



MACRO-MODELLING OF QUASI-BRITTLE CONSTRUCTIONS UNDER LATERAL IN-PLANE LOADS

A. Baratta⁽¹⁾, I. Corbi⁽²⁾, O. Corbi⁽³⁾

⁽¹⁾ Full professor, University of Naples Federico II, alessandro.baratta@unina.it

⁽²⁾ Researcher, University of Naples Federico II, ileana.corbi@unina.it

⁽³⁾ Associate professor, University of Naples Federico II, ottavia.corbi@unina.it

Abstract

In the paper one addresses the problem of the evaluation of contribution of the masonry infill embedded into structural elastic rc or steel frames to the overall lateral resistance of the structure to lateral loads. Actually the infill may contribute to the resistance of the primary structure even significantly ([1]-[7]), and the evaluation of its static operation requires to abandon purely elastic models, which, although they result easier to be handled and implemented into commercial software, may give results very far from the real situation.

The approach proposed in the paper involves the proper modeling of the infill, which is usually made of masonry or brick elements, and it thus requires an ad hoc set up for getting reliable forecasts on its influence on the main structure's behavior. To this aim in the paper, one first synthetically introduces the procedure required by the proper implementation of the problem for infill made of masonry and thus modelled through a non-linear mechanical hypothesis about the constituent material. In order to get a proper modelling of the masonry infill embedded in the contouring structural frame, and then a suitable analysis of the coupled frame/infill system, one considers to model the infill under the No Tension assumption ([8]-[16]), instead of the linear elastic hypothesis usually adopted in current structural analyses. Since this approach requires an ad-hoc numerical implementation in calculus codes which is able to provide more reliable results with respect to available commercial software, one develops and compiles some specific codes able to reproduce the behaviour of the infill and of the complete frame/infill system. The developed calculus codes are more complete, and they also allow to forecast the stress distribution within the infill and the fractures developed in the masonry infill panel.

In the second part of the paper, the results from the numerical investigation are then reported in Sect. 3, for varying horizontal loads. Focus is addressed as well on the effects of the detachment of the infill from the structural frame and the consequent loss of overall lateral resistance of the structure, allowing to further emphasize the influence of the presence of the infill on the overall response of the structure; the case of minimum anchoring between the infill and the structural frame, which is closer to the case when the presence of the infill is completely ignored in calculations, essentially shows a much lower performance of the structure against horizontal loads. Although this result is to be expected, the reported subsequent comparison with results from modelling provided in commercial software under varying horizontal loads allows to figure out how usual analyses that do not make recourse to appropriate modelling of masonry may largely underestimate the action of the infill, and push towards misleading conclusions.

Keywords: 2D Elements, Structural frames, Quasi-Brittle material, Interactions, Numerical implementation

1. Introduction

The seismic performance assessment of existing buildings and constructions is often required to be conceived and tailored on the structure characteristics and behavior, which are exhibited in the static and dynamic environments. Many secondary elements such as infill walls in structural elastic frames are not considered structural elements although they may significantly contribute to the overall resistance of the structure. Their influence is often neglected in structural calculations, with some safety margins but, on the other hand, ignoring an important feature of the real behavior of the construction.

Infill panels made by brick masonry and representing the interior partitions and exterior walls in RC and steel frame buildings, that can be commonly found in many countries of the world as Italy, Greece, Turkey, China, Usa, Mexico, etc., showed in past earthquakes an observed performance ranging from satisfactory to poor, mainly depending on the level of anchoring, sometimes having a beneficial additional effect and a detrimental one in other cases where loss of anchoring or not proper anchoring was encountered (e.g., [6]). Although the infill walls are considered as non-structural elements, at the state they interact with the frame and contribute to the global behaviour of the building, increasing the strength and stiffness of the structure, which can induce diagonal shear failures along the infill surface with the possibility of masonry collapse. A number of studies in literature have been mainly devoted to the execution of experimental tests even on shaking table facilities, also in case of different types of retrofitting (see e.g. [2],[4],[5],[7]), showing different cracking patterns depending on the test layout as in Figure 1 for tests in [7].

