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SHEAR CAPACITY EVALUATION OF RC FOOTING BEAMS WITH LARGE WEB-OPENING FOR MAINTENANCE BASED ON STRESS TRANSFER MECHANISM

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Abstract

If the openings are made on the web of footing beams for the maintenance purpose, the shear capacity is reduced. The shear capacity design formula is absolutely needed for the seismic design. AIJ code [1] includes the evaluation of shear capacity around the web-opening. However, as it limits the opening's diameter to less than 1/3 of the beam depth, the beam depth is sometimes determined by this limitation. Unnecessary large beam depth is not economical. Recently the footing beams with large web-opening of which diameter is more than 1/3 of the beam depth are experimentally discussed [2]-[7] and adopted in the structural design with aid of AIJ code [1]. The problems are that AIJ design formula [1] is an empirical equation and that it does not give the constant safety margin to the actual shear capacity.

In this study, the shear force transfer mechanism around the web-opening was experimentally discussed, and the shear force transfer model was proposed. The diagonal bars form a truss structure and carry a part of shear force independently. The truss and arch mechanism of two stage beams upper and lower side of the opening carry the other part of shear force. The stirrups on the side of the opening balance with the strut of the arch mechanism. The shear capacity was successfully evaluated with the constant safety margin based on the proposed shear force transfer model.

Keywords: RC beams with web-opening; Shear capacity; Stress transfer mechanism; Strut and tie model

1. Introduction

The large openings are made on the web of RC beams connecting pile caps or spread footings (Hereinafter referred to as the "footing beams") for checking the facility piping or beam conditions. Because the existence of web-opening reduces the shear capacity, the precise evaluation of shear capacity is absolutely needed for the seismic design. AIJ design code [1] includes a shear capacity design formula for the beams with web-opening. However it limits the opening's diameter to less than 1/3 of the beam depth to secure the enough shear capacity. Because of this limitation, the beam depth is sometimes determined by the diameter of opening. Unnecessary large beam depth is not economical. Then, recently the footing beams with large web-opening of which diameter is more than 1/3 of beam depth are experimentally discussed [2]-[7] and adopted in the structural design.

At least 600mm of the opening in diameter is required from the inspector's body size. In this case, no stirrups can be placed in the region of the web-opening. In order to secure the shear capacity around the web-opening, the reinforcing arrangement as shown in Fig.1 is conventionally provided.

1) The horizontal bars are placed upper and lower side of the opening. The longitudinal main bars of the original beam and these horizontal bars are surrounded by stirrups, and two stage beams are constituted (Hereinafter referred to as "sub-beams"). It looks like a Vierendeel girder.
2) The stirrups, which cannot be placed in the region of opening, are intensively placed at the both right and left side of the opening.
3) The diagonal bars are placed around the opening.

AIJ design formula [1] is an empirical equation. It normally gives safety side evaluation even if it is applied to the beams with large web-opening of which diameter is more than 1/3 of the beam depth. The problem is that it does not give the constant safety margin to the actual shear capacity. For example as shown in Fig.12 (b), the plots of the calculated values by AIJ design formula [1] vs. the test result in this study are randomly distributing.

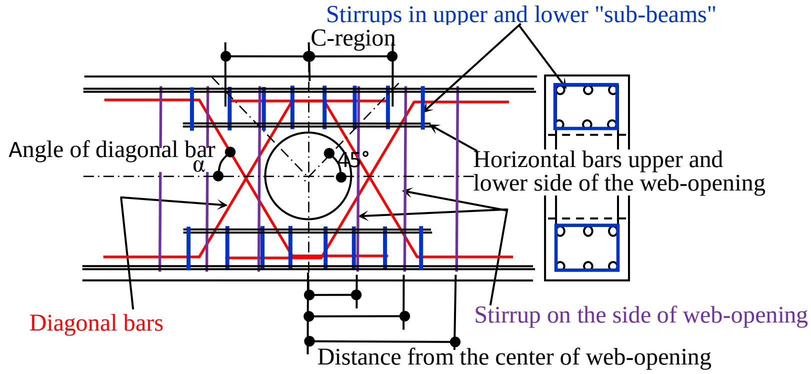


Fig.1 Reinforcement arount the web-opening

The objective of this study is to propose the theoretical shear capacity evaluation method based on the shear force transfer mechanism around the web-opening to rationalize the seismic design.

2. Loading tests of footing beams with large web-opening

Sixteen 1/3 scaled specimens in Table 1 were tested. The beam section is 500mm in depth (D) and 180mm in width (b). The span length is 1200mm. One web-opening with 200mm in diameter (H) was made at mid-span. The conventional reinforcement was provided around the web-opening. The stirrups are the closed type with 135° hook, and they were placed at the position shown in Table 1. The detail of the reinforcement is as follow considering their role for the contribution to the shear capacity.

1) The horizontal bars to constitute "sub-beams" upper and lower side of the opening were extended from the opening edge to the 20 times the rebar diameter in order to ensure the anchor length.
2) The stirrups in "sub-beams" were placed up to the horizontal portion of the diagonal reinforcement.
3) The diagonal reinforcement has the angle of α against the horizontal direction, and the length of the horizontal portion was 25 times the rebar diameter in order to ensure the anchor length.

An example of reinforcing arrangement (No.2) is shown in Fig.2.

In Series I, the amount of the diagonals bars and the stirrups on the side of the opening was decided so that the calculated shear capacity by AIJ design formula [1] is approximately constant. The shear reinforcement ratio in "sub-beams" of No.1-No.4 is same as that of the original beam. The shear reinforcement ratio in "sub-beams" of No.5 is twice as that of No.3 expecting the increase of the shear capacity.

