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THE SEISMIC ASSESSMENT OF THE ASINELLI TOWER IN BOLOGNA

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Abstract

The Asinelli tower located in the city center of Bologna (northern Italy), whose construction began at the beginning of the XII century, is one of the tallest masonry towers in Italy, reaching a height of almost 100 m. Due to the large amount of stress induced at the basement, it experienced some soil settlement which leaded to a tilt toward West of 2.23 m. For its structural configuration, the tower appears prone to seismic-induced damages and, therefore, an assessment of its dynamic properties is of primary importance in order to evaluate its own safety against seismic actions. In the present paper, first an assessment of the main dynamic properties of the Tower (i.e. natural frequencies and mode shapes) is carried out by cross-correlating the response of different structural models (from simple continuum analytical models to more complex finite element models) with the results of a continuous dynamic monitoring performed in 2012 and 2013-2014 by the Italian Institute of Geophysics and Volcanology (INGV). Then, specific hazard analyses have been developed in order to identify the most probable earthquake scenarios which (i) occurred in the past and (ii) may probably occur in the future in the site of the monument. By making use of a nonlinear finite element models, the seismic vulnerability assessment of the actual state of the Asinelli tower has been conducted.

Keywords: masonry tower, dynamic monitoring, seismic vulnerability assessment

1. Introduction

The Asinelli Tower and generally all masonry buildings are typically featured by inherent seismic vulnerability due to the negligible tensile resistance of the masonry material and the peculiar geometrical configuration. The evaluation of the seismic behavior of historical masonry structures is particularly difficult to evaluate due to their uncertainty on structural schemes and mechanisms, materials properties and state of conservation. Moreover, limited experimental tests should be performed on historical buildings in order to preserve their architectural integrity. This means that the approach commonly used for the assessment of ordinary buildings cannot be simply adopted to these kinds of structures. In order to perform structural (static or seismic) analyses consistent with the real structural behavior is essential a thorough knowledge of the buildings. In the light of these aspects, the seismic assessment of the Asinelli tower has been investigated starting from: (i) the topographic survey of the geometry of the superstructure (ii) the materials characterization (typologies and mechanical properties); (iii) the historical reconstruction of the main interventions and modifications of the structural system; and (iv) an accurate description of the actual state of degradation (main cracks, tilts of the external walls).

An approach to obtain an exhaustive mechanical material characterization, in lack of sufficient experimental data performed on the structure, has been proposed and adopted by the authors [1]. Moreover, the knowledge of dynamic properties and site seismicity are important for an accurate estimation of the seismic safety of the historical structures. Thus, different structural models (from simple continuum analytical models to 3D finite element models of increasing complexity: models with fixed base, models accounting for the soil-structure interaction) have been developed in order to evaluate the main dynamic properties of the tower. The numerical results are compared with experimental measurements obtained from the dynamic monitoring carried out by the INGV after the 2012 Emilia Earthquake. Specific hazard analyses have also been developed in order to identify the most probable earthquake scenarios and the historical seismicity for the site of theAsinelli tower [2]. Based on these analyses, an assessment of the tower's structural response is carried out on a simple nonlinear 3D finite element model.

2. The Asinelli tower

2.1 The geometry

Between the 12th and the 13th century, a large number of masonry towers were built in Bologna. The two most prominent ones (Asinelli and Garisenda), known as the Two Towers, are the landmark of the city [3]. The Asinelli tower is the higher of the two with 97 m high and it has an inclination of 1.7° (corresponding to an overhanging of 2.5m) in the West direction (Fig. 1a). The cross section of the tower is approximately square for the whole height with a gradual decrease (almost linear) of the side width from 8.5m at the base to 6.0m at the top, excepting a sudden discontinuity at a height of 34m. The tower's base was made of big blocks of selenite stone. The remaining walls were realized in so-called "a sacco" masonry: with a thick inner wall and a thinner outer wall, with the gap being filled with stones and mortar (Fig. 1b). The total thickness of the masonry (the two outer walls plus the inner walls) decreases almost linearly from 3.15 m at the base to 0.45 m at the top. Three main discontinuities are present at 11.5 m, 34.0 m and 56.0 m. The masonry assemblies are not regular, with variations in both the width of the bricks and the thickness of the mortar (from 1.0 cm to 3.0 cm).