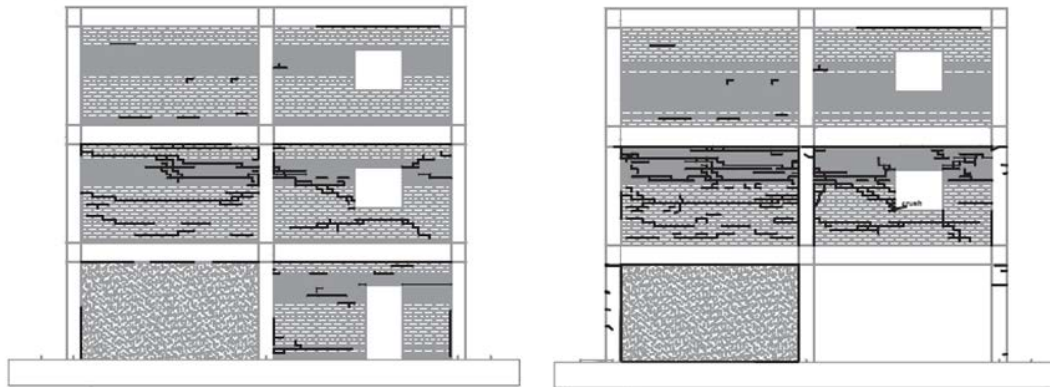


Fig. 1 - Cracking patterns for the 3D-prototype tested on the outdoor shake table at the University of California at San Diego (by Koutromanos et al., 2013 [7]).

A proper evaluation of the contribution to the absorption especially of horizontal loads and to the overall increase of the lateral resistance of the structure is then to be performed for a more realistic forecast. This objective would require the proper modeling of the infill, taking into account the properties of the basic masonry material.

Quasi-brittle existing constructions or structural and non-structural components present in existing structures, which exhibit an heterogeneous resistance in tension and compression, are usually requiring and pushing towards the formulation and adoption of suitable mechanical models able to capture the non-linear nature of their behaviour ([17]-[28]). Current modelling issues for masonry constructions and components are usually relying on mechanical models of the basic materials, where the tensile resistance is almost completely, or completely, neglected (see [12]-[15] for scientific references on the topic by the authors).

In this case, a linear elasticity law is assumed under compression stress states, coupled to a null or low resistance in tension, thus resulting in an overall fragile non-linear behaviour. It has been shown that, in the case at hand, the loading capacity might be properly investigated through a suitable extension and specialization of



limit analysis tools. On the other hand, the study of the intermediate crack situation cannot be performed through limit analysis techniques for masonry structures, while the elastic analysis of the masonry tissue under the assumptions of perfect integrity of the structure and of purely compressive stresses can lead to significant results.

Optimisation (stress or strain) procedures, deriving from the implementation of the fundamental variational methods suitably extended and specialized to such models, can, then, be successfully developed. Since the inner constraint is mainly concerned with the stress or the fracture strain components (according to the used method), a discretization with constant stress/constant strain elements is to be adopted for modelling masonry structures.

In the following the problem of the interactions and coupling of masonry or brick elements exhibiting a non-linear behaviour with linear elastic structural components belonging to the main resisting structure is investigated, also with reference to the evaluation of the overall lateral resistance of mixed elements, with the objective of deepening the incidence of this feature on the structural resistance to horizontal loads.

2. The overall setup

When considering infill made of masonry or bricks embedded in elastic structural frames, like rc or steel frames, which is very common in civil buildings, one may wish to properly model the interactions of the infill with the frame and to give more appropriate forecasts about the behaviour of the infill. Actually the infill may contribute to the resistance of the primary structure even significantly, and the evaluation of its static operation would require to abandon purely elastic models, which, although they result easier to be handled and implementable in commercial software, may give results very far from the real situation.

To this aim in the following one first synthetically introduces the procedure required by the proper implementation of the problem for infill made of masonry and thus modelled through a non-linear mechanical hypothesis about the constituent material; thereafter some results from numerical investigation are reported in Sect. 3. In the last section of the paper, focus is addressed to the effects of the detachment of the infill from the structural frame and the consequent loss of overall lateral resistance of the structure.

In order to get a proper modelling of the masonry infill embedded in the contouring structural frame, and then a suitable analysis of the coupled frame/infill system, let consider to model the infill itself according to the above referred NT assumption, instead of the linear elastic hypothesis usually adopted in current structural analyses. This modelling, as mentioned, requires an ad-hoc implementation of the problem in ad hoc calculus codes, since commercial software are not available to this purpose; as a counterpart, the approach is able to give more reliable results about the real behaviour of the infill and , therefore, of the complete frame/infill system, by properly reproducing the behaviour of the infill masonry. Moreover results are more complete, also allowing to forecast the development of fractures in the masonry within the infill plane.

The treatment of the problem requires to be set up in terms of Total Potential Energy (TPE) approach; this should be fitted to the characteristics and properties of the NT material and continuum as regards to the infill elements of the mesh, which is constrained in the sign of developed stresses and fractures strains, and also at this time the flexural/extensional properties of the contouring frame beams and piles should be taken into account. The kinematic approach is thus referred to, by selecting primary kinematic variables.

