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# PLASTIC HINGE LENGTH IN COLUMNS – DEFINITION THROUGH CONSIDERATION OF YIELD PENETRATION EFFECTS

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## *Abstract*

The plastic hinge length is defined as the length over a seismically swaying column, where flexural moments exceed the yielding capacity. This length, measured from the critical section towards the shear span, signifies the region where intense inelasticity occurs during the earthquake, and is determined in design codes through calibrated empirical relationships that account primarily for the length of the shear span and the diameter of primary reinforcing bars. The latter is meant to account in a simplistic manner for the effects of bar yielding penetration from the critical section towards both the shear span and the support of columns. Contrary to the fixed design values adopted by codes of assessment, a consistent definition of the notion of the plastic hinge length is with reference to the actual state of reinforcement – as the length over which bar strains exceed yielding. Note that the development of flexural yielding and large rotation ductilities in the plastic hinge zones of frame members is essentially synonymous with the spread of bar reinforcement yielding. Yield penetration in the anchored reinforcing bar inside the shear span of the column where it occurs, destroys interfacial bond between bar and concrete and reduces the strain development capacity of the reinforcement. This affects the plastic rotation of the member by increasing the contribution of bar slippage. In order to establish the plastic hinge length in a manner consistent to the above definition, this paper pursues the explicit solution of the field equations of bond over the shear span of a column. Through this approach, the bar strain distributions and the extent of yield penetration from the yielding cross section towards the shear span are resolved and calculated analytically. By obtaining this solution the aim is to establish a consistent definition of plastic hinge length and to illustrate the true parametric sensitivities of this design variable for practical use in seismic assessment of existing structures. Results obtained from the analytical procedures are compared with data from selected tests on reinforced concrete columns under seismic loading reported in the literature.

*Keywords: shear span; bond; yield-penetration; plastic hinge length; disturbed region*

## 1. Introduction

A large component of the deformation capacity of reinforced concrete (RC) columns is owing to pullout rotation which occurs in the critical sections near the end supports as a result of the penetration of strains both inside the support of the member (*e.g.* footing) but also inside the shear span. In columns that do not fail by web crushing, this mechanism of deformation increases gradually with imposed drift, claiming a predominant share of the members' deformation capacity near the ultimate limit state. In order to evaluate this aspect of the response, it is necessary to establish and solve the field equations of bond along the principal reinforcement of the deformed member under lateral sway, with particular emphasis on the part of the reinforcement that is strained beyond the limit of yielding into the hardening range. The regions where large inelastic strains may develop are within the so-called plastic hinge length and the anchorage length, both being adjacent to the critical section. Due to inelastic strain development, the reinforcement experiences length change; this elongation accounts for the large flexural crack that opens up at the base of the member. Other implications of rebar elongation are, (a) the vertical displacement which is reported to occur at the tip of the cantilever column during cycling under lateral loading (b) the acceleration of crushing of the concrete cover in the compression zone due to the local increase in compression strains [1].

Strain penetration occurs in the bars beyond the critical section due to the degradation of bond beyond a critical magnitude of slip that marks the initiation of the descending branch in the local bond-slip law. Analytical models representing the state of bond along the lateral surface of an embedded reinforcing bar are intended for interpretation and simulation/prediction of the behavior of structural concrete in a manner consistent with first principles. Previous studies have illustrated how detailed bond models may be used in the study of stress states arising in the assessment of the rotation capacity of RC members [2]. Through evaluation of the strain distribution it is possible to estimate the localization of excessive strain magnitudes in the critical zones, thereby enabling a novel approach for evaluation of the plastic hinge length in flexure-shear members. Additionally, the reinforcement stress and strain response and its displacement with respect to the surrounding concrete can be explicitly described through the solution of the equations of bond in the shear span of the member; this enables a detailed study of the tension stiffening phenomena, and how these affect the behavior of cracked concrete.

In this paper, a unidirectional model of bond is considered as a basis for the evaluation of the longitudinal strain distribution of the primary reinforcement of the column. Although several solutions that refer to the problem of force development along the anchorage have been proposed, yet the problem of strain penetration in the anchorage has received limited attention from researchers [3, 4]. Related studies have been conducted for lap splices developed in a region of constant moment (no shear) [5]. On the other hand, the problem of strain penetration in the shear span of the member has not been addressed explicitly yet.

## 2. Constitutive Relationships for Reinforcement to Concrete Bond

The basic equations that describe force transfer lengthwise from a bar to the surrounding concrete through bond are derived from force equilibrium applied to an elementary bar segment of length $dx$ and from compatibility between bar translation (slip), axial bar strain $\varepsilon$, and concrete strain $\varepsilon_c$ over $dx$, namely [6,7]:

$$df/dx = -(4/D_b)f_b\,,\ ds/dx = -(\varepsilon - \varepsilon_c) \cong -\varepsilon \tag{1}$$

where $f$ is the axial stress of the bar; $D_b$ is the bar diameter ; $f_b$ is the local bond stress and $s$ is the relative slip of the bar with respect to the surrounding concrete. The terms in Eq. (1) are related through the bond-slip law, $f_b = f_b(s)$ and the bar material stress-strain relationship, $f=f(\varepsilon)$. The concrete contribution to relative slip is $\varepsilon_c dx$ ; this term is neglected when dealing with normal-weight concrete, considering that the average concrete strain is an order of magnitude smaller than the average bar strain. Solution of Eq. (1) is possible though exact integration, resulting in closed-form solutions for the state of stress and strain along the anchorage, through pertinent selection of simple models for the material laws (*e.g.* piecewise linear relations). This approach has a clear advantage over the numerical solution alternative in that it enables transparent insight into the role of the various design parameters on the behavior of bar anchorages.

