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On the Fundamental Periods of Vibration of Flat-Bottom Ground-Supported Circular Silos containing Gran-like Material

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Abstract

Despite the significant amount of research effort devoted to understanding the structural behavior of grain-silos, each year a large number of silos still fails due to bad design, poor construction, with a frequency much larger than other civil structures. In particular, silos frequently fails during large earthquakes, as occurred during the 1999 Chi-Chi, Taiwan earthquake when almost all the silos located in Taichung Port, 70 km far from the epicenter, collapsed. The EQE report stated that "*the seismic design of practice that is used for the design and construction of such facilities clearly requires a major revision*". The fact indicates that actual design procedures have limits and therefore significant advancements in the knowledge of the structural behavior of silo structures are still necessary. The present work presents an analytical formulation for the assessment of the natural periods of grain silos. The predictions of the novel formulation are compared with experimental findings and numerical simulations.

Keywords: flat-bottom on-ground circular grain-silo, seismic response, fundamental period, code-like formula

1. Introduction

The structural design of grain-silos requires accounting for the effect of the ensiled grain on the wall both under static and under dynamic conditions. Grain-silos are considered different to many other civil structures [1] and are usually classified as "non-building structures" [2, 3]. In particular, given that the weight of the silo structure is sensibly lower than the full loads from particulate solids ensiled mass, in case of strong earth motion, the grain-structure interaction plays a fundamental role on the global dynamic response.

As widely known, the identification of the natural periods is the basic step for any seismic design. Unfortunately, whilst for ordinary civil structures (e.g. frames structures), the fundamental periods can be easily evaluated using consolidated code-like formulas or by means of simple elastic finite element models, for flat-bottom grain-silos either reliable formulas nor simple finite element procedure for modelling models are available for the evaluation of the natural periods to date.

During the last century, few experimental tests (most of them via shaking-table on silo specimens), numerical simulation via Finite Element (FE) modelling and analytical studies were performed in order to investigate the dynamic behavior of circular ground-supported grain-silos and the interaction between cylindrical shell and granular ensiled content. Although different test protocol have been generally adopted by different Authors for the dynamic characterization of grain-silo systems, it appears that grain-silos present a marked non-linear dynamic behavior. Therefore, the common methods adopted for the dynamic analysis of common civil structures cannot be straightly applied to grain-silos.

The present study aims at providing an analytical formulation for the estimation of the fundamental period of vibration of flat-bottom circular grain-silos referable to the class of silo with isotropic continuous wall (such as rolled steel plate silos). Starting from the analytical framework proposed by [4] and [5] and adopting the same idealized model, the dynamic behavior of grain-silos is re-conducted to that of an equivalent linear-elastic system. In addition, a simple procedure for the numerical estimation via FE modelling of the dynamic properties of more complex typology of grain-silos, e.g. with orthotropic (corrugated) or stringer stiffened wall, composed by bolted component, is proposed.

In the first part of the present paper, a brief review of the experimental, theoretical and numerical research works conducted by many Authors related to the dynamic behavior of grain-silos is briefly presented. In the second part of the paper, the theoretical framework adopted, the assumptions, the closed-form expressions for the analytical evaluation of the fundamental period of vibration and the proposed code-like formula for design purposes are presented. Finally, the theoretical estimation is compared with the experimental data gathered via shaking-table tests on a silo specimen containing Ballottini-glass and data available from the scientific literature. In the last part, a procedure for the analysis of the dynamic behavior of circular on-ground grain-silo via FE model is also proposed.

2. A review of the research on the seismic behavior of grain-silos

The knowledge of the dynamic properties (at least natural periods of vibration, equivalent damping ratios) of a structural system is the basic step toward its reliable seismic design. They are fundamental for the assessment of the structural response under strong earth-motion and thus the sizing of the structural elements. The dynamic behavior of common linear-elastic system, such as frame structures, is well established in structural dynamics and their design methods are nowadays well consolidated in the design codes. Nonetheless, for other structural systems, such as grain-silos, the prediction of the seismic behavior is still a relevant challenge, due to the strong non-linear behavior under seismic excitation and complex mass-structure interaction phenomena. Consequently, the lack of a general and universally accepted theoretical framework for the dynamic behavior of grain-silos reflects in important shortcomings in actual seismic design provisions [6, 7].

A conceptual schematization that allows at appreciating the mutual relations in terms of complexity in the actual dynamic behavior, advancement in the scientific knowledge and related code provisions for grain-silos as compared to frame structures is provided in Fig. 1.

While for frame structures the scientific knowledge and code provisions are reasonably close to their actual dynamic behavior (the gap is relatively small), in the case of grain silos the gap between the code provisions and actual dynamic behavior is still remarkable and a significant scientific advancement of knowledge is desirable as recognized by some of the most eminent researchers [6, 8] and practitioners [7] in the field.