Besides the nodal displacement $\mathbf{u}$ vectors in the infill elements of the FE-model, one should also consider the fracture strain possibly arising $\boldsymbol{\varepsilon}_f$ in the NT elements, which results in the introduction of some constraints related to the fracture strain tensor, that is required to be positive semi-definite. The energetic functional should in this case embed the two contributions to the total potential energy deriving from the flexural/extensional structural components of the frame on one side, and from the inner plane infill on the other side. The search of the minimum of the energetic functional finally allows to identify the solution, together with the current stress and fracture distribution at any point of the frame and of the infill.

The problem is then set, for the coupled system, by introducing the jointed functional in the form



$$\mathcal{E}(\mathbf{u}, \boldsymbol{\varepsilon}_f) = \frac{1}{2} \int_{\Omega} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_f)^T \mathbf{C} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_f) dV + \frac{1}{2} \int_{\Sigma} (EI \mathbf{u}^T \mathbf{B}^T \mathbf{B} \mathbf{u} + EA \mathbf{u}^T \mathbf{D}^T \mathbf{D} \mathbf{u}) ds - \frac{1}{2} \int_{\Omega} \mathbf{F}^T \mathbf{u} dV - \frac{1}{2} \int_{\Sigma} \mathbf{p}^T \mathbf{u} ds \quad (1)$$

where $\mathbf{C}$ denotes the compliance NT tensor for the infill, $\mathbf{B} \mathbf{u}$ denotes the curvature and $\mathbf{D} \mathbf{u}$ the elongation relevant to the linear beam/pile elements of the frame with $\mathbf{B}$, $\mathbf{D}$ suitable matrixes, EI the flexural stiffness and EA the extensional stiffness of the frame linear elements; Ω denotes the infill body whose contour Σ is superposed to the linear beam/pile elements of the frame.

Thereafter one sets up the constrained optimization problem as follows

$$\begin{aligned} \text{Find } \min_{\mathbf{u}, \boldsymbol{\varepsilon}_f} \mathcal{E}(\mathbf{u}, \boldsymbol{\varepsilon}_f) = \min_{\mathbf{u}, \boldsymbol{\varepsilon}_f} & \left\{ \frac{1}{2} \int_{\Omega} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_f)^T \mathbf{C} (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_f) dV + \right. \\ & \left. + \frac{1}{2} \int_{\Sigma} (EI \mathbf{u}^T \mathbf{B}^T \mathbf{B} \mathbf{u} + EA \mathbf{u}^T \mathbf{D}^T \mathbf{D} \mathbf{u}) ds - \frac{1}{2} \int_{\Omega} \mathbf{F}^T \mathbf{u} dV - \frac{1}{2} \int_{\Sigma} \mathbf{p}^T \mathbf{u} ds \right\} \quad (2) \\ \text{Sub } & \begin{cases} J_{1f} \geq 0 \\ J_{2f} \geq 0 \end{cases} \quad \text{in } \Omega \end{aligned}$$

where J_{1f} and J_{2f} represent the first and second fracture strain invariants respectively to be not negative in the infill volume.

3. Numerical investigation

The numerical investigation is developed on a structural frame embedding a secondary brick panel, with dimensions 5.0×3.0 m.

The structural frame is characterized by extensional stiffness of the piles $EA = 0.3 \times 10^9$ kg, and of the beams $EA = 0.45 \times 10^9$ kg and flexural stiffness of the piles $EI = 0.4 \times 10^{11}$ kg·cm², and of the beams $EI = 0.135 \times 10^{12}$ kg·cm², while the infill is characterized by compressive Young's modulus $E_{NT} = 0.5 \times 10^5$ kg·cm⁻² and Poisson's coefficient $\nu_{NT} = 0.1$. Acting forces are due to the self-weight and to a varying pushing horizontal load applied at the left top side of the frame.

One first considers results under an horizontal load of 3000 kg, when the anchoring of the infill panel is complete, which means that the infill is attached to the structural frame at any point of its perimeter. In Figures 2-4 the results are reported concerning the case when the wall is then considered to be perfectly connected to the structural frame at any point on the contact lateral surfaces along its thickness. Thereafter the detached case when only some joints around the corners of the infill panel are kept connected with the structural frame, and some loss in the anchoring has occurred, is considered. In this case the results are reported in Figures 5-6.

In Figure 2 one depicts the model referred to for the mesh adopted in the numerical investigation with the layout of active loads. The deformed configuration of the combined structural component for the first case of undamaged anchoring is reported in Figure 3, where an amplification factor of 500 is adopted in order to

emphasize the induced displacements. The compressive stress distribution in the infill is then depicted in Figure 4.