Solution of Eq. (1) requires that the general form of the constitutive relationships of the bar and the local bond-slip law are known (Fig. 1). Here the reinforcing bar stress-strain relationship is considered elastoplastic with hardening (representing conventional steel reinforcement) (Fig. 1a). Without loss of generality, and to facilitate derivation of closed-form solutions, a linear elastic, perfectly plastic local bond-slip relationship with residual bond is assumed. The last branch represents the residual friction between the concrete cover and the steel bar after failure of the rib interlocking mechanism (Fig. 1b). The difference between the characteristic local bond strength and the average bond strength deduced from test data is also depicted in Fig. 1b. The plateau in the local bond-slip law implies sustained bond strength. This feature is not always manifested in the test data; to be measured it requires redundancy in the anchorage (i.e., availability of longer anchorages to enable force redistribution before failure). In the assumed law the end of the plateau is marked by abrupt loss of bond strength to a residual value $f_b^{res}$. (Note that $f_b^{res}$ is taken nonzero only in the case of ribbed steel bars, but not for smooth steel bars.)

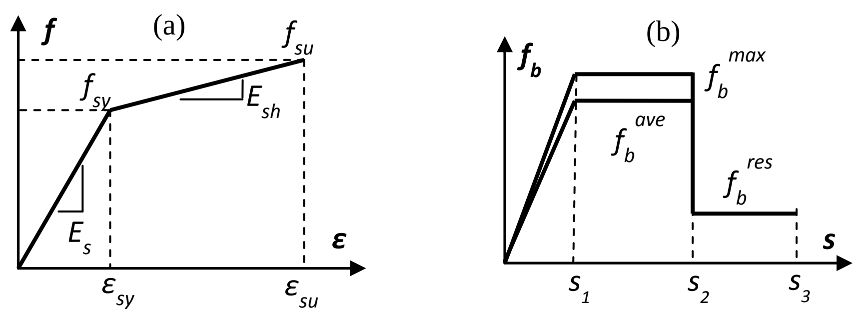


Fig 1 - (a) Stress-strain law of steel bar and (b) local bond law

## 3. Tension-Stiffening Model

It was mentioned earlier that spread of inelastic strains occurs on both sides of a critical section (*e.g.* at the base of a column). The process of inelastic strain penetration in the anchorage of a reinforcing bar has already been demonstrated in [8]. This section is dedicated to solving the same problem in the other side of the critical section, that is, along the shear span of a column. Here the problem is different from that of the anchorage only in the type of boundary conditions that may be enforced (in other words, the governing differential equation is the same); with regards to the bond-slip law, although the general form of the multilinear envelope may be taken the same, the bond strength value, $f_b^{max}$, may be less in the shear span as compared to the anchorage due to the reduced confinement available. Considering the column under lateral sway, the moment-shear relationship in the span of a cantilever RC column under horizontal loading is identical to that occurring over the length of the actual frame member extending from the inflection point at midheight (this is the point of zero moment, zero curvature) to the fixed end support.

Before any kind of cracking takes place along the length of the flexural member, the bar strain is estimated from the flexural analysis of the uncracked column cross section (*i.e.* the moment-curvature analysis) as per the Eq. (2) where $M(x)$ is the moment at distance $x$ from the support, $E$ is the elastic modulus of concrete, $I_g$ is the moment of inertia of the uncracked section with area $A$ (referred to as gross section), $N$ is the axial load, $h$ is the section height and $c$ is the cover (Fig. 2a):

$$\varepsilon(x) = \left(M(x) \cdot y_{s.na}/\left(E \cdot I_g\right)\right) - N/(E \cdot A)\,,\; y_{s.na} = (h/2) - c - 0.5D_b \qquad (2)$$

Or more generally:

$$\varepsilon(x) = \varphi(x) \cdot y_{s.na} \tag{3}$$

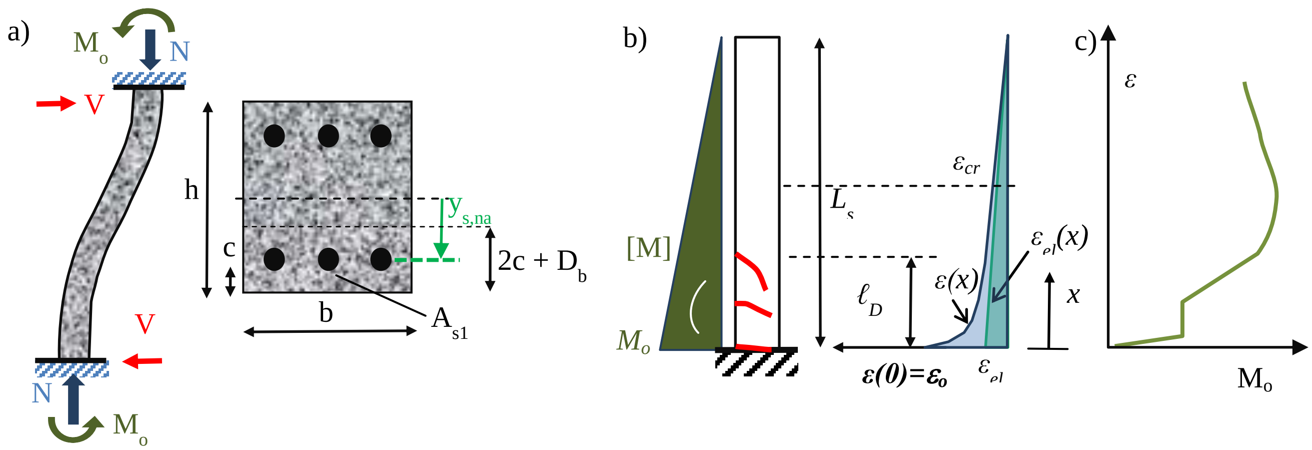


Fig 2 – a) Definition of terms for lateral swaying. b) Moment distribution along the shear span $L_s$ and definition of disturbed region, $l_D$. c) The bar strain at the critical section experiences a significant jump upon cracking even though the moment change from the uncracked to the cracked stage may be imperceptible.

