



Study of Controlling Vertical Structural Stiffness for the Frame Structure Supported by Foundations with Different Elevations

Y.Y. Tang⁽¹⁾, Y.M. Li⁽²⁾, B.L. Jiang⁽³⁾, S.Y. Ji⁽⁴⁾, X.Y. Zhou⁽⁵⁾, G.J. W⁽⁶⁾

⁽¹⁾ Ph.D. student, School of Civil Engineering, Chongqing University, tyang90@126.com

⁽²⁾ Professor, School of Civil Engineering, Chongqing University, liyingmin@cqu.edu.cn

⁽³⁾ Ph.D. student, School of Civil Engineering, Chongqing University, jiangbaolong@cqu.edu.cn

⁽⁴⁾ Associate Professor, School of Construction Management and Real Estate, Chongqing University, jishuyan@cqu.edu.cn

⁽⁵⁾ Assistant, Gansu Construction Vocational Technical College, lzlqxiaoyan@163.com

⁽⁶⁾ Ph.D. student, School of Civil Engineering, Chongqing University, wangguojue@gmail.com

Abstract

Foundations located at different elevations will result in vertical structural irregularities of mountain structures. Usually, the vertical structural irregularities can be reduced through the control of vertical structural stiffness. The special seismic response of mountain structures lead to the failure of existing Chinese codes of controlling the vertical structural irregularities. This paper describes the methods of controlling vertical structural irregularities of mountain structures, which would result in better seismic performance of mountain structures. Based on existing methods of controlling vertical structural stiffness, mechanical characteristics of mountain structures, theoretical derivation and engineering experience, this paper offers three methods of controlling vertical structural stiffness in order to reduce the vertical structural irregularities: 1) mutual control with intra-story to story stiffness ratio; 2) equivalent stiffness ratio; 3) corresponding part stiffness ratio. The comparisons among methods pertinent to the performances of frame structure supported by foundations with different elevations (FSSFDE) are demonstrated.

Keywords: *Frame Structure Supported by Foundations with Different Elevations; Vertical structural Stiffness; Stiffness Ratio; Seismic Behavior*

1. Introduction

Vertical regularity is an important indicator in seismic design of buildings. According to Chinese codes [1, 2], the elevation configuration of buildings should be uniform and the lateral stiffness should be gradually reduced from bottom to top. Moreover, stiffness ratio is recommended as the major index to estimate vertical structural irregularity in Chinese codes [1, 2]. For frame structure supported by foundations with different elevations (FSSFDE), vertical structural stiffness varies obviously at floors near upper embedding floor. The special definitions in FSSFDE are as shown in Fig. 1. Due to the reduction of lateral force-resisting members in the floor below upper embedding floor, it is difficult to meet the requirements of vertical regularity in the existing seismic codes [1, 2]. In addition, it's unreasonable to confirm the soft story of FSSFDE completely adopting the method of controlling stiffness ratio in existing seismic codes [1, 2] according to current studies [3, 4]. New method should be presented to control vertical irregularity of FSSFDE.

For frame structure supported by foundations with different elevations (FSSFDE), not all shear force of upper embedding floor can be transferred to the floor below upper embedding floor, therefore, the force-transferring mechanism of FSSFDE is different from that of common structures. According to previous studies of seismic response of FSSFDE, Wang et.al [4, 5] proved that: 1) the deformation and internal force of FSSFDE were affected by both distribution of intra-story stiffness and distribution of story stiffness near upper embedding floor; 2) the methods of estimating vertical irregularity which provided in existing national codes were not suitable for upper embedding floor and its adjacent floors. Thus, it is necessary to research the methods of controlling the vertical structural stiffness near upper embedding floor. For other floors, the methods provided in existing codes [1, 2] are still effective.

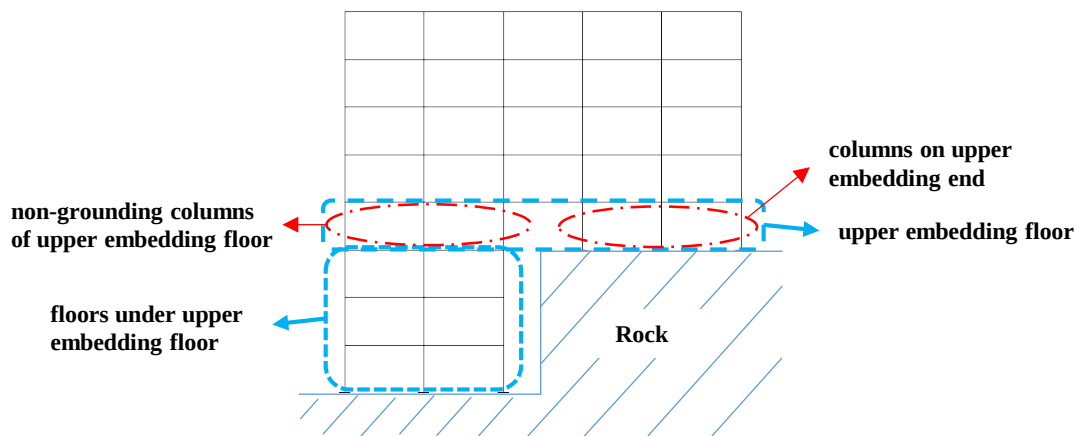


Fig. 1 – The special definitions

Based on existing methods of controlling vertical structural stiffness, mechanical characteristics of mountain structures, theoretical derivation, and engineering experience, this paper offers three methods of controlling vertical structural stiffness in order to reduce the vertical structural irregularities: 1) mutual control with intra-story to story stiffness ratio; 2) equivalent stiffness ratio; 3) corresponding part stiffness ratio. Frame structures supported by foundations with different elevations are comparatively analyzed to assess these methods.

