



ENERGY-BASED SEISMIC DESIGN METHODOLOGY: A PRELIMINARY APPROACH

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Abstract

By using energy-based methods to design earthquake-resistant structures, the effect of seismic action in terms both of force and displacement demands is taken into account, as well as the cumulative effect of damage produced by cyclic loading. Energy-based methods are effective tools for seismic design, especially when control techniques such as base isolation or energy dissipation systems are used to protect the structure. Although they were established in the 1950's, design methods based on the energy balance equation require further investigation and development for use in the framework of a Performance Based Earthquake Engineering (PBEE) design approach. In particular, research efforts should address the characterization of uncertainties of the energy-based parameters involved in the design process. This paper proposes an energy-based general methodology to design seismic-resistant structures according to a PBEE approach. Ground motion prediction equations and optimal intensity measures to be applied within the proposed methodology are discussed, as well as ongoing research on key parameters used in energy-based methods such as the equivalent number of cycles and the shear strength coefficient.

Keywords: energy; performance based earthquake engineering; intensity measures



1. Introduction

Performance Based Earthquake Engineering (PBEE) aims at designing structures that are able to satisfy multiple target performance levels at different ground motion intensities. Performance levels may be introduced into the overall design process through energy concepts. By using an energy-based approach, the design of structures protected by control systems such as base isolation or energy dissipation devices can be efficiently optimized.

The fundamentals of the energy-based design approach were established by Housner in 1956 [1], when he proposed an equation to estimate the input energy for structures subjected to earthquakes. He concluded that a safe design could be achieved if the sum of elastic energy and plastic energy (energy supply) is greater than or equal to the total input energy (energy demand).

Housner made a theoretical prediction of the applicability of the energy concept to generic structures, but did not provide detailed formulations for the seismic design of multi-story frames. In the 1970's, Akiyama developed Housner's approach and devised a deterministic earthquake-resistant design method that can be applied from one-story buildings to high-rise buildings. Akiyama's developments were published in 1980 in the book titled "Earthquake resistant limit state design of buildings" (English version published in 1985) [2]. Later, Uang and Bertero [3, 4] re-examined and studied an alternative definition of the input energy to that used by Housner and Akiyama, based on using the absolute motion instead of the relative motion. Even though both approaches are derived from the same equation for dynamic equilibrium, they lead to a different interpretation of the kinetic and input energies. Further studies [5-13] developed in the late 80's and 90's confirmed that the most rational and reliable way to estimate the seismic damage to a structure is through the evaluation of the amount of energy imparted to the structure from the earthquake ground shaking.

It is worth emphasizing that the energy concept originally proposed by Housner and developed by Akiyama, Bertero and others is different from the energy concept used later by Veletsos and Newmark, and following researchers. The misunderstanding that Housner's energy concept was inherited by Veletsos and Newmark hindered sound development of the energy-based methodology. Veletsos and Newmark investigated the ratio of maximum response deformation in elastic-perfectly plastic systems to the maximum response deformation in elastic systems; and suggested making an estimate of its upper bound value by assuming the equivalence between the plastic strain energy dissipated by the elastic-perfectly plastic system subjected to monotonic loading, and the elastic strain energy stored in the elastic system. The "monotonic" energy used by Veletsos and Newmark is not the cumulated input energy by the earthquake through cyclic oscillations that is used by Housner.

Monotonic energy has received attention in the literature. Leelataviwat et al. [14] and Goel et al. [15], for example, presented a seismic evaluation procedure based on a monotonic energy balance concept. The method is based on a framework that utilizes energy demand and capacity diagrams obtained from a pushover analysis. The skeleton capacity curve of the structure is converted into an energy capacity plot that is superimposed over the corresponding energy demand plot for a given hazard level to determine the expected peak displacement demand.

