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SAFETY ASSESSMENT OF QUASI-BRITTLE CRACK STRUCTURES BY SIMILARITY

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Abstract

In the paper one addresses problems concerning the analysis in the static and dynamic ranges of structures made by masonry material. The analysis of quasi-brittle constructions is usually requiring an appropriate formal set up and analytical treatment [1]-[10]. About this topic a number of studies on the statics and dynamics of structural systems made of materials exhibiting quite poor, null in the limit, tensile strength, have been extensively developed in literature [11]-[24].

The further assumption, that considerably simplifies the solution of the problem, based on some agreement with current observation, is that the compressed material exhibits and indefinitely linear elastic behaviour, apart from local problems that are not included in a first level approach.

These assumptions lead to rather reliable modelling for safety assessments and succeeds in giving reason of the main features of the structural behaviour of masonry buildings, finally pushing towards the No-Tension fundamental hypothesis. It follows that this, relatively simple, mechanical model yields a firm reference point for technical well-founded assessment, as well as the current reinforced-concrete theory does.

In the paper one thus refers to a masonry material modelled through such hypothesis in order to study existing buildings and objects, based on their geometrical similarity as regards to their own safety and dynamic response. Some developments are reported allowing to draw some conclusions about the correlation in the response of similar masonry solids with different size, under the action of specified, possibly time-varying, loads.

The treatment is intentionally confined to the case in which the original solid and the model are geometrically affine, since structural systems are most often very complex, and more elaborated transformations may be rather hard to be realised.

The main focus is on the circumstance that, although some preliminary result could be drawn from simple dimensional analysis, a specialised equation approach is to be preferred, as developed in the paper, since it yields much more details allowing to identify possible strategies to be investigated in order to improve the similitude between the models.

Keywords: Brittle behaviour, Crack, Dimensions, Models, Statics, Dynamics,

1. Introduction

The recourse in studies on structural design to similarity and proportionality dates back to the first conception of ancient constructions, when formulae deducted on the basis of the similitude rule where usually referred to and the geometrical shape was kept independently of the object size. Starting from the former Galilei's study about the effects of changes of size and the square-cube law, to the formalization of the *Dimensional Analysis*, where, based on the *principle of similitude*, laws of dimensional analysis are applied in order to compare geometrically similar systems, this approach has proved very useful in structural design.

In most cases, the structural analysis of quasi-brittle constructions is properly approached through a suitable theoretical set up. About this topic a wide literature has been extensively developed on the statics and dynamics of structural systems made of materials exhibiting rather poor, null in the limit, tensile strength, modeling masonry. Even as an idealization, the NT (No-Tension) model offers a quite reliable representation of the real behaviour of materials exhibiting light tensile resistance such as masonries or soils in general ([1]-[10] and [19]-[23]).

Analysis of NT continua proves that the stress, strain and displacement fields obey extremum principles of the fundamental energy functionals.

The solution of both problems can be numerically pursued by means of Operational Research methods (see i.e. [24]-[26]) suitably operating a discretization of the analysed NT continuum.

The theorems of Limit Analysis can be specialized to NT continua after defining the classes of kinematically sufficient mechanisms and statically admissible stress fields.

Few studies are, on the contrary, available when referring to structures with different sizes assumed to behave according to the NT assumption, and the influence of the geometrical dimensions on their behavior, loading capacity and strength.

This represents a quite primary feature in the analysis of masonry structures, also considering that monumental and historical constructions were usually conceived, as well know, according to a number of art rules about the geometrical proportions between the single structural components, codified during centuries in order to ensure the overall stability.

In this context, the importance of the geometrical dimensions in analyses conducted on masonry constructions has been confirmed by some researches. Direct proportionality may be demonstrated to not hold for all the variables when dealing with masonry constructions of different sizes, where collapse essentially depends on the geometry of the system. This pushes to emphasize the need of taking into account the geometrical change due to deformation of the structure when developing some considerations about their response to solicitations. One fundamental issue is represented by the circumstance that, if in a masonry construction with contained dimensions a deformation situation of some importance appears, it may be nevertheless still neglected because of the dimensions of the structure. The same phenomenon may on the contrary assume a very critical character when transposed on a structure with large dimensions, even leading to its failure and to the overall collapse of the structure. The dimensional effect should thus be carefully accounted for in current analyses, avoiding the methodic application of the theoretical formulations that ignore in course deformation phenomena.