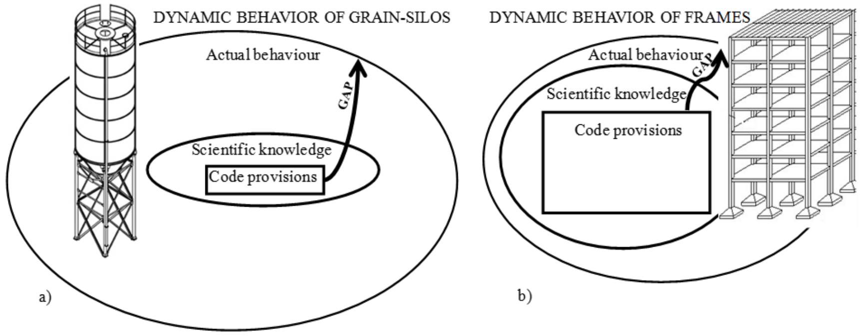


Fig. 1 - Actual behavior vs scientific knowledge vs code provisions: (a) grain silos; (b) frame structures

In this context, design based on experience of previous successes and, most of all, on failures appears more robust and sound with respect to the mere application of code prescriptions, which are actually mostly based on empirical rules.

From a scientific point of view one of the main challenge to be faced deals with the evaluation of the period of vibration, which involves both the assessment of the *effective mass* involved in the dynamic response and the contribution of the silo wall in terms of stiffness. Clearly, a scientific advancement in such direction would also benefit the development of more consistent design rules. As matter of fact, even though many current design codes deal with the design of elevated and on-ground circular grain-silos, their provisions do not explicitly give formulas for the evaluation of the fundamental period of vibration of grain-silos [1], or suggest to simply consider a rigid-body response [3] (see §15.7.9.2).

Extensive experimental tests have been conducted during the last decades aimed to the study of the dynamics of flat-bottom ground-supported silos and to fully understand the complex interaction between cylindrical shell and ensiled content under earthquake excitation. Almost all the investigations were performed through shaking-table tests by applying different dynamic excitations (white noise signal WN, impulsive load IL, stationary harmonic signal HS with increasing frequency until resonance of the grain-silo occurs, earthquake recorded signals EQK). Few free-vibrations tests are also available in the scientific literature. In detail, experimental dynamic tests were conducted by [9, 10, 11, 12, 13, 14, 15, 16, 6, 17, 18]. Generally, on the basis of the instrumentation used by different Authors, the main results obtained by means of dynamic tests are: (i) the fundamental natural frequency f_1; (ii) the resonance curve (i.e. the curves providing the dynamic amplifications as a function frequency f and amplitude of acceleration a); (iii) the value of the *effective mass* m_{eff} (i.e. the grain mass participating to the motion as expressed in term of fraction of the total ensiled mass); (iv) the dynamic amplification of the horizontal accelerations measured at different distance z; (v) the damping ratio.

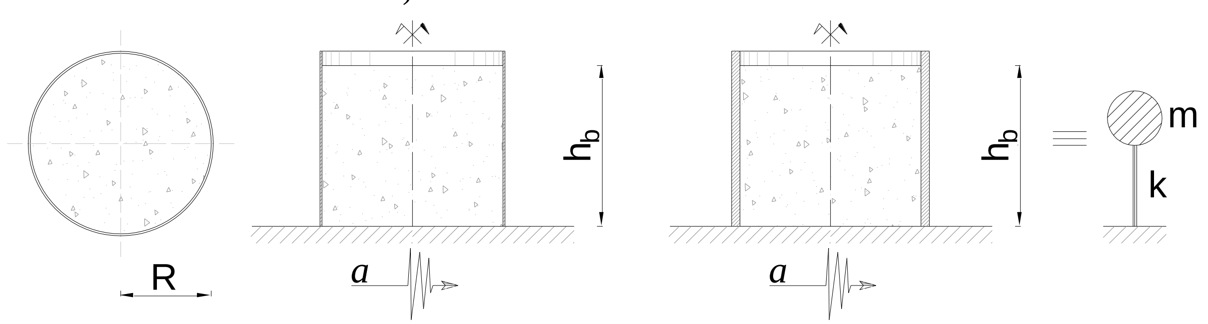
In general terms, from the analysis of the aforementioned experimental works, it appears that:

- The dynamic response of grain silos is significantly affected by the nature of the dynamic input and by the properties of the ensiled material (rough material may lead to both large equivalent damping ratio and large *effective mass*);
- The values of the *effective mass* seems to be significantly influenced by the excitation: large values (around 0.8) are obtained when the silo in excited close to its natural frequency; lower values are obtained under with noise, seismic excitation, and also under harmonic excitation far from the resonance;
- The natural frequencies are largely influenced by the excitation type (white noise or harmonic signal) and acceleration amplitude (values larger or smaller than the critical acceleration a_{crit} at which horizontal grain sliding occurs, around 0.30 g). In particular, under harmonic excitations a larger amount of grain mass tend to be involved in the motion with respect to the mass typically involved during earthquake excitation. On the contrary, under WN excitation the amount of grain mass involved in the motion is similar to that excited under strong earth motion. Those facts suggest that the dynamic identification