Recently, Gosh and Collins [16] proposed a probabilistic design methodology, considering the hysteretic energy demand within the framework of performance-based seismic design of buildings. This methodology is based on replacing the actual multi-degree of freedom (MDOF) structure with an equivalent single degree of freedom (SDOF) system having an equal roof displacement and a uniform hazard spectrum (UHS) for the hysteretic energy, which relies on the yield-base shear force. The objective is to obtain the minimum yield-base shear force corresponding to a given probability p_i that the hysteretic energy is exceeded to some desired level, attaining a target roof displacement (performance point).

In developing an energy-based design approach, first, the earthquake input energy shall be defined. Given the energy demand, its distribution within the structure shall be determined. The design shear force at the base of the structure for dissipating the earthquake input energy must be estimated. Finally, requirements of maximum interstory drifts not exceeding given thresholds with an optimal distribution of the damage within the structure (to avoid spurious damage concentration on any story) shall be satisfied. In view of this general description of



the steps needed to develop an energy-based design approach within the probabilistic framework of the performance based design, further investigation of the following four issues is clearly required:

1. The input energy into a structure during earthquake ground motion;
2. The distribution of the input energy throughout the structure;
3. The energy absorption capacity of structural members;
4. The relationship between the cumulative plastic strain energy and the maximum inelastic displacement, that is, the equivalent number of cycles.

2. Energy-based intensity measures

In PBEE, seismic hazard is computed through a parameter that is usually indicated as Intensity Measure (IM), which should comprehensively define seismic input to the structure. In modern seismic provisions, seismic hazard is usually expressed in the form of a design spectrum where the IM is evaluated through an attenuation relationship or ground motion prediction equations (GMPEs). A GMPE is an equation for calculating the IM value as a function of different variables representative of the earthquake properties, such as magnitude, fault mechanism, source-to-site distance, and soil condition. The GMPE, used in Probabilistic Seismic Hazard Analysis (PSHA), provides a prediction of the expected (mean) value and standard deviation of the IM at a given site, and thus can be used to calculate the annual rate of exceeding a specific earthquake intensity level of interest.

Most of the GMPEs currently available in the literature provide estimates of peak ground motion values or spectral responses expressed in terms of acceleration, velocity or displacement-based parameters. Compared to the amount of research conducted in these fields, there are very few works focused on energy-based IMs for PBEE [17-22]. Recent studies [23-28] have re-evaluated energy-based IMs and showed their significant predictive capability with respect to some types of structures and Engineering Demand Parameters (EDPs) typically used to measure damage. It is also important to underline that beyond amplitude and frequency content, the input energy reflects the duration of ground motion directly through time domain integration, providing an improved basis for defining seismic hazard.

According to the energy balance equation, the input energy (absolute or relative) $E_I(t)$ can be expressed as [4] $E_I(t) = E_k(t) + E_s(t) + E_H(t) + E_\xi(t)$, where $E_k(t)$, $E_s(t)$, $E_H(t)$ and $E_\xi(t)$, denote the kinetic energy (absolute or relative), the elastic strain energy, the hysteretic energy and the damping energy, respectively. The seismic input energy, $E_I(t)$, represents seismic demand for the structure. In particular, two different input energies can be defined [4]: the absolute input energy E_{Ia} (equal to the work done by the total force applied to the base of the SDOF system in the ground displacement u_g), and the relative input energy E_{Ir} (equal to the work done by the static equivalent force in the displacement of the equivalent fixed-base SDOF system relative to the ground u). The kinetic energy, $E_k(t)$ is related to the response of the structure at time t . Depending on whether E_{Ia} or E_{Ir} is used in the energy balance equation, two different kinetic energies must be defined: the absolute kinetic energy, $E_{ka}(t)$ expressed in terms of the absolute or total velocity, or the relative kinetic energy, $E_{kr}(t)$ expressed in terms of the relative velocity. The damping energy, $E_\xi(t)$, which can be physically interpreted as the energy dissipated by inherent viscous damping of the system, is a cumulative quantity, ever increasing with time. The elastic strain energy, $E_s(t)$, is a quantity that depends on the current elastic deformation level at time t . Finally, the hysteretic energy, $E_H(t)$, is a cumulative quantity related to plastic deformations. Kinetic and elastic strain energy vanish at the end of the vibration. Absolute and relative energies input, for constant displacement ductility, are very close in the period range of practical interest (0.3s-5s). At the end of ground motion duration, $E_{Ia} = E_{Ir}$ and $E_{ka} = E_{kr}$ because ground velocity is equal to zero (as often happens at points in the ground motion duration). The difference between E_{Ia} and E_{Ir} is not relevant from the seismic resistant design standpoint, and unless very accurate values of the ground velocity are available, the evaluation of E_{Ia} is prone to significant errors. Because of this, E_{Ir} is usually preferred to E_{Ia} .