2. Problem Setup

Correlation in the dynamic behaviour of similar objects made by NT materials may be investigated with reference to a rheological model in which dissipation occurs as the result of viscosity. It is assumed that the strain $\boldsymbol{\varepsilon}$ is the sum of the *linearly elastic* strain $\boldsymbol{\varepsilon}_e$ and the inelastic component $\boldsymbol{\varepsilon}_f$, the *fracture* strain.

The stress $\boldsymbol{\sigma}$ is the resultant of the *elastic* and the *viscous* components, $\boldsymbol{\sigma}_c$ and $\boldsymbol{\sigma}_v$, that are respectively proportional to the elastic strain $\boldsymbol{\varepsilon}_e$ and the elastic strain rate, denoted by the superimposed dot. If $\mathbf{C}$ and $\mathbf{V}$ are the tensors of elastic and viscous constants, with $\mathbf{C} = \mathbf{D}^{-1}$, one has

$$\begin{aligned} &\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_f \\ &\boldsymbol{\sigma}_c = \mathbf{C}\boldsymbol{\varepsilon}_e \\ &\boldsymbol{\sigma}_v = \mathbf{V}\dot{\boldsymbol{\varepsilon}}_e \\ &\boldsymbol{\sigma}_c = \boldsymbol{\sigma}_c + \boldsymbol{\sigma}_v \end{aligned} \tag{1}$$

By the NT assumption, $\boldsymbol{\sigma}_c$ and $\boldsymbol{\varepsilon}_f$ are defined in sign and the energy related to fractures is null

$$\begin{aligned} &\boldsymbol{\varepsilon}_f \geq \mathbf{0} \\ &\boldsymbol{\sigma}_c \leq \mathbf{0} \\ &\boldsymbol{\sigma}_c \cdot \boldsymbol{\varepsilon}_f = 0 \\ &\boldsymbol{\sigma}_c \cdot \dot{\boldsymbol{\varepsilon}}_f = 0 \\ &\boldsymbol{\sigma}_v \cdot \boldsymbol{\varepsilon}_f = 0 \\ &\boldsymbol{\sigma}_v \cdot \dot{\boldsymbol{\varepsilon}}_f = 0 \end{aligned} \tag{2}$$

It follows that the material obeys the Drucker's postulate, and therefore that all results of classical perfect elasto-plasticity with associate flow laws hold, including Limit Analysis. On the other side, dissipation is null, and the material is also (non-linearly) elastic.

Let now consider two continuous bodies with different size depending on a factor $\lambda > 0$, i.e. the body $\mathsf{B} = \{\mathbf{x} \in \mathrm{R}^3 : \mathrm{f}(\mathbf{x}) \leq 0\}$ with boundary Ω in Figure 1, and the body $\mathcal{B}^\lambda$, since $\mathbf{x} \in \mathsf{B} \Leftrightarrow \lambda\mathbf{x} \in \mathsf{B}^\lambda$, one has

$$\begin{aligned} &\mathrm{f}(\mathbf{x}) \leq 0 \Leftrightarrow \mathrm{f}(\lambda\mathbf{x}/\lambda) \leq 0 \\ &\Leftrightarrow \mathrm{f}(\mathbf{x}^\lambda/\lambda) \leq 0 \Leftrightarrow \mathrm{f}^\lambda(\mathbf{x}^\lambda) \leq 0 \end{aligned} \tag{3}$$

The *correspondence between two points* P and P^λ in the two bodies respectively is then established in Eq. (3).

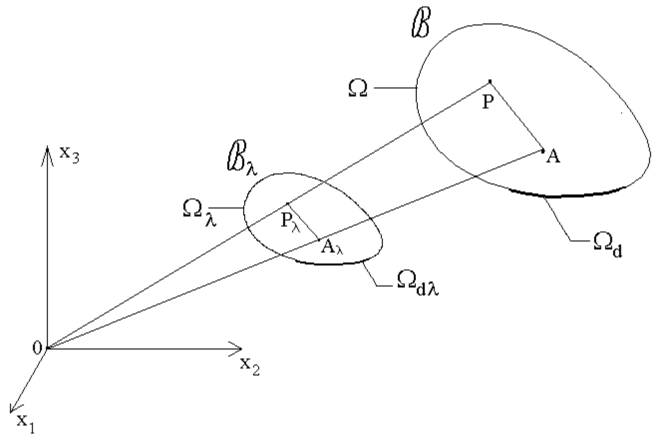

Fig. 1 – The two bodies $\mathcal{B}$ and $\mathcal{B}^\lambda$.