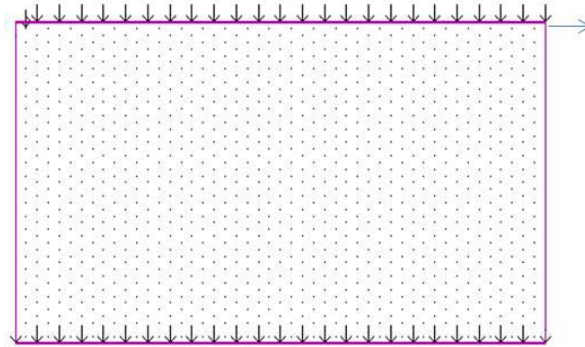


Fig. 2 – The elastic frame with masonry infill, with the applied load pattern.

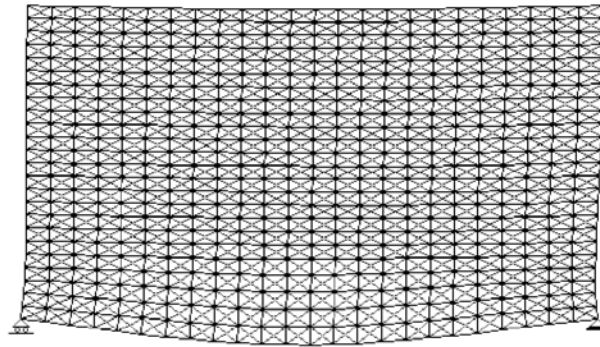


Fig. 3 – The deformed infill under applied loads.

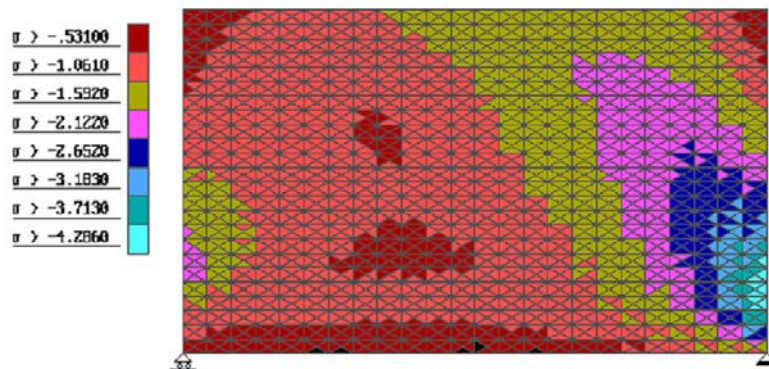


Fig. 4 – The compressive stress distribution in the infill.

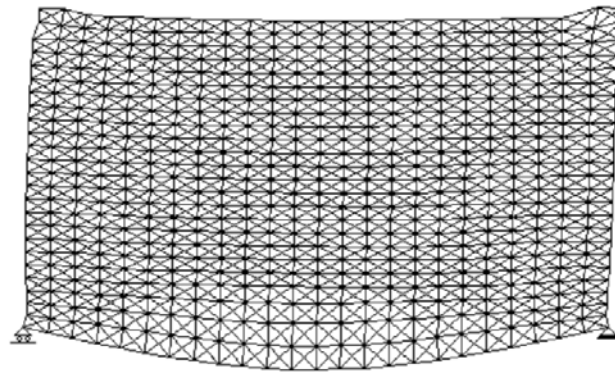


Fig. 5 – The deformed infill under applied loads.

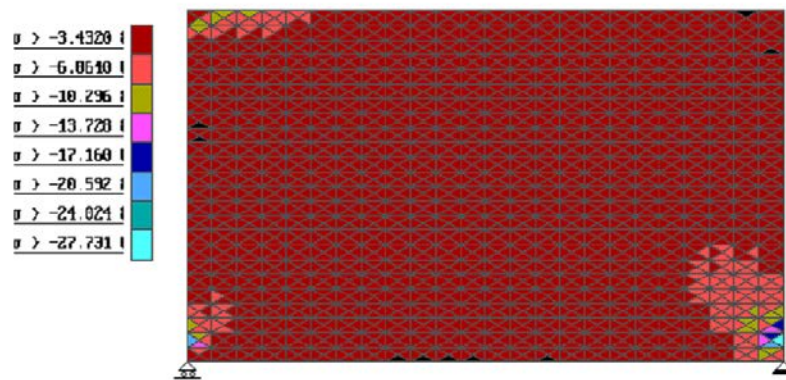


Fig. 6 – The compressive stress distribution in the infill.