Chou and Uang [18] considered the input energy in the form of absorbed energy spectra ($E_a = E_s + E_H$). For an elastic system $E_H = 0$ and the absorbed energy (E_a , or in terms of equivalent velocity, $V_{Ea} = \sqrt{2E_a/m}$) would be equal to the recoverable strain energy, which is related to the pseudo-velocity (S_{pv}) as $E_a = mS_{pv}^2/2$. For an inelastic system, the absorbed energy is composed of the recoverable elastic strain energy, E_s , and the irrecoverable hysteretic energy, E_H . Chou and Uang [18] also suggested expressing the absorbed energy in a non-dimensional form, N_a , defined as the absorbed energy divided by the recoverable strain energy at yielding. For a given period of vibration, N_a can be represented by:

$$N_a = \frac{E_a}{E_y} = \frac{mV_{Ea}^2/2}{mg^2C_y^2T^2/8\pi^2} = \frac{4\pi^2}{T^2} \frac{V_{Ea}^2}{C_y^2g^2} \quad (1)$$

where C_y is the yield force normalized by the weight, and $E_y (= mg^2C_y^2T^2/8\pi^2)$ represents the maximum strain energy that can be stored in an elastic-perfectly-plastic SDOF system.

The standard approach to develop a GMPE is to carry out a regression analysis on IM values calculated from a database of earthquake records, by using a fixed- or a mixed-effects model. In fixed-effects models, parameters are assumed to be the same each time data is collected, while in random-effects models they are considered sample-dependent random variables. In mixed-effects models, both fixed and random effects are accounted for. Considering the example case of E_I or V_{EI} , the two models can be expressed as follows:

- $\ln(E_{Ii}, V_{EIi}) = f(M_i, R_i, \theta) + \varepsilon_i$ fixed-effects model
- $\ln(E_{Iij}, V_{EIij}) = f(M_i, R_{ij}, \theta) + \eta_i + \varepsilon_{ij}$ mixed-effects model

where $f(M_i, R_i, \theta)$ is a functional form consisting of the ground motion prediction equation, M_i is the earthquake magnitude of the i -th record, R_i is the source-to-site distance, θ is a model coefficient matrix, and ε_i is an error term that is usually assumed to be normally distributed with zero mean. The main weakness of the fixed-effects model is that it can lead to bias if data are dominated by many records from few earthquakes or recording sites. In order to overcome this drawback and to reduce the bias, a mixed-effects model can be adopted. In this model, ε_{ij} is the error term for the j -th ground motion record from the i -th earthquake, and η_i is the random effect for the i -th earthquake.

Although some ground motion prediction equations have been developed for E_I and V_{EI} , they only provide marginal distributions without information about the joint occurrence of the spectral values at different periods. In order to build new prediction models, Cheng et al. [22] evaluated the correlation coefficients between the response spectral values corresponding to different periods and components of the ground motion.

A comparison among the proposals of input energy spectra of Decanini and Mollaioli [13] (black line), Benavent-Climent et al. [29] (grey line) and Cheng et al. [22] (blue and red lines) is shown in Fig. 1. The first two are elastic input energy design spectra (EIEDS), representing an envelope of spectra. The latter considers the mean and the 84% percentiles of the absolute (V_{Ela}) and relative (V_{EIr}) input energy spectra, respectively, obtained with a SDOF subjected to a set of records. The properties of the different records used to derive the three proposals are: (i) magnitude ranged between $5.4 < M < 6.2$, soil type S2 and focal distance $5 < D_f < 12$ km for the first; (ii) peak ground acceleration equal to 0.23g, soft-medium soil and corner input energy spectrum period $T_g = 0.40$ for the second; and (iii) magnitude $M = 6.5$, fault distance 10 km and $V_{s30} = 250$ m/s for the last.