with $y_{s.na}$ the distance from the neutral axis to the centroid of tension reinforcement (Fig. 2a), and $\varphi(x)$ the curvature on the cross section at distance $x$ from the support. The distance to the neutral axis changes significantly from the initial linear elastic state $y_{s.na}{}^{gr}$, to the cracked state of a cross section $y_{s.na}{}^{cr}$. If the concrete tension zone of the member is uncracked, the position of the neutral axis may be estimated from equilibrium requirements; same holds in locations where distinct cracks have formed if it may be assumed that "plane sections remain plane". Based on classical flexural analysis concepts, a RC member may be considered "cracked" in regions where the flexural moment exceeds the cracking moment. Although a large region may satisfy this definition, however, cracks occur at discrete locations $x_{icr}$. Thus, if an analysis of the cracked cross section is available, the reinforcement strains $\varepsilon(x_{icr}.)$ that occur in the crack locations may be calculated from Eq. (3). However, it is clear that in the segment between cracks, where moment may exceed the cracking value, bar strains cannot be estimated from flexural analysis as prescribed by Eq. (3). The reason is that due to reinforcement slip, the degree of strain compatibility between steel and concrete in these locations is not well understood, as would be required by the "plane-sections remain plane" assumption, nor can the concrete be considered inert as would happen in a fully cracked tension zone. Because it takes some distance from a crack location before the reinforcement may fully engage its concrete cover in tension so as to satisfy the conditions of strain compatibility, it is clear that Eq. (3) may be invalid even in regions adjacent to a flexural crack, even if the moment in these regions falls below the cracking limit. The bar strain in these regions may be estimated from solution of the differential equation of bond. To address all the possible exceptions to the validity of the flexural requirement stated by Eqs. (2,3), here the term "undisturbed" is used as a qualifier to "uncracked" in order to refer to sections that satisfy the plane sections remain plane compatibility requirement, where concrete and reinforcement strains at the same distance from the neutral axis may be assumed equal. Thus, in regions where strains are obtained from solution of the bond equation, this requirement is not valid – therefore even if apparently uncracked, the region may be "disturbed" according with this definition.

The length of shear span is referred to henceforth as $L_s$. The flexural moment in any cross section $x$ (Fig. 2b), where $x$ is measured from the face of the support, may be obtained from equilibrium with reference to the flexural moment occurring at the support, $M_o$ ($\varepsilon_o$ is the corresponding steel strain), according with:

$$M(x) = M_0 \cdot (1 - x/L_s) \tag{4}$$

As the sequence of crack formation is critical for the occurrence of disturbed regions and for the problem of strain penetration that will be subsequently addressed, in the present discussion the static problem represented by Eq. (4) will be solved for a gradually increasing moment at the support. As a starting point in the following derivation, it is assumed that the characteristic flexural resistance curve of any cross section along the shear span (*i.e.* the moment – curvature and moment – bar strain diagram) are available from classical flexural analysis (plane-sections) over the entire range of the response.

For a member with uniform primary reinforcement over its length, the moment distribution that follows Eq. (4) will cause first cracking at the face of the support. According with the preceding discussion, the bar strain at the base of a cantilever column with shear span $L_s$ experiences a significant jump upon cracking of the tension zone to maintain equilibrium. For example, if the cracked section stiffness is about 1/3 of the uncracked value, the bar strain at the critical section is expected to increase threefold by the mere occurrence of the crack even though the moment change from the uncracked to the cracked stage may be imperceptible (Fig. 2c). Thus suddenly the whole region adjacent to the cracked location becomes "*disturbed*". Over the length of the disturbed region, the reinforcement strain is described by the solution of the bond equation [4] *i.e.*:

$$\varepsilon(x) = C_1 \cdot e^{-\omega x} + C_2 \cdot e^{\omega x}\ ,\ \ \omega = [4 f_b^{max}/(E_s \cdot D_b \cdot s_1)]^{0.5} \tag{5}$$

The solution of Eq. (5) is valid provided bond is in the elastic range (ascending branch in the bond slip law). Before the creation of any other crack, the disturbed region extends over a distance $\ell_D$ from the critical section. What characterizes the end of the disturbed region is that a) at that point the gradient of the bar strain distribution, $\psi = d\varepsilon(x)/dx$, obtained from Eq. (5), matches the slope of the strain diagram as would be obtained from the flexural analysis of the member, whereas b) the bar strain $\varepsilon(x)$ at that location satisfies simultaneously Eqs. (2,3,5). Therefore, from Eq. (4) it follows that the slope of the strain gradient owing to flexural moments at uncracked location $\ell_D$ is (Fig. 3):

$$\psi = d\varepsilon(x)/dx = -\left[\underbrace{\left(M_0 \cdot y_{s.na}^{gr}\right)/EI_{gr}}_{\varepsilon_{el}}\right] \cdot 1/L_s = \omega \cdot (-C_1 \cdot e^{-\omega \ell_D} + C_2 \cdot e^{\omega \ell_D}) \tag{6}$$

$$\varepsilon(\ell_D) = C_1 \cdot e^{-\omega \ell_D} + C_2 \cdot e^{\omega \ell_D} = \varepsilon_{el} \cdot (1 - \ell_D/L_s) - N/(E \cdot A) \tag{7}$$

