



## SIMPLIFIED RELIABILITY ASSESSMENT OF RC FRAMES EQUIPPED WITH VISCOUS DAMPERS

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### Abstract

The object of this paper is the study of simplified probabilistic procedures for the seismic assessment of nonlinear structures equipped with nonlinear fluid viscous dampers. The considered reference probabilistic approach is the 2000 SAC-FEMA method, which allows to obtain the probability of exceeding a specified performance level. In particular the purpose is to study the correlation between the results of the probabilistic seismic assessment method for structures without and with dampers, with emphasis on the results in terms of dispersion. To this aim a wide set of recorded ground motions has been selected and applied to the considered RC frames. The study has been performed without applying scaling factors to the earthquake records, but selecting different sets of records for increasing values of seismic intensity. All the obtained results have been examined considering different criteria for the determination of the set of time-history analyses to be used for the probabilistic evaluation and different methods to obtain the dispersion of seismic demand. Moreover, with reference to the application of the SAC-FEMA method, a sensitivity analysis has been performed considering different procedures for the interpolation of the hazard curve. From the analyses it has been possible to derive expressions to correlate the results for structure without and with dampers and suggestions for the application of the SAC-FEMA method. Finally, a direct procedure previously proposed by some of the authors for the response assessment of nonlinear structures with nonlinear viscous dampers has been applied for the probabilistic assessment of the case study as an alternative to nonlinear dynamic analyses.

*Keywords: nonlinear fluid viscous dampers, probabilistic assessment, nonlinear dynamic analyses*



## 1. Introduction

In the last fifty years a large part of the research has been devoted to the development of earthquake-resistant systems in order to raise seismic performance levels while keeping construction cost reasonable. One of the innovative technique of seismic retrofit is the insertion in existing buildings of nonlinear fluid viscous dampers. Their advantages are the reduction of damper forces at high velocities, the supply of higher damper forces at low speed and the dissipation of a larger amount of energy than the other dampers [1]. In general the assessment of the seismic response of a structure is affected by several uncertainties so that it seems appropriate to work in a probabilistic framework. For this reason this research has followed a probabilistic approach, namely the 2000 SAC/FEMA method [2]. This approach provides a closed form expression to evaluate the annual probability of exceeding a specified performance level for a given structure. The purpose of the research has been to study the correlation between the results of the probabilistic assessment method for structures without and with dampers, with particular emphasis on the results in terms of the dispersion due to ground motion variability. In particular, the variability and influence of the terms inside the closed form expression, as the dispersion of seismic demand, have been studied here, considering the near collapse limit state, a wide set of ground motions and different methods to approximate the hazard curves. The analyses have been performed without applying scaling factors to the earthquake records, but considering different records for increasing values of seismic intensity.

In addition, a direct procedure previously proposed by some of the authors [3] for the response assessment of nonlinear structures with nonlinear viscous dampers has been applied to the case study as an alternative to nonlinear dynamic analyses. This procedure, called DAM (direct assessment method), requires only a single pushover analysis of the structure and can be applied also to derive the response for increasing values of seismic intensity (IDAM). Avoiding nonlinear dynamic analyses allows to obtain a significant simplification of the SAC/FEMA method. In this case the method has been applied considering the dispersion values obtained in the nonlinear dynamic analyses.

The considered case study is a RC frame, characterized by three bays and six floors, designed to resist only gravity loads; nonlinear fluid viscous dampers have been inserted for the seismic retrofit. The seismic demand parameters here considered are the maximum displacement at the top of the structure and the maximum interstory drift. The probabilistic assessment on the basis of the results obtained from nonlinear dynamic analyses has been performed for those parameters. Nine return periods have been chosen to identify nine values of seismic intensity and twenty ground motions have been selected for each of them. The analyses have been reported considering two different models for the plastic hinges behaviour: the first model with post peak strength deterioration, the second model without it. In the first case the results have been obtained only for the records for which the analyses converged for both structures, in the second case the results have been obtained for all the considered records, that is 180 for both structures.