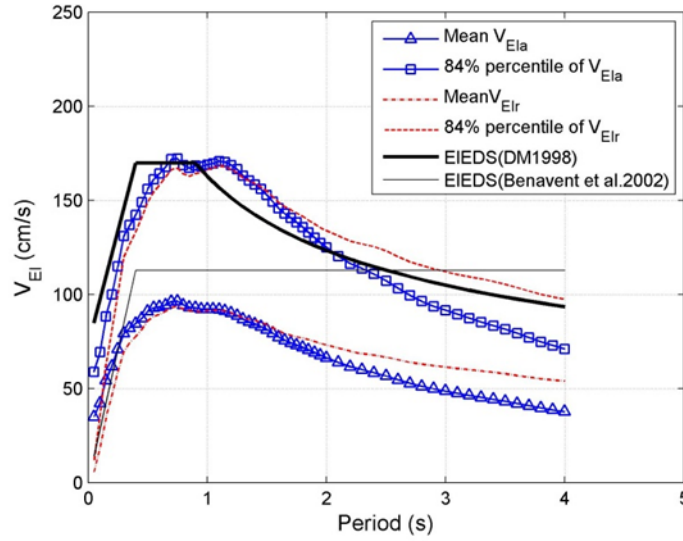


Fig. 1 – Comparison among different proposals for the input energy spectrum

Once the prediction equation of the input energy is available, it is also possible to estimate the inelastic input energy ($E_{I\mu}$) and the hysteretic energy (E_H). It was shown, in fact, that the reduction factor from the elastic input energy to the inelastic input energy is related to the force reduction factor, being slightly dependent on the hysteretic model. The hysteretic energy, on the contrary, strongly depends on the hysteretic model. In order to obtain a stable relationship, Fajfar et al. [30, 31] proposed a non-dimensional parameter, γ , defined as $\gamma = \frac{\sqrt{E_H/m}}{\omega\delta}$, where E_H is the dissipated hysteretic energy, m is the mass of the system, ω is the natural circular frequency and δ is the maximum displacement of the system, which represents the ratio between two equivalent velocity amounts.

A more stable relationship is obtained [32] if the parameter ζ defined as $\zeta = \frac{\sqrt{E_I/m}}{\omega\delta}$ [12] is used, due to the fact that input energy, as already underlined, is a parameter that scarcely depends on the hysteretic properties of the structure. Both parameters γ and ζ relate the energy demand directly to the displacement demand.

Given the inelastic input energy spectrum E_I , the dissipated energy, E_H , can be obtained from the ratio $\chi = E_H/E_I$. [31, 33, 34]. In PBEE, this parameter should be evaluated consistently with the estimation of the input energy.

To conclude, in PBEE an energy-based characterization of the ground motion intensity requires the evaluation of:

- the elastic input energy E_I (through a prediction equation) or the equivalent velocity $V_{EI} = \sqrt{2E_I/m}$, with the influence of the hysteretic model assumed to be limited;
- the hysteretic energy in accordance with the input energy spectra, obtained as follows:

$$\chi = \frac{E_H}{E_I} = f(\zeta, \text{hysteretic model}, \mu) \quad \Rightarrow E_H = \chi E_I \quad (2)$$