(natural period and damping ratio) should be conducted by mean of WN test and varying the amplitude acceleration up to the critical value;

- As expected, the frequencies and equivalent damping ratios substantially changes from empty to full filled conditions. In detail, the ratios between the first frequency of the empty and full-filled silos varies between 1.5 and 4.0 .Correspondently, the damping ratios increases from 1-4% up to 20%.
- The values of the maximum dynamic amplifications are significantly influenced by the excitation type: under harmonic excitation, at the resonance, the maximum dynamic amplifications achieve values around 20-25 for empty conditions or very low acceleration amplitudes (a<0.05) and around 5-10 for full filled conditions. Under white noise and earthquake excitations, the maximum dynamic amplifications are between 2 and 5;
- At the resonance (under harmonic excitations) and for acceleration larger than a_{crit} (under earthquake), the dynamic amplifications of the grain tend to be larger than the dynamic amplifications of the silo wall, thus indicating a relevant horizontal grain sliding.

In addition to experimental tests, various analytical and numerical models have been proposed in the scientific literature for the prediction of the dynamic response of vibration of circular grain-silos. Most of them presents modeling techniques, which are validated based on some of the experimental results reported in the previous sections. For what concerns the theoretical studies, [19] studied the dynamic behavior of cylindrical shell filled with liquid (even if focused on fluid-liquid storage tanks, this research work provides an analytical framework for the evaluation of fundamental period of such cylindrical shell structures). [20] proposed the first analytical formulation for the prediction of additional grain-pressures distribution acting on the silo wall in accelerated conditions. [21] analyzed the dynamic response of vertical, rigid circular cylindrical tanks filled with a homogeneous, linear viscous-elastic solid medium (Fig. 2a). [4] considers an idealized system to model ground-supported flat-bottom circular silos filled with grain-like material under dynamic conditions. The original theory (further refined by [5]), starting from the [22] formulation, accounts the effect of horizontal and vertical accelerations. [23] proposed an analytical formulation for the estimation of the fundamental period of vibration of circular flat-bottom silos containing elastic material by means of an equivalent Single Degree of freedom system (Fig. 2b). For what concerns the numerical simulations, many Authors [10, 11, 13, 16, 26, 6, 24]) studied the dynamic and seismic response of grain-silos by explicitly modeling both the silo wall and the ensiled material through FE (shell and solid elements for wall and grain are generally used, respectively). [25] followed a different approach by modelling the silo wall with shell elements having a fictitious mass density in order to account for the contribution of the ensiled grain mass participating to the motion (which is not directly considered within the numerical model).



(a) *(b)*

Fig. 2 - Analytical model by [21] and (b) [23] (adapted)

3. An analytical formulation for the fundamental period of grain-silos

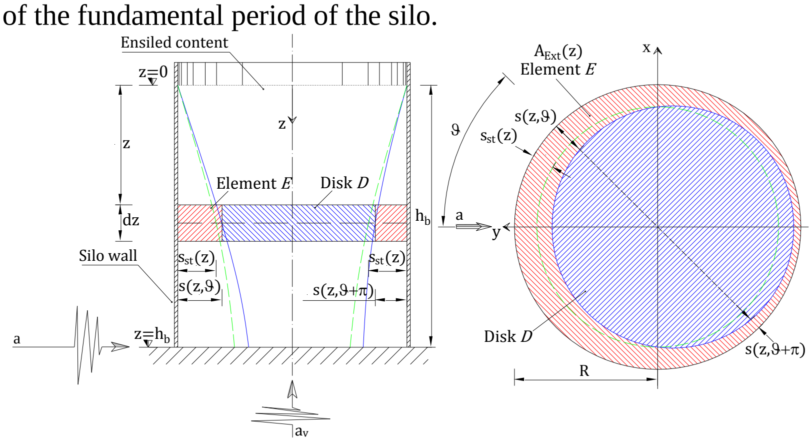
3.1. The basic assumptions

The formulation that is here proposed for the evaluation of the fundamental period of flat-bottom on-ground circular grain-silos is grounded on the Silvestri-Pieraccini theory assumptions, and make use of the approach by

[19], which is specialized for the case of grain-silo systems. For the sake of clearness, the fundamental assumptions of the Silvestri-Pieraccini theory are here briefly summarized: (i) a portion of the mass of the ensiled grain leans against the silo wall, whilst the reaming mass does not interact with the silo wall during the ground shaking; (ii) the grain-wall friction and the grain-grain friction are fully exploited during the ground shaking; (iii) no horizontal grain sliding is considered. The Silvestri-Pieraccini theory states that only the mass of the ensiled material leaning against the wall, corresponding to the *effective mass*, is activated during the horizontal shaking. The geometrical shape of the mass leaning against corresponds to the external torus of variable thickness *s* as represented in Fig. 3. The mass interacting with the silo wall moves together with the silo (i.e. no horizontal sliding occur) as observed during the experiments.