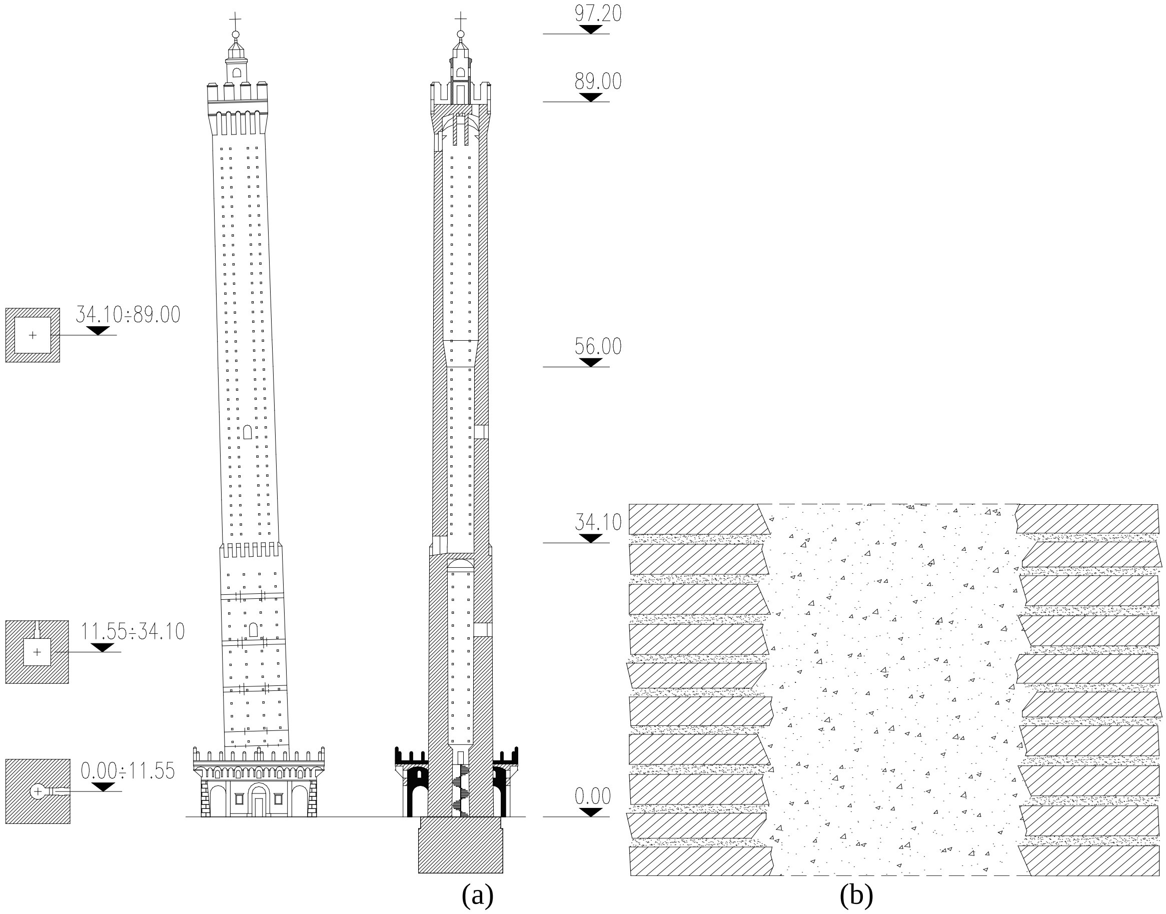


Fig. 1 – The tower elevation with the indication of the main discontinuities. (b) A schematic view (vertical cross-section) of the “a sacco” masonry.

2.2 The material properties

Limited experimental tests (in situ and laboratory tests) were performed on the Asinelli tower in order to preserve its architectural integrity. In situ tests included: (i) flat jack deformability tests (one compression test and two shear test) performed on the South side of the Tower at a height of approximately 7 m and (ii) pointing hardness tests (six tests: one on the internal wall and 5 on the external walls). The compression tests allow evaluating the masonry Young’s modulus (E) and the compression strength (f), while the shear tests allow estimating the masonry shear strength (f_v). During the pointing hardness tests, the penetration of the drill is measured and used for correlations with the previous mechanical properties. The information obtained from these tests are not sufficient for a complete characterization of the mechanical properties of the tower. An approach for the mechanical characterization in presence of insufficient experimental data has been studied and applied to the Asinelli tower in a previous work by the authors [1]. This approach is based on the integration of the material properties through the values of similar structures, as suggested by the scientific literature or by codes, and the validation of these values through analytical and numerical models [4 10]. Table 1 summarized the values obtained by the experimental tests (in bold) and the assumed properties (within brackets).