3. Energy-based design methodology

The Energy-Based Design Methodology is based on the energy balance equation, assessed when the seismic action has faded away. The workable form of this equation (already introduced in the previous section of the paper) is:

$$E_I = E_e + E_H + E_\xi \quad (3)$$

where $E_e = E_k + E_s$ is the elastic vibrational energy. The research carried out by Akiyama [2] showed that the first and second member of the equation could be treated as uncoupled. In other words, the first member is associated with the input energy introduced by the earthquake in the structure (demand, D) and can be computed independently of the second member, related to the energy dissipated in the structure (capacity, C). Therefore, the system can survive the seismic action if $C > D$. This is a key point in the energy-based design methodology; one of its advantages is that it lends freedom to structural engineers in their selection of a mechanism to dissipate the input energy, and therefore to quantitatively control the damage. That is, if damage is to be prevented, $E_e + E_\xi$ must be larger than E_I , with $E_H = 0$. If an amount of damage characterized by E_H is allowed, the sum $E_e + E_H + E_\xi$ must be larger than E_I . A major aspect of PBEE is controlling the degree of damage for each seismic hazard level. Energy-based methods can play an important role within this framework, as anticipated by SEAOC [35].

Rearranging terms in Eq. (3), the following expression is obtained:

$$E_D = E_I - E_\xi = E_e + E_H \quad (4)$$

where E_D was referred to by Housner [1] as the energy that contributes to damaging the structure. According to this author, E_D expressed as an equivalent velocity by $V_D = \sqrt{2E_D / M}$ is close to the maximum response velocity of an elastic SDOF, V_{max} , the latter being the upper limit of the former. The pseudo-velocity obtained from the elastic response spectrum, $S_{pv}(T)$, could be a good estimator of V_D . Alternatively, V_D can be obtained from E_I , throughout relationships proposed in the literature. E_H/E_I relationships are used for medium-high levels of damage in which the vibrational energy can be neglected in comparison with hysteretic energy ($E_H \gg E_e$) and, thus, $E_D \approx E_H$. Furthermore, the seismic demand in terms of V_E or V_D can be reasonably well estimated, depending mainly on: (i) the total mass of the structure, M ; (ii) the fundamental period of the structure, T ; and (iii) the expected level of plastic deformation.

As for the capacity of the structure, it is necessary to estimate both E_e and E_H . For E_e , Akiyama [2] put forward an equation based on the equivalence between the lateral elastic behavior of the structure and that of a continuum shear-strut model in free vibration as follows:

$$E_e = \frac{Mg^2 T^2 \alpha_1^2}{8\pi^2} \quad (5)$$

where $\alpha_1 = Q_{y,1} / Mg$ is the base yield-shear force coefficient, that is, the base yield-shear force, $Q_{y,1}$, normalized by the total weight of the structure. This equation is equivalent to $E_e = Q_{y,1} \delta_{max,1} / 2$, where $\delta_{max,1}$ is the maximum interstory drift at the ground floor.

The capacity of the structure to dissipate energy by means of plastic deformations can be assessed using: (i) equivalent viscous damping models; (ii) estimations based on math models that provide the hysteretic energy dissipated by the structure depending on the value of a required engineering demand parameter. For the latter case, it is worthwhile to define two outstanding ratios for energy-based design methods.

The first one is the ductility ratio, μ , representative of the apparent plastic deformation of the structure in each domain (positive and negative), and is defined as follows:



$$\mu = \frac{\delta_{\max} - \delta_y}{\delta_y} \quad (6)$$

where $\delta_{\max}$ and δ_y are the maximum and yield displacements, respectively, of a SDOF system. This definition can be applied to a MDOF system, by using the maximum and yield interstory drift, $\delta_{\max,i}$ and $\delta_{y,i}$, respectively, to define the ductility ratio μ_i of the generic i -th story.

The second is the cumulative hysteretic ratio of a SDOF usually related with the cyclic behavior, also known as the normalized cumulative damage owing to plastic deformations, η , which is defined as:

$$\eta = \frac{E_H}{Q_y \delta_y} \quad (7)$$

where Q_y is the yield shear force. Similarly to what was stated before, this ratio can be easily applied for the i -th story of a MDOF system, η_i , taking into account the hysteretic energy, yield shear force and yield displacement, that is, $E_{h,i}$, $Q_{y,i}$ and $\delta_{y,i}$, respectively.