## 2. Probabilistic approach

A widespread simplified approach for the probabilistic seismic assessment of structures is the 2000 SAC/FEMA method [2]. This method provides a closed form expression to evaluate the seismic risk of a structure in terms of  $P_{PL}$ , the annual probability of exceeding a specified performance level (e.g., the annual probability of collapse or the annual probability of exceeding the life safety level). Three approximations of the probabilistic representation of ground motion intensity, displacement demand and displacement capacity have been proposed in order to obtain a closed form expression of  $P_{PL}$ . The first assumes that the site hazard curve can be approximated in the region around  $P_{PLSa}$  (in the region of hazard levels close to the limit state probability  $P_{PL}$ ) by the following relation:

$$H(s_a) = P[S_a \geq s_a] = k_o s_a^{-k} \quad (1)$$

where  $H(s_a)$  is the annual probability of exceeding  $s_a$ ,  $S_a$  is the spectral acceleration at the fundamental period (assumed as intensity measure),  $k$  and  $k_o$  are constants depending on the interpolation of the hazard function in a



log-log plot in the region of interest. The second approximation assumes that the median drift demand  $\hat{D}$  can be represented, in the region around  $P_{PLSa}$ , by the following relation:

$$\hat{D} = a(S_a)^b \quad (2)$$

where  $a$  and  $b$  are constants depending on the interpolation of the results in terms of seismic demand. Lastly, the third approximation assumes that the drift demand  $D$  is lognormally distributed about the median with the standard deviation of the natural logarithm,  $\beta_{D|S_a}$ ; this definition will be considered as dispersion. Also the drift capacity  $C$  is assumed to be lognormally distributed with dispersion  $\beta_C$ . With the previous approximations it is possible to derive the following expression:

$$P_{PL} = H\left(s_a^{\hat{C}}\right) \exp\left[\frac{1}{2} \frac{k^2}{b^2} \left(\beta_{D|S_a}^2 + \beta_C^2\right)\right] \quad (3)$$

where  $s_a^{\hat{C}}$  is the spectral acceleration associated to the attainment of the capacity.

### 3. The considered case study

The considered case study is a configuration representative of typical RC frames, with three bays and six floors (Fig. 1). This frame has been designed to resist only gravity loads. Nonlinear fluid viscous dampers have been inserted in order to increase the level of seismic action bearable by the structure. The following properties of dampers have been considered as given parameters in this paper [3]: exponent of velocity  $\alpha=0.5$ , supplemental damping provided by the damping system equal to 24.5% and damping coefficient equal to  $556 \text{ kN (s/m)}^{0.5}$ . Regarding the geometry of the structure, each bay is 6 m wide and each interstorey is 3.2 m high. The beams are 30 cm wide and 60 cm deep in all floors. The columns have a square cross section. On the ground floor, those at the edges have a 40 cm side length, while the central ones have a 45 cm side length; on the first floor all columns have square cross section of  $40 \times 40 \text{ cm}$ ; on the third floor  $35 \times 35$  and on the last three floors  $30 \times 30 \text{ cm}$ . A concrete with a cylinder strength equal to 28 MPa and a steel with a yield strength equal to 450 MPa have been assumed in this study. The seismic weights are 516.6 kN for the sixth floor, 833.4 kN for the fifth and fourth floor, 838.6 kN for the third floor, 849.8 kN for the second floor and 859.2 kN for the first floor. The structure is assumed to be located in Santa Sofia, Italy.

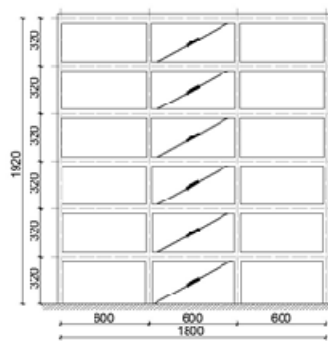


Fig. 1 – Geometrical characteristics of the considered RC frame (dimensions in cm)

Since the aim of this paper is the probabilistic assessment of the seismic response of RC structures equipped with nonlinear fluid viscous dampers, it is necessary to carry out a wide number of nonlinear dynamic analyses and consequently to select a high number of spectrum-compatible recorded ground motions. Nine return periods have been considered ( $T_R=30, 50, 101, 201, 475, 664, 975, 1950$  and  $2475$  years) and for each of them, 20 recorded ground motions characterized by an elastic acceleration response spectrum compatible with the code spectrum (Italian building code, [4]) have been selected through the software Rexel [5]. The code



elastic response spectrum for the assumed site and for a soil type C has been determined for each considered return period. Given the site and the soil type, the PGA for  $T_R=475$  is equal to 0.29 g. Each record has been scaled to the code design value of PGA. Fig. 2 shows the spectra [6] of the 20 selected records for  $T_R=475$  years and their average spectrum compared to the code spectrum.

