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SIMULTANEOUS SHEAR AND AXIAL FAILURE OF REINFORCED CONCRETE COLUMNS OF EXISTING SUBSTANDARD BUILDINGS

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Abstract

Previous experience from earthquake events has shown that existing substandard Reinforced Concrete (R/C) buildings are particularly vulnerable to strong ground motions. One of the main causes of collapse of such structures is the axial failure of shear or flexure-shear critical vertical load bearing elements. In this context, the phenomenon of simultaneous shear and axial failure is of really high significance, because the collapse mode of the whole structure can be adversely affected by such members leading to redistribution of vertical loads at a very low lateral deformation level and the structure's collapse probability can thus be greatly increased. Therefore, the cardinal priority in a pre-seismic retrofit programme for existing structures should be to prevent failure of these particularly vulnerable elements.

Observations have been made in previous studies on possible circumstances leading to such a failure mode. However, the development of a reliable criterion is still an open issue. The main contribution of the present work is the proposal of the necessary conditions for the simultaneous shear-axial failure of a column in the form of a two-parameter classification criterion, constituting a handy tool for engineers working on assessment and retrofit of existing R/C structures. If not all member's details are known or if a preliminary classification is only required, a simplified one-parameter criterion can be used, instead; nonetheless, the latter comes with a high uncertainty. Furthermore, criteria for "extra-safe zones" have also been extracted, in case a higher inter-storey drift capacity is to be ensured.

Last but not least, two well-established predictive models are compared as to their ability to predict simultaneous failure of an R/C member. The one previously developed by the authors performs far better and would be recommended for use in relevant applications, with the aforementioned two-parameter criterion still being the optimal choice for a reliable classification of a column as safe from simultaneous failure.

Keywords: Reinforced Concrete, Seismic Assessment, Column, Post-peak response, Simultaneous shear and axial failure

1. Introduction

Previous experience from earthquake events has shown that existing Reinforced Concrete buildings, which have not been adequately designed against seismic actions, are particularly vulnerable to strong ground motions [1]. One of the main causes of total or partial collapse of such structures is the axial failure of shear or flexure-shear critical vertical load bearing elements [2]. Most of these members possess some displacement capacity even after they fail in shear - subsequent or prior to yielding of the longitudinal reinforcement; in other words, after the onset of shear failure, they can reach higher ductility levels accompanied by the corresponding deterioration of their lateral strength [3] (Fig. 1a).

Nonetheless, there is a subset of columns that can collapse axially immediately after their shear failure, thus providing no warning to occupants prior to axial failure [4] (Fig. 1b and 1c). This most daunting phenomenon is commonly referred to as "simultaneous shear and axial failure" in the literature. It is of paramount significance, because the collapse mode of the whole structure can be adversely affected by such members leading to redistribution of vertical loads at a very low lateral deformation level, and the structure's collapse probability can thus be greatly increased. Therefore, the highest priority in a pre-seismic retrofit attempt of an existing structure should be to secure these particularly vulnerable elements.