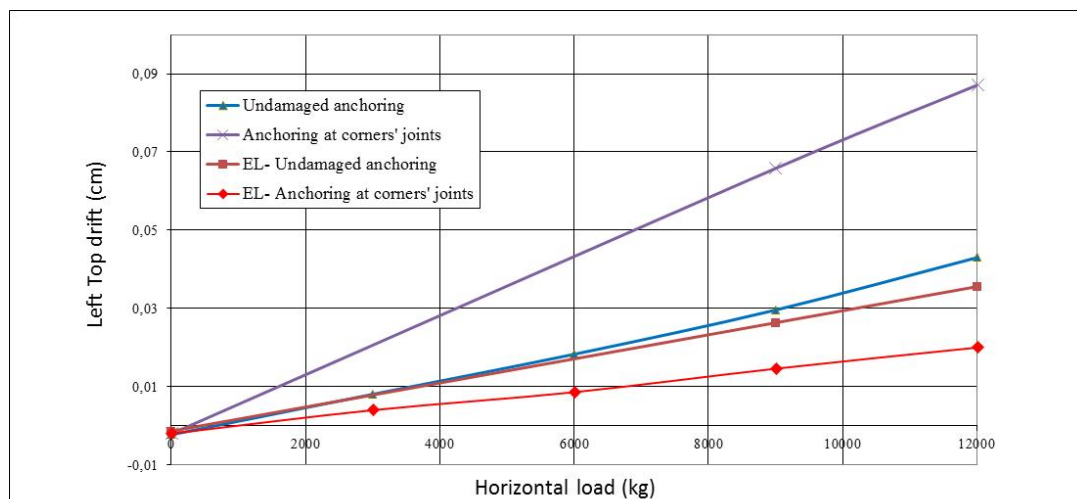




Fig. 7 – The top drifts for the frame with undamaged anchoring and anchoring limited at joints around the frame corners.

The results concerning the loss of anchoring, which remains limited to the joints around the frame corners, are illustrated in Figures 5-6, where the deformed configuration and the distribution of the compressive stresses are reported respectively.

Comparison between the undamaged anchoring case and the case with significant detachment of the infill wall from the structural frame as regards the attained stress intensities and the deformed configurations may be inferred by the direct comparison of Figures 3-4 with 5-6 respectively.

As regards the deformed patterns, the observation of Figures 3 and 5 allows to notice that a quite more significant deformation of the frame occurs in case of the damaged anchoring. Attained compressive stresses are much larger in the average in the detached infill, but, also as concerns maximum achieved intensities, these are higher in the case of loss of anchoring, as clear by looking at Figure 4 and 6, whilst of course tensile stresses remain practically null at any point of the infill for both cases.

In Figure 7 one may observe the decrease in the lateral resistance of the coupled frame/infill system for varying values of the horizontal load, caused by the detachment of the infill from the structural frame. The loss of anchoring in this case under a lateral force of 10000 kg results approximately equal to 50%, demonstrating the cooperation of the infill to the structural function.

From the same graph one can also look at results obtained under the elastic assumption for the undamaged anchoring and the almost completely detached infill; whilst the entity of the decrease of lateral resistance is kept almost the same, still approximately of 50% after the infill detachment, the forecasts about the attained drifts are quite far from those obtained by referring to more accurate modelling, which thus confirms to be preferable.

4. Conclusions

Modeling issues about masonry analysis are shown to be of fundamental importance in order to provide adequate forecasting. In the paper one focuses on the problem of proper modelling and quasi-static analysis of a coupled system consisting of a 2D panel with inner unilateral constraint and a contouring frame; this situation may find application in a number of practical cases, such as, for example, in masonry or brick infill embedded in rc or steel structural frames. Although structural investigations limited to the properly structural organisms are usually faster, require a lower computational effort, and act with some safety advantage, the proper evaluation of the contribution of infill to the structural function is very useful, especially as regards to horizontal loads, in order to provide some more realistic forecasts about the behaviour of the entire building. Nevertheless, an appropriate modelling of the non- structural elements represents a fundamental issue.

In the paper the proper treatment of the complete problem embedding the NT infill panel is set up in energetic terms, suitably specialized to the problem under exam. The approach is then fitted to the characteristics and properties of the NT material and continuum as regards to the infill elements of the mesh, which is constrained in the sign of developed stresses and fractures strains; moreover the flexural/extensional properties of the contouring frame beams and piles should be embedded in the problem through the proper definition of the energetic objective functional. Major contribution of the infill, when properly modelled, is finally emphasized.

5. Acknowledgements

The present research has been supported by the Dept of Civil Protection of the Italian Government through the ReLuis Pool (convention signed 27/12/2013).



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