The nonlinear dynamic analyses have been performed using a concentrated plasticity model implemented in a finite element computer program (SAP2000). A moment-rotation curve has been assigned to the plastic hinges, located at the ends of each element. The moment-rotation curve has been identified by assigning the yielding and the ultimate bending moments and the corresponding chord rotations, which have been provided by empirical relations given in the Commentary to the National code and similar to those of Panagiotakos and Fardis [7]. Two different moment-rotation curves have been considered. At first, a moment rotation curve with post peak strength deterioration has been used (trilinear moment rotation curve, Fig. 3a); then a moment-rotation curve without post peak strength deterioration (bilinear moment rotation curve, Fig. 3b) has been considered to ease the convergence of the large number of analyses and to obtain a greater number of results for the probabilistic assessment.

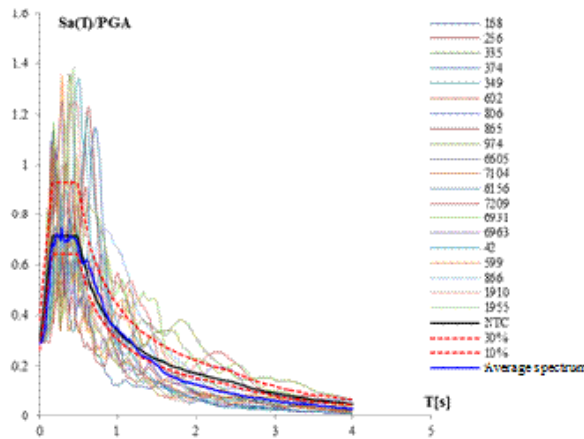


Fig. 2 – Spectra of the 20 selected compatible records for  $T_R=475$  years, average spectrum of the selected records, code spectrum and tolerance limits (30 % and 10%)

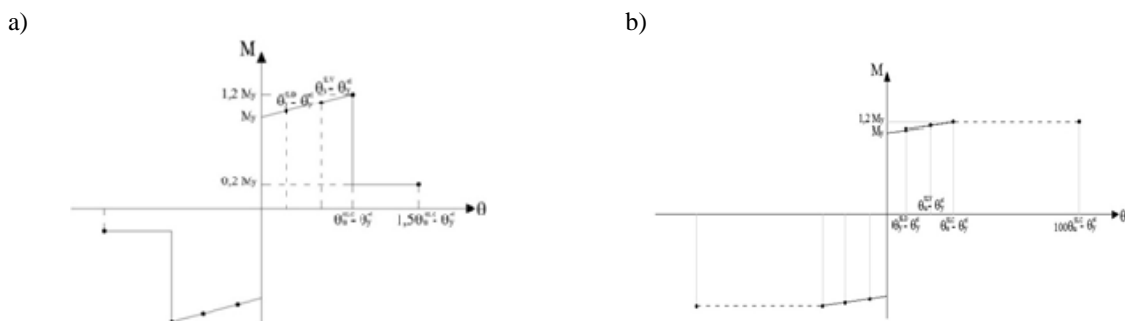


Fig. 3 – a) trilinear moment-rotation curve with post peak strength deterioration; b) bilinear moment-rotation curve without strength deterioration

The obtained results are grouped in three cases, as shown in Table 1. In the first case a trilinear moment rotation curve (Fig. 3a) is considered and the probabilistic assessment is made on 170 and 104 records (those for which converged the analyses) respectively for the structure with and without dampers. In the second case the same plastic hinge model of the first case is adopted, but the probabilistic assessment is made on the same number of records, i.e. 104, for both structures. In the third case a bilinear moment rotation curve (Fig. 3b) is assumed and the probabilistic assessment is performed on 180 records for both structures.