From the system of Eqs. (6, 7) the length of disturbed region adjacent to the crack may be determined if the moment at the support $M_o$ is known. The solution given by Eq. (5) is also subject to the following boundary condition (Fig. 2b):

$$\varepsilon(0) = C_1 + C_2 = \varepsilon_o \tag{8}$$

In an algorithm developed to solve Eq. (6,7,8) numerically, the controlling parameter is $\varepsilon_o$; therefore, at each incremental step which begins by selecting the value of $\varepsilon_o$, the corresponding moment $M_o$ is uniquely determined from the moment- bar strain diagram of the member cross section under study. Equations (6,7,8) are a system of three equations having three unknowns – given the value of $\varepsilon_o$ and $\omega$ the unknowns are $C_1$, $C_2$ and $\ell_D$. At this point it is relevant to determine the location of the next crack formation. Whether the next crack will form within the undisturbed or the disturbed region depends on the magnitude of tensile stress transferred through bond to concrete:

a) A check is performed regarding whether next cracking will occur in the disturbed region. The least value of coordinate $x < \ell_D$ should be determined that also satisfies the requirement:

$$\left[(E_s \cdot A_{s1})/(f_{ct} \cdot A_{c.eff})\right] \cdot [\varepsilon_o - \varepsilon(x)] = 1, \;\; A_{c.eff} = b \cdot (2c + D_b) - A_{s1} \tag{9}$$

In Eq. (9) $A_{s1}$ is the area of the tensile reinforcement, $A_{c.eff}$ is the area of concrete effectively engaged in tension, $f_{ct}$ is the tensile concrete strength, $b$ is the width of the section of the column (Fig. 2a). For a crack to be formed into $\ell_D$ the force undertaken by bond mechanism (i.e. $E_sA_{s1}[\varepsilon_o-\varepsilon(x)]$) should exceed the force of the effectively engaged in tension concrete (i.e. $f_{ct}A_{eff}$); in this case the left-hand expression of Eq. (9) should be >1 else no further cracking is possible in the disturbed zone as long as the reinforcement remains elastic.

b) A check is performed regarding whether next cracking will occur in the undisturbed region. Therefore the coordinate $x \geq \ell_D$ should be determined that satisfies the following requirement ($\varepsilon_{c.cr}$ is the cracking concrete strain) based on Eqs. (2,4):

$$\varepsilon(x) = \varepsilon_{el}(1 - x/L_s) - N/(E \cdot A) = \varepsilon_{c.cr} \Rightarrow x = L_s \cdot [1 - \varepsilon_{c.cr}/\varepsilon_{el} - N/(EA\,\varepsilon_{el})] \tag{10}$$

This process is repeated as the value of the strain $\varepsilon_o$ in the support is increased. If the criterion (b) controls, *i.e.* the next crack forms in the undisturbed region, then from there on this becomes the controlling strain value and the next disturbed region that begins from that point and extends away from the support is calculated. The new disturbed region is defined for this crack, $\ell_{D2}$; <u>the total disturbed region</u> of the cantilever extends from the support to the end of $\ell_{D2}$ beyond the second crack. This is denoted henceforth as $\ell_D$ (Fig. 3a). As the support strain increases this process is continued with more cracks forming towards the tip of the cantilever, with the disturbed region spreading further over the shear span. Its significance is that *over the total disturbed zone* $\ell_D$*, bar strains are calculated from the solution of the bond equation, as in this region the assumption of plane-sections remaining plane is no longer valid.*

After stabilization of cracking (no more primary cracks develop) and beyond elasticity of the steel bar the yielded segment of the disturbed region undergoes simultaneous degradation of bond. Thus, of the total length $\ell_D$, there is a segment $l_r$ where yielding has penetrated (Fig. 3b). For that portion of the disturbed zone, bar strains increase without a commensurate increase of stress: this means that bond must have degraded to zero as a consequence of Eq. (1), since $df_s/dx=0$ and thus $f_b$=0. Even if the yield-plateau is neglected, and the bar stress-strain diagram is considered bilinear with hardening, it is clear that the small hardening slope may only be supported by the residual bond strength – in other words in order for a bar to yield, it must have slipped beyond the limit $s_2$ in the bond - slip law (Fig. 1). Note that limit $s_2$ is not an intrinsic property of the bar – concrete interface as several Codes define, rather it depends on the available bonded length [9].

Similar to the derivation of strain, slip and bond plastification for the yield penetration length of an elastoplastic bar in the anchorage [8], the following equations are defined for a yielded bar in a shear span. Since hardening is included in the steel's constitutive law a residual bond strength is obtained through the application of Eq. (1). Again the solution of Eq. (1) consists of the yield penetration length $l_r$, the bond plastification length $l_p$ and the elastic bar length:

For $0 \leq x \leq l_r$:

$$\varepsilon(x) = \varepsilon(0) - \frac{4f_b^{res}}{E_{sh}D_b}x \tag{11}$$

$$s(x) = s_2 + 0.5(l_r - x)\left[\varepsilon(x) + \varepsilon_{sy}\right] \tag{12}$$

$$f_b(x) = f_b^{res} \tag{13}$$

For $l_r \leq x \leq l_r + l_p$ :

$$\varepsilon(x) = \varepsilon_{sy} - \frac{4f_b^{max}}{ED_b}(x - l_r) \tag{14}$$

$$s(x) = s_1 + 0.5\left(l_r + l_p - x\right)[\varepsilon(x) + \varepsilon_{el}^3] \tag{15}$$

$$f_b(x) = f_b^{max} \tag{16}$$

$$\varepsilon_{el}^3 = \varepsilon_{sy} - \frac{4f_b^{max}}{ED_b} l_p \tag{17}$$

For $l_r + l_p \leq x \leq \ell_D$:

$$\varepsilon(x) = C_1 \cdot e^{-\omega(x-l_p-l_r)} + C_2 \cdot e^{\omega(x-l_p-l_r)} \tag{18}$$

$$C_1 = \frac{\varepsilon(\ell_D)-\varepsilon_{el}^3 \cdot e^{\omega(\ell_D-l_p-l_r)}}{e^{-\omega(\ell_D-l_p-l_r)}-e^{\omega(\ell_D-l_p-l_r)}} \tag{19}$$

$$C_2 = \varepsilon_{el}^3 - C_1 \tag{20}$$

$$s(x) = \frac{1}{\omega}\left(C_1 \cdot e^{-\omega(x-l_p-l_r)} - C_2 \cdot e^{\omega(x-l_p-l_r)}\right) \tag{21}$$

$$f_b(x) = \frac{f_b^{max}}{s_1} \cdot s(x) \tag{22}$$

The length of yield penetration $l_r$ (Eq. 23) may be estimated if continuity of strain is considered at $x = l_r$.

$$l_r = \left(\varepsilon(0) - \varepsilon_{sy}\right) \cdot \frac{E_{sh}D_b}{4f_b^{res}} \tag{23}$$

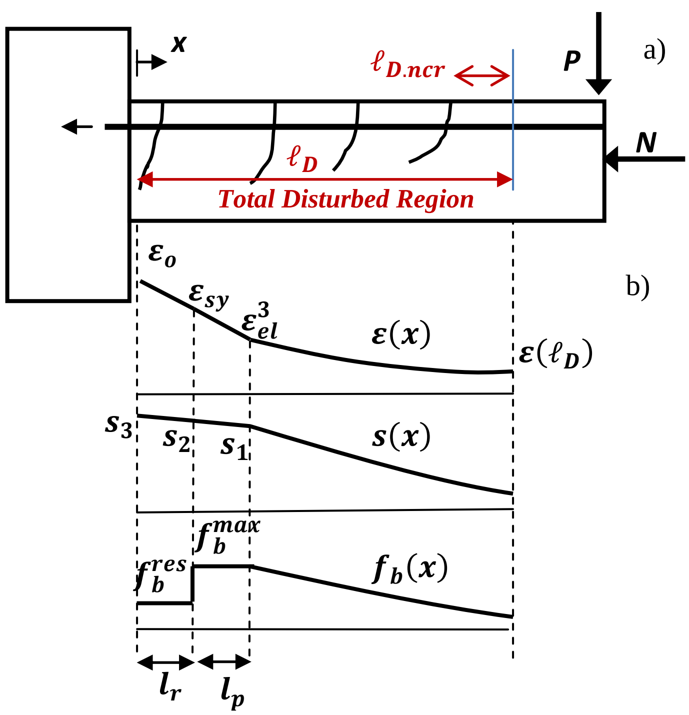


Fig.3 – a) Definition of the ongoing development of the disturbed region. b) Plastic tensile bar response in the shear span of a cantilever RC column.

The following algorithm is thus established in order to define the locations of cracks and the bar strain, slip and bond distribution along the shear span of a cantilever reinforced concrete column as well as the yield penetration length (which, in the context of the present paper, coincides with the plastic hinge length):

**1st step:** Define moment – curvature and moment – tensile bar strain diagrams for the section of the reinforced concrete column under study (identical reinforcing detailing along the shear span).

**2nd step:** Select value of bar strain after crack formation at the support, $\varepsilon_o$, (Eqs. 2,4).

**3rd step:** Find the corresponding moment, $M_o$ at the support, from moment-bar strain diagram. Solve for the length of the disturbed region emanating from the first crack.

**4th step:** Check if next crack will occur inside the disturbed region according to Eq. (9).

**5th step:** Check if next crack will occur inside the undisturbed region according to Eq. (10).

**6th step:** Define the bar strain, slip and bond distribution for the segment between crack at the support and next crack (into $\ell_D$, Fig. 3). The total disturbed region begins from the support and extends to the end of the disturbed region of the last crack. This entire zone is then represented by the solution of the bond equation according to Eqs. (11-22). If no bond plastification or yielding of the bar is present the distribution is described only by the elastic part (Eq. 18-22) with zero $l_r$ and $l_p$.. The bar distribution for the remaining segment ($L_s$-$\ell_D$) where the column remains elastic, uncracked and undisturbed is described by Eqs. (2,4) (linear).

**7th step:** Repeat steps 2 to 6 until the stabilization of cracking (no more primary cracks develop).

**8th step:** Increase in steps the bar strain at the support until the one corresponding to the ultimate moment from moment-bar strain diagram and define the bar strain, slip and bond distribution for the total disturbed region.

**9th step:** The plastic hinge length is the yield penetration length that is marked by a bond distribution segment with residual bond strength inside the total disturbed region ($l_r$ in Fig. 3b).