For energy-design methods the ratio between η and μ is a key parameter which enables one to take into account not only the value of the input energy, but also the way that it is absorbed/dissipated by the structure. It is known as the equivalent number of cycles [2, 36], and for a SDOF system it is:

$$n_{eq} = \frac{E_H}{Q_y (\delta_{\max} - \delta_y)} = \frac{\eta}{\mu} \quad (8)$$

Furthermore, n_{eq} can be defined for each i -th story of the MDOF system as follows:

$$n_{eq,i} = \frac{E_{h,i}}{Q_{y,i} (\delta_{\max,i} - \delta_{y,i})} = \frac{\eta_i}{\mu_i} \quad (9)$$

where $E_{h,i}$ is the hysteretic energy dissipated in the i -th story. The value of n_{eq} depends mainly on both the elastic and plastic deformation capacity of the structure, as well as on the seismological characteristics of the earthquake. For this reason, it is essential to distinguish between near- and far-field ground motions.

Rearranging Eq. (8) for a SDOF, the following expression is obtained:

$$\frac{E_H}{n_{eq}} = Q_y \delta_{\max} - \frac{Q_y^2}{k} \quad (10)$$

where k is the lateral stiffness of the system. Thus, E_H/n_{eq} represents the monotonic plastic strain energy associated to the maximum displacement $\delta_{\max}$. Once E_H , n_{eq} and $\delta_{\max}$ are established, Q_y can be obtained from Eq. (10). n_{eq} is lower for near-field earthquakes than for far-field earthquakes (especially for pulse-like ground motions) and this leads to larger values of Q_y . Moreover, according to Eq. (10), the ratio E_H/n_{eq} has only one maximum for $Q_y = k\delta_{\max}/2$. Hence, if the demand E_H/n_{eq} is higher than this maximum value, the response displacement required by the structure to dissipate the energy introduced by the earthquake will exceed the $\delta_{\max}$ design value.

Therefore, n_{eq} is an outstanding parameter which relates the strength and the maximum displacement of the system with the hysteretic energy demand of the earthquake. This can be used from the design standpoint to obtain the required lateral strength, Q_y , for which the structure is capable of dissipating the hysteretic input energy without exceeding the design lateral displacement $\delta_{\max}$. Substituting E_H and E_e given by Eq. (10) and Eq. (5) in Eq. (4), one can easily obtain the required Q_y so that the system survives an input energy E_D without exceeding the maximum displacement $\delta_{\max}$.



In applying this proposal to MDOF systems, it is necessary to know how the hysteretic energy is distributed among the different stories. Akiyama [2] proposed an expression to predict the hysteretic energy accumulated in a given story i that depends on the deviation of the actual lateral strength Q_{yi} of this story in relation to an “optimum” value $Q_{yi,opt}$ that makes η_i approximately equal for all stories (i.e. $\eta_i = \eta$). Lateral strengths are expressed in terms of dimensionless yield-shear force coefficients $\alpha_i = Q_{yi} / \sum_{j=1}^N m_j g$, where N is the number of stories and m_i the i -th story mass. The optimal yield-shear force coefficient $\alpha_{i,opt}$ is normalized by the yield-shear force coefficient at the first story, formally expressed by:

$$\bar{\alpha}_i = \alpha_{i,opt} / \alpha_1 \quad (11)$$

Several expressions for $\bar{\alpha}_i$ have been proposed in the literature [2, 37] and in recent seismic codes [38].