Table 1 – The three cases in which the performed analyses have been grouped

	<b>Case 1)</b> Trilinear moment-rotation curve with post peak strength deterioration	<b>Case 2)</b> Trilinear moment-rotation curve with post peak strength deterioration	<b>Case 3)</b> Bilinear moment-rotation curve without post peak strength deterioration
Structure with dampers	170 results	104 results	180 results
Structure without dampers	104 results	104 results	180 results

## 4. Results and comments

### 4.1. Probabilistic seismic demand analysis

The following parameters have been examined for each nonlinear dynamic analysis:

- profiles of maximum displacement, obtained by the envelope of the maximum displacements which occur on each floor during the seismic event;
- profiles of maximum interstorey drift, which represent the envelope of maximum interstorey drifts occurring during the seismic event.

Once obtained the maximum displacement and maximum interstorey drift profiles, the maximum displacement on the top of the building ( $D_{roof}$ ) and the maximum interstorey drift ( $\delta_{max}$ ) along the height have been determined for each record and each return period. The values thus obtained have been plotted in graphs, which have a parameter representing the seismic intensity ( $S_a(T_1)$ , spectral acceleration at the first natural period of the structure and for 5% damping) as abscissa and a parameter representing the seismic demand ( $D_{roof}$  or  $\delta_{max}$ ) as ordinate.

In order to perform the probabilistic assessment, the median and the dispersion values have been determined. As for the median, it is assumed from the scientific literature [2] that the distribution of median values of the seismic demand parameters follows the relation:

$$MeD = a(S_a(T_1))^b \quad (4)$$

where  $MeD$  is the median value of demand parameter  $D$ ,  $S_a(T_1)$  is the spectral acceleration and  $a$ ,  $b$  are constants deriving from a regression analysis. These constants have been identified for  $D_{roof}$  and  $\delta_{max}$  once the points  $MeD_{roof}-S_a(T_1)$  and  $Me\delta_{max}-S_a(T_1)$  have been determined for each return period.

As for the dispersion, two different dispersion formulations have been considered. The first considers a variable dispersion with the seismic intensity. The dispersion for each return period is obtained through the formulation proposed from scientific literature: standard deviation of demand parameters of natural logarithm; it is indicated with the notation  $\beta_{reg}$  and it is obtained through the expression:

$$\beta_{D|S_a} = \sqrt{\frac{\sum_k (\ln x_k - \hat{\mu})^2}{n}} \quad (5)$$

where  $n$  is the number of values and  $\hat{\mu}$  is the median, determined as the mean of the natural logarithm of the results:

$$\hat{\mu} = \frac{\sum_k \ln x_k}{n} \quad (6)$$

A regression analysis has been carried out on the obtained dispersion values determining the parameters of the straight line which best interpolates the dispersion values:

$$\beta_{D|S_a} = a + bS_a(T_1) \quad (7)$$



In order to evaluate the accuracy of the obtained correlation, the parameters  $R^2$  (determination index) and the errors (mean and standard deviation of residues) have been considered. The second formulation, denoted with  $\beta_{cost}$ , considers a parameter of constant dispersion with seismic intensity. This is obtained performing a regression analysis of  $\ln D$  on  $\ln S_a$  on the totality of the results. The value thus obtained is the standard deviation of the residues.

Table 2 shows the expressions of median and dispersion of demand parameters for the analyses of Case 1 (see Table 1). Fig. 4 and 5 illustrates for the same case the graphs of median and dispersion of demand parameters as a function of seismic intensity. The same results are reported in Table 3, Fig. 6 and 7 for the analyses of Case 2, and in Table 4 and Fig. 8 and 9 for the analyses of Case 3. Fig. 4 shows that the median roof displacements ( $D_{roof}$ ) of the structure without dampers are greater than those of the structure with dampers, as expected. With regard to the dispersion values ( $\beta_{regr}$ , Fig. 4b) both for the structure with dampers and without dampers, it increases with seismic intensity.