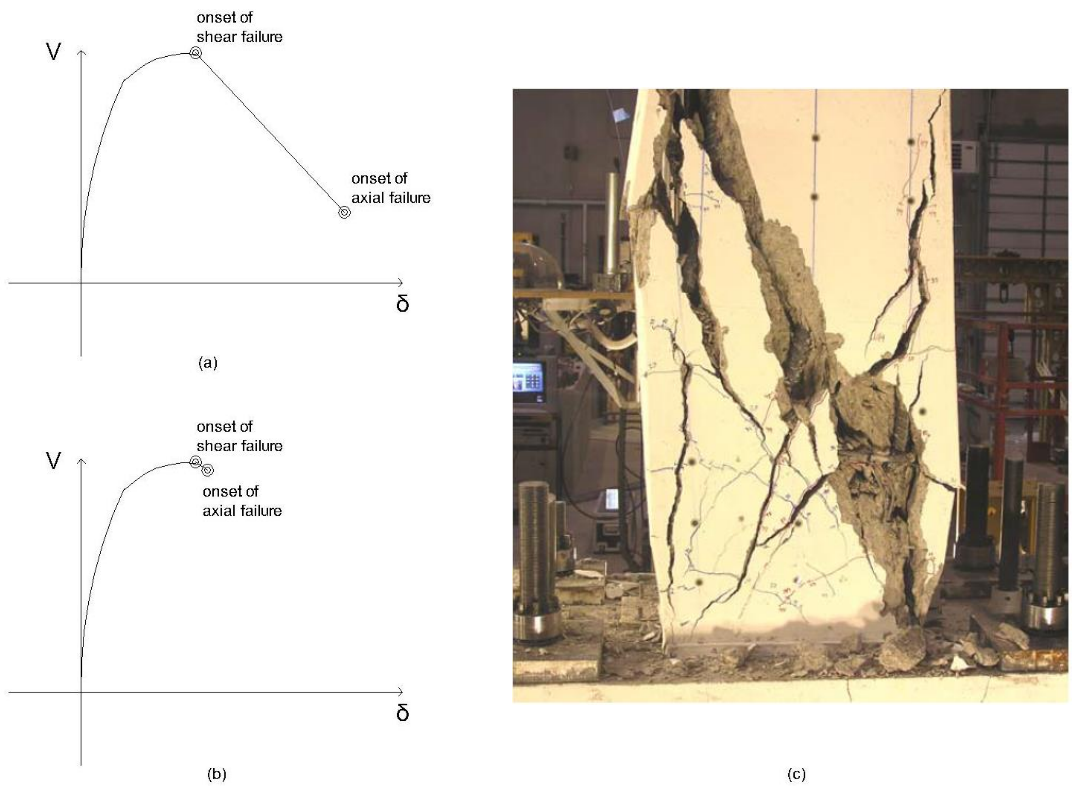

Fig. 1 – Indicative sketches of cyclic envelopes of the hysteretic lateral response of columns failing axially after shear failure (a), or failing in simultaneous shear and axial failure (b). Specimen failing in simultaneous shear and axial failure [5] at the point of failure (c).

Determining which columns are susceptible to such failure, however, is not such a straightforward task. Observations have been made from researchers in the past on possible circumstances leading to such a phenomenon, but no definitive criterion has been proposed so far. Some experimental evidence suggests that simultaneous shear and axial failure occurs when the applied axial load is (about) equal or greater than the axial capacity of the longitudinal reinforcement [5]. Elsewhere, it was shown that columns that experienced such a failure had a ratio of axial load over yield strength of longitudinal reinforcement greater than 0.65 and had the highest ratios of axial load to buckling capacity of the longitudinal reinforcement in a subset of 11 columns examined [6]. The former agrees with a remark of a recent study by the authors, based on a database of nearly 90 specimens [3]. Henkhaus et al. [4] studied a dataset of 40 columns cycled up to axial failure. They noted that the 4 of them that failed in simultaneous shear and axial failure had transverse reinforcement ratios of $\rho_w \geq 0.06\%$ and axial load levels of $v \leq 0.3$, contrary to what ASCE/SEI 41-06 [7] predicted for the specific members. Yoshimura et al. [8] observed that axial failure occurred simultaneously with shear failure in shear-flexure critical elements, quite suddenly and pertaining to crushing of concrete and buckling of longitudinal bars at the plastic hinge region, as opposed to shear critical specimens, where axial collapse took place when shear strength degraded almost totally; their finding was based on their limited experimental series, comprising 8 columns. Eurocode 8-3 [9], on the other hand, does not provide for post-peak response of elements undergoing shear failure, considering it as a brittle failure and solely using a force criterion; this is equivalent to the conservative assumption that every shear or flexure-shear critical R/C member fails in simultaneous shear-axial failure.

The main goal of the current paper is to explore the validity of these observations and propose the empirically observed necessary conditions of simultaneous shear and axial failure of R/C columns. Subsequently, two well-established relevant models will be compared regarding their ability to predict this type of failure.

2. Database and Parameter of Interest

A large database of shear and flexure-shear critical specimens of rectangular section, exhibiting post-peak behaviour and/or axial failure was compiled in a previous work by the authors [3]. A slightly extended database will be used herein, in order to explore the conditions resulting in simultaneous shear and axial failure. In total, 89 specimens of the database have sustained axial failure; another 44 have been included in the database due to their post-peak behaviour, which means that at least 30% lateral strength degradation has been recorded, so definitely no simultaneous failure has occurred. Hence, 133 specimens will be employed in total.

The main parameter employed herein is the post-peak Inter-storey Drift Ratio, $IDR_{,pp}$, which is equal to the post-peak lateral displacement up to the onset of axial failure normalised by the length of the specimen. This is a parameter expressing a column's post-peak deformability, which the earthquake engineering community is very familiar with, making it an optimal choice.

When this quantity is zero or close to zero in a given specimen, its post-peak response is practically non-existent, sustaining axial failure simultaneously with shear failure. However, how much could be considered as "close to zero"? The post-peak Inter-storey Drift Ratio ranges from 0.0% to almost 12.0% in this database, with the main bulk of specimens lying between 0.6% and 3.2% (first and third quartile, respectively). Taking into account the typical lengths of existing building storeys, even a 0.5% drift ratio would translate into 15 to 20 mm; this is a very large value with regard to post-peak response. Consequently, a value of 0.2%, i.e. up to 6-8 mm post-peak displacement for typical building columns, was chosen as a conventional upper limit in order to consider a specimen as having failed simultaneously in shear and axial load. Out of the 133 specimens of the database, about 9.0% have sustained simultaneous shear and axial failure following this definition.