If the strength distribution follows the optimum distribution $\bar{\alpha}_i$, the hysteretic energy distribution in the structure (termed as the standard damage distribution law) is given by:

$$\frac{E_{h,i}}{E_H} = \frac{s_i}{\sum_{i=1}^N s_i} \quad (12)$$

where the factor s_i is defined as follows:

$$s_i = \left(\sum_{j=1}^N \frac{m_j}{M} \right)^2 \bar{\alpha}_i \left(\frac{k_1}{k_i} \right) \quad (13)$$

Here k_i is the i -th story lateral stiffness and M the total mass. The expression proposed by Akiyama [2] to predict the hysteretic energy accumulated in a given story i , E_{hi} , is as follows:

$$\frac{E_{h,i}}{E_H} = \frac{s_i p_i^{-n}}{\sum_{i=1}^N (s_i p_i^{-n})} = \frac{1}{\gamma_i} \quad (14)$$

where $p_i = \alpha_i / (\alpha_1 \bar{\alpha}_i)$ and n is the so-called damage concentration index, whose value ranges between 6 and 12 depending on the susceptibility of the structure to concentrate damage, while γ_i is the damage dispersion index. The large n is, the more prone the structure is to concentrate damage. Thus, if $p_i < 1$ for the i -th story, the value of the ratio $E_{h,i}/E_H$ (damage concentration) will depend strongly on the value of n . The opposite occurs for $p_i > 1$. The Building Standard Law of Japan [38] also uses Eq. (11) to Eq. (14).

The value of the hysteretic energy of a structure whose lateral strength follows the optimum distribution $\bar{\alpha}_i$ (and therefore $\eta_i = \eta = \text{constant}$) can be obtained using Eq. (7) and Eq. (14) as follows:

$$E_H = \gamma_1 E_{h,1} = \gamma_1 Q_{y,1} \delta_{y,1} \eta = \gamma_1 \frac{Q_{y,1}^2}{k_1} \eta = \gamma_1 M^2 g^2 \frac{\alpha_1^2}{k_1} \eta \quad (15)$$

Therefore, using Eq. (5) and Eq. (15) in Eq. (2) gives the following expression:

$$\frac{Mg^2 T^2 \alpha_1^2}{8\pi^2} + \gamma_1 M^2 g^2 \frac{\alpha_1^2}{k_1} \eta = E_D \quad (16)$$



Taking into account for the i -th story: (i) the equivalent number of cycles given by Eq. (9); (ii) the value of the interstory drift given by $\delta_{y,i} = Q_{y,i} / k_i = \sum_{j=i}^N (m_j g) \alpha_i / k_i$; and (iii) the yield-shear force coefficient $\alpha_i = \bar{\alpha}_i \alpha_1$, the following expression is obtained:

$$\eta_i = \eta = n_{eq,i} \mu_i = n_{eq,i} \left(\frac{k_i \delta_{max,i}}{\bar{\alpha}_i \alpha_1 \sum_{j=i}^N m_j g} - 1 \right) \quad (17)$$

Therefore, substituting Eq. (17) in Eq. (16) gives the base yield-shear force coefficient, $\alpha_{1,i}$, required to attain a target interstory drift $\delta_{max,i}$ at the i -th story when the system is subjected to the input energy E_D . If this procedure is repeated for each i -th story, it is possible to obtain the design base yield-shear force coefficient as the maximum of all of them as follows:

$$\alpha_{1,max} = \max \{ \alpha_{1,i} \}_{i=1}^N \quad (18)$$

Once the base yield-shear force is obtained ($Q_{y,1} = \alpha_{1,max} Mg$), the values of $Q_{y,i}$ and $\delta_{y,i}$ for the upper stories are determined using Eq. (11) as follows:

$$Q_{y,i} = \bar{\alpha}_i \alpha_{1,max} \sum_{j=i}^N m_j g \quad (19)$$

$$\delta_{y,i} = \frac{Q_{y,i}}{k_i} \quad (20)$$

Figure 2 offers a flow chart that summarizes the energy-based design methodology put forward here.