Let Ω_d be the kinematically constrained part of the surface contour Ω of the body $\mathcal{B}$ with larger dimension, and the remaining $\Omega_p = \Omega - \Omega_d$ its contour area acted on by surface loads.

After some developments, one may show that directions and dimensional ratios are kept unaltered in $\mathcal{B}$ and $\mathcal{B}^\lambda$. If the bodies are made of the same material, which is indefinitely elastic in compression and not resisting tension, the material constitutive relationships between stresses $\boldsymbol{\sigma}$ and strains $\boldsymbol{\varepsilon}$, in $\mathcal{B}$ are given as follows

$$\begin{aligned} \boldsymbol{\sigma} &= \mathbf{C}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_f) \\ \boldsymbol{\varepsilon} &= \mathbf{D}\boldsymbol{\sigma} + \boldsymbol{\varepsilon}_f \end{aligned} \tag{4}$$

with $\boldsymbol{\varepsilon}_f$ the fracture strain tensor, $\mathbf{C}$ the elastic compliance tensor, and $\mathbf{D} = \mathbf{C}^{-1}$

Whilst, as regards to $\mathcal{B}^\lambda$, one has

$$\begin{aligned} \sigma^\lambda &= \mathbf{C}^\lambda(\varepsilon^\lambda - \varepsilon_f^\lambda) \\ \varepsilon^\lambda &= \mathbf{D}^\lambda \sigma^\lambda + \varepsilon_f^\lambda \end{aligned} \tag{5}$$

Since the material in the two bodies is kept the same one has

$$\begin{aligned} \mathbf{C}^\lambda(\mathbf{x}^\lambda) &= \mathbf{C}(\mathbf{x}^\lambda/\lambda) = \mathbf{C}(\mathbf{x}) \\ \mathbf{D}^\lambda(\mathbf{x}^\lambda) &= \mathbf{D}(\mathbf{x}^\lambda/\lambda) = \mathbf{D}(\mathbf{x}) \end{aligned} \tag{6}$$

As regards to forces, one may consider that the distributions of the mass forces in the volumes of the two bodies is governed by the same rule; then the force acting on the volume unit is the same in the two bodies

$$\mathbf{F}^\lambda(\mathbf{x}^\lambda) = \mathbf{F}(\mathbf{x}^\lambda/\lambda) = \mathbf{F}(\mathbf{x}) \tag{7}$$

The distribution of the surface loads on the free parts of the contours of the two bodies follows the λ-ratio, and one has as regards to the load per surface unit

$$\mathbf{p}^{\lambda}\left(\mathbf{x}^{\lambda}\right)=\lambda\mathbf{p}\left(\mathbf{x}^{\lambda}/\lambda\right)=\lambda\mathbf{p}(\mathbf{x}) \tag{8}$$

Under these assumptions, the relations governing the elastic equilibrium problem in the two cases hold as follows as regards stresses

$$\boldsymbol{\sigma}^{\lambda}\left(\mathbf{x}^{\lambda}\right)=\lambda\boldsymbol{\sigma}\left(\mathbf{x}^{\lambda}/\lambda\right)=\lambda\boldsymbol{\sigma}(\mathbf{x}) \tag{9}$$

With reference to strains one has

$$\boldsymbol{\varepsilon}^{\lambda}\left(\mathbf{x}^{\lambda}\right)=\lambda\boldsymbol{\varepsilon}\left(\mathbf{x}^{\lambda}/\lambda\right)=\lambda\boldsymbol{\varepsilon}(\mathbf{x}) \tag{10}$$

Whilst the following relations hold for displacements

$$\mathbf{u}^{\lambda}\left(\mathbf{x}^{\lambda}\right)=\lambda^2\mathbf{u}\left(\mathbf{x}^{\lambda}/\lambda\right)=\lambda^2\mathbf{u}(\mathbf{x}) \tag{11}$$

The latter of Eqs (11) concerning the relation between displacements in the two bodies, shows how the two differently sized NT structures are characterized by displacements that do not vary in a proportional way but according to a quadratic rule, and are increased by the factor λ^2 when passing to the larger structure.