## 4. Numerical Results

The plastic hinge length $\ell_{pl}$ is defined in the literature as the length over which the flexural moments exceed the yielding capacity: $\ell_{pl} = (M_u-M_y) \cdot L_s/M_u$ ($M_u$ is the ultimate moment and $M_y$ is the yielding moment). However this theoretical definition does not comply with the experimental evidence; it is inconsistent too, since it would lead to a zero plastic hinge length region in the absence of hardening (when $M_y=M_u$). The plastic hinge length measured from the critical section towards the shear span signifies the region where intense inelasticity occurs during the earthquake. Despite the shortcomings associated with the mathematical definition of $\ell_{pl}$, it was considered as a convenient artifact in earthquake engineering, necessary in order to conduct calculations of plastic rotation capacity due to flexure (according with $\theta_{pl}=(\varphi_u-\varphi_y)\cdot \ell_{pl}$ [10], $\theta_{pl}$ is the plastic rotation and $\varphi_u$, $\varphi_y$ is the ultimate and yielding curvature). To avoid the inaccuracies associated with the mathematical expressions above, $\ell_{pl}$ is determined in design code procedures through calibrated empirical relationships that account primarily for the length of the shear span and the diameter of primary reinforcing bars (Eq. (24) [11], Eq. (25) [10], $h$ = depth of the member):

$$\ell_{pl} = 0.08 \cdot L_s + 0.022 \cdot D_b \cdot f_y \tag{24}$$

$$\ell_{pl} = 0.1 \cdot L_s + 0.17 \cdot h + 0.24 \cdot D_b \cdot f_y/\sqrt{f_c} \tag{25}$$

In the context of the present paper, the length of plastic hinge is by definition the length of yield penetration (thus $\ell_{pl}=l_r$), occurring from the critical section towards the shear span; physically it refers to the extent of the nonlinear region and it may be used to calculate the inelastic rotation capacity of the column in the critical section. Contrary to the fixed design values adopted by codes of assessment, the former is actually the only consistent definition of the notion of the plastic hinge length.

An example of strain, slip and bond distribution in the shear span and in the anchorage of the cantilever column is reported here, defining the plastic hinge length of the column through the proposed procedure. The result is compared with that of the empirical relationships for the plastic hinge length. The anchorage distributions are defined according to the theory established in [4].

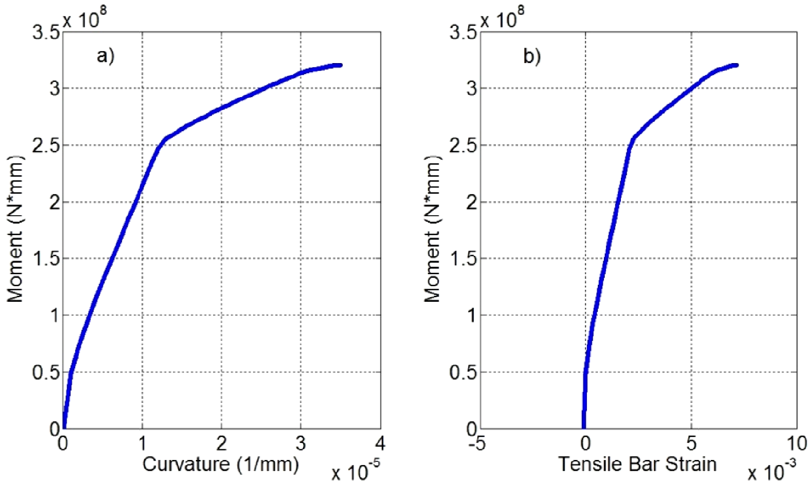


Fig. 4- a) Moment - curvature and b) moment - tensile bar strain diagrams for the column under study.

The square section of the column under consideration has a width of 350 mm with eight $D_b$=25 mm longitudinal reinforcing bars and stirrups of $D_{b,st}$=10 mm spaced at 75 mm as transverse reinforcement. The concrete strength is 34.8 MPa and the longitudinal steel yielding strength is 430 MPa with a 5% hardening. The hoops' yielding strength is 470 MPa. Concrete cover dimension is 45 mm. The shear span is one meter ($L_s$=1000 mm). The results of the moment curvature analysis are depicted in Fig 4. For the moment-curvature analysis a fiber section has been employed. The assigned to the fibers constitutive models are the modified Kent and Park model [12] for concrete and a bilinear stress-strain law with hardening (5%) for longitudinal steel reinforcement. For the foundation the bond strength was defined based on [13] as $f_{b.max}=1.25\sqrt{f_{ck}}$ (7.37 MPa) where $f_{ck}$ is the characteristic concrete strength. Instead, for the shear span in order to take into account the contribution of stirrups in bond strength the following equation is applied:

$$f_{b.max} = \frac{2\mu_{fr}}{\pi D_b}\left(2c \cdot f_{ctk} + 0.33 \cdot \frac{A_{st} f_{yw}}{N_b s}\right) \tag{26}$$

where $N_b$ is the number of tension bars (or pairs of tension spliced bars if reinforcement is spliced) laterally restrained by the transverse pressure exerted in the form of confinement by the stirrups, $c$ is the concrete cover, $A_{st}$ is the area of stirrup legs enclosing the $N_b$ lapped bars (the area of legs crossing the splitting plane), $s$ is the stirrup spacing along the member length, $\mu_{fr}$ is coefficient of friction, $f_{ctk}$ is characteristic concrete tensile strength and $f_{yw}$ is the yielding strength of stirrups. Therefore the maximum bond strength for the shear span is 7.22 (In Eq. (26): $\mu_{fr}$ =1, $f_{ctk} = 0.33\sqrt{f_{ck}}$, $N_b$ = 3). The residual bond strength $f_b^{res}$ is defined as 20% of the maximum bond strength and $s_1$ = 0.2 mm ($s_1$ is an intrinsic property of the bar-concrete interface whereas $s_2$ mainly depends on the anchorage length). The process of detecting the crack formation is described already and the results of this procedure in terms of strain distribution for the column under consideration are presented in Figs. 5, 6. It should be noted that stabilization of cracking occurs before yielding of the tensile bars but additional secondary cracks may occur near the tip of the column as the ultimate moment is approached since shifting of the cracking moment takes place. In Figure 7a the slip distribution after formation of the last crack (ultimate moment) is depicted where it can be seen that the crack width at the support is 3 mm (as the sum of slip values calculated from the anchorage and the shear span).