3. Assessment of Existing Observations

The first observation that is tested is the claim that flexure-shear – as opposed to shear – critical specimens sustain simultaneous failure [8]. This can readily be shown not to apply in all cases, since:

1. about half the specimens sustaining such failure herein are shear critical, and
2. the majority of flexure-shear critical specimens (88.1%) have not failed simultaneously in shear and axial failure.

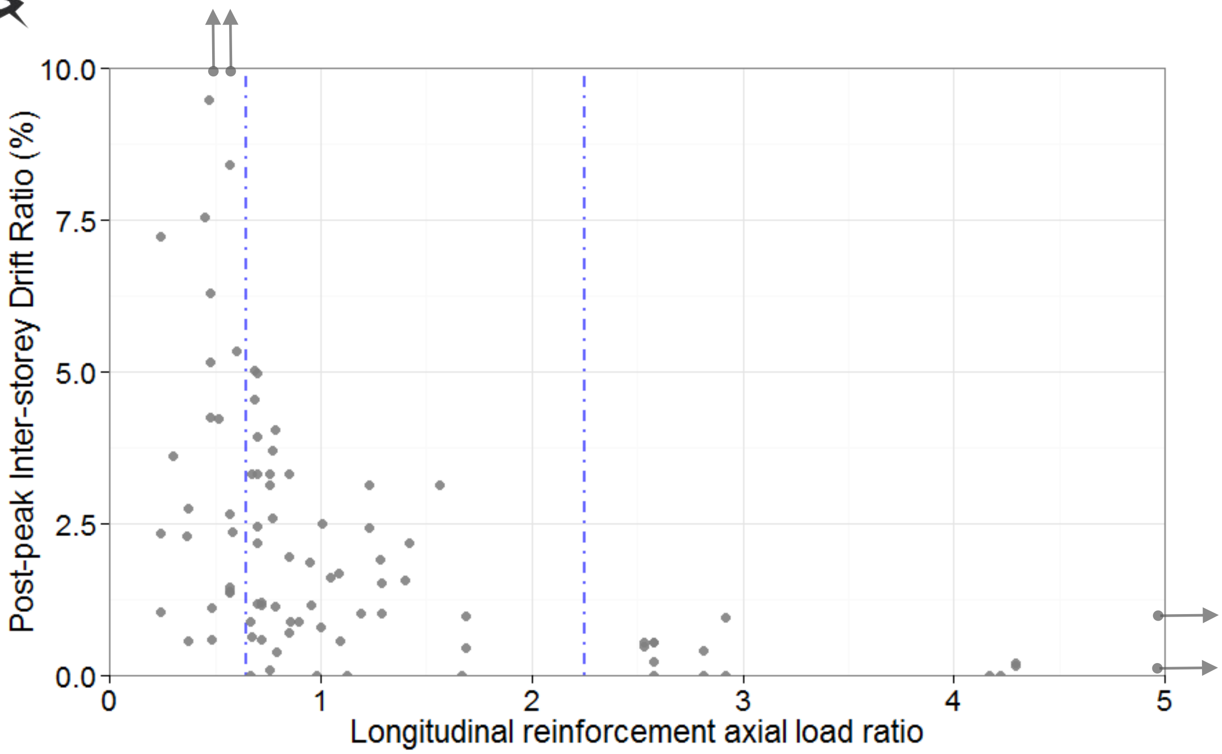

Fig. 2 – Correlation of the post-peak Inter-storey Drift Ratio with the axial load ratio based on the axial capacity of the longitudinal reinforcement. Few points are shown with arrows, indicating accurately one of their coordinates and that the other one is higher than the maximum of the range of the respective axis.

The next observations tested are the ones pertaining to the ratio of axial load over yield strength of longitudinal reinforcement, i.e. $v_l = N / (A_{sl} \times f_{yl})$. Based on a previous study [3], Matchulat's [5] finding about the ratio of 1.0 being the determining criterion of simultaneous failure cannot be generalised; however, Matamoros's & Woods's [6] remark about the ratio of 0.65 seems to provide a lower threshold for the occurrence of the phenomenon. The upper threshold, above which the occurrence is highly probable seems to be the ratio of 2.25, as all of the specimens with greater v_l exhibit *IDR,pp's* less than 1.0% and nearly half of them fail in simultaneous failure. These patterns can be more clearly seen in Fig. 2 (note that the 44 specimens that have not failed axially at all have values of $0 \leq v_l \leq 2.25$ and are not depicted in the diagram), where the lower (v_l = 0.65) and upper thresholds (v_l = 2.25) for the occurrence of simultaneous shear and axial failure are depicted. Nonetheless, it is clear that using only this parameter cannot provide a comprehensive criterion, since the range of specimens in-between the two limits - where the behaviour is unpredictable – is quite extensive.