Table 1 Test specimens

Series	Specimen	Stirrups: Distance from the center of the web-opening (mm), C=215mm										Diagonal bars		Stirrups in sub-beams	Reinforcement ratio by AIJ design formula			Concrete strength N/mm²
		120	134 (130)	168 (160)	200	225	240	320	400	480	560				p_{sv}(%)	p_{sd}(%)	p_s(%)	
I	No.01						□D6	□D6	□D6	□D6	□D6	60°	3-D10	D6@80	0	0.752	0.752	21
	No.02	□D6					□D6	□D6	□D6	□D6	□D6	60°	2-D10	D6@80	0.165	0.501	0.666	23
	No.03	□D6	□D6		□D6		□D6	□D6	□D6	□D6	□D6	60°	1-D10	D6@80	0.496	0.251	0.747	25
	No.04	□D6	□D6	□D6	□D6		□D6	□D6	□D6	□D6	□D6	60°	—	D6@80	0.661	0	0.661	25
	No.05	□D6	□D6		□D6		□D6	□D6	□D6	□D6	□D6	60°	1-D10	D6@40	0.496	0.251	0.747	25
II	No.06						□D6	□D6	□D6	□D6	□D6	60°	—	D6@80	0	0	0	20
	No.07						□D6	□D6	□D6	□D6	□D6	60°	3-D10	—	0	0.752	0.752	20
	No.08	□D6	□D6	□D6	□D6		□D6	□D6	□D6	□D6	□D6	60°	—	—	0.661	0	0.661	20
III	No.09		(□D10)					□D6	□D6	□D6	□D6	45°	1-D10	D10@50	0.367	0.259	0.626	26
	No.10					□D10		□D6	□D6	□D6	□D6	45°	1-D10	D10@50	0	0.259	0.259	27
	No.11							□D10	□D6	□D6	□D6	45°	1-D10	D10@50	0	0.259	0.259	25
	No.12	□D6						□D6	□D6	□D6	□D6	45°	2-D10	D10@50	0.165	0.519	0.684	25
IV	No.13						□D6	□D6	□D6	□D6	□D6	60°	1-D10	D6@80	0	0.251	0.251	20
	No.14						□D6	□D6	□D6	□D6	□D6	60°	2-D10	D6@80	0	0.501	0.501	20
	No.15							□D6 □D6	□D6	□D6	□D6	60°	1-D10	D6@80	0	0.251	0.251	21
	No.16			(□D10)				□D6	□D6	□D6	□D6	60°	1-D6	D6@80	0.165	0.113	0.278	22

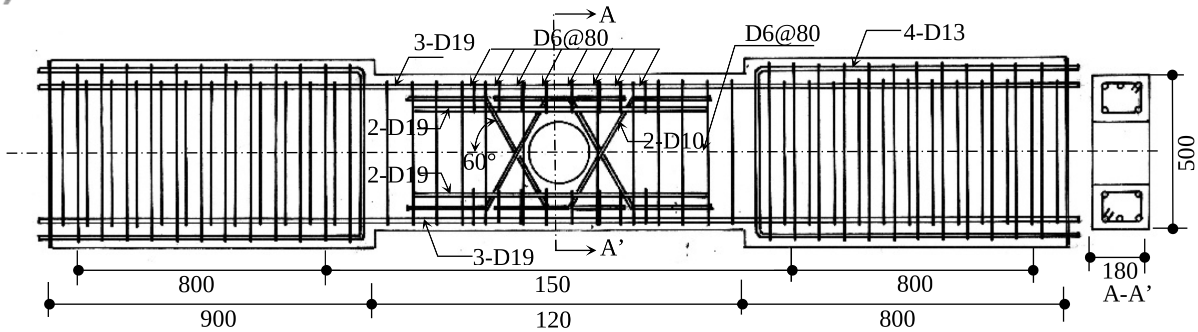


Fig.2 An example of reinforcing arrangement (e.g. No.2 specimen)

In Series II, assuming the shear force transfer by truss action of "sub-beams", by the stirrups in C-region of Fig.1 [1] and by the diagonal bars, one of them was considered in each specimen. The stirrups in C-region and the diagonal reinforcement were removed in No.6, and just "sub-beams with stirrups" were constituted. In No.7 and No.8, the truss action in "sub-beams" was eliminated by removing the bond of the horizontal bars in mid 560mm portion and the stirrups in "sub-beam". In No.7, no stirrup in C-region was provided and just the diagonal bars were placed. In No.8, no diagonal bar was placed and just the stirrups in C-region were placed.

In Series III, much stirrup (□D10@50) was provided in "sub-beams". If the concrete would reach the failure criteria by only the truss action of "sub-beams", the stirrups in C-region of Fig.1 [1] would no longer have contribution to the shear capacity. The position of the stirrup on the side of the opening was adopted as a parameter expecting no influence to the shear capacity. In No.9-No.11, one stirrup 1-□D10 was placed at the position of 130mm, 225mm and 320mm from the center of the web-opening. In No.12, the stirrup on the side of the opening of No.9 was replaced by 1-□D6 and the diagonal bar was increased to 2-D10, so the reinforcement ratio p_s (%) of AIJ design formula [1] is almost same as that of No.9.

Series IV was planned to know that the stirrups beyond C-region of Fig.1 [1] would be involved in the shear force transfer mechanism around the web-opening. In No.13-No.15, the stirrups in C-region were removed. In No.16, the diagonal bar of No.13 was replaced by 1-D6 and one stirrup 1-□D6 in C-region was added, so the reinforcement ratio p_s (%) of AIJ design formula [1] is almost same as that of No.13.

The compressive strength of concrete in each specimen is shown in Table 1. The yield strength of the used re-bars is listed in Table 2. The test setup is shown in Fig.3. The shear force was given by four points loading to create the double curvature moment conditions. Two loading cycles were given at each loading stage increasing the applied load. On the halfway, the loading was switched to the deformation control.