The approach by [19] consists in modeling the cylindrical shell with its content as a uniform linear-elastic shear-flexural cantilever beam. Additional assumptions are here necessary to extend the [19] approach to grain-silos: (1) horizontal input is applied only; (2) the *effective mass* is independent on the profile and amplitude of the horizontal accelerations; (3) in the deformed configuration, plain section remain plain and no section vocalizations occur; (4) the stiffness of the system is provided by the silo wall only; (5) the overall mass of the equivalent beam consists of 2 contributions: the grain mass corresponding to the *effective mass* and the mass of the silo structures, and is considered as uniformly distributed along the height.

Assumption 1 considers the scenario in which only a horizontal motion is applied, since the effects of the vertical component a_v appears to be negligible. Assumption 2 states that the *effective mass* does not depend on the intensity of the shaking until no significant grain sliding occurs. For values of μ_{GW} , λ and Δ typical of as-built silos, the variation of the volume of the external torus for amplitudes of a up to the critical value a_{crit} is, for engineering purposes, negligible. Assumption 3 states that no cross section ovalizations occur due to the presence of the ensiled grain material that prevent for local deformations. The experimental observations by [18] indicates that, for $a \le a_{crit}$, the assumption is roughly verified. Assumption 4 states that the grain does not offer an additional contribution to the lateral stiffness of the silo wall, apart preventing from local cross deformations. In other words, the lateral stiffness of the system is coincident with the silo wall stiffness. This assumption is in agreement with experimental evidences by [9] and numerical results as deduced by the work of [23]. Assumption 5 states that a uniform mass per unit length is assumed. This is necessary in order to obtain an analytical expression of the fundamental period of the silo.



(a) (b)

Fig. 3 – (a) External torus *E* (red hatching) and internal disk *D* (blue hatching) of the grain layer. (b) Vertical section (adapted)

3.2. The analytical estimation of the fundamental period of vibration of grain-silos

Based on the aforementioned assumptions, the fundamental period of the realistic flat-bottom on-ground circular grain-silo of Fig. 4a is evaluated with reference to the idealized equivalent uniform shear-flexural cantilever beam model, as represented in Fig. 4b. The silo of Fig. 4a has smooth wall with stepwise variable thickness $t_{w,i}$ (*i* is the *i*-th wall portion characterized by constant thickness $t_{w,i}$ and length Δz_i, r is the total number of wall portions). A conical roof with an angle measured with respect to the horizontal plane equals to α_r and uniform thickness t_r covers the silo. All the other relevant geometrical properties of the silo are indicated in Fig. 4a. The equivalent cantilever beam of Fig. 4b, has a height H_{beam} (vertical length between the silo bottom and the highest solid-wall contact, for a full-filled silo is identified as height of overfull filling), an hollow uniform circular cross-section of diameter d_c and thickness $\bar{t}$, and is clamped at the base. The value $\bar{t}$ varies with respect to the homogenization criteria: equal mass, equal shear frequency, equal flexural frequency. The three criteria will be specified in the following sections.

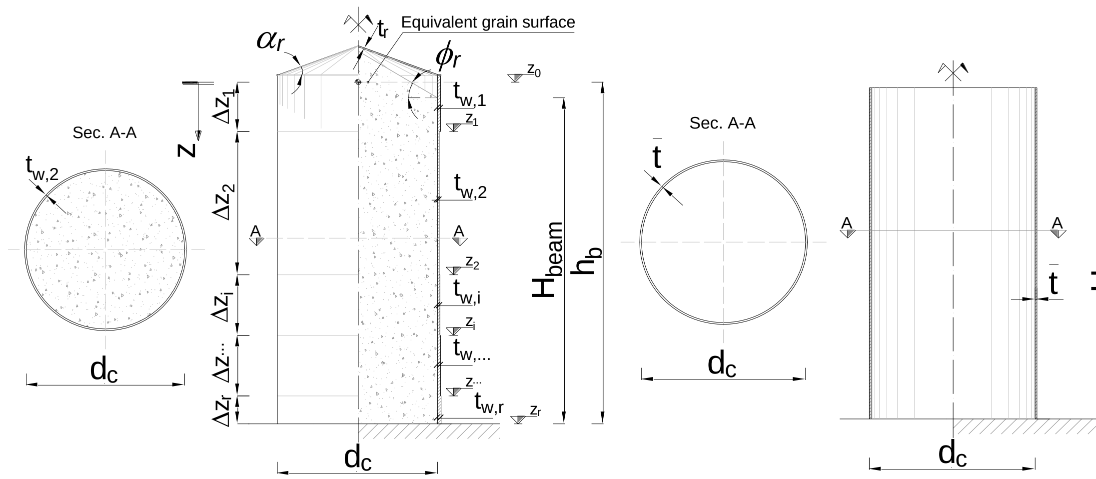


Fig. 4 – (a) Geometry of a realistic flat-bottom ground-supported circular grain-silo;(b) Geometry of the corresponding equivalent beam

According to assumptions 1, 2, 5 and with reference to the silo configuration represented in Fig. 4, the mass per unit length to be used for the estimation of the fundamental period of vibration is made of the following contributions: (i) the *effective mass* of the grain; (ii) the mass of the silo wall; (iii) the mass of the silo roof.