Table 1 – Experimentally measured (in bold) and assumed (within bracket) material properties.

Material		Elastic modulus (MPa)	Poisson coefficient (-)	Compressive strength (MPa)	Tensile strength (MPa)
Outer wall	Brick	**8000**	(0.1)	**9.0**	(0.8)
	Mortar	**2000**	(0.3)	**1.9**	(0.5)
	Masonry assembly	**3000**	(0.2)	**4.0**	(0.5)
Internal fill	Brick	(8000)	(0.1)	(8.0)	(1.0)
	Stone	(8000)	(0.1)	(8.0)	(1.0)
	Mortar	(1500)	(0.3)	(1.9)	(0.5)
	Masonry assembly	**3000**	(0.2)	**4.0**	(0.5)
Selenitic basement	Brick	(15000)	(0.1)	(12.0)	(0.8)
	Mortar	(2000)	(0.3)	(1.9)	(0.5)
	Masonry assembly	**4000**	(0.2)	**6.0**	(0.7)

3. The dynamic properties of the tower

The dynamic properties of the Asinelli tower (natural periods and mode shape) have been identified through a natural frequency analysis performed on different models of increasing complexity (from simple continuum analytical models to more complex finite element models) studied in detail by the authors in previous works [1]. All the models are based on the assumption of linear elastic material. The finite element models have been developed using the commercial software SAP2000. The analytical and numerical results, presented below, have also been compared to the experimental measurements of the free vibration response of the tower performed by the INGV.

3.1 Structure models

First, the fundamental flexural frequency of the tower has been studied schematizing the tower as a cantilever beam with fixed base conditions and making use of the Rayleigh's method [11]. The fundamental period obtained is equal to 3.1 sec. Then, different finite element models have been developed with the purpose of investigate the influence on the dynamic property of the tower due to: (i) soil-structure interaction, (ii) P-Δ effects and (iii) masonry orthotropic. The following models have been developed:

- 1D-FB models: monodimensional (beam) models with fixed base conditions.
- 1D-SS models: monodimensional (beam) models including soil-structure interaction.
- 1D-SS-PD models: monodimensional (beam) models including soil-structure interaction and P-Delta effects.
- 2D-FB models: bidimensional (isotropic shell) models with fixed base conditions.
- 2D-SS models: bidimensional (isotropic shell) models including soil-structure interaction.
- 2D-O-SS-PD models: bidimensional (orthotropic shell) models including soil-structure interaction and P-Delta effects

The values of the first three flexural periods and the first torsional periods as obtained from the finite element models are collected in the Table 2.

Table 2 –Flexural periods and torsional periods obtained from the different finite elements models.

Model name	Finite element	Base conditions	P-Δ	T1,f (s)	T2,f (s)	T3,f (s)	T1,t (s)
1D-FB	beam	fixed base	no	3.37	0.73	0.28	-
1D-SS	beam	soil-structure interaction	no	3.58	0.79	0.31	-
1D-SS-PD	beam	soil-structure interaction	yes	3.68	0.79	0.31	-
2D-FB	shell	fixed base	no	3.48	0.79	0.30	0.34
2D-SS	shell	soil-structure interaction	no	3.69	0.85	0.33	0.34
2D-O-SS-PD	shell	soil-structure interaction	yes	3.69	0.83	0.34	0.40