2. Methods of controlling vertical structural stiffness

2.1 Mutual control with story and intra-story stiffness ratio

For FSSFDE, upper embedding floor and floors under upper embedding floor are deemed to be the equivalent bottom floor. Based on the concept of mutual control with story and intra-story stiffness ratio proposed by Wang [4], story stiffness ratio is defined as the stiffness of the equivalent bottom floor to stiffness of the floor above, and intra-story stiffness ratio is as shown in Fig. 2.

In Fig. 2, part1 and part 2 are simplified to column AC and column BD respectively. Due to the existence of large relative error [6], the stiffness of column AC and column BD shall not be directly calculated by D-value method. Nevertheless, Wang [4] has modified the D-value method for this kind of frame and verified its feasibility. The improved D-value method is adopted to calculate stiffness of column AC and column BD.

For frame shown in Fig. 2, part 1 is a n-bay y-story frame that has bays with width of $L1$, stories with height of h ; Part 2 is a n-bay one-story frame that has bays with width of $L2$, story with height of h .

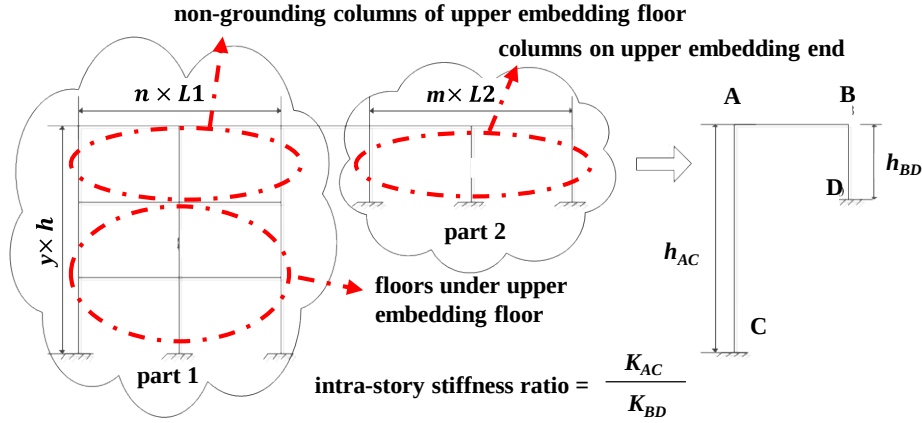


Fig. 2 – The illustration of equivalent model of bottom floor

The improved D-value method can be represented by Eq. (1) to Eq. (10). The lateral stiffness of i^{th} floor in part 1 is represented by Eq. (1), where B_{ij} is the lateral stiffness of j^{th} column at i^{th} floor.

$$D_{fi} = \sum_{j=1}^{n+1} B_{ij} \quad (1)$$

The top displacement of part 1 is the sum of every story drift of this part, so the equivalent stiffness of part 1 (D_{p1}) can be expressed as Eq. (2).

$$\frac{1}{D_{p1}} = \sum_{i=1}^y \frac{1}{D_{fi}} \quad (2)$$

The lateral stiffness of part 2 (D_{p2}) is calculated by Eq. (3).

$$D_{p2} = \sum_{k=1}^{m+1} B_{1k} \quad (3)$$

As is mentioned before, the frames were simplified to two columns AC and BD, so the stiffness of column AC and BD can be calculated by Eq. (4) and Eq. (5), where h_{AC} and EI_{AC} are the height and sectional flexural stiffness of column AC, h_{BD} and EI_{BD} are the height and sectional flexural stiffness of column BD. When EI_{AC} and EI_{BD} are assigned, h_{AC} and h_{BD} can be obtained.

$$D_{p1} = \frac{12EI_{AC}}{h_{AC}^3} \quad (4)$$

$$D_{p2} = \frac{12EI_{BD}}{h_{BD}^3} \quad (5)$$

Based on the improved D-value method [4], the lateral stiffness of column AC (K_{AC}) and column BD (K_{BD}) can be calculated by Eq. (6) and Eq. (7), and the stiffness of equivalent bottom floor is the sum of them. The beam AB is not simplified in the improved D-value method.

$$K_{AC} = (1 + \alpha_{ac}\xi) \cdot \frac{12i_{AC}}{h_{AC}^2} \quad (6)$$

$$K_{BD} = (1 + \alpha_{BD}\xi) \cdot \frac{12i_{BD}}{h_{BD}^2} \quad (7)$$

$$\xi = -\frac{3}{4 + 4 \cdot \frac{EI_{AB} \cdot h_{AC}}{EI_{AC} \cdot h_{AB}} + 4 \cdot \frac{EI_{AB} \cdot h_{BD}}{EI_{BD} \cdot h_{AB}} + 3 \cdot \frac{EI_{AB} \cdot h_{AC}}{EI_{AC} \cdot h_{AB}} \cdot \frac{EI_{AB} \cdot h_{BD}}{EI_{BD} \cdot h_{AB}}} \quad (8)$$

$$\alpha_{BD} = 1 + \frac{EI_{AB}}{EI_{AC}} \cdot \frac{h_{AC}}{h_{AB}} - \frac{1}{2} \cdot \frac{EI_{AB}}{EI_{BD}} \cdot \frac{h_{BD}^2}{h_{AC} \cdot h_{AB}} \quad (9)$$

$$\alpha_{AC} = 1 + \frac{EI_{AB}}{EI_{BD}} \cdot \frac{h_{BD}}{h_{AB}} - \frac{1}{2} \cdot \frac{EI_{AB}}{EI_{AC}} \cdot \frac{h_{AC}^2}{h_{BD} \cdot h_{AB}} \quad (10)$$

Where h_{AB} is the length of the beam AB, EI_{AB} is the sectional flexural stiffness of beam AB, ξ is the proportion coefficient of shear force produced from the rotation of column and lateral top displacement, α_{AC} and α_{BD} are the column rotation capacity coefficient of column AC and column BD respectively.