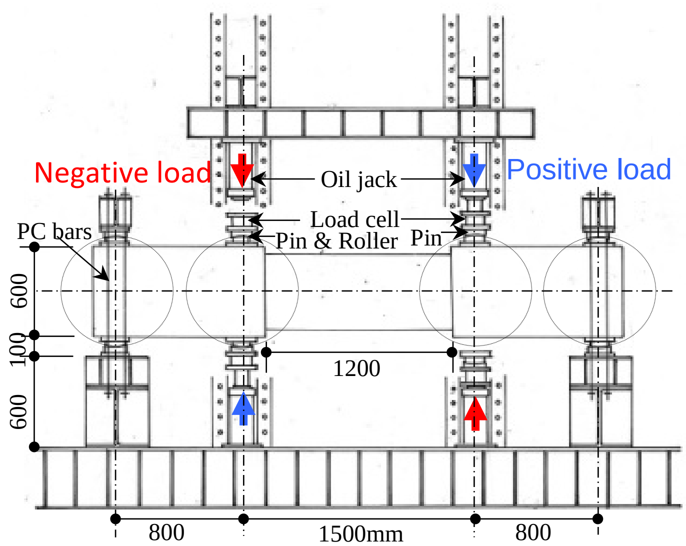


Fig. 3 Test setup

Table 2 Yield strength of the used re-bars (N/mm^2)

Series	D19 for main bars	D10	D6
I	512	379	371
II	512	379	371
III	413	337	387
IV	413	337	387

3. Role of the diagonal reinforcement

The relation between shear force (V) and the rotation angle of member (R) of No.1-No.16 are shown in Fig.5. The ultimate failure mode is also shown in Fig.5. The filled triangular marks (▼) show the points where the diagonal bars reached the yield strain. Except for No.7, the diagonal bars yielded before reaching shear capacity. No.7 has the same amount of diagonal reinforcement as No.1. However the the diagonal bars of No.7 did not show the yield strain. It should be noted that No.7 has no stirrups in C-region of Fig.1 [1] and no stirrup in "sub-beams", and the shear failure occurred earlier at small rotation angle of member. Though the diagonal reinforcement is supposed to transfer the shear force independently, some magunitude of rotation angle of member should occur when the diagonal bars show the full strength.

The difference of No.1, No.6, No.13 and No.14 is only the amount of diagonal bars. Assuming that the diagonal bars form a truss structure as shown in Fig.7, the shear forces transferred by the diagonal bars and the other mechanisms were obtained from the test results as shown in Fig.4. The reference [7] is saying that the diagonal bars exert the full tensile force and its vertical component may be added to the shear capacity. If the amount of diagonal bars is less than a certain amount, this idea would be reasonable. The shear cracks firstly occurs in nearly 45° direction from the opening edge. It does not dictate the ultimate state, but it relates to the damage control limit under the short-term load condition. Because the diagonal bars are placed almost orthogonally to the first shear crack and they exist near the opening edge, much amount of the diagonal bars is desirable in terms of the shear crack width control. However, there must be some condition where the diagonal bars exert full tensile force as mentioned above.

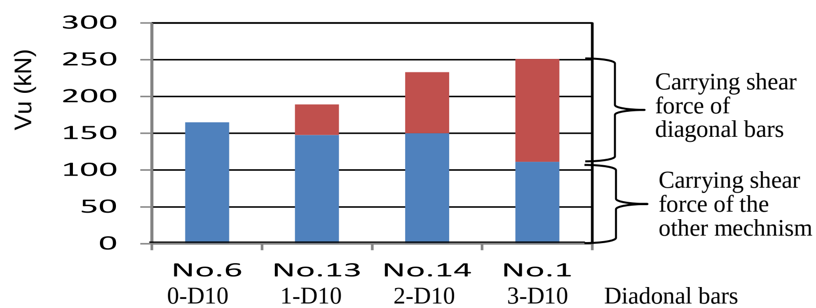


Fig.4 Carrying shear force by the diadonal bars and that by the other mechnisms

4. Role of the stirrups on the side of the web-opening

From the ultimate cracking pattern shown in Fig.5, the angle of shear crack in "sub-beams" changes depending on the position of the stirrups on the side of the web-opening. There is a tendency for the angle to be large when more stirrups are placed intensively near the opening edge. Series IV have no stirrups in C-region of Fig.1 [1], and the angle of the shear crack in "sub-beams" is small and directs to the position of the first stirrup or to the point beyond the first stirrup. It is considered that the stirrups beyond C-region as well as those in C-region would have some relation to the angle of the shear crack in "sub-beams."

Hirase et al. [8] cut out a part of the concrete strut of the truss mechanism of the original beam that appeared upper and lower side of the opening, and likened it to the strut of the arch mechanism. Their idea is reasonable because it corresponds to the shear crack in "sub-beams". However, something that balances with this strut is needed. It may be the stirrups on the side of the opening including those in and beyond C-region of Fig.1 [1]. In this paper, the idea of this strut upper and lower side of the opening is same as that of Hirase et al. [8] and the stirrups on the side of the opening are supposed to balance with this strut. This paper calls it "local arch mechanism". The ultimate failure pattern in Fig. 5 looks as the shear failure in the length LL of Fig.9. As the