Henkhaus et al.'s [4] study claims that the axial load ratio and the transverse reinforcement ratio limits of ASCE/SEI 41-06 [7] are not sound, since all of the specimens sustaining simultaneous failure examined in their dataset would be deemed safe from it. It is established in the present study that a transverse reinforcement ratio of $\rho_w \geq 0.05\%$ and an axial load ratio of $v \leq 0.6$ – or even $\rho_w \geq 0.06\%$ and $v \leq 0.3$ – do not ensure that simultaneous failure will not occur, as can be seen in Fig. 3. The vast majority (83.3%) of the specimens that failed simultaneously satisfy the former limits (v = 0.60 and ρ_w = 0.05%, shown with blue dashed lines). One third of them are even inside the latter limits (v = 0.30 and ρ_w = 0.06%, shown with green dashed lines).

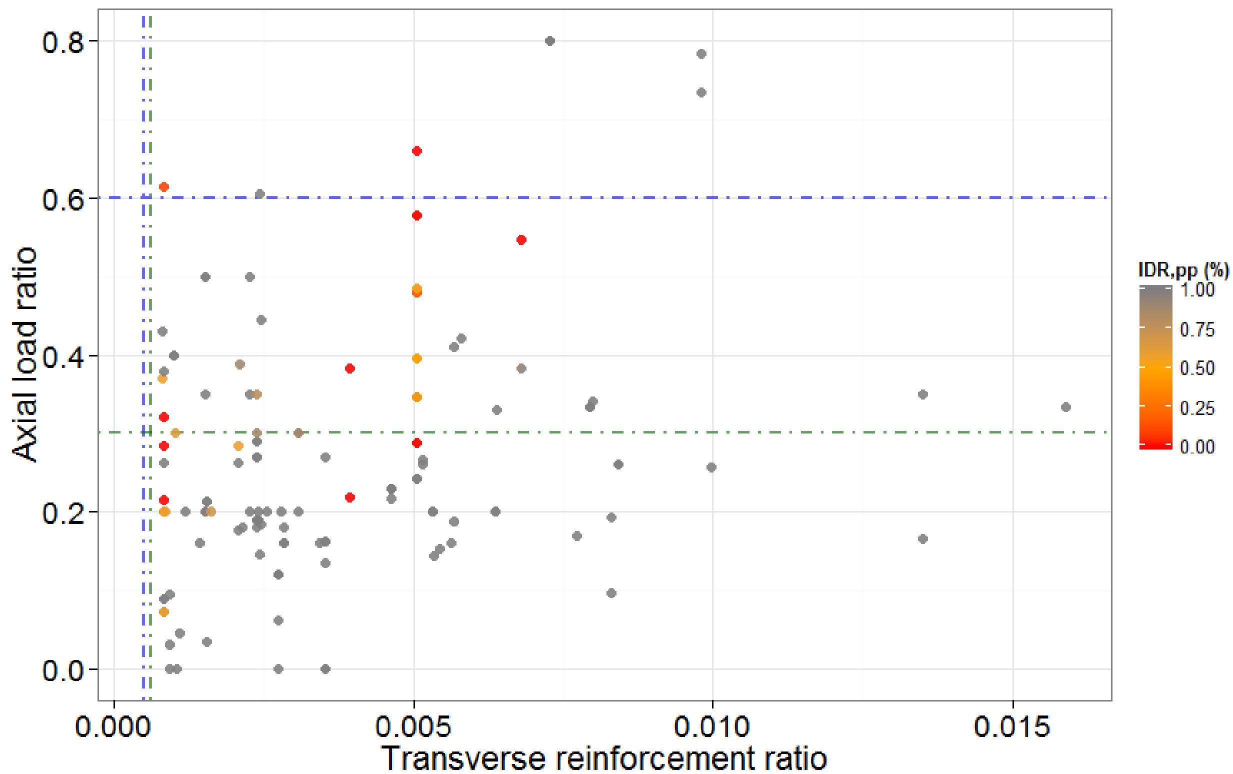

Fig. 3 – Values of post-peak Inter-storey Drift Ratio (*IDR,pp*) depicted on a plane with the axial load ratio on the vertical axis and the transverse reinforcement ratio on the horizontal. Values greater than 1% are in grey colour.

4. Proposed Criteria

Based on the previous section, a simplified criterion is extracted, which can be used if a limited amount of data is known for a given column or if only a preliminary estimation is to be made. This criterion is based on the ratio of axial load over yield strength of longitudinal reinforcement ($v_l = P / (A_{sl} X f_{yl})$) and reads as follows (Fig. 2):

- If $v_l \leq 0.65$ (compression positive), the column can be considered safe.
- If $v_l \geq 2.25$, the column will most likely fail in simultaneous shear and axial failure.