As it is evident from Figs. 7a,b the yield penetration length or plastic hinge length based on the proposed procedure is 217 mm (0.71$d$ or 0.22$L_s$) in the shear span and by including the yield penetration in the footing as it happens with Eqs. (25,26) is 429 mm (the yield penetration length inside the footing is 212 mm or 0.02$D_b f_y$).

Figure 8 depicts the comparison of this length with Eqs. (24), (25) as well as with the empirical expression of the plastic hinge length being equal to half of the effective depth of the column. Moreover, comparison with the classic definition of plastic hinge length (the length where the yielding moment is exceeded) is also included. It should be mentioned that the selected column has properties similar to the specimen U3 of the experimental study in [14]. The reported damage of the column can be seen in Fig. 8. It seems that the EC8-III [10] expression is closer to the reported damage and to the proposed method. The other empirical expressions for the evaluation of the plastic hinge give lower values than the proposed method.

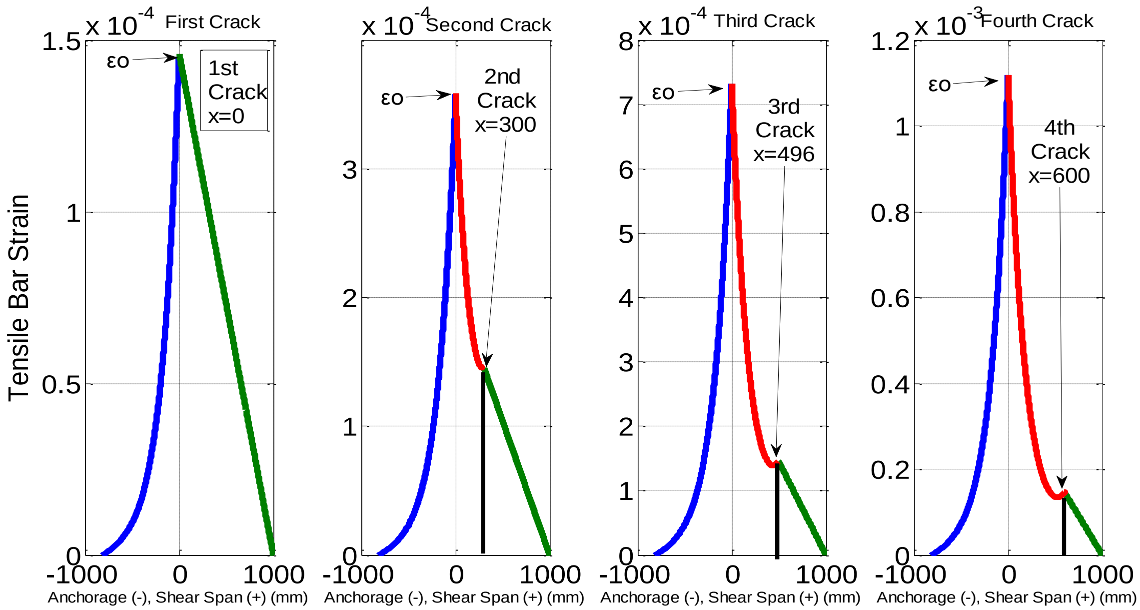


Fig. 5 - Tensile bar strain distributions along the anchorage (blue curves) and the shear span (red-green curves) where the position of the successive cracks is indicated (from 1st to 4th crack).

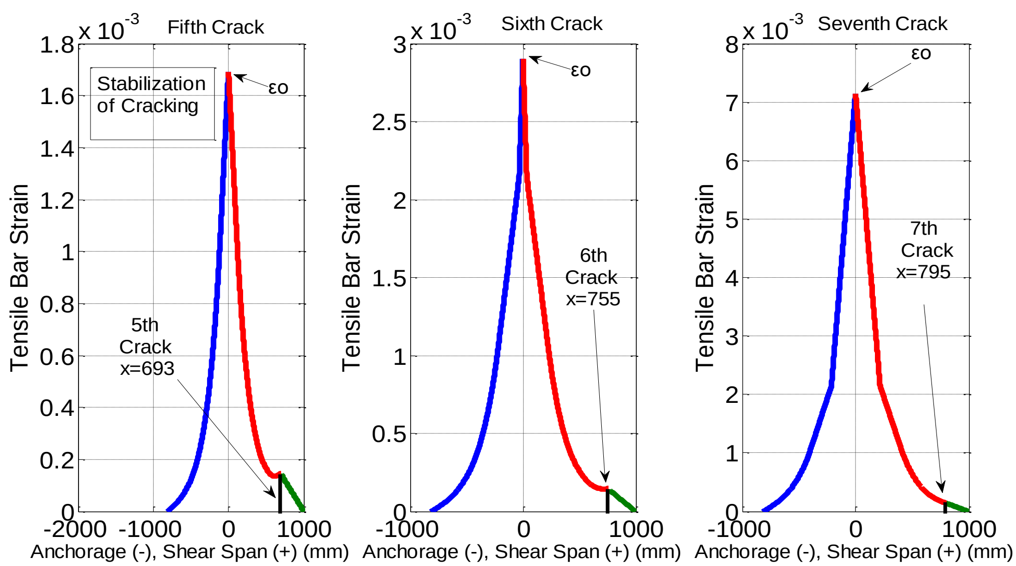


Fig. 6 - Tensile bar strain distributions along the anchorage (blue curves) and the shear span (red-green curves) where the position of the successive cracks is indicated (from 5th to 7th crack).