2.2 The method of equivalent stiffness ratio

The national code of China [2] has provided another method of controlling vertical structural stiffness which is called the method of equivalent stiffness ratio. The equivalent lateral stiffness ratio of bottom can be calculated by Eq. (11).

$$\gamma_e = \frac{\Delta_1 H_2}{\Delta_2 H_1} \quad (11)$$

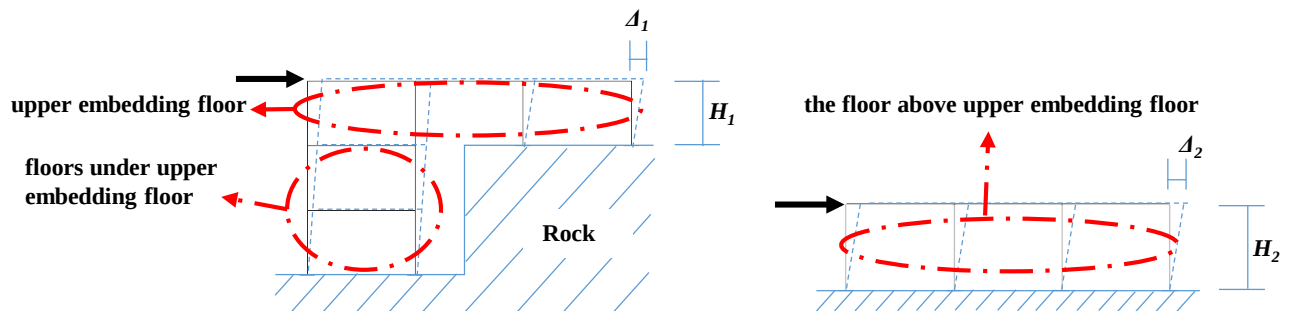


Fig. 3 – Calculation model of equivalent stiffness ratio



Where γ_e is the equivalent stiffness ratio, H_1 is the height of upper embedding floor, H_2 is the height of the floor above equivalent bottom floor, Δ_1 and Δ_2 are the lateral displacements caused by unit lateral load force at the top.

2.3 The method of corresponding part stiffness ratio

Considering mechanical characteristics of structure supported by foundations with different elevations and engineering experience, the seismic story shear force of floors under upper embedding floor is relatively small and it is not necessary to limit the abrupt change of vertical structural stiffness of the floor under upper embedding floor. Therefore, the method of controlling corresponding part stiffness ratio is proposed, in which the ratio is defined as the stiffness of non-grounding columns in upper embedding floor to counterpart stiffness in floor above upper embedding floor, and the abrupt change between upper embedding floor and the floor under upper embedding floor is not limited. For other floors, stiffness ratio is controlled according to Chinese code [1, 2].

3. Models for comparisons and analyses

In order to study the sensitivity of the methods mentioned in chapter 2, several models with the same structure were created (see Fig. 4). For each model, the only difference is the different sections of beams and columns which resulted in different stiffness of the models. These models were designed according to Chinese code [1, 2].

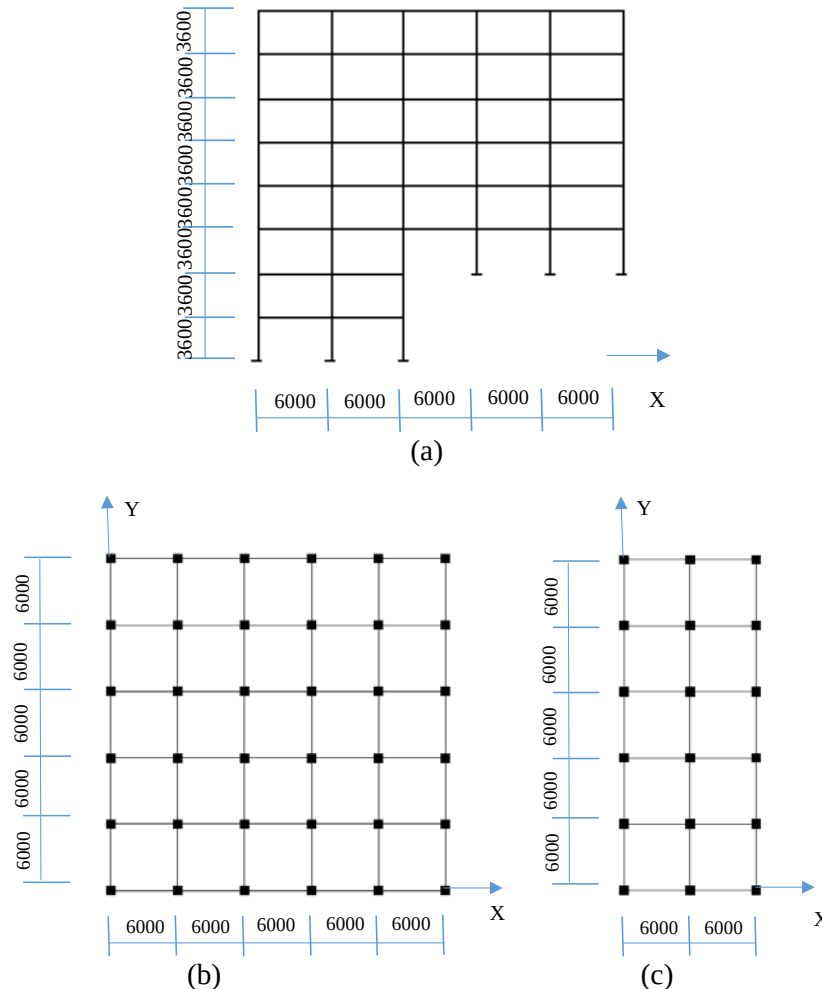


Fig. 4 – Elevation and plan of the example building(all dimensions are in mm); (a) elevation of the example building; (b) third to eighth floor plan; (c) first to second floor plan