In the intermediate zone ($0.65 \leq v_l \leq 2.25$), the behaviour cannot be reliably determined, as there are specimens belonging to both categories.

Subsequently, a more reliable and accurate criterion is searched. Besides v_l and v, other parameters are studied, in order to discover suitable classification variable(s). The transverse reinforcement was used in the ASCE/SEI 41-06 [7] classification and is generally believed to play an important role in this phenomenon. Therefore, many different parameters including s/d, ρ_w, f_{yw} and combinations of these variables are tried out, since the transverse reinforcement ratio is shown not to be adequate by itself (Fig. 3).

Eventually, the combination of the parameters v_l and $\rho_w / (s/d)$ was found to be the best one to classify the specimens. Its classification capability can be seen in the scatterplot of Fig. 4. The latter parameter expresses the content of transverse reinforcement further reduced or increased by sparse or dense hoop spacing, respectively.

The criterion defines a "safe area" for columns and reads as follows (Fig. 4, with blue dashed lines showing the limits of the safe area at v_l = 1.65 and ρ_w / *(s/d)* = 0.1%.):

- If $v_l \leq 1.65$ and ρ_w / *(s/d)* $\geq$ 0.1%, the column is considered safe, i.e. it is not going to fail in simultaneous shear and axial failure.
- If $v_l > 1.65$ or ρ_w / *(s/d)* < 0.1%, the column might fail in simultaneous shear and axial failure, i.e. the necessary conditions are met. Nevertheless, there are specimens falling into this category that do not fail thus, as the conditions are not sufficient and necessary at the same time.

Summarising, having knowledge of the axial load carried by a given column, its longitudinal and transverse reinforcement, one can define if it is safe from this type of failure or if it meets the necessary conditions for it.

Leaning towards the conservative side, if the goal is to avoid a post-peak Inter-storey Drift Ratio lower than 1.0%, one can use one of the two following "extra-safe zones" criteria:

- $v_l \leq 1.65$ and ρ_w / *(s/d)* $\geq$ 1.15%, or
- $v_l \leq 0.85$ and ρ_w / *(s/d)* $\geq$ 0.3%.