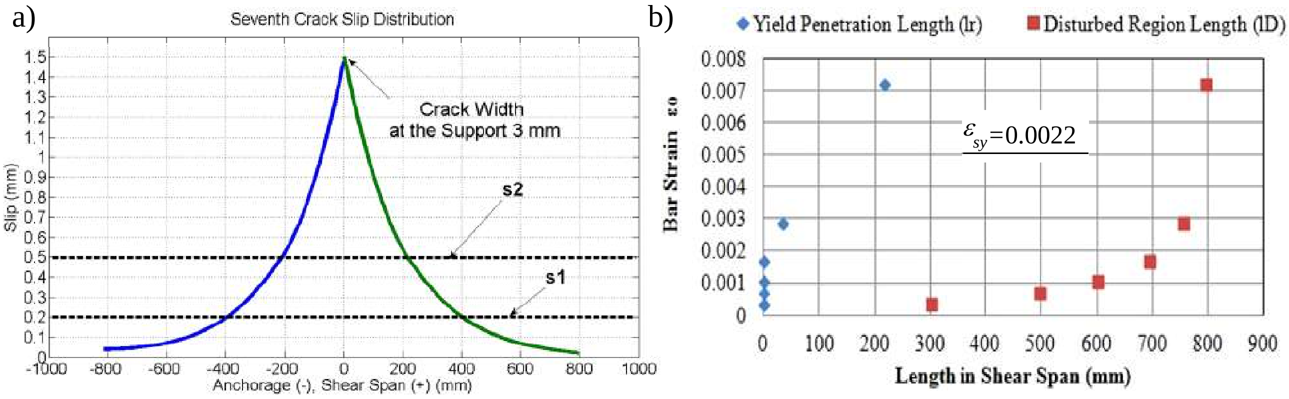


Fig.7 – a) Slip distribution in the anchorage and in the shear span of the cantilever column at the ultimate moment. b) Progress of the yield penetration and disturbed region lengths with increased tensile bar strains.

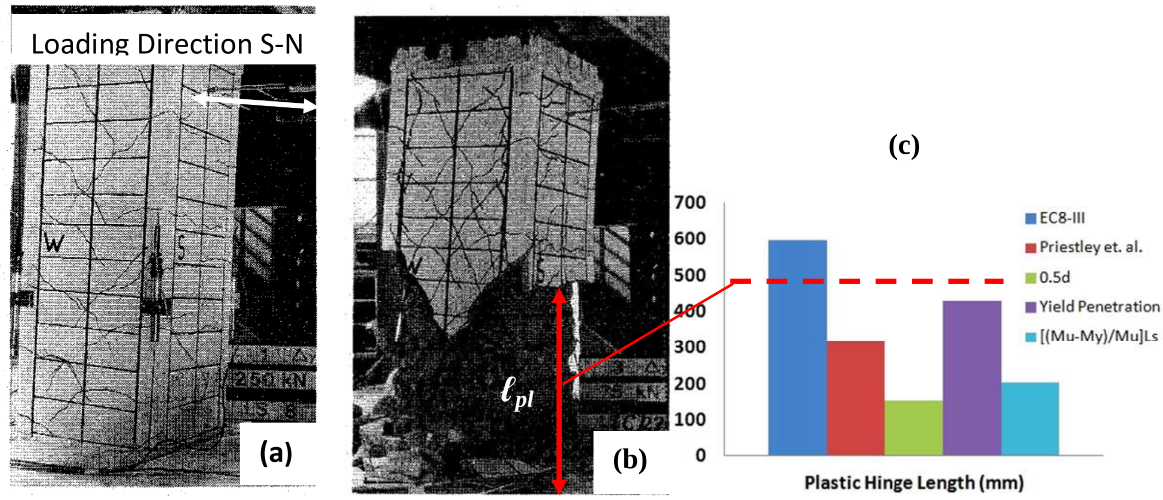


Fig. 8. a) The crack pattern of specimen U3 at yielding, b) the plastic hinge length $\ell_{pl}$ with the extended damage at $3\Delta_y$ ($\Delta_y$ the specimen's displacement at yielding) (loading at direction S-N, [14]) and c) correlation with different equations that determine $\ell_{pl}$.

## 5. Conclusions

Yield penetration occurs from the critical section towards both the shear span and the support of columns; physically it refers to the extent of the nonlinear region and determines the pullout slip measured at the critical section. Contrary to the fixed design values adopted by codes of assessment, the former is actually the only consistent definition of the notion of the plastic hinge length, whereas the latter determines the contribution of pullout rotation to column drift and column stiffness. In order to establish the plastic hinge length in a manner consistent to the above definition, this paper pursued the explicit solution of the field equations of bond over the shear span of a column. Through this approach, the bar strain distributions and the extent of yield penetration from the yielding cross section towards the shear span were resolved and calculated analytically. By obtaining this solution a consistent definition of plastic hinge length is established. The numerical results show good agreement with the experimental evidence.

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