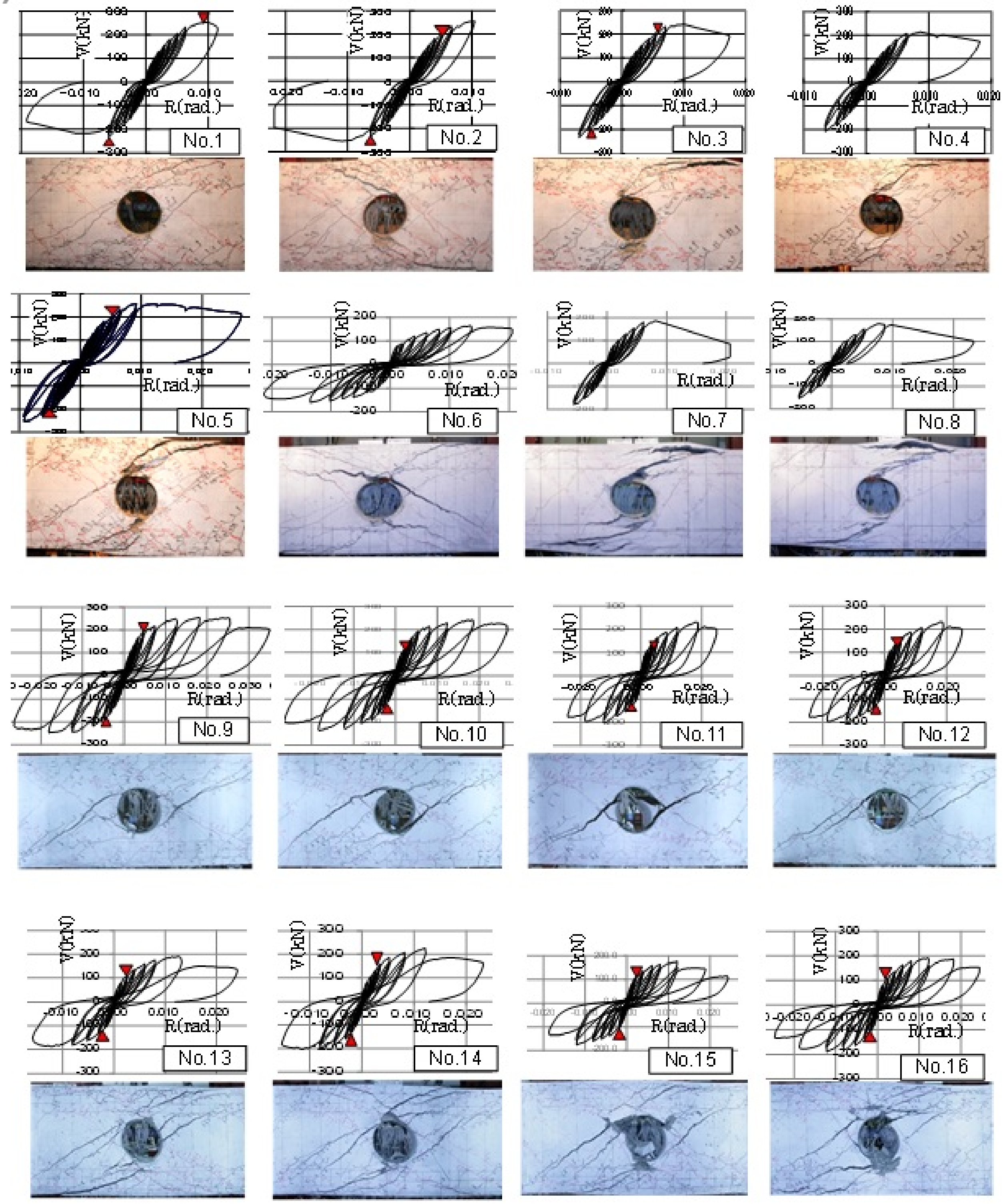


Fig.5 Relations between shear force (V) and the rotation angle of member (R) and the ultimate failure mode

stirrups are intensively placed close to the opening edge, LL becomes smaller. As a result, the span of "sub-beams" is shortened and the carrying shear force increases, and the shear failure occurs in more brittle manner.

If there is no stirrups in C-region of Fig.1 [1], it is considered that the stirrups beyond C-region would play the role of the element that balances with the strut of the local arch mechanism as seen in No.6 or series IV. The angle of the strut becomes small, and its shear capacity becomes smaller. However, the carrying shear force would decrease more gradually after the maximum load, because the span length LL of Fig.9 becomes large. To enhance the shear capacity, the intensively placed stirrups close to the opening edge is effective. They are also effective for the first shear crack width control. However it should be noted that the brittle failure will occur.

5. Role of "sub-beams" upper and lower side of the web-opening

The test results of shear capacity and the calculated values by AIJ design formula [1]are shown in Table 3. The safety margin of shear capacity of No.6 without the diagonal reinforcement and the stirrups in C-region of Fig.1[1] to the calculated value by AIJ design formula[1] is 4.27, and it is apparently expected that "sub-beams" over and lower side of the opening will carry the large shear force. Figure 6 shows that the shear capacity increases according to the increase of the amount of stirrups in "sub-beams" that is not considered in AIJ design formula[1]. Hirase et al. [8] proposed a shear capacity evaluation formula combining the truss mechanism in the upper and lower part of the web-opening with their proposed arch mechanism mentioned above, regardless of the presense or absense of the detail of "sub-beams". However, in order to build up the detail of "sub-beams" it is necessary that the horizontal bars are placed upper and lower side of the opening and the stirrups are provided so as to surround this horizontal bars and the longitudinal main bars of the original beam as the detail of the specimens in this study.

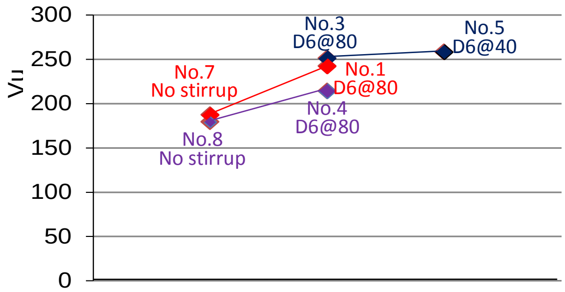


Fig.6 Deference of shear cpacity due to the diference of the amount of stirrups in "sub-beams"

Series III has large amount of stirrups in "sub-beams", and they did not yield before reaching the maximum load. The intensively placed stirrup 1-□D10 on the side of the opening was the parameter of the experiment. The difference of the maximum load was much less than the tensile capacity of 1-□D10, and the maximum load is almost same regardless of the position of the intensively place stirrup, when the concrete strength was taken into account. The carrying shear force decreased gradually after the maximum load. It is considered that the ultimate failure mode and the post shear capacity behavior would change depending on the amount of stirrups in "sub-beams", even if the local arch mechanism would be performed by the stirrups on the side of the web-opening. Also the core concrete of "sub-beams" should be confined in order to ensure the anchorage of the diagonal bars. Anyway, it is better to provide the stirrups in "sub-beams" as much as possible.

6. Shear force transferred by diagonal bars

The diagonal bars form the independent truss structure as shown in Fig.7. From the force balance condition at the points A or B in Fig.7,

$$T = C \quad (1)$$

$$\begin{aligned} V_d &= (T + C) \cdot sin\alpha \\ &= 2T \cdot sin\alpha \\ &= 2\,\sigma_{dy} \cdot a_d \cdot sin\alpha \end{aligned} \quad (2)$$

V_d : Carrying shear force of the truss structure by the diagonal bars
α : Angle of the diagonal bars to the longitudinal direction of the original beam
σ_{dy} : Yield strength of the diagonal bars
a_d : Sectional area of diagonal bars

