72600	114470	117890
Initial cost, control [USD]			53100	50592	20089	20089
Total initial $C_0(\mathbf{x}_d)$ [USD]	968660	961730	975723	947752	959119	962539
Repair struct cost [USD]	427	307	239	330	262	79
Repair control cost [USD]			8587	18797	78630	75636
Repair no struc cost [USD]	47803	24592	26	171	4184	5254
Repair content cost [USD]	16213	16006	9802	10532	6276	4272
Social cost $C_S(\mathbf{x}_d)$ [USD]	3360	2752	1829	4124	2515	827
Total cost $C(\mathbf{x}_d)$ [USD]	1036463	1005387	996206	981706	1050986	1048607

c) Results about the costs

The optimization process looks for minimum total cost solution, which can be achieved for different combinations of the design parameters. Furthermore, the cost function is quite flat near the minimum, with very similar costs for different solutions.

The implemented process obtains, for each type of structure, lower costs when no reliability constraints are imposed. This is clearly seen in the structure with dissipating devices, with lower initial costs for the solution



without restrictions but higher repair costs and social costs, reaching a total of USD 981700, which is less than USD 996200 when restrictions are imposed. However, in the three types of structures analyzed, the total cost differences are small between the cases with and without restrictions, indicating that optimal solutions are very similar, or in other words, the limits of failure probabilities used are satisfactory.

The incidence of repair costs for damages and social costs in the optimal solution is small, with average values of 6% for uncontrolled portal frames, 3% for portal frames with dissipating devices, and 8.5% for portal frames with base isolators. These results indicate that it is economically more convenient to build structures to which the earthquakes cause minor damage. Otherwise, repair damage costs and social costs rapidly increase the total cost.

The comparison between the total costs of the three types of structures analyzed shows that the lowest total cost is obtained for the structure with energy dissipation devices based on steel yielding. Also, it should be considered that the cost of the solution with base isolators could be improved if smaller isolators were utilized. Anyway the three alternatives are applicable because the maximum differences in total cost are about 6%.

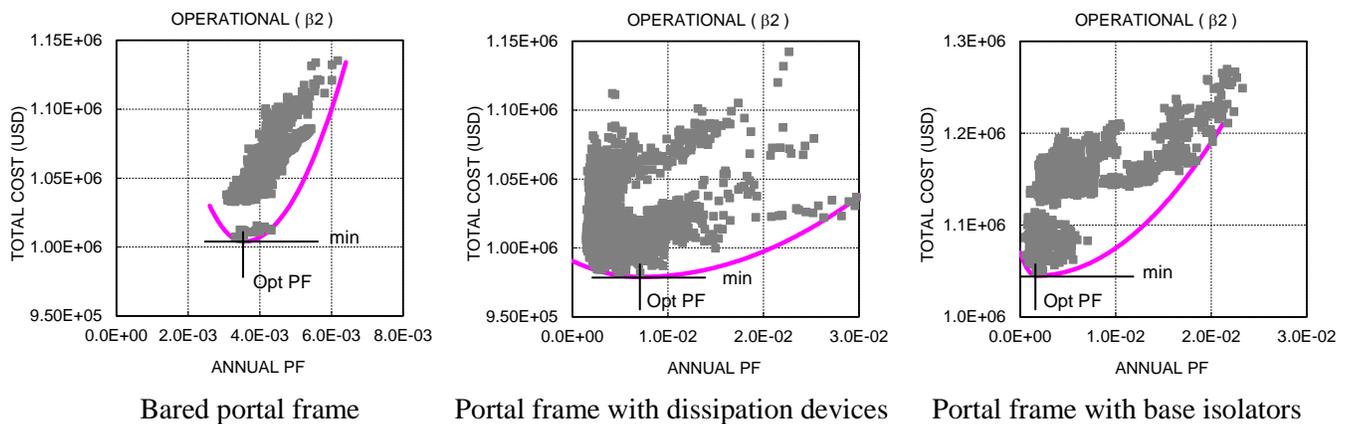


Fig. 3 – Minimum cost and optimal probability of failure

The Fig.3 shows the results (without minimum reliability constraints) from the evolution of the total cost objective function during the optimization process, presented in correspondence with the calculated associated annual failure probability for operational performance level for each type of structure analyzed. The minimum cost is associated with an optimum annual failure probability and, for these example, these optima are somewhat lesser than the constraint  $2 \times 10^{-2}$ . The lower envelope of the results in Fig.3 (Pareto front) clearly shows that, up to a point, the total cost decreases as the annual failure probability increases. At small probabilities of failure, the total costs are controlled by the initial costs. These can be decreased if higher failure probabilities are accepted. Beyond the minimum point, increases in annual failure probability correspond to increases in costs, as a result of repair and social consequences becoming more dominant.

## 5. Conclusions

The general optimization process organized in three main blocks is appropriate and consistent, taking into account uncertainties in the variables and several limit states in each performance level considered. Through minimizing the total cost, including construction costs, repair costs for damages and social costs over the service life, it is shown to be a suitable tool for making decisions regarding the most convenient type of structure and vibration control devices.

For the office building studied in this paper, the three structural solutions analyzed are acceptable, with total costs not differ by more than 6% between them, while a structure with portal frames including energy dissipation devices based on steel yielding has the lowest total cost.



The structures with control have smaller dimensions of beams and columns than the structure without control, but offset by the cost of the devices. The reliability indices for optimal solutions of each type of structures, or annual failure probability, are similar when considering restrictions or not, because the recommended constraints are slightly higher than the optimal values.

For the three cases, the incidence of repair costs for damages and social costs is less than 9%, indicating that the optimal designs must have little damage when earthquakes occurs, because beyond this point, the incidence of such costs growing rapidly. Also, for the three type of structure the total cost is slightly lower when no restrictions are imposed.

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