The uniform dead load and the uniform live load, which are 4.5 kN/m^2 and 2.0 kN/m^2 respectively from 1st floor to 7th floor, are 5 kN/m^2 and 0.5 kN/m^2 respectively in the 8th floor. The line load of infilled wall is 7 kN/m . It is assumed that the predominant period of the site is 0.4 s , the design spectral response acceleration at short periods are 35 cm/s^2 and 220 cm/s^2 under frequent earthquake and rare earthquake respectively. The period time deduction factor is assumed as 0.7 in order to consider the influence of infilled wall.

Fig. 5 demonstrates the normalized response spectra of three natural ground motion records (USA01361, USA01923, USA02619) and two artificial ground motion records (ACC1, ACC2) chosen for this study. It can be seen that the normalized response spectra of chosen waves are pretty close to the target spectrum.

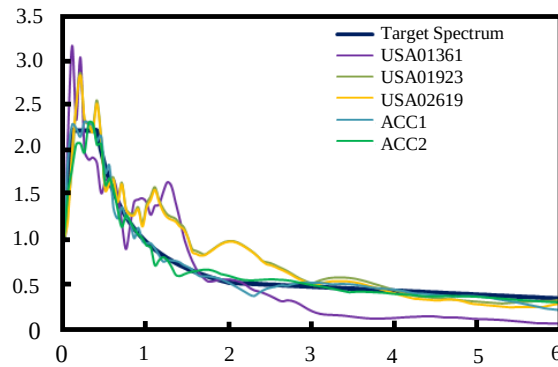


Fig. 5 –Normalized response spectra of ground motion records selected

4. Results and discussions

Deformation characteristics and distribution of plastic hinges were analyzed to assess the effectiveness of three methods described in previous chapters.

4.1 Deformation behavior

The effect of variation in equivalent stiffness ratio on elastic and plastic story drift is shown in Fig. 6. Story drift are not linear correlation with equivalent stiffness ratio as shown in Eq. (11). Owing to different lateral stiffness and fundamental frequency, the trend of changes in maximum story drift is strong model dependent, and it varies for different models. The maximum elastic story drifts are almost found at 4th floor, however the maximum plastic story drifts are found at 5th floor or 6th floor and plastic story drifts tend to be comparatively uniform.

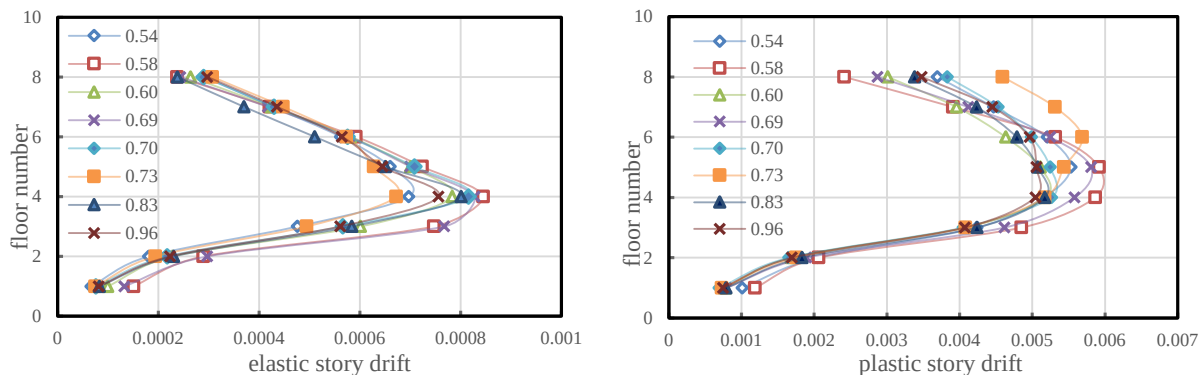


Fig. 6 –Effect of variation in equivalent stiffness ratio on story drift

The deformations of structures established by other methods is similar with that presented in Fig. 6, therefore, their deformations will not be listed any more.

4.1.1 Elastic deformation behavior

The ratio of maximum story drift $\theta_{\max}$ to average story drift θ_{avg} represents the degree of abrupt change of structural displacement curve, and it is defined as maximum relative deformation, which is always greater than 1. The larger the ratio is, the sharper the abrupt change of structural displacement is and the easier the concentration of deformation occurs.