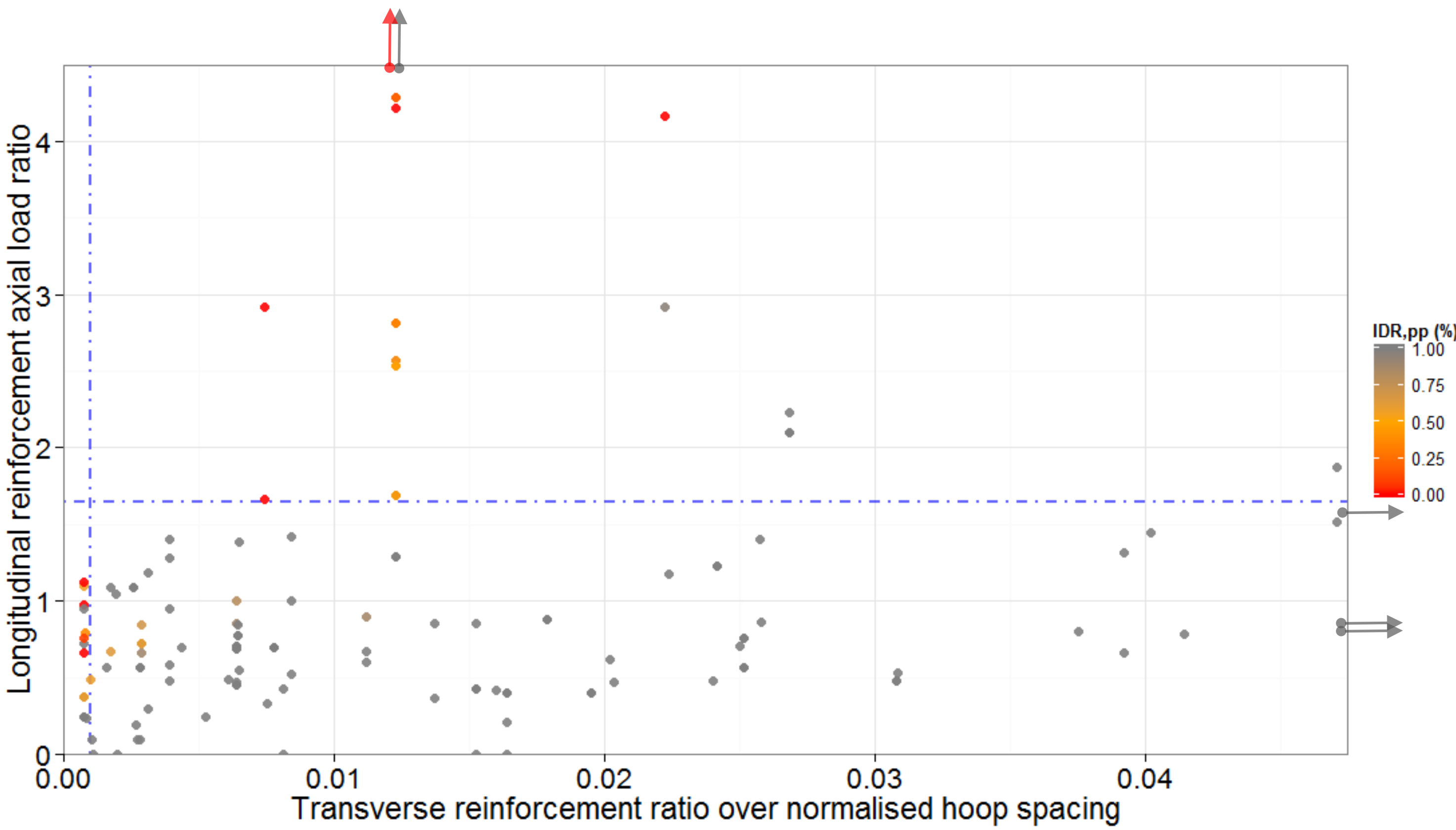

Fig. 4 – Values of the post-peak Inter-storey Drift Ratio (*IDR,pp*) depicted on a plane with the ratio of axial load over longitudinal reinforcement yield strength on the vertical axis and the transverse reinforcement ratio over normalised hoop spacing on the horizontal. Values greater than 1% are in grey colour. Few points are shown with arrows, indicating accurately one of their coordinates and that the other one is higher than the maximum of the range of the respective axis.

5. Evaluation of Predictive Models

In this section, two recent empirical models predicting the post-peak response of R/C members will be tested against the experimental observations of the database. The first one comprises the Elwood & Moehle equations [10], [11] that calculate the drift ratio at the onset of shear and the onset of axial failure, respectively, and which are used in several member-type constitutive models:

$$\frac{\Delta_s}{L} = 3 + 400\rho_w - \frac{1}{5}\frac{\tau_{\max}}{\sqrt{f_c}} - \frac{1}{0.4}\frac{N}{A_g f_c} \geq 1 \tag{1}$$

$$IDR,_{pp} = \frac{\Delta_a}{L} - \frac{\Delta_s}{L} = 4\frac{1+(\tan 65)^2}{\tan 65 + N\left(\dfrac{s}{A_{sw} f_{yw} d_c \tan 65}\right)} - \frac{\Delta_s}{L} \geq 0 \tag{2}$$

where $IDR,_{pp}$ is calculated in %, ρ_w is the transverse reinforcement ratio introduced with its actual value (not in %), τ_{max} is the maximum nominal shear stress, f_c is the concrete compressive strength, N is the axial load applied on the column, A_g is the cross-section area, A_{sw} and f_{yw} are the area and yield strength of the transverse reinforcement, the tangent of the angle is calculated in degrees (65^o), s is the spacing between the transverse reinforcement ties and d_c is the depth of the column core from centreline to centreline of the ties.

The second is a model previously developed by the authors [3], which predicts the post-peak shear strain of an R/C member as part of a stand-alone beam-column model:

$$IDR,_{pp} = 100\frac{L_{cr}}{L}\left[0.65\left(\frac{\rho_l}{A_{conf,\%}}\right)^{1.2}\sqrt{\frac{\rho_w f_{yw}}{v_l \, {}^{s}\!/_{d} \, {}^{\tau_{\max}}\!/_{\sqrt{f_c}}}}\right] \geq 0 \tag{3}$$

where $IDR,_{pp}$ is calculated in %, ρ_l (longitudinal reinforcement ratio) and ρ_w are introduced with their actual value (not in %), $A_{conf,\%}$ is the percentage of the cross-section area that is confined and is introduced with its actual value (not in %), $v_l = N / (A_{sl} \times f_{yl})$ is the longitudinal reinforcement axial load ratio and L_{cr} is the shear critical length defined by the critical shear crack angle (see [3] for more details). For consistency, the predicted post-peak displacements of the model have been normalised by the whole length of the member, instead of the shear critical length.