Table 2 – Expressions of median and dispersion for  $D_{roof}$  and  $\delta_{max}$ , Case 1

$D_{roof}$		$\delta_{max}$	
Structures with dampers 170 records	Structures without dampers 104 records	Structures with dampers 170 records	Structures without dampers 104 records
$MeD_{roof} = 0.2114 \cdot S_a^{0.9872}$	$MeD_{roof} = 0.2129 \cdot S_a^{0.8059}$	$Me\delta_{max} = 2.041 \cdot S_a^{1.0755}$	$Me\delta_{max} = 4.1216 \cdot S_a^{1.1327}$
$\beta_{regr} = 0.3385 + 0.3894 S_a$	$\beta_{regr} = 0.3274 + 0.2272 S_a$	$\beta_{regr} = 0.3359 + 0.4341 S_a$	$\beta_{regr} = 0.4016 - 0.2375 S_a$
$\beta_{cost} = 0.4523$	$\beta_{cost} = 0.5252$	$\beta_{cost} = 0.4595$	$\beta_{cost} = 0.56296$

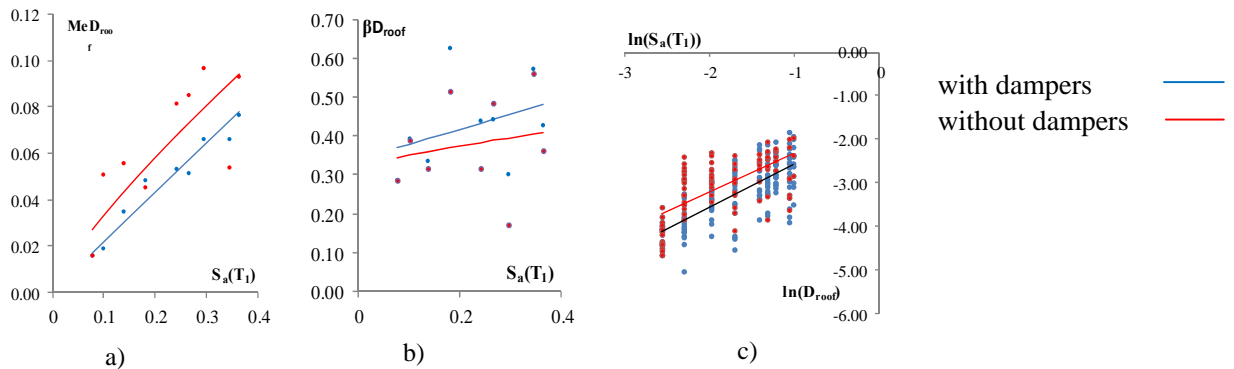


Fig. 4 – Case 1: a)  $MeD_{roof}$  [m]-  $S_a(T_1)$  [s]; b)  $\beta_{D_{roof}}$  -  $S_a(T_1)$ ; c)  $\ln D_{roof} \sim \ln S_a(T_1)$

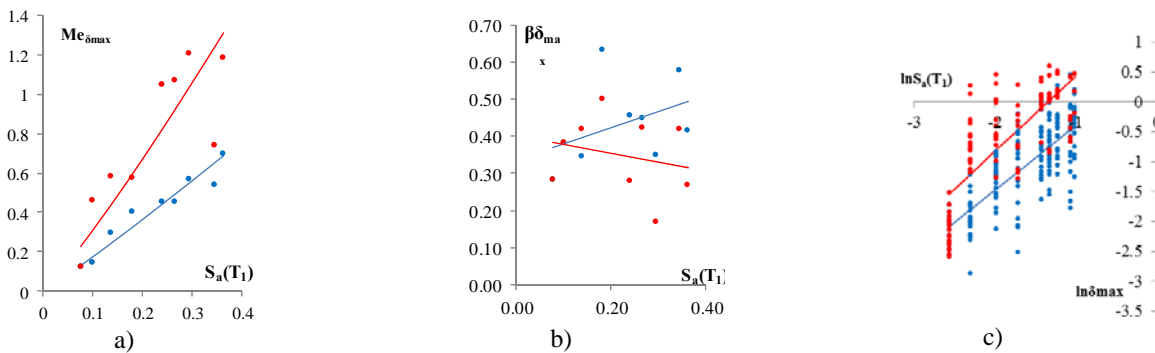


Fig. 5 – Case 1: a)  $Me\delta_{max}$  [%]-  $S_a(T_1)$  [s]; b)  $\beta_{\delta_{max}}$  -  $S_a(T_1)$ ; c)  $\ln\delta_{max} \sim \ln S_a(T_1)$