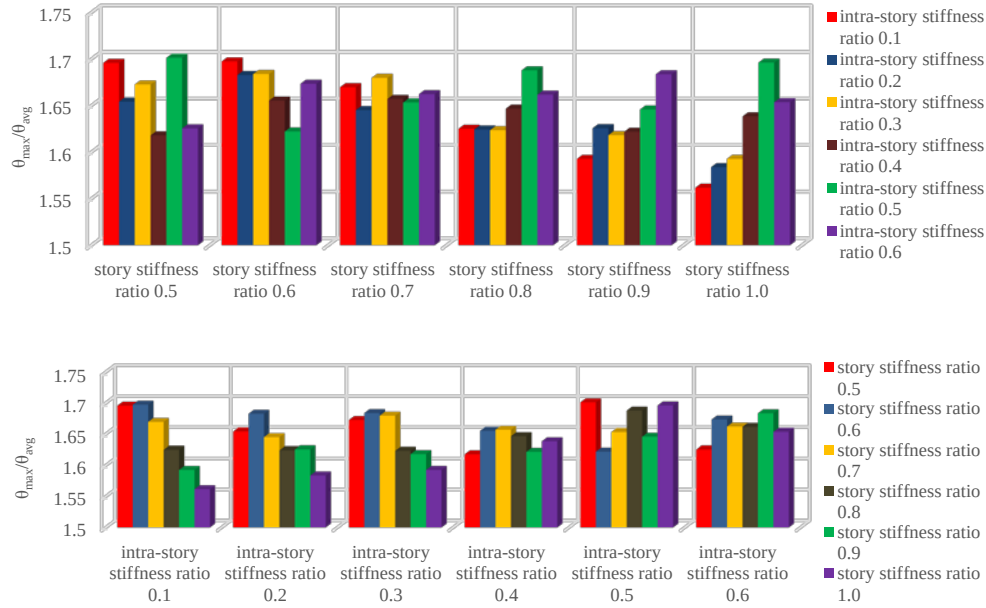


Fig. 7 – Effect of variation in intra-story stiffness ratio and story stiffness ratio on elastic maximum relative deformation

Fig. 7 shows the effect of variation in intra-story stiffness ratio and story stiffness ratio on elastic maximum relative deformation. The intra-story stiffness ratio of the models is assigned with values of 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6 to examine the effect of variation in intra-story stiffness ratio. Small story stiffness ratio (less than 1.0) imply severely stiffness weakened of bottom floor because of the foundations located at different elevations. From Fig. 7, it is found that: 1) the value of $\theta_{\max}/\theta_{\text{avg}}$ have no obvious trend with the increase of intra-story stiffness ratio when story stiffness ratio is small, however, the influence of intra-story stiffness ratio on $\theta_{\max}/\theta_{\text{avg}}$ tends to be apparent when story stiffness ratio is greater than 0.8; 2) when story stiffness ratio is smaller than 0.3, $\theta_{\max}/\theta_{\text{avg}}$ decreases as the story stiffness ratio increases, but no consistent trend of changes in $\theta_{\max}/\theta_{\text{avg}}$ is presented with increase in story stiffness ratio when intra-story stiffness ratio is larger than 0.4; 3) among all analyzed models, the smallest $\theta_{\max}/\theta_{\text{avg}}$ is achieved when story stiffness ratio is 1.0 and intra-story stiffness ratio is 0.1, in which case the deformation is considered to be the most uniform in the models.

Combining the influence rules of story stiffness ratio and intra-story stiffness ratio, the trend of structural deformation is gentle with large story stiffness ratio and small intra-story stiffness ratio.

Effect of variation in equivalent stiffness ratio on elastic maximum relative deformation is shown in Fig. 8. From this figure, it is found that the growth of equivalent stiffness ratio basically results in the growth of elastic maximum relative deformation except for individual case.

Moreover, for a series of models, their story stiffness ratios, intra-story stiffness ratios and corresponding part stiffness ratios are obtained and indicated in Fig. 9, meanwhile, values of $\theta_{\max}/\theta_{\text{avg}}$ are represented. As shown in Fig. 9, the trend of changes in corresponding part stiffness ratio is a contrast to that in equivalent stiffness ratio, that is, corresponding part stiffness ratio decreases when equivalent stiffness ratio increases. For these models, on one hand, as the increase in equivalent stiffness ratio, structural intra-story stiffness ratio basically keeps unchanged and the trend of changes in story stiffness ratio is not agree with that in equivalent



stiffness ratio. On the other hand, the trend of changes in maximum relative deformation conforms to that of story stiffness ratio. The index of equivalent stiffness ratio is incapable to represent the deformation behavior of FSSFDE.

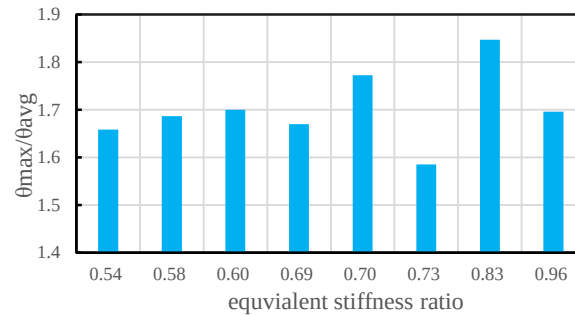


Fig. 8 –Effect of variation in equivalent stiffness ratio on elastic maximum relative deformation

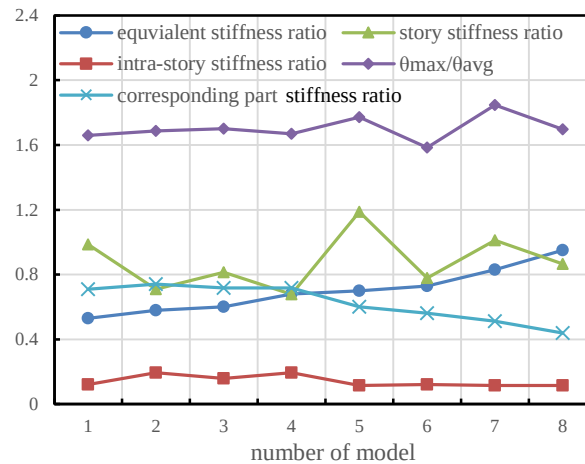


Fig. 9 – Stiffness ratio and elastic maximum relative deformation of a series of models

As to the location of elastic maximum relative deformation developed, when story stiffness ratio is less than 1.0, elastic maximum relative deformation is found at 4th floor, however, when the story stiffness ratio is equal to 1.0, the location is at the 5th floor. In models assigned with equivalent stiffness ratio and corresponding part stiffness ratio, elastic maximum relative deformation mainly is developed at 4th floor even though story stiffness ratio of some models is greater than 1.0.