The first model seems to underestimate greatly the deformability of the database's specimens, predicting a 41.4% frequency of simultaneous shear and axial failure, instead of the actual 9.0%. This is much more vividly shown in Fig. 5a (with blue dashed lines showing the limits of the previously defined safe area, for comparison), where specimens way into the safe zone are predicted to have post-peak drifts equal to or close to zero. On the other hand, the second model seems to predict quite accurately the pattern of occurrence of simultaneous failure, producing a diagram very similar to the experimentally obtained one (Fig. 5b, with blue dashed lines showing the limits of the previously defined safe area, for comparison).

For an objective comparison, initially the models' predicted values are compared to the observed values and comparison statistics are produced (Table 1), including Mean Absolute Error (*MAE*), Root Mean Squared Error (*RMSE*) and Coefficient of Variation (R^2). Reading the statistics, one realises that the first model is doing worse in explaining the variation of the observed values than their mean value (i.e. negative R^2), compared to the far superior performance of the second one. The difference in the other two statistics complements this finding, with the first model producing more than double the error of the second on average.

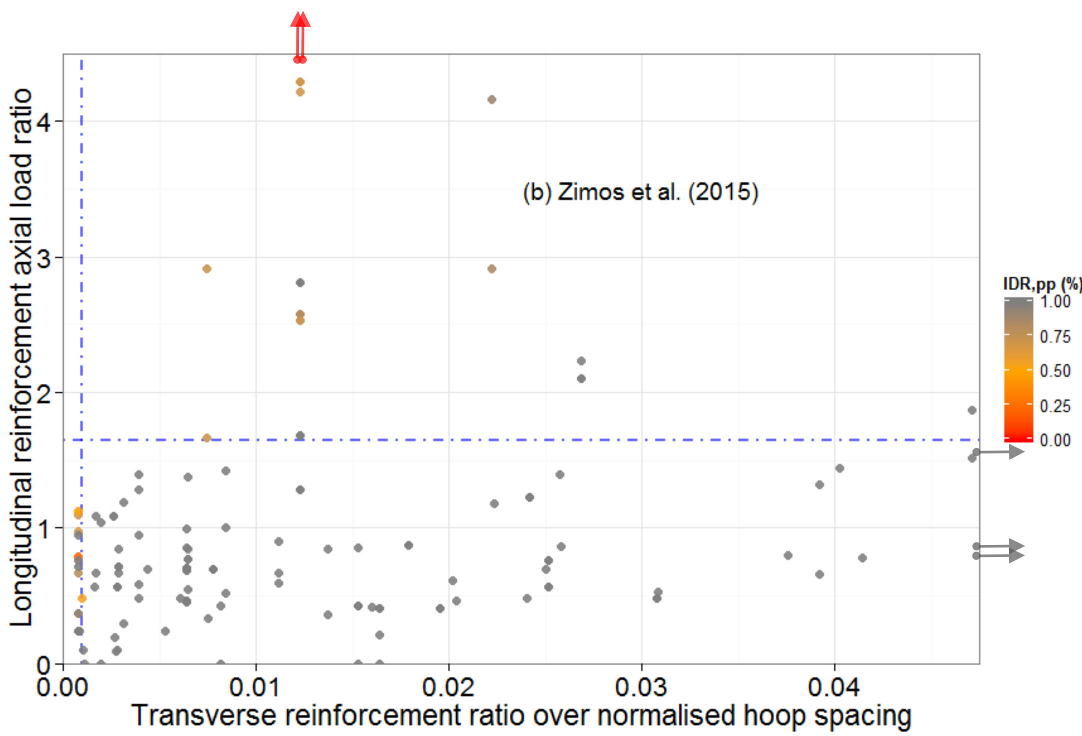

Fig. 5 – Values of the post-peak Inter-storey Drift Ratio ($IDR_{,pp}$), predicted by (a) the Elwood & Moehle model [10], [11], and (b) the Zimos et al. [3] model, depicted on a plane with the ratio of axial load over longitudinal reinforcement yield strength on the vertical axis and the transverse reinforcement ratio over normalised hoop spacing on the horizontal. Values greater than 1% are in grey colour. Few points are shown with arrows, indicating accurately one of their coordinates and that the other one is higher than the maximum of the range of the respective axis.

Table 1 – Comparison statistics of the two predictive models against the experimentally observed values.

Model	R^2	MAE	RMSE
Elwood & Moehle	-0.37	1.814	2.791
Zimos et al.	0.69	0.976	1.330

Subsequently, the models' classification capabilities are compared based on the observed values and confusion matrices are constructed (Table 2). Therein, the number of cases that a model predicted correctly that a specimen is safe or that it will fail simultaneously lie on the diagonal (classification as "safe" and "failed" is based on an $IDR_{,pp} \leq 0.2\%$, as previously mentioned in section 2). Conversely, in the other cells one can spot the number of specimens that were falsely predicted as due to fail in shear-axial failure but were actually safe or the opposite. The classification with the abovementioned two-parameter classification criterion (section 4) is also included, for comparison purposes. From these, statistics are computed to compare the classification strength of each model (Table 3).