Moreover, the dispersion of the structure with dampers is greater than that without dampers; this is a trend not in line with the expectations. Lastly, Figure 4c illustrates that, in accordance with Fig. 4a, the displacements of the structure with dampers are smaller than those obtained for the structure without dampers. Considering the parameter of constant dispersion ( $\beta_{cost}$ , Table 2), the dispersion of the structure without dampers is greater than that with dampers with a trend in line with the expectations. Regarding Fig. 5, the same remarks are made for the maximum interstorey drifts ( $\delta_{max}$ ); in addition it is possible to observe that: the variable dispersion ( $\beta_{reg}$ ) decreases for the structure without dampers when seismic intensity increases and this trend is not in line with the expectations; the values of constant dispersion ( $\beta_{cost}$ ) are slightly higher than those obtained for  $D_{roof}$ .

Fig. 6a shows, for the structure with damper and case 2 (smaller number of records), smaller values of median  $D_{roof}$  than for case 1. Fig. 6b confirms that the dispersion ( $\beta_{reg}$ ) always increases for both structures (with and without dampers) when the seismic intensity increases. Moreover, the values of dispersion ( $\beta_{reg}$ ) decrease for the structure with dampers if we consider a smaller number of records. Fig. 6c and Table 3 confirm the same trend also for the dispersion ( $\beta_{cost}$ ). Fig. 7 shows that the same remarks made for the maximum roof displacement ( $D_{roof}$ ) can be made for the maximum interstorey drift. However, passing from the structure with damper to the one without damper, a trend in line with the expectations is not obtained yet for  $\beta_{reg}$ .

Table 3 – Expressions of median and dispersion for  $D_{roof}$  and  $\delta_{max}$ , Case 2

$D_{roof}$		$\delta_{max}$	
Structures with dampers 104 records	Structures without dampers 104 records	Structures with dampers 104 records	Structures without dampers 104 records
$MeD_{roof} = 0.1127 \cdot S_a^{0.7398}$	$Me\delta_{max} = 0.2129 \cdot S_a^{0.8059}$	$Me\delta_{max} = 1.0595 \cdot S_a^{0.8221}$	$Me\delta_{max} = 4.1216 \cdot S_a^{1.1327}$
$\beta_{reg} = 0.3306 + 0.2143 S_a$	$\beta_{reg} = 0.3274 + 0.2272 S_a$	$\beta_{reg} = 0.3254 + 0.257 S_a$	$\beta_{reg} = 0.4016 - 0.2375 S_a$
$\beta_{cost} = 0.42236$	$\beta_{cost} = 0.5252$	$\beta_{cost} = 0.42239$	$\beta_{cost} = 0.56296$

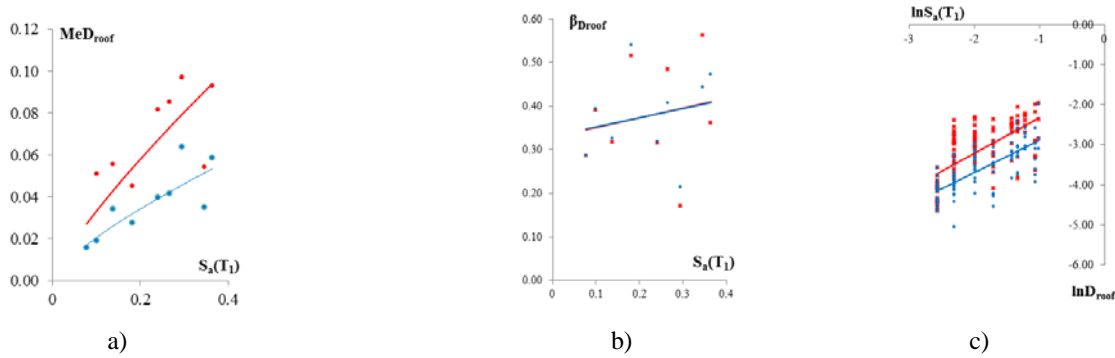


Fig. 6 – Case 2: a)  $MeD_{roof}$  [m]-  $S_a(T_1)$  [s]; b)  $\beta_{D_{roof}}$ -  $S_a(T_1)$ ; c)  $\ln D_{roof}$ - $\ln S_a(T_1)$