On average, the influence of story stiffness ratio is the most remarkable on elastic deformation behavior of FSSFDE.

4.1.2 Plastic deformation behavior

The effect of variation in intra-story stiffness ratio and story stiffness ratio on plastic maximum relative deformation is represented in Fig. 10. It is confirmed that the effect of variation in story stiffness ratio is not consistent with each other when intra-story stiffness ratio is assigned. However, when story stiffness ratio is ensured, plastic maximum relative deformation increases as intra-story stiffness ratio increases.

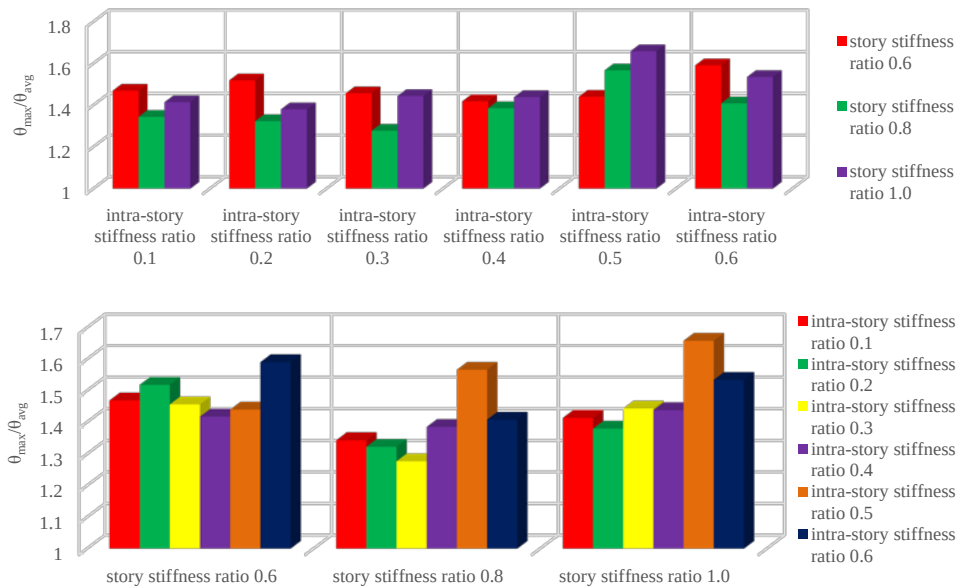


Fig. 10 –Effect of variation in intra-story stiffness ratio and story stiffness ratio on plastic maximum relative deformation

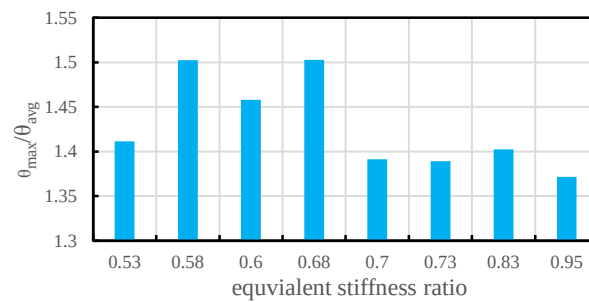


Fig. 11 –Effect of variation in equivalent stiffness ratio on plastic maximum relative deformation

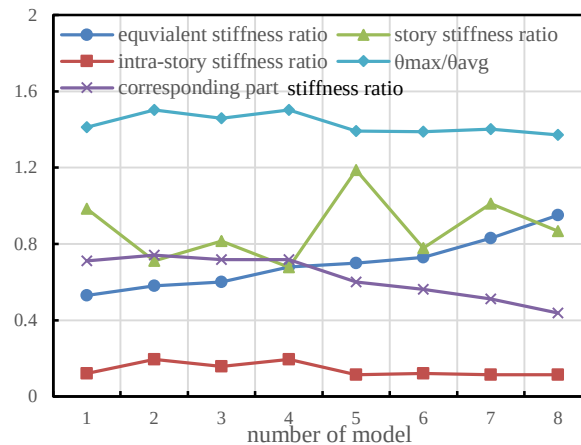


Fig. 12 – Stiffness ratio and plastic maximum relative deformation of a series of models

Effect of variation in equivalent stiffness ratio on plastic maximum relative deformation is shown in Fig. 11. Generally speaking, the larger the equivalent stiffness ratio is, the smaller the plastic maximum relative deformation is. Fig. 12 demonstrates that the plastic maximum relative deformation changes slightly and the

trend of changes in plastic maximum relative deformation conforms to that in corresponding part stiffness ratio. There is no significant relationship between story stiffness ratio and plastic maximum relative deformation.

Comparing elastic maximum relative deformations with plastic maximum relative deformations, the former are always greater than 1.5 while the latter are mostly less than 1.5, which indicates the deformation of structure tends to be uniform in elastic-plastic state. Meanwhile, the location of plastic maximum relative deformation shift upward no matter which method is based on.