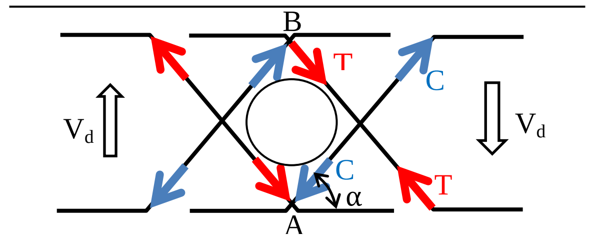


Fig.7 Truss structure by the diagonal bars

The horizontal components of the axial forces of diagonal members in Fig .7 cause the compressive stress σ_0 in "sub-beams" in horizontal direction.

$$\sigma_0 = \frac{2T \cdot cos\alpha}{b \cdot \frac{D-H}{2}} \tag{3}$$

D : Beam depth $\quad$ b : Beam width

H : Diameter of the opening

7. Shear force transferred by the truss mechanism of "sub-beams"

The horizontal bars are placed upper and lower side of the opening and the stirrups are provided surrounding these horizontal bars and the longitudinal main bars of the original beam. Then, "sub-beams" are formed upper and lower side of the opening. It looks like a Vierendeel girder. It was proved from the loading tests that the stirrups in these "sub-beams" contributes to the shear capacity gain as shown in Fig.6.

From the force balance condition of the compressive strut and the stirrup at the point A in Fig.8,

$$(\sigma_t \cdot s \cdot sin\emptyset \cdot b) \cdot sin\emptyset = a_s . \sigma_{sy} \tag{4}$$

$$\sigma_t = p_{sw} \cdot \sigma_{sy} \cdot (1 + cot^2\emptyset) \tag{5}$$

s : Spacing of stirrups $\quad$ σ_t : Compressive stress in the strut by the truss mechanism

b : Beam width $\quad$ a_s. : Sectional area of a stirrup

σ_{sy} : Yield strength of stirrups $\quad$ p_{sw} : Shear reinforcement ratio of "sub-beams"

$\emptyset$: Angle of the strut to the longitudinal direction of "sub-beams"

The carrying shear force of "sub-beams" is calculated by Eq.(6). If σ_t doesn't reach the failure criteria, the stirrups show the yield strength. If σ_t reaches the failure criteria first, the stirrups do not show the yield strength and the web concrete of "sub-beams" fails in shear compression mode.

$$V_s = p_{sw} \cdot \sigma_{sy} \cdot j_s \cdot b \cdot cot\emptyset \tag{6}$$

$$V_t = 2V_s \tag{7}$$

V_t : Carrying shear force by the truss mechanism of the upper and lower "sub-beams"

j_s : Distance between the longitudinal bars in "sub-beam"

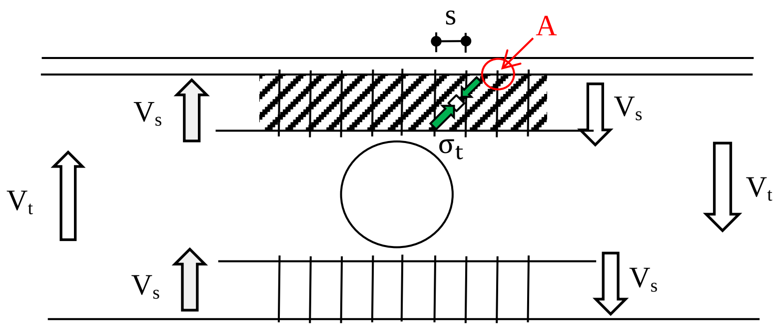


Fig. 8 Truss mechanism in "sub-beams"

8. Shear force transferred by the local arch mechanism

When there is the web-opening, the concrete strut of the truss mechanism of the original beam bends escaping the opening. A part of this strut, that appears upper and lower side of the opening, is regarded as the strut of the local arch mechanism in "sub-beams" as mentioned in the section 4. In Fig.9, the vertical component of the strut balances with the tensile force of the stirrups on the side of the web-opening. From Fig.9,

$$tan\theta_1 = \frac{\left(\frac{D-H}{2}-y\right)+\left(\frac{H}{2}-\frac{H}{2}cos\theta_1\right)}{\frac{H}{2}sin\theta_1+LL+y\cdot tan\theta_1} \tag{8}$$

The carrying shear force of the local arch mechanism is,

$$\begin{aligned} V_{LA} &= \sigma_a \cdot \frac{y}{cos\theta_1} \cdot b \cdot sin\theta_1 \\ &= \sigma_a \cdot y \cdot b \cdot tan\theta_1 \end{aligned} \tag{9}$$

If $\theta_1 = 0$, the sectional area of the strut is maximum, but $V_{LA} = 0$. If θ_1 increases, $sin\theta_1$ increases, but "y" approaches zero. Then, V_{LA} aproaches zero. It means that there is some "y" that gives the maximum V_{LA}. "y" can be approximately regarded as $\frac{1}{2}\left(\frac{D-H}{2}\right)$ in order to obtain the maximum V_{LA}.

$$\mathrm{y} = \frac{D-H}{4} \tag{10}$$

Here, approximately $tan\theta_1$ is given as follow from Eq. (8).

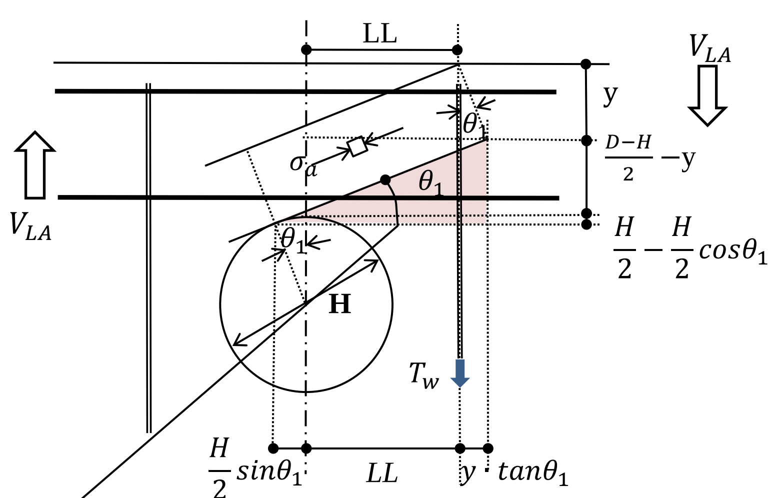
$$tan\theta_1 = \frac{D-H}{4\cdot LL} \tag{11}$$