The mass per unit length corresponding to the *effective mass* of the grain (or bulk solid) $m_b(z)$ is given by:

$$m_b(z) = \frac{2\pi R}{g} \cdot p_{wf}(z) \tag{1}$$

where $p_{wf}(z)$ is the wall frictional traction at a distance *z* under static condition according to [27] (referred also to as $\tau_{v,GW,st}(z)$ in [5]). For slender silos, or when the grain surface may be considered almost flat, the [12] formulation of $p_{wf}(z)$ is suitable. On the contrary, for squat silos, the contribution of the upper conical portion of the ensiled grain may become significant, and the semi-empirical [28] formulation of $p_{wf}(z)$ is preferable. In particular, making use of the two above mentioned formulations of $p_{wf}(z)$, the expression of $m_b(z)$ as given by Eq. (1) specifies as follows:

$$\text{Janssen (1895): } m_b(z) = \frac{2\pi R}{g} \cdot \left[\mu_{GW} \cdot p_0(z)\right] = \frac{\gamma_b}{g} \cdot \pi R^2 \cdot \left[1 - e^{-\frac{2\cdot\mu_{GW}\cdot\lambda\cdot z}{R}}\right] \tag{2}$$

$$\text{Reimbert (1976): } m_b(z) = \frac{2\pi R}{g} \cdot \left[\mu_{GW} \cdot p_{hf}(z)\right] = \frac{\gamma_b}{g} \cdot \pi R^2 \cdot \left\{1 - \left[\left(\frac{z - h_0}{z_0 - h_0}\right) + 1\right]^N\right\} \tag{3}$$

where $p_0(z)$ is the horizontal pressure given by [22], $p_{hf}(z)$ is the horizontal pressure given by [28], whilst h_0, z_0 and *N* are given by Eq. (5.77), (5.75) and (5.74) of [27], respectively.

The mass per unit length of the silo wall $m_w(z)$ can be expressed as:

$$m_w(z) = 2\pi R \cdot t_w(z) \cdot \frac{\gamma_w}{g} \tag{4}$$

where $t_w(z)$ is the thickness of the silo wall and γ_w is the unit weight of the wall material.
The mass of the conical roof M_r is equal to:

$$M_r = \pi R^2 \cdot \sqrt{1 + tg(\alpha_r)^2} \cdot t_r \cdot \frac{\gamma_r}{g} \tag{5}$$

Where γ_r is the unit weight of the roof. The equivalent uniform mass per unit length $\overline{m}$ of the equivalent beam (accounting for the three contributions $m_b(z)$, $m_w(z)$ and M_r) results equal to:

$$\overline{m} = \frac{\int_0^{h_b} m_b(z) \cdot dz + \int_0^{h_w} m_w(z) \cdot dz + M_r}{H_{beam}} \tag{6}$$

Making use of Eqs. (2) and (3), $\overline{m}$ (Eq. 6) specifies as follows:

$$\text{Janssen (1895): } \overline{m} = \frac{\gamma_b}{g} \cdot \pi R^2 \cdot m_{eff} + 2\pi R \cdot \overline{t_w} \cdot \frac{\gamma_w}{g} + \frac{\pi R^2 \cdot \sqrt{1 + tg(\alpha_r)^2} \cdot t_r}{H_{beam}} \cdot \frac{\gamma_r}{g} \tag{7}$$

$$\text{Reimbert (1976): } \overline{m} = \frac{\gamma_b}{g} \cdot \pi R^2 \cdot m_{eff} + 2\pi R \cdot \overline{t_w} \cdot \frac{\gamma_w}{g} + \frac{\pi R^2 \cdot \sqrt{1 + tg(\alpha_r)^2} \cdot t_r}{H_{beam}} \cdot \frac{\gamma_r}{g} \tag{8}$$

where $\overline{t_w}$ is the uniform thickness of the equivalent beam leading to the same wall mass of the silo: $\overline{t_w} = \left[\sum_{i=1}^{r} \Delta z_i \cdot t_{w,i}\right] / H_{beam}$ (equal mass criterion) and the analytical expression of the *effective mass* m_{eff} according to the Janssen and the Reimbert formulation inside Eqs. (7) and (8) results, respectively:

$$\text{Janssen (1895): } m_{eff} = 1 + \frac{1 - e^{\omega}}{\omega} \tag{9}$$

$$\text{Reimbert (1976): } m_{eff} = \frac{1}{H_{beam}} \cdot \left[h_b - z_V(z = h_b)\right] \tag{10}$$

where $\omega = -4 \cdot \mu_{GW} \cdot \lambda \cdot \Delta$, $z_V(z)$ is given by Eq. (5.80) of [27].