In elastic-plastic state, structural deformation tends to be uniform no matter which method the structure established by.

4.2 Plastic hinges

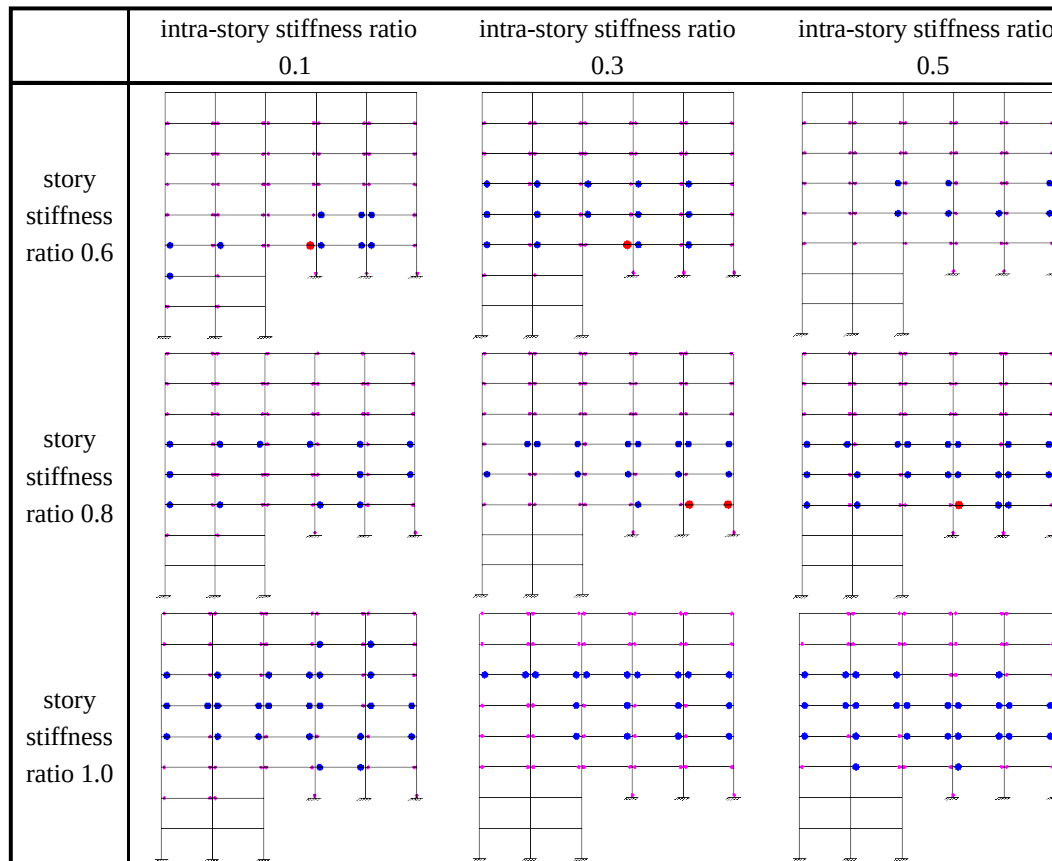


Fig. 13 – Plastic hinges of structures established by mutual control with intra-story to story stiffness ratio

The analysis of plastic hinges contains the locations and degrees of structural damage under seismic action, which reveals the failure mechanism of FSSFDE to some extent. The reasonability of structural mechanical behavior can be assessed by the analysis. Generally, ductility coefficient of sectional curvature is recommended to indicate the degree of plastic hinge. When the sectional curvature of beam or column is greater than its yield curvature, in other words, ductility coefficient of sectional curvature is greater than 1, plastic hinge is developed.

The degree of plastic hinge is divided into three levels in this article. Three colors, purple, blue and red, represent the range of ductility coefficient of sectional curvature are (1, 2], (2, 2.5] and (2.5, +∞) respectively,

Plastic hinges of models are displayed in Fig. 13. Structures subjected to rare earthquake have plastic hinges at column ends, which are mainly developed at the bottom of grounding columns of upper embedding floor and their ductility coefficients vary in (1, 2]. Mostly plastic hinges at beam ends are developed in upper

embedding floor and floors above, and ductility coefficients of plastic hinges at beam ends are generally greater at lower floors than those at higher floor. Mixed hinge failure mechanism is formed in the overall structure.

In the method of mutual control with intra-story to story stiffness ratio, plastic hinges at beam ends are developed in floors under upper embedding floor when intra-story stiffness ratio is small. When story stiffness ratio is 0.6, the shift of plastic hinges at beam ends is not significant. It is worthwhile noting that, the sensibility of story stiffness ratio on shift of plastic hinges at beam ends is more remarkable than that of intra-story stiffness ratio. With the increase of story stiffness ratio, energy dissipation of plastic hinges at beam ends at upper embedding floor decrease, and the locations of plastic hinges at beam ends are shifted to 5th and 6th floor, which is consistent with the transfer of floor of maximum relative deformation.

Plastic hinges of models with different equivalent lateral stiffness ratio are shown in Fig. 14. Plastic hinges at beam ends are developed at floors under upper embedding floor. Ductility coefficients of plastic hinges at beam ends of upper embedding floor are relatively greater. With the increase of equivalent stiffness ratio, the trend of plastic hinges shifting towards higher floors is proved to be observable. Meanwhile, the degree of plastic hinges at beam ends of upper embedding floor becomes seriously.