Considering the upper and lower "sub-beams",

$$\begin{aligned} V_a &= 2 \cdot V_{LA} \\ &= 2 \cdot \sigma_a \frac{D-H}{4\cdot cos\theta_1} \cdot b \cdot sin\theta_1 \end{aligned} \tag{12}$$

Here, if V_{LA} is less than T_w $(= a_w \cdot \sigma_{wy})$, it means that the stirrups do not reach the yield strength and the concrete reaches the failure criteria first. If V_{LA} is greater than T_w $(= a_w \cdot \sigma_{wy})$, the stirrups reach the yield strength first and the concrete strut does not fail. a_w is the sectional area of stirrups and σ_{wy} is the yield strength of stirrups. T_w is the resultant force of some stirrups on the side of the opening. Here, T_w is supposed to be the resultant of stirrups *1, 2,* ･･････ , *N* in Fig. 10 (a). From Fig.10 (a),

$$LL = \frac{\sum_{i=1}^{N}(a_{wi}\cdot\sigma_{wyi}\cdot x_{wi})}{T_w} \tag{13}$$



D : Overall beam depth
H : Diameter of web-opening
LL: Distance of stirrups from the center of web-opening
θ_1 : Angle of concrete strut to the longitudinal direction
σ_a : Compressive stress in the strut by the local arch mechanism
T_w : Tensile force of stirrups
V_{LA}: Carrying shear force by the local arch mechanism in the upper or the lower side of web-opening

Fig.9 Concrete strut of the local arch mechanism

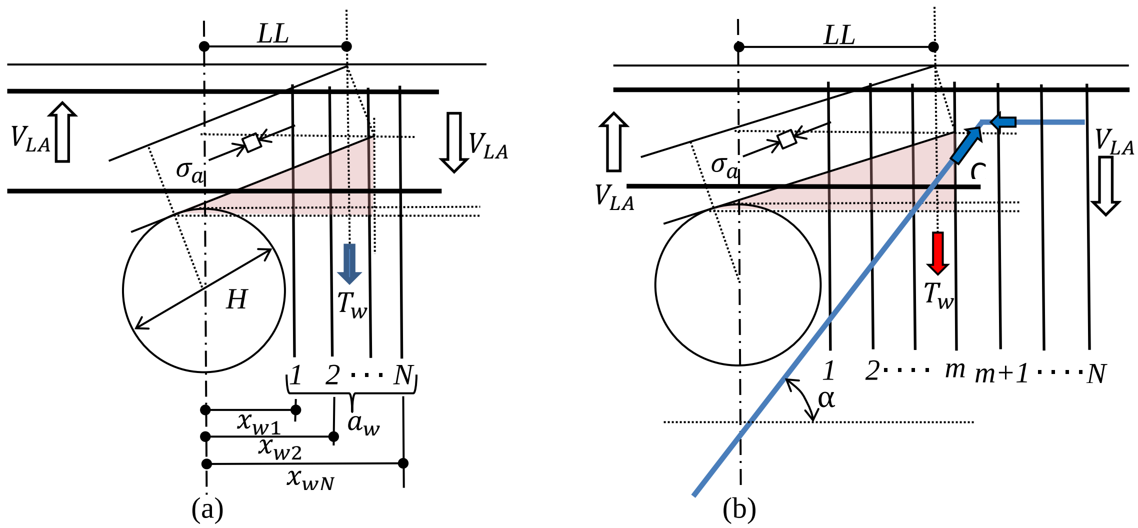


Fig.10 Stirrups that balance with the concrete strut of the local arch mechnism

a_{wi} : Sectional area of stirrup-i σ_{wyi} : yield strength of stirrup-i
x_{wi} : Location of stirrup-i from the center of the web-opening

$$a_w = \sum_{i=1}^{N} a_{wi} \tag{14}$$

$$T_w = \sum_{i=1}^{N} a_{wi} \cdot \sigma_{wyi} \tag{15}$$

If *N* increases, a_w , T_w and and σ_a increase. However, θ_1 decreases because *LL* increases. Then, there is "*N*" that gives the maximum V_{LA} . If σ_a reaches the failure criteria of concrete, no more *N* is needed. When the location of stirrups-(*m*+*1*) to *N* is beyond the bending point of the diagonal bars, a part of stirrups-(*m*+*1*) to *N* has to balance with the vertical component of diagonal compression member of Fig.7 as well.

$$C = a_d \cdot \sigma_{dy} \tag{16}$$

$$Cv = a_d \cdot \sigma_{dy} \cdot sin\alpha \tag{17}$$

Cv : Vertical component of the diagonal compression member of the truss structure
α : Angle of diagonal bars to the longitudinal direction of the original beam

The resultant tensile force of stirrups-(*m*+*1*) to *N* is,

$$T_{w2} = \sum_{i=m+1}^{N} a_{wi} \cdot \sigma_{wyi} \tag{18}$$

If T_{w2} is less than Cv, the stirrups-(*m*+*1*) to *N* are not related to the local arch mechanism. If T_{w2} is greater than Cv, the stirrups-*1* to *m*, and $(T_{w2} - C_v)$ of the stirrups-(*m*+*1*) to *N* balance with the vertical component of the strut of the local arch mechanism. *LL* and *Tw* are calculated as follow,

$$T_{w1} = \sum_{i=1}^{m} a_{wi} \cdot \sigma_{wyi} \tag{19}$$

$$LL_1 = \frac{\sum_{i=1}^{m} a_{wi} \cdot \sigma_{wyi} \cdot x_{wi}}{T_{w1}} \tag{20}$$

$$LL_2 = \frac{\sum_{i=m+1}^{N} a_{wi} \cdot \sigma_{wyi} \cdot x_{wi}}{T_{w2}} \tag{21}$$

$$LL = \frac{T_{w1} \cdot LL_1 + T_{w2} \cdot LL_2}{T_w} \tag{22}$$

$$T_w = T_{w1} + T_{w2} - C_v \tag{23}$$

9. Failure criteria of concrete

Ignoring the direction of σ_t , σ_a and σ_0 , the shear capacity is supposed to be obtained when the summation of the scalar quantities of these compressive stresses reach the failure criteria.

$$\sigma_t + \sigma_a + \sigma_0 = v_0 \cdot {}_c\sigma_B \tag{24}$$

${}_c\sigma_B$: Compressive strength of concrete v_0 : Effective coefficient

According to the reference [1], the effective coefficient is assumed to be given as follow,

$$v_0 = 0.7 - \frac{{}_c\sigma_B}{200} \tag{25}$$

If much amount of the stirrups in "sub-beams" and much diagonal bars are provided and $(\sigma_t + \sigma_0)$ is greater than $(v_0 \cdot {}_c\sigma_B)$, the local arch mechanism does not exist. If $(\sigma_t + \sigma_0)$ is less than $(v_0 \cdot {}_c\sigma_B)$, σ_a of Eq.(9) and Eq.(12) is replaced by $(v_0 \cdot {}_c\sigma_B - \sigma_t - \sigma_0)$.