It should be recognized that, for common steel real silos, the mass contribution of the conical steel roof is negligible.

The main elastic properties (wall cross section shear area A_w and wall cross section moment of inertia I_w) of the equivalent beam model as represented in Fig. 4b , which are necessary to evaluate the fundamental shear and flexural frequencies, can be explicated as follows:

$$A_w' = \frac{2\pi R \cdot \overline{t_{w,sh}}}{\chi} \tag{11}$$

$$I_w = \pi R^3 \cdot \overline{t_{w,flex}} \tag{12}$$

where χ represents the shear coefficient; $\overline{t_{w,sh}}$ and $\overline{t_{w,flex}}$ are the thickness of the uniform shear and flexural beam satisfying the following criterion:

$$\text{Equal shear frequency } \overline{t_{w,sh}} = \frac{H_{beam}^{\ 2}}{\sum_{i=1}^{r}\left[\frac{z_i^{\ 2} - z_{i-1}^{\ 2}}{t_{w,i}}\right]} \tag{13}$$

$$\text{Equal flexural frequency } \overline{t_{w,flex}} = \frac{H_{beam}{}^4}{\sum_{i=1}^{r}\left[\dfrac{z_i{}^4 - z_{i-1}{}^4}{t_{w,i}}\right]} \tag{14}$$

The *n*-th natural frequency of a continuous uniform linear elastic cantilever shear-beam are given by [29]:

$$f_{n,sh} = \frac{(2n-1)}{4 \cdot H_{beam}} \cdot \sqrt{\frac{G_w \cdot A_w^{'}}{\overline{m}}} \tag{15}$$

Where G_w is the shear modulus of the wall material.

The *n*-th natural frequency of a continuous uniform linear elastic cantilever flexural-beam can be expressed according to the formulation by [30]:

$$f_{n,flex} = \frac{(\phi \cdot H_{beam})_n^2}{2\pi} \cdot \sqrt{\frac{E_w \cdot I_w}{\overline{m} \cdot H_{beam}{}^4}} \tag{16}$$

Where E_w is the Young's modulus of the wall material, $(\phi \cdot H_{beam})_n^2$ is the second power of the product between the *n*-th root of the secular equation and the beam length, which can be found in [30].

According to Dunkerley's approximation [31] the fundamental frequency $f_{n,sh+flex}$ accounting for both shear and flexural deformations of an equivalent shear-flexural beam can be computed as follows:

$$\frac{1}{\left(f_{n,sh+flex}\right)^2} = \frac{1}{\left(f_{n,sh}\right)^2} + \frac{1}{\left(f_{n,flex}\right)^2} \tag{17}$$

Combination of Eqs. (15), (16) and (17) leads to the following expression of $f_{n,sh+flex}$:

$$f_{n,sh+flex} = f_{n,sh} \cdot \sqrt{\frac{1}{1+\left(\dfrac{f_{n,sh}}{f_{n,flex}}\right)^2}} = f_{n,sh} \cdot \sqrt{\frac{1}{1+\psi_n \cdot \left[\dfrac{\overline{\Delta}^2}{\chi \cdot (1+\upsilon_w)} \cdot r_t\right]}} \tag{18}$$

Where $\psi_n = \left[\dfrac{\pi \cdot (2n-1)}{(\phi \cdot H_{beam})_n^2}\right]^2$ is a function of *n*, $\overline{\Delta} = \dfrac{H_{beam}}{d_c}$ is the filling slenderness ratio and $r_t = \dfrac{\overline{t_{w,sh}}}{\overline{t_{w,flex}}}$ is the ratio of the thickness of the uniform shear beam on the thickness of the uniform flexural beam.
In detail, the first frequency of vibrations (*n*=1) specifies as follows:

$$f_{1,sh+flex} \cong \sqrt{\frac{1}{\overline{m}} \cdot \frac{\pi \cdot E_w}{\chi \cdot (1+\upsilon_w)} \cdot \left(\frac{1}{32 \cdot \overline{s_w} \cdot \overline{\Delta}^2}\right)} \cdot \sqrt{\frac{1}{1+0.90 \cdot \left[\dfrac{\overline{\Delta}^2}{\chi \cdot (1+\upsilon_w)} \cdot r_t\right]}} \tag{19}$$