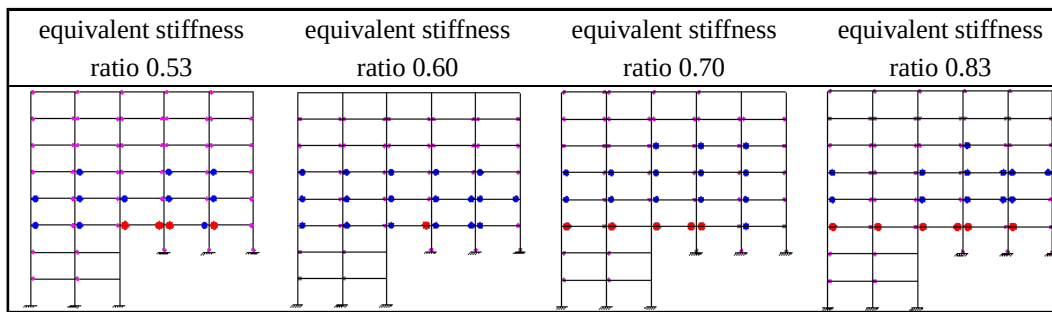


Fig. 14 – Plastic hinges distributions of structures established by equivalent stiffness ratio

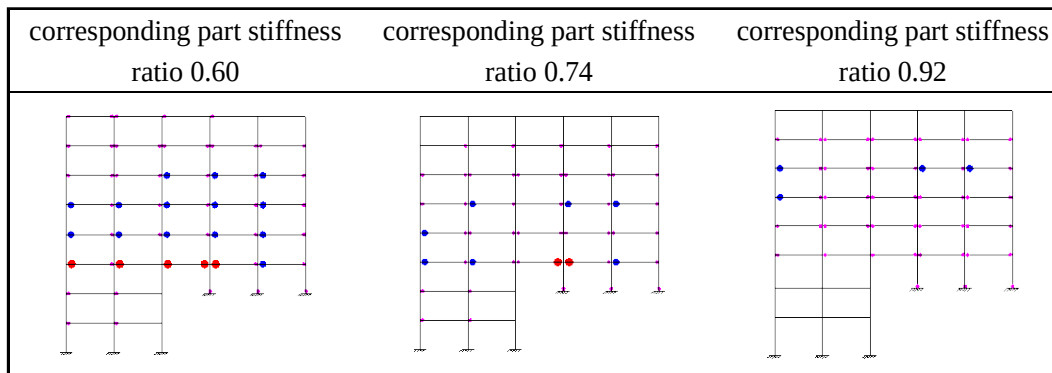


Fig. 15 – Plastic hinges of structures established by corresponding part stiffness ratio

For structure models with different corresponding part stiffness ratios, plastic hinges are presented in Fig. 15. When corresponding part stiffness ratio is 0.92, plastic hinges at beam ends with large ductility coefficients are mainly developed at 5th floor and 6th floor, and there is nearly no plastic hinges at beam ends in floors under upper embedding floor. The shift of plastic hinges is the most remarkable comparing with that of structures established by the other two control methods.

Summing up the discussions about plastic hinges, mixed hinge failure mechanism is formed in structures. The shift of plastic hinges at beam ends is obvious when story stiffness ratio is large, while it may shift to floors under upper embedding floor when intra-story stiffness ratio is small. When structures are established using the method of equivalent stiffness ratio, the plastic hinges at beam ends are occurred at higher floors and more



obvious in upper embedding floor. The shift of plastic hinges at beam ends is the most significant for models controlling corresponding part stiffness ratio.

5. Conclusions

According to the analysis of elastic deformation behavior, plastic deformation behavior and plastic hinges of FSSFDE, the following conclusions can be drawn:

1. It is necessary to control the vertical distribution of lateral stiffness for structure supported by foundations with different elevations. The method of vertical distribution of lateral stiffness is not unique, and the three methods suggested in this paper have their respective advantages.

2. No matter which control scheme is adopted, it was found that the deformation is not concentrated on specific floor, and the mixed hinge failure mechanism formed in the FSSFDE is similar to that of common structures.

3. According to the comparative analysis of structures established by different control methods, the influence of story stiffness ratio is the most remarkable to elastic deformation behavior of structure, however in elastic-plastic state, structural deformation tends to be uniform, no matter which method is used. Meanwhile, the shift of plastic hinges at beam ends is the most significant for models controlling corresponding part stiffness ratio.

4. All these three methods can keep the performance of structures in satisfactory range when appropriate ratio is adopted. Considering the calculation of first two methods is complicated, the method of corresponding part stiffness ratio is suggested to be used in practical design.

6. Acknowledgements

The research in this paper was supported by the National Natural Science Foundation of China (51638002, 51478067 and 51478068).

7. References

- [1] GB 50011-2010 (2010): *Code for seismic design of buildings*. The Press of China Building Industry, Beijing.
- [2] JGJ 3-2010 (2010). *Technical specification for concrete structure of tall building*. The Press of China Building Industry, Beijing.
- [3] B. Long (2010): Study on Several Problems Related to the Seismic Design of Structure Supported by Foundations with Different Locations. Master's thesis, Chongqing University, Chongqing, China. (in Chinese)
- [4] L.P. Wang (2010): Design ground motion input and control method of lateral stiffness for building structures on the slope, PhD thesis, Chongqing University, Chongqing, China. (in Chinese)
- [5] L. He (2010): Research on the Calculate Method of Stiffness on Step-Terrace Structure and the Plastic Seismic Performance. Master's thesis, Chongqing University, Chongqing, China. (in Chinese)
- [6] C. Zhang, W. Wang, J.F. Chen (2002): Application of D-value method in structural analysis of frames with uneven height columns in the first story under lateral forces. *World Earthquake Engineering*, **18** (4), 116-122. (in Chinese)