3.2 The dynamic experimental tests by the INGV

In 2012, following the seismic sequence of Emilia Romagna, started on 20th May with an M 6.2 earthquake, the Istituto Nazionale di Geofisica e Vulcanologia (INGV) was called by the local authorities to design an experiment for the dynamical monitoring of the Towers [12]. Four seismic stations were installed inside the Asinelli Tower. All the stations were equipped with triaxial seismometers. Data, sampled at 200 sps, were continuously recorded from 22th June 2012 to 17h September 2012. The seismic stations were located at 0 m, 35 m, 70 m and 86 m. Although the seismic sequence in Emilia-Romagna was active during the monitoring interval no earthquake was effectively recorded due to the strong seismic noise background of the city center. Therefore, the used dataset consists essentially of seismic ambient noise: the ground vibration induced by natural or artificial sources that propagate along the towers. The analysis of ambient noise allowed to identify the normal modes of oscillation of the towers and to distinguish between flexural, rotational and axial modes. Average hourly Fast Fourier Transform (FFT) has been computed in order to estimate the Tower frequencies of vibration (Fig. 2). The experimental frequencies show a good agreement with those estimated through FE models. The first three flexural frequencies fall within the range 0.32-0.33 Hz, 1.3-1.5 Hz and 3.0-3.3 Hz, respectively (Table 3).

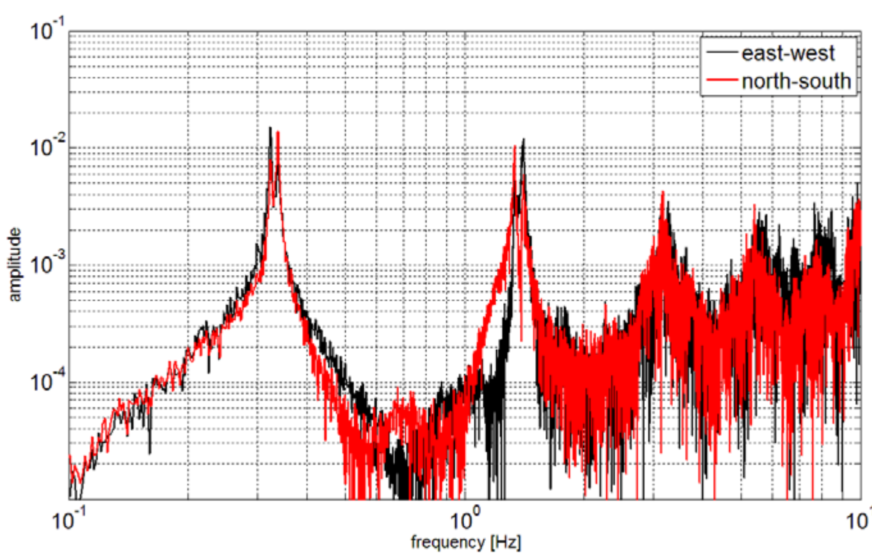


Fig. 2 – FFT of the recorded signal

Table 3 –Experimentally measured period (range)

Periods	$\mathbf{T_{1,f}}$ **(s)**	$\mathbf{T_{2,f}}$ **(s)**	$\mathbf{T_{3,f}}$ **(s)**	$\mathbf{T_{1,t}}$ **(s)**
MSS	3.0-3.3	0.72-0.76	0.30-0.32	0.42-0.45

4. Seismic hazard analysis

The main objective of the seismic hazard analysis is to compute, for a given site over a given observation time, the probability of exceeding any particular value of a specified ground motion parameter (commonly the Peak Ground Acceleration, PGA).

Typically, the seismic hazard analysis can be subdividing into:

- Probabilistic seismic hazard analysis (PSHA)
- Deterministic seismic hazard analysis (DSHA)

The probabilistic seismic hazard analysis (as performed according to the approach suggested by Cornell in 1968) assume that, in each point of the seismic zone area, the probability of occurrence of an earthquake is uniform [13]. Thus, this approach is suitable for designing new buildings and for regional planning. However, it is not appropriate for the identification of the seismic input to be adopted in the studies of monumental buildings, where the consequences of failure are intolerable and protection is needed against the worst that can be reasonably expected to occur. In these cases, the deterministic method is strongly recommended [14]. Two kinds of deterministic seismic hazard analyses have been performed for the site of the Asinelli tower:

1. Historical Deterministic Seismic Hazard Analysis (HDSHA);
2. Maximum Historical Earthquake Analysis (MHEA);

These analyses have been based on the following data:

- the ZS9 zoning (subdivision of the Italian Territory): the Asinelli tower is located in the zone 913 (http://zonesismiche.mi.ingv.it/);
- the CPTI04 earthquake catalogue (http://emidius.mi.ingv.it/CPTI04/);
- the Sabetta-Pugliese attenuation law [15];
- the Gutenberg-Richter recurrence law [16].