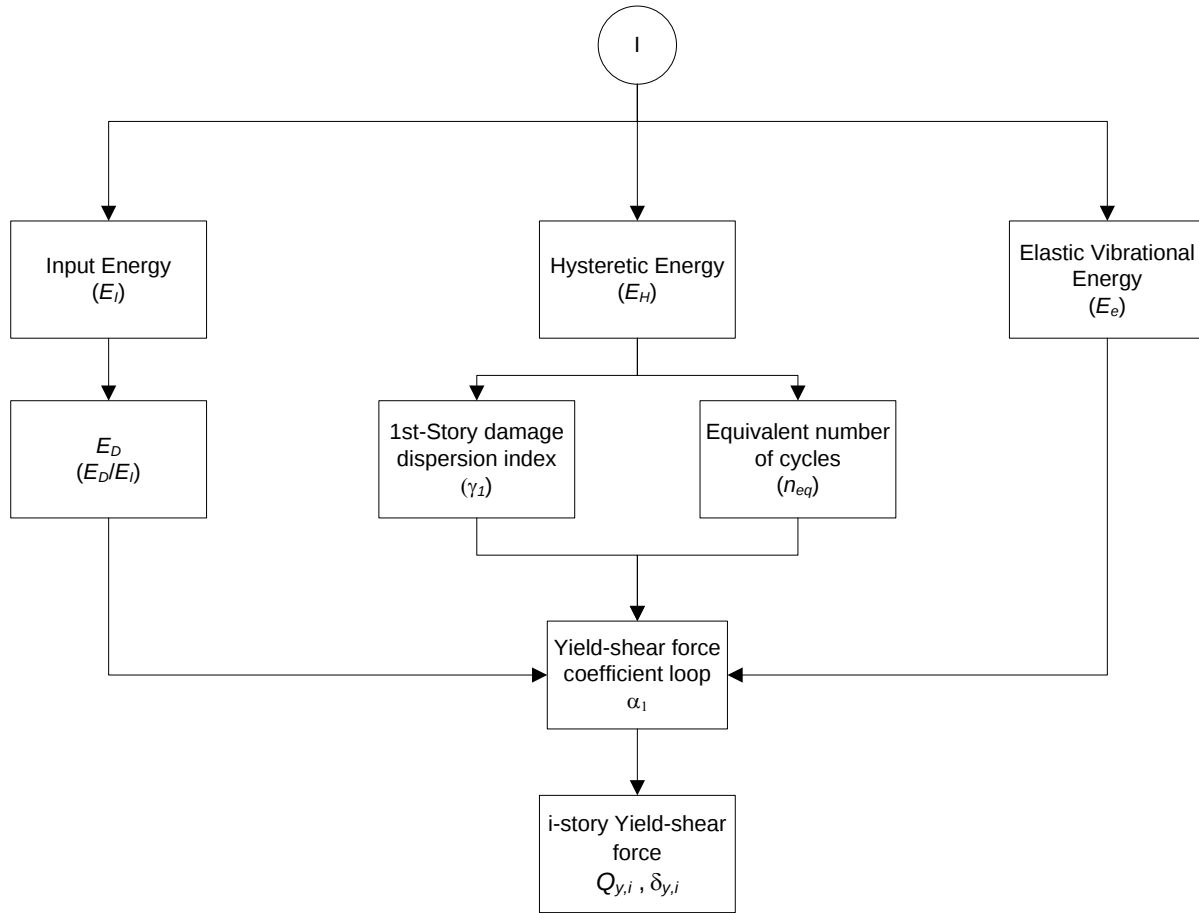


Fig. 2 – Flow chart of the energy-based design methodology

4. Conclusions

This paper has identified and discussed key issues of the energy-based methodology that need to be investigated within a probabilistic framework for the further development of the seismic design approach based on the energy concept. The probabilistic framework makes it possible to explicitly account for the uncertainties of the parameters involved in the design process and can thereby make energy-based seismic design methods the preferable design tool to attain the goals of Performance Based Earthquake Engineering. The energy-based design procedure discussed in this paper uses the maximum interstory drift as a main engineering demand parameter. It is concluded that the key issues calling for a probabilistic characterization for a further characterization of energy-based methods are: the input energy E_I , the amount of input energy to be dissipated through hysteretic action E_H , the number of equivalent cycles of plastic deformation n_{eq} , and the distribution of damage among the stories γ_i . Ongoing research explores these key issues under a probabilistic approach.

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