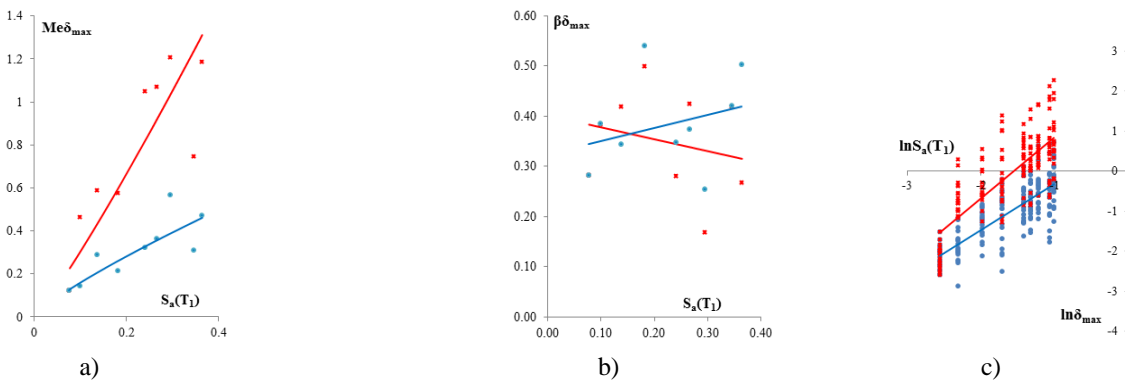


Fig. 7 – Case 2: a)  $Me\delta_{max}$  [%]-  $S_a(T_1)$  [s]; b)  $\beta_{\delta_{max}}$ -  $S_a(T_1)$ ; c)  $\ln \delta_{max}$ - $\ln S_a(T_1)$





If we consider a greater number of records for both the structures (Case 3), a trend in line with the expectations is obtained. As for the maximum roof displacement ( $D_{roof}$ ) one can note that (Fig. 8): the median values for the structure without dampers are greater than those obtained for the structure with dampers (Fig. 8a); the dispersion  $\beta_{regr}$  always increases for both the structures when seismic intensity increases and the expected trend is obtained, that is the dispersion of the structure without dampers is greater than that with dampers (Fig.8b); the dispersion  $\beta_{cost}$  is greater for both structures than the previous cases but it always maintains the same trend, that is the dispersion for the structure without dampers is greater than that for the structure with dampers (Fig. 8c). The same remarks can be made for the maximum interstorey drift ( $\delta_{max}$ , Fig. 9), with the additional observation that the decreasing trend (observed in cases 1 and 2) disappears for  $\beta_{regr}$ , which always increases with seismic intensity.

Table 4 – Expressions of median and dispersion for  $D_{roof}$  and  $\delta_{max}$ , Case 3

$D_{roof}$		$\delta_{max}$	
Structures with dampers 180 records	Structures without dampers 180 records	Structures with dampers 180 records	Structures without dampers 180 records
$MeD_{roof} = 0.2421 \cdot S_a^{1.0523}$	$MeD_{roof} = 0.44 \cdot S_a^{1.1357}$	$Me\delta_{max} = 2.2724 \cdot S_a^{1.1285}$	$Me\delta_{max} = 9.8605 \cdot S_a^{1.5088}$
$\beta_{regr} = 0.3145 + 0.5809 S_a$	$\beta_{regr} = 0.2626 + 1.0257 S_a$	$\beta_{regr} = 0.2975 + 0.7001 S_a$	$\beta_{regr} = 0.2906 + 1.1428 S_a$
$\beta_{cost} = 0.4696$	$\beta_{cost} = 0.5651$	$\beta_{cost} = 0.4803$	$\beta_{cost} = 0.6389$