4.1 Historical Deterministic Seismic Hazard Analysis

HDSHA has the objective to reconstruct the intensity of historical earthquakes that have actually affected the tower in the past centuries. Significant historical earthquakes have been selected from the CPTI04 earthquake catalogue, through the following criteria:

- earthquakes that occurred within 10 km from the tower;
- earthquakes characterized by the greater magnitude that occurred in the ZS9 seismogenetic zones near to the site of the tower;
- significant earthquakes in relation to the historical information.

Table 4 shows these significant earthquakes of the past and the reconstruction of their Peak Ground Accelerations, in correspondence of the site of the tower, as obtained using the Sabetta-Pugliese attenuation law. Fig. 3 shows the reconstruction of the median of the PGA, obtained considering the epistemic uncertainty associated to the Sabetta-Pugliese ground motion prediction model, for all earthquakes of the CPTI04 earthquake catalogue. The figure indicates that during 1000 years the Asinelli tower has been hit by 2 earthquakes with acceleration around 0.20 g and 7 earthquakes with acceleration bigger than 0.12 g.

Table 4 – Reconstruction of peak Ground Acceleration (PGA) in correspondence of the site of the Asinelli tower for the selected earthquakes

Selection criteria	N.	Year	Location Name	Seismogenetic zone (ZS9)	Msp	PGA mode	PGA median	PGA mean value	PGA percentile 80%
Earthquakes that occurred within 10 km from the tower	84	1323	Bologna	913	4.25	0.120	0.145	0.160	0.220
	106	1365	Bologna	913	4.80	0.190	0.230	0.253	0.340
	142	1433	Bologna	913	4.80	0.190	0.230	0.253	0.340
	202	1505	Bologna	913	5.41	0.140	0.172	0.189	0.250
	203	1505	Bologna	913	4.25	0.130	0.154	0.169	0.230
	368	1666	Bologna	913	4.53	0.150	0.187	0.206	0.280
	692	1801	Bologna	913	4.25	0.130	0.154	0.169	0.230
	1144	1889	Bologna	913	4.53	0.060	0.073	0.081	0.110
Earthquakes characterised by the greater M that occurred near to the site of the tower	393	1688	Romagna	912	5.85	0.040	0.044	0.049	0.070
	30	1117	Veronese	906	6.49	0.040	0.054	0.059	0.080
	776	1828	Valle dello Staffora	911	5.55	0.010	0.009	0.010	0.050
	195	1501	Appennino modenese	913	5.82	0.040	0.052	0.057	0.080
	278	1584	Appennino tosco-emiliano	914	5.99	0.030	0.033	0.037	0.060
	1708	1920	Garfagnana	915	6.48	0.030	0.041	0.045	0.070
	988	1873	Liguria Orientale	916	5.47	0.020	0.018	0.020	0.110
Historical information	47	1222	Basso bresciano	906	6.05	0.020	0.027	0.029	0.480

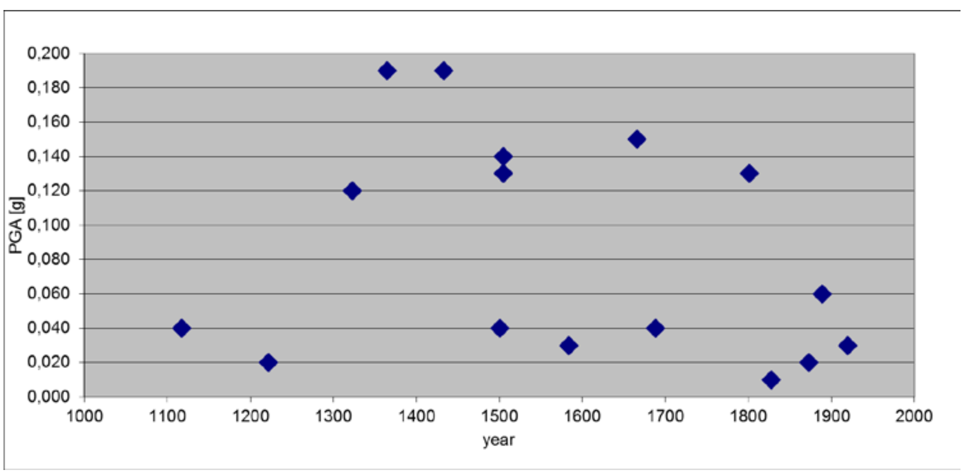


Fig. 3 – Reconstruction of the median of the PGA, obtained considering the epistemic uncertainty associated to the Sabetta-Pugliese ground motion prediction model, for all earthquakes of the CPTI04 earthquake catalogue.