Where $\overline{s_w} = d_c / \overline{t_{w,sh}}$ is the ratio of the diameter on the uniform shear thickness. The approximation is related to the value of parameter ψ_1 (approximated to 0.90). In addition, for the specific, but usual, case of a thin-walled cylindrical metal silo ($\upsilon_w = 0.30$, $\chi = 2$) Eq. (19) simplifies to:

$$f_{1,sh+flex} \cong \frac{0.2}{\overline{\Delta}} \cdot \sqrt{\frac{E_w}{\overline{m} \cdot \overline{s_w}}} \cdot \sqrt{\frac{1}{1+0.35 \cdot \overline{\Delta}^2 \cdot r_t}} \tag{20}$$

Or in terms of first natural period of vibration:

$$T_{1,sh+flex} \cong 5\overline{\Delta} \cdot \sqrt{\frac{\overline{m} \cdot \overline{s_w}\left(1+0.35 \cdot \overline{\Delta}^2 \cdot r_t\right)}{E_w}} \tag{21}$$

By making use of Eqs. (7) or (8) for $\overline{m}$ (which may depend on the slenderness of the silo and/or free grain surface configuration) a fully-analytical expression of $f_{1,sh+flex}$ (or $T_{1,sh+flex}$) could be derived. The expressions are too heavy and are here not explicitly provided. For thin-walled steel silos, a further simplification could be made by neglecting the wall and roof mass contributions, thus considering only the bulk mass inside Eqs. (7) or (8). Nevertheless, in the practice the formulas can be easily implemented in a spreadsheet. The expression of Eq. (21) depends on: filling slenderness ratio $\bar{\Delta}$, wall geometrical and elastic properties (E_w, r_t, $\overline{s_w}$), and the ensiled material properties and *effective mass* (ρ_b, μ_{GW}, λ).

3.3. A simple code like-formula steel silos

For steel silos containing common grain-like materials (wheat is considered suitable to be representative of different granular bulk solids [33], ρ = 900 kg/m3, μ_{GW} = 0.38 and λ = 0.54 for wall type D2 as per Table E.1 of [27]) designed according to [27] ($\overline{s_w}$ = 5000, 2000 and 1000 for squat, intermediate slender and slender silos, respectively) the following simple code-like formula for the first natural period can be used:

$$T_{1,sh+flex} \cong \left(0.0036\bar{\Delta}^2 + 0.006\bar{\Delta}\right)\cdot d_c \text{ (m)} \tag{22}$$

4.A modeling technique based on the analytical formulation

A modeling technique for the evaluation of the natural period of grain-silos is proposed and implements the analytical formulation introduced in section 3. The silo wall are explicitly modelled, while the effect of the ensiled material is accounted by increasing the wall density by adding the *effective mass* of the grain uniformly on the wall. The *effective mass* has to be explicitly introduced by the user. Additional constraints should be included within the numerical model in order to ensure that no wall ovalizations arise. Such approach is suitable for seismic equivalent static and response spectrum analyses. Such approach, with respect to the analytical formula, allows to encompass more complex structural configuration, such as horizontally-corrugated silos with vertical stiffeners, typically used for agricultural silos (e.g. silos containing maize, grain, soya beans). The modeling technique here proposed is based on the following steps: (1) Development of the actual FE model of the silo structure including: the shell (smooth or corrugated, with the actual stepwise variations of the thickness), horizontal stringer, vertical stiffeners, the roof. A regular mesh of quad elements is suggested for the wall. In case of smooth wall, all elements are modeled with their own elastic properties (elastic modulus and Poisson's coefficient). In case of corrugated wall, the orthotropic behavior should be considered; (2) application of rigid horizontal diaphragms at each vertical shell mesh level in order to prevent from section ovalizations (no local modes arises); (3) assignment of a uniform equivalent material density $\overline{\rho_{eq}}$ (according to Eq. 23).

$$\overline{\rho_{eq}} = \frac{\gamma_w}{g}\cdot\left\{1+\frac{\gamma_b}{\gamma_w}\cdot\frac{R}{2\cdot\overline{t_w}}\cdot\left(\frac{1}{1-\frac{tg\left(\phi_r\right)}{6\bar{\Delta}}}\right)\cdot m_{eff}+\frac{\gamma_r}{\gamma_w}\cdot\sqrt{1+tg\left(\alpha_r\right)^2}\cdot\frac{t_r}{2\cdot\overline{t_w}\cdot H_{beam}}\right\} \tag{23}$$

5.Experimental verification of the analytical formulation and modeling technique

5.1 Experimental results from the scientific literature

Table 1 compares the values of the first natural frequencies of grain silos available from the scientific literature with those obtained according to Eq. (19). Almost all the test are performed on squat and intermediate slender silos with coal as ensiled material subjected to a harmonic signal HS at the resonance. For this reason, the analytical values are calculated by using a fixed *effective mass* value of 0.8. The only exception is represented by the WN test of [18], whose analytical frequency is estimated by using the *effective mass* value given by Eq. (9). On average, excluding one of the test by [11] (relative error 100 %), the relative error in the prediction of the first natural frequency is of the order of 15%. For the single test of [18], the relative error is of the order of 5%.