Table 2 – Confusion matrices of the predicted values (predicted) of the two predictive models compared to the observed values (true) of the database.

Elwood & Moehle Model		
true \ predicted	"safe"	"failed"
"safe"	77	44
"failed"	1	11
Zimos et al. Model		
true \ predicted	"safe"	"failed"
"safe"	120	1
"failed"	11	1
Classification Criterion Model		
true \ predicted	"safe"	"failed"
"safe"	96	25
"failed"	0	12

It's observed that the Zimos et al. model is the most accurate one having the highest accuracy and producing the lowest error rate, i.e. it predicts the most correct cases compared even to the classification criterion. The Elwood & Moehle model produces the highest error rate and has the lowest accuracy.

However, the classification criterion results in zero Type I error, which is exactly its purpose of providing the necessary conditions of failure, i.e. each specimen that it predicts as safe is indeed safe. If the conditions were necessary and sufficient, Type II error would also be zero, i.e. each specimen that it would predict as due to fail in axial-shear failure would indeed fail. The first model results in very low Type I error, largely because it systematically underestimates tremendously the deformability of all specimens (see Fig. 5a). The second model results in very high Type I error, because it slightly overestimates the deformability systematically, hence many specimens have a post-peak Inter-storey Drift very close to the actual one but do not qualify for the "failed" category marginally; this is apparent graphically in Fig. 5b with several specimens in the "failed" area being closer to orange than red (i.e. slightly higher than the limit of 0.2%). Nonetheless, this slight overestimation leads to nearly zero Type II error, i.e. hardly any safe specimen is falsely predicted as due to fail.

Table 3 – Comparison statistics of the predictive models' classification capabilities based on the experimentally observed values.

Model	Accuracy	Error Rate	Error I	Error II
Elwood & Moehle	0.66	0.34	0.08	0.36
Zimos et al.	0.91	0.09	0.92	0.01
Classification criterion	0.81	0.19	0	0.21

In conclusion, the Zimos et al. model [3] seems to be much better in predicting the post-peak deformability of an R/C member, compared to Elwood & Moehle's model [10], [11]. However, one should keep in mind that the onset of shear failure of the latter is assumed to be the point of 20% strength degradation instead of the peak point, therefore being expected to result in slightly lower deformability than the experimental observations. Furthermore, the ranges of parameters of the initial limited databases to which these empirical equations were calibrated were much narrower compared to the one used herein.

On the other hand, when it comes to reliably predicting the safety of a specimen, i.e. each specimen predicted as safe being indeed safe, the two-parameter classification criterion is the best, producing no error, with the Elwood & Moehle model following.

6. Conclusions

The problem of determining which columns in existing R/C buildings are susceptible to simultaneous shear and axial failure has been comprehensively investigated herein, based on previous observations by researchers and a database of 133 specimens, about 9.0% of which failed simultaneously. No reliable criterion had been proposed for this phenomenon thus far, albeit remarks have been made in several past studies.

The main contribution of the present work is the proposal of the necessary conditions for the simultaneous failure of a column in the form of a two-parameter classification criterion. It requires knowledge of the axial load carried by the column, its longitudinal and its transverse reinforcement characteristics and achieves the reliable classification of a column as safe from simultaneous shear and axial failure or as likely to fail in such mode. This is believed to be a very useful tool for engineers working on assessment and retrofit of existing R/C structures.

In case the transverse reinforcement details are unknown or if a preliminary estimation is to be made, a simplified one-parameter criterion can be used, instead. Nonetheless, this comes with a high uncertainty.

Being on the safe side, any column falling outside of the safe area of the two-parameter criterion or lying above the upper threshold of the simplified one-parameter criterion should be assigned a foremost priority in a pre-earthquake retrofit scheme for an existing R/C building. Failing to prevent the simultaneous shear-axial failure of such vulnerable element(s) can lead to redistribution of vertical loads at a very low lateral deformation level, hence adversely affecting the collapse mode of the whole structure and severely increasing the structure's collapse probability.

Criteria for "extra-safe zones" have also been extracted, which ensure a column's post-peak inter-storey drift capacity of at least 1.0%.

Two recent predictive models have been compared, also with the proposed two-parameter classification criterion, with regard to their ability of predicting simultaneous failure of an R/C member. The one previously developed by the authors performs far better in predicting the post-peak deformability of a member, in general. The classification criterion is still the most reliable, however, in reliably classifying a column as safe from simultaneous shear-axial failure.

Further experimental work pertaining to this phenomenon would be desirable in order to enrich the existing database of axially failing R/C columns and validate or further refine the proposed criteria.

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