4.2 Maximum Historical Earthquake Analysis

The MHEA allows to estimating the most violent earthquake that could occur in the future on the specific site of the Asinelli tower. The PGA recorded in a specific site during an earthquake depends on two factors: the magnitude and the distance between the epicentre and the site. Therefore, the worst seismic scenario for a specific site occurs with the combination of the high magnitude and null epicentre-site distance. The maximum magnitudes recorded in the past in the seismic zone (913) of the tower and also in the adjacent zones (912, 914, 915, 916,911 and 906) were obtained from the earthquake catalogue. Then, it is assumed that earthquakes of such magnitudes could occur at zero distance from the tower, and the intensity of the earthquake worse future is reconstructed considering the epistemic uncertainty associated to the Sabetta-Pugliese ground motion prediction model. Table 5 displays the list of the highest magnitudes occurred in all the considered zones and the reconstructed median, mode, mean values and 80% percentile values of the PGA variable. According to seismic activity of the two areas 913 and 912, it can be stated that a future earthquake with acceleration of about 0.50 g can occur.

Table 5 – Reconstruction of peak Ground Acceleration (PGA) in correspondence of the site of the Asinelli tower for the selected earthquakes

ZS zoning	Rmin from Cathedral	Mas max	Msp max	Mode	Median	Mean value	80% percentile
913 (zone of tower)	0.00	5.82	5.82	0.480	0.580	0.639	0.840
912	3.73	5.85	5.85	0.390	0.477	0.525	0.690
914	18.74	5.99	5.99	0.140	0.172	0.190	0.250
915	39.85	6.48	6.48	0.100	0.125	0.138	0.190
916	60.45	5.32	5.47	0.030	0.036	0.039	0.060
911	143.93	5.55	5.55	0.010	0.016	0.018	0.080
906	93.12	6.49	6.49	0.050	0.054	0.060	0.080

5. Seismic vulnerability analyses

The behavior of masonry structures, especially historical buildings, under seismic load is difficult to understand. The inherent complexity of most of historical structures, together with the natural aging due to material degradations, render the assessment of the "actual state" of the building extremely difficult and associated with high uncertainties. Moreover, the masonry is difficult to model because its properties are strongly dependent upon the properties of its constituents. Thus, the behavior of the masonry is characterized by a nonlinear response and very row resistance in presence of tensile stresses. The linear finite element models presented in the previous section are not able to take into account the opening of the cracks during the earthquakes. A simple nonlinear 3D finite element model has been developed in order to identify the most vulnerable sections of the tower.

5.1 Nonlinear 3D finite element model

The tower has been schematized by several shell elements (16 sections with a height approximately equal to the width), characterized by linear elastic behavior, with the nonlinear behavior concentrated only in the interface elements (Fig. 4a). Considering the hypothesis that the masonry has not tensile strength, the interface has been modeled through gap elements characterized by an axial behavior able to resist only compression load (Fig.4b) [17]. The distance between the sections is equal to 0.10 m. The gap elements are characterized by the following mechanical properties: the mass (m), the normal stiffness (k_1) and the shear stiffness (k_2). The mass of each gap element turns out to be:

$$m = \rho \cdot A_{\inf} \cdot 0.1m \qquad (1)$$

where ρ is the density of the masonry and A_{inf} the influence area corresponding to each gap elements (1/8 of the total area in this case). The normal and shear stiffness are obtained by the Eq. (2) and Eq. (3):

$$k_1 = \frac{E \cdot A_{\inf}}{0.1m} \qquad (2)$$

$$k_2 = \frac{G \cdot A_{\inf}}{\chi \cdot 0.1m} \tag{3}$$

where E is the Young modulus, G is the shear modulus and χ is the shear coefficient.

Table 6 collects these properties for the gap elements of each section. The soil is simulated by a system of equivalent linear springs and dampers (translational and rotational springs and dampers) where the stiffness are obtained from the soil characteristics in terms of soil density, soil elastic properties (Young's modulus and Poisson's coefficient) and shear wave velocity [1].