Then it is possible to assess that the solution [$\mathbf{u}^{\lambda}(\mathbf{x}^{\lambda})$, $\boldsymbol{\varepsilon}_f^{\lambda}(\mathbf{x}^{\lambda})$, $\boldsymbol{\sigma}^{\lambda}(\mathbf{x}^{\lambda})$] of the equilibrium problem in the body $\mathcal{B}^{\lambda}$ is related to the solution [$\mathbf{u}(\mathbf{x})$, $\boldsymbol{\varepsilon}(\mathbf{x})$, $\boldsymbol{\sigma}(\mathbf{x})$] in the original body $\mathcal{B}$ through the relationships in Eqs (9)-(11), which means that *full similitude* is to be expected from tests on NT models.

With reference to the dynamic equilibrium problem of the two bodies made by NT-elastic material, with given boundary and initial conditions, when considering the applied loads they are considered to composed by the permanent mass and surface loads $\mathbf{f}_o$ ($\mathbf{x}$), $\mathbf{p}_o$ ($\mathbf{x}$) and the time-varying actions $\mathbf{f}_t$ ($\mathbf{x}$,t), $\mathbf{p}_t$ ($\mathbf{x}$,t). Also in this case, it is assumed that the body $\mathcal{B}^{\lambda}$ body is made by the same material as the original body, and that $\mathbf{C}^{\lambda}(\mathbf{x}^{\lambda}) = \mathbf{C}(\mathbf{x}^{\lambda}/\lambda)$ whilst $\mathbf{V}^{\lambda}(\mathbf{x}^{\lambda}) = \lambda\mathbf{V}(\mathbf{x}^{\lambda}/\lambda)$ with $\lambda \geq 0$. Permanent body forces are the same both for the two bodies, and permanent surface loads are reduced by λ; as regards to variable loads, in the second body the body forces are contracted in the time scale and corrected by the superposition of an additional term proportional to the divergence of the viscous stress, whilst surface loads are reduced by the scale factor λ, are contracted in the time scale and are corrected by a term proportional to the surface dissipative stress vector.

Under these assumptions, one can assess that the solution $\left[\mathbf{u}^{\lambda}\left(\mathbf{x}^{\lambda},t^{\lambda}\right),\boldsymbol{\varepsilon}_f^{\lambda}\left(\mathbf{x}^{\lambda},t^{\lambda}\right),\boldsymbol{\sigma}^{\lambda}\left(\mathbf{x}^{\lambda},t^{\lambda}\right)\right]$ of the dynamic problem is related to the solution $\left[\mathbf{u}(\mathbf{x},t),\boldsymbol{\varepsilon}_f(\mathbf{x},t),\boldsymbol{\sigma}(\mathbf{x},t)\right]$ in the original body , as regards to stresses, by the relationships

$$\boldsymbol{\sigma}^{\lambda}\left(\mathbf{x}^{\lambda},t^{\lambda}\right)=\boldsymbol{\sigma}_c\left(\mathbf{x}^{\lambda},t^{\lambda}\right)+\boldsymbol{\sigma}_v\left(\mathbf{x}^{\lambda},t^{\lambda}\right)=\lambda\boldsymbol{\sigma}_c(\mathbf{x},t)+\beta\boldsymbol{\sigma}_v(\mathbf{x},t) \tag{12}$$

with $t^{\lambda} = \lambda t$.

With reference to strains one has

$$\boldsymbol{\varepsilon}_f^{\lambda}\left(\mathbf{x}^{\lambda},t^{\lambda}\right)=\lambda\boldsymbol{\varepsilon}_f\left(\mathbf{x}^{\lambda}/\lambda,t^{\lambda}/\lambda\right)=\lambda\boldsymbol{\varepsilon}_f(\mathbf{x},t) \tag{13}$$

Whilst the following relations hold for displacements

$$\mathbf{u}^{\lambda}\left(\mathbf{x}^{\lambda},t^{\lambda}\right)=\lambda^2\mathbf{u}\left(\mathbf{x}^{\lambda}/\lambda,t^{\lambda}/\lambda\right)=\lambda^2\mathbf{u}(\mathbf{x},t) \tag{14}$$

Finally, one can assess that, under the given conditions about the loads and imposed displacements, the response of the two models are related by the above reported relationships.

3. Conclusions

In the paper one refers to masonry constructions and addresses the problem of their safety assessment based on studies about the dimensional similarity between objects. This issue appears of fundamental importance when dealing with structural models for providing reliable and adequate forecasting of their behaviour in the dynamic and static ranges. In particular, one presents an analytical setup based on the assumption of No-Tension material of the masonry structure, and reports some developments that allow to draw some conclusions about the correlation in the response of similar masonry solids with different size, under the action of specified, possibly time-varying, loads.

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