10. Angle of concrete strut of the truss mechanism in "sub-beams"

From the maximum $cot\emptyset$,

$$cot\emptyset \leq 2 \tag{26}$$

$(\sigma_t + \sigma_0)$ has to be less than $v_0 \cdot {}_c\sigma_B$. From Eq. (5),

$$cot\emptyset \leq \sqrt{\frac{v_0 \cdot {}_c\sigma_B - \sigma_0}{p_{sw}} - 1} \tag{27}$$

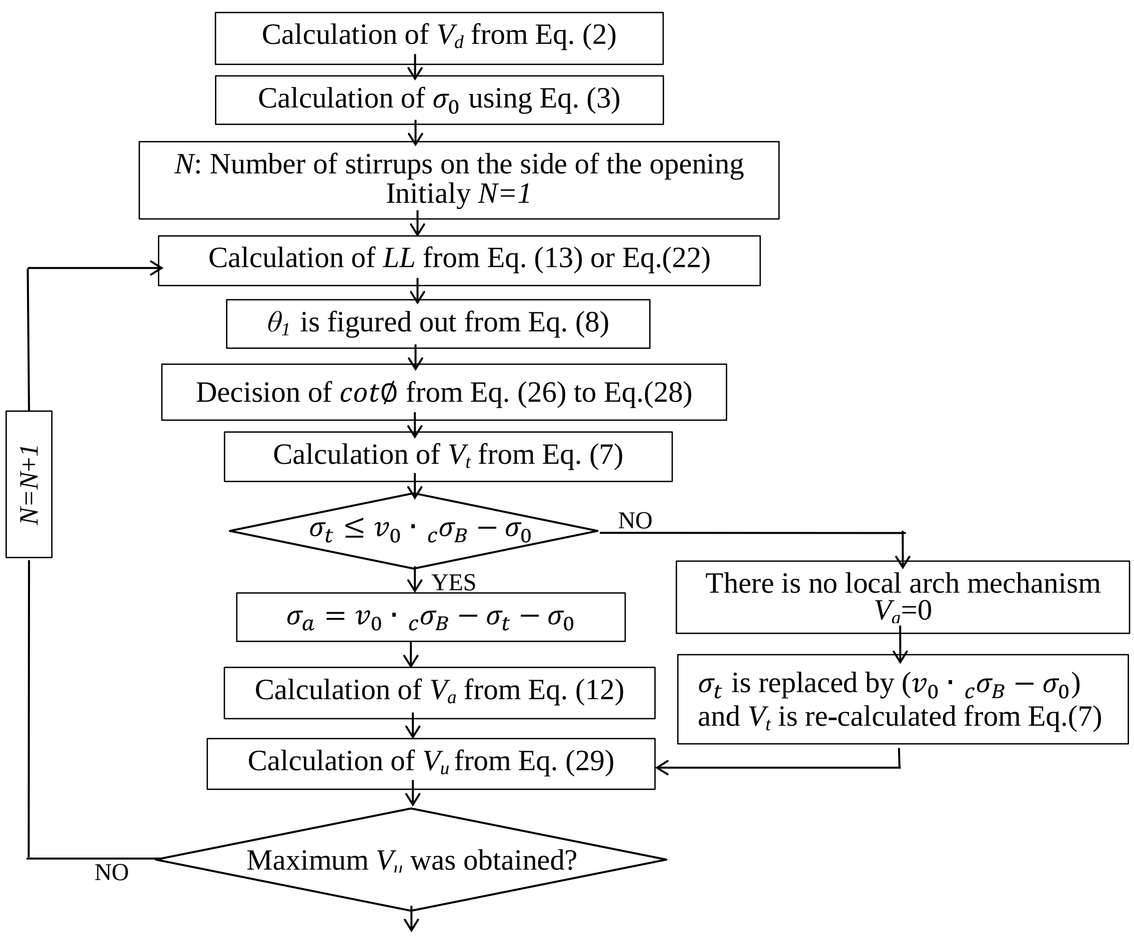


Fig. 11 Calculation flow of the shear capacity

The shear capacity of "sub-beams" is $(V_s + V_{LA})$ from Eq.(6) and Eq.(9). This is the quadratic equation of $cot\emptyset$. $(V_s + V_{LA})$ increases according to the increase of $cot\emptyset$ in the following range.

$$cot\emptyset \leq \frac{j_t}{(\frac{D-H}{2})\cdot tan\theta_1} \qquad (28)$$

From the lower boundary theorem of the theory of plasticity, $cot\emptyset$ can be selected so as to give the maximum $(V_s + V_{LA})$. In another words, the maximum $cot\emptyset$ can be selected satisfying Eq. (26), Eq. (27) and Eq. (28)

11. Shear capacity of beams with large web-opening

The ultimate shear capacity is the summation of the carrying shear force by three mechanisms.
From Eq. (2), Eq. (7) and Eq. (12)

$$V_u = V_d + V_t + V_a \qquad (29)$$

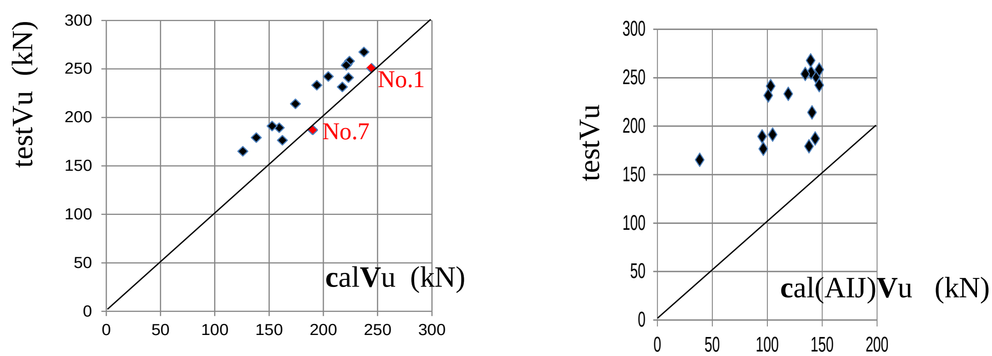
The shear capacity is calculated following the flow in Fig 11.