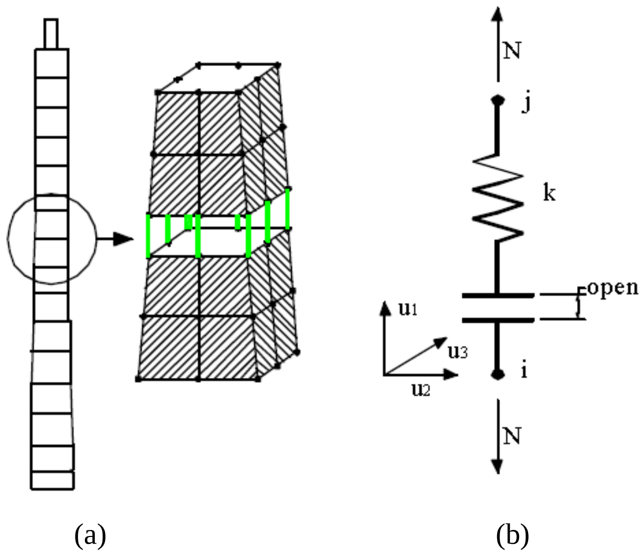


Fig. 4 – (a) The schematization of the tower (b) The gap element used in the interface of the different sections.

Table 6 – The mass (m), the normal stiffness (k_1) and the shear stiffness (k_2) for the gap elements of each sections

Section	**Quote (m)**	**m (kg)**	**k_1 (N/m)**	**k_2 (N/m)**
1	0.000	1924	$3.69 \cdot 10^{11}$	$1.28 \cdot 10^{11}$
2	3.300	1392	$2.12 \cdot 10^{11}$	$0.74 \cdot 10^{11}$
3	9.000	1170	$1.77 \cdot 10^{11}$	$0.61 \cdot 10^{11}$
4	15.275	1142	$1.72 \cdot 10^{11}$	$0.60 \cdot 10^{11}$
5	21.550	1113	$1.68 \cdot 10^{11}$	$0.58 \cdot 10^{11}$
6	27.825	1084	$1.64 \cdot 10^{11}$	$0.57 \cdot 10^{11}$
7	34.100	818	$1.24 \cdot 10^{11}$	$0.43 \cdot 10^{11}$
8	39.575	810	$1.23 \cdot 10^{11}$	$0.43 \cdot 10^{11}$
9	45.050	793	$1.20 \cdot 10^{11}$	$0.42 \cdot 10^{11}$
10	50.525	775	$1.18 \cdot 10^{11}$	$0.41 \cdot 10^{11}$
11	56.000	760	$1.16 \cdot 10^{11}$	$0.40 \cdot 10^{11}$
12	59.300	605	$0.89 \cdot 10^{11}$	$0.31 \cdot 10^{11}$
13	65.240	578	$0.94 \cdot 10^{11}$	$0.33 \cdot 10^{11}$
14	71.180	553	$0.90 \cdot 10^{11}$	$0.31 \cdot 10^{11}$
15	77.120	526	$0.86 \cdot 10^{11}$	$0.30 \cdot 10^{11}$
16	83.060	500	$0.82 \cdot 10^{11}$	$0.28 \cdot 10^{11}$

5.2 Nonlinear time history analysis

Based on the results obtained from the seismic hazard analysis and in order to develop nonlinear time history analysis on the model, two different earthquakes have been considered: 1940 EI Centre and 1980 Calitri earthquakes with a PGA around 0.313 g and 0.177g respectively. In order to identify the position of the cracks due to the seismic loads, the normal stress in the interface links has been checked. The percentage of the crack portions of each section (psf) is obtained by the ratio between the not compressed links and the total number of the links of the section. Fig 5 shows the time history of the axial force recorded by a gap element located at the section 3 (9 m). It can be noticed that the measured axial force reaches zero between 10 and 20 seconds, that means a new crack has been identified. The time history analysis has also been performed scaling the PGA of the two earthquakes of 75%, 50%, 25%. The graphs in Fig 6 show the psf for both the two earthquakes analysed and the different PGA. It is clear that with a PGA lower than 0.08 g the tower does not present damages. Instead, with a PGA bigger than 0.09 g, the tower presents several cracks mainly located at the base and the 3/4 of the height.