Table 1 - Comparison of the experimental fundamental frequencies of flat-bottom ground-supported circular silo specimen filled with granular material and the analytical prediction by Eq. (19)

Reference	Specimen			Frequencies				
	Wall material	Δ [-]	Ensiled material	Type	a [g]	Experimental	Analytical [Eq. 19] (*)	Relative error [%]
[10]	Acrylic resin	1.01	Coal	HS	0.05	19 (*)	22.5	-18
[11]	PVC resin	1.01	Coal	HS	0.30	13 (*)	13.5	-4
	PVC resin				0.10	20 (*)	20.5	-3
	PVC resin				0.10	22 (*)	29	-32
	Steel				0.10	23 (*)	46	-100
[13]	Acrylic plastic	1.33	Coal	HS	0.05	28.6 (*)	36	-26
					0.10	31.0 (*)	36	-16
					0.20	33.7 (*)	36	-7
					0.30	28.6 (*)	36	-26
			Slag		0.10	24.5 (*)	25	-2
[18]	Polycarbonate (roughneed)	1.0	Ballottini glass	WN	0.10	15.6	14.9	4

(*): for the harmonic tests (HS) m_{eff} is set equal to 0.80

5.2 The natural periods of realistic grain-silos

The silos already analyzed by [32] are here considered in order to compare the analytical and numerical estimations of the natural periods for realistic cases. Five cases are analyzed. All the studies refer to flat-bottom silos filled with wheat. The slenderness ratios Δ varies between 0.65 and 5.2, while the silo diameter varies between 5 m and 10 m. The silos have stepwise wall thickness variation (increasing from the top to the bottom). For each specimen, a FE model has been developed following the modeling technique described in section 4. Table 2 summarizes the values of the first period of vibration according to the analytical Eq. (19), the code-like Eq. (22) and the FE models. The level of accuracy decreases going from the FE model (uniform), to the rigorous formula Eq. (19), and finally to the simple code-like Eq. (22). It can be noted that: (i) going from the FE model to the rigorous analytical estimation of Eq. (19) differences are appreciated especially for larger slenderness ratios (order of 50%); (ii) going from the rigorous analytical estimation of Eq. (19) to the simple code-like formula more significant discrepancies appear. Nonetheless, the simple equation seems adequate to capture the essence of the response and thus a potential code-like candidate.

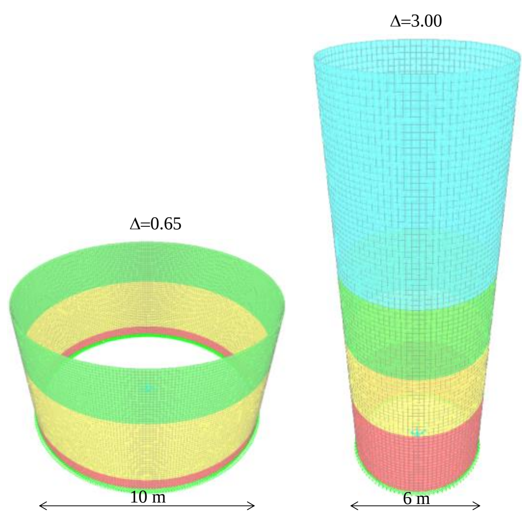


Fig. 5 - FE models for the squat silo (Δ =0.65) and the slender silo (Δ =3.00) with stepwise variation of the wall thickness and uniform equivalent wall density

Table 2 - Comparison of the fundamental period of realistic flat-bottom ground-supported circular silos filled with wheat with various slenderness ratios, according to the proposed analytical formulation, the code-like formula and FE simulations

Geometrical properties		First natural period		
$\bar{\Delta}$ [-]	d_c [m]	Code-like formula Eq. (22) [s]	Rigorous formula Eq. (19) [s]	FE model [s]
0.55	10.0	0.04	0.06	0.06
1.37	7.6	0.11	0.12	0.14
1.94	6.8	0.17	0.16	0.15
2.88	6.0	0.28	0.27	0.23
5.08	5.0	0.62	0.60	0.46

6. Conclusions

In the present paper, an analytical formulation for the estimation of the natural periods of grain silos is proposed. The formula is grounded on the Silvestri-Pieraccini theory. The silo is modelled as an equivalent shear-flexural cantilever beam with an applied mass equal to the mass of the silo structure plus the mass corresponding to the portion of the ensiled mass which is activated during the earthquake ground motion. Doing so, a fully analytical formula has been derived. The fully analytical expression of the natural period is quite heavy and it has not been explicitly provided. Nevertheless, it can be easily implemented even in a simple excel spreadsheet. In addition to the fully analytical formula, an approximate code-like formula for steel silos containing common grain-like materials has been finally derived. Finally, a modeling technique to be easily implemented in a commercial finite element software has been proposed.

7. References

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