12. Verification of the proposed model with test results

Table 3 and Fig.12 show the calculated and experimental values of shear capacity. The calculated values by this study well correspond to the experimental values with the almost constant safety margin except for No.1 and No.7. Both No.1 and No.7 have much amount of diagonal reinforcement. In this paper, the diagonal

Table 3 Experimental and calculated values of shear capacity

Series	Specimen	Shear capacity by the test $_{test}Vu$(kN)	Calculated Shear capacity $_{cal}Vu$(kN)	$\frac{_{test}Vu}{_{cal}Vu}$	Shear capacity by AIJ code[1] $_{cal(AIJ)}Vu$ (kN)	$\frac{_{test}Vu}{_{cal(AIJ)}Vu}$	Diagonal bars
I	No.1	251	244.1	1.03	144.4	1.74	3-D10, 60°
	No.2	255	222.1	1.15	140	1.82	2-D10, 60°
	No.3	242	204.7	1.18	147.4	1.64	1-D10, 60°
	No.4	214	174.3	1.23	140.9	1.52	No reinforcement
	No.5	258	224.1	1.15	147.4	1.75	1-D10, 60°
II	No.6	165	125.8	1.31	38.6	4.27	No reinforcement
	No.7	187	190.2	0.98	143.7	1.30	3-D10, 60°
	No.8	179	138.3	1.29	138.2	1.30	No reinforcement
III	No.9	253.6	221.2	1.15	134.7	1.88	1-D10, 45°
	No.10	241	223.3	1.08	103.2	2.34	1-D10, 45°
	No.11	231.4	217.5	1.06	101.1	2.29	1-D10, 45°
	No.12	267.4	237.4	1.13	139.7	1.91	2-D10, 45°
IV	No.13	189.2	159.5	1.19	95.5	1.98	1-D10, 60°
	No.14	233.1	194.1	1.20	119.2	1.96	2-D10, 60°
	No.15	176.3	162.4	1.09	96.5	1.83	1-D10, 60°
	No.16	191	152.9	1.25	105	1.82	1-D6, 60°



(a) Verification by the proposd model in this study (b) Verification by AIJ code [1]

Fig.12 Experimental and calculated values of shear capacity reinforcement is assumed to transfer the shear force by the independent mechanism as shown in Fig.7. However, the amount of the diagonal bars should be carefully checked when it increases. About 15-20% of the constant safety margin to the test results occurs in the other specimens, probably because the minimum depth at the upper and lower side of the opening is adopted for the depth of "sub-beams". The calculated values may be affected by the failure criteria of concrete as well.

13. Conclusions

The loading test was conducted targeting the footing beams with large web-opening of which diameter is 40% of beam depth. The contribution of "sub-beams" upper and lower side of the opening, the stirrups on the side of the web-opening and the diagonal reinforcement to the shear capacity was experimentally discussed. Since the region, where the stirrups cannot be placed because of the large web-opening, increases, it is most important to constitute "sub-beams" upper and lower side of the opening providing the horizontal bars and the stirrups in "sub-beams" as much as possible. The stirrups on the side of the opening balance with the strut of the local arch mechanism. If there is no stirrups in C-region of Fig.1 [1], the stirrups beyond C-region will play the role of the reinforcement that balances with the strut of the local arch mechanism. It may be considered that the diagonal reinforcement transfer the shear force by the independent mechanism as far as the anchor is secured. However, it shoud be noted that the diagonal reinforcement has possibility not to excert the full tensile force if its amount is increased.

Based on the test results, a shear force transfer model was proposed. The calculated shear capacity well agreed with the test results with almost constant safety margin to the test results. The shear span to depth ratio is not taken into account in the proposed model unlikely AIJ design formula [1]. If the shear span is shorten beyond the position of stirrups that balance with the strut of the local arch mechanism, the strut must be directed to the original beam end and the angle of the strut increases. As a result, the vertical component of the compression force of the strut increases, that is, the shear force transferred by the local arch mechanism may increase. In another words the shear span to depth ratio can be indirectly taken into account in the proposed model. The shear capacity could be calculated assuming the strong stirrup at the original beam end.

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References

[1] Architectural Institute of Japan (2000): AIJ Standard for Structural Calculation of Reinforced Concrete Structures.

[2] Takenaka Corporation (2009): Takenaka reinforcing method for the footing beams with web-opening. *Bulletin of General Building Research Corporation,* **34** (2), 89.

[3] Hazama Corporation et al. (2011): Reinforcing method of the footing beams with large web-opening. *Bulletin of General Building Research Corporation,* **36** (4), 42.

[4] Haseko Corporation (2012): Haseko reinforcing method of the footing beams with large web-opening. *Bulletin of General Building Research Corporation,* **37** (1), 63.

[5] Kiuchi Construction (2012): Reinforcing method of the footing beams with large web-opening. *Bulletin of General Building Research Corporation,* **37** (3), 64.

[6] Toda Corporation (2012): Reinforcing method for the footing beams with large web-opening. *Bulletin of General Building Research Corporation,* **37** (4), 74.

[7] Architectural Institute of Japan (1999): Design guidelines for earthquake resistant reinforced concrete buildings based on inelastic displacement concept.

[8] Hirase T, Hibino Y, Shinohara Y, Hayashi S (2011): Experimental study on the shear force transfer mechanism of RC beams with web-opening. *Proc. of Japan Concrete Institute,* **33(2),** 259-264