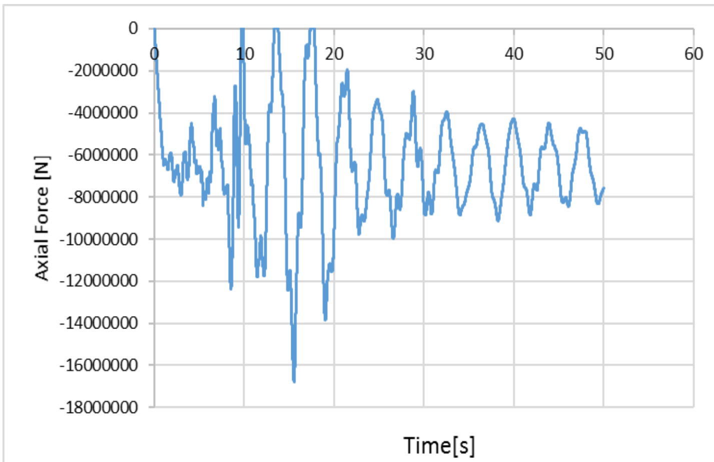
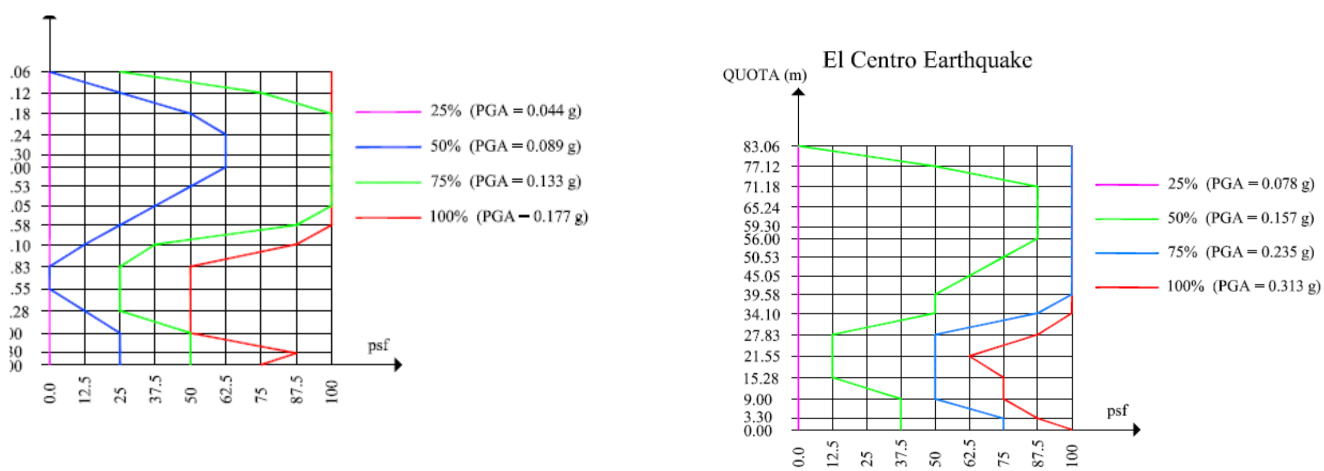


Fig. 5 – Time history of the axial force recorded by a gap element located at the section 3 (9 m).



(a) (b)

Fig. 6 – (a) Percentage of crack portions due to the Irpinia earthquake input (b) Percentage of crack portions due to the EI centro earthquake input

6. Conclusion

In the present paper, the seismic response of the Asinelli tower has been investigated. Following a reliable mechanical material characterization, the dynamic properties of the tower have been identified through different elastic structure models.

The numerical results are also compared with the fundamental frequencies as obtained from recent dynamic measurements performed by the INGV. The measured ranges of the first three lateral periods are in good agreements with the results of the numerical model.

Then, a seismic hazard analyses have been developed in order to obtain information on the characteristics of past earthquakes and predict the characteristics of possible future ones. The results of the past seismicity reveal that during 1000 years the Asinelli tower has been hit by 2 earthquakes with acceleration around 0.20 g and 7 earthquakes with acceleration bigger than 0.12 g. Moreover, from the study of the possible future earthquake it can be stated that an earthquake with acceleration of about 0.50 g could occur. Making use of the information obtained, a time history analyses have been performed on a nonlinear 3D finite element model in order to identify the more vulnerable portions of the tower under seismic load. These analyses relevel that the cracks due to seismic load are located mainly at the base and at 3/4 of the height of the tower.

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