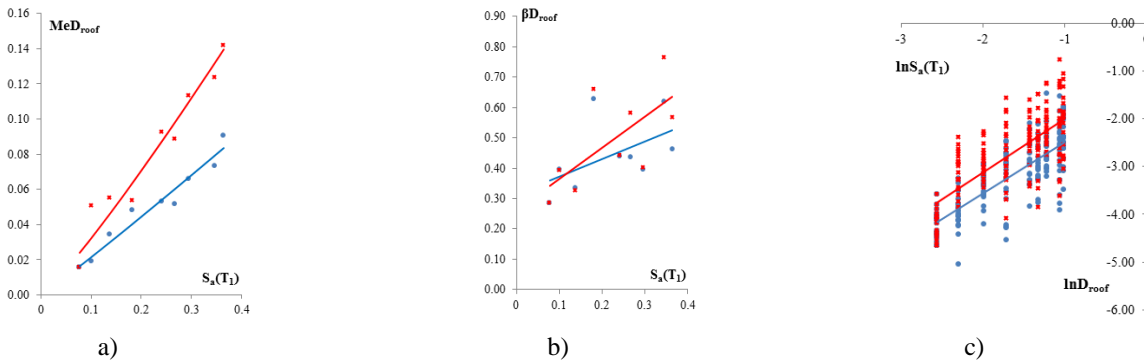


Fig. 8 – Case 3: a)  $MeD_{roof}$ [m]-  $S_a(T_1)$  [s]; b)  $\beta D_{roof}$ -  $S_a(T_1)$ ; c)  $\ln D_{roof}$ - $\ln S_a(T_1)$

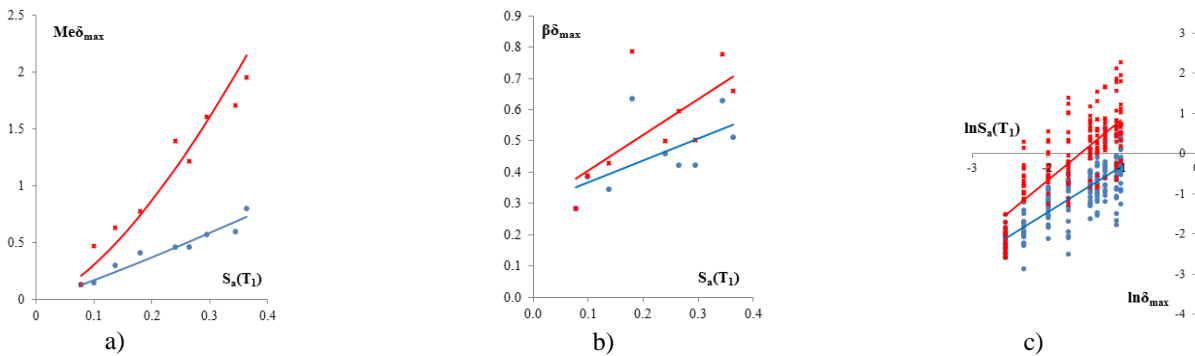


Fig. 9 – Case 3: a)  $Me\delta_{max}$  [%]-  $S_a(T_1)$  [s]; b)  $\beta\delta_{max}$ -  $S_a(T_1)$ ; c)  $\ln\delta_{max}$ - $\ln S_a(T_1)$

The direct assessment method [3] previously mentioned has been applied to estimate the median values of  $D_{roof}$  for case 3. The method has been applied using the code expression for the damping reduction factor and the area-based criterion for the equivalent damping ratio. In Fig. 10 the median curves of  $D_{roof}$  from nonlinear dynamic analyses are compared with those obtained by applying the direct assessment method [3] for increasing





seismic intensities (IDAM). It should be noted that the IDAM curves are conservative, especially for the structure with dampers; this is due to the conservative nature of the code expression of the damping reduction factor. Anyway the IDAM curves are consistent with those from nonlinear incremental dynamic analyses.

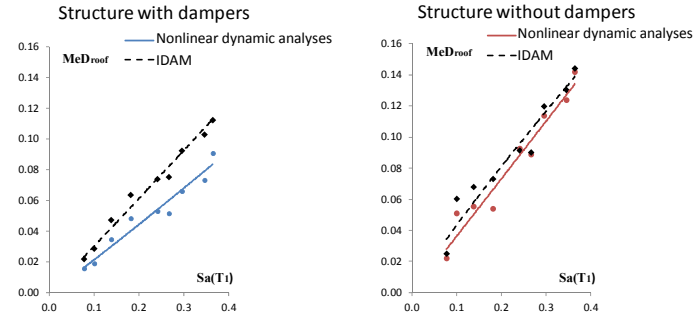


Fig. 10 – Case 3:  $MeD_{roof}$  [m]-  $S_a(T_1)$  [s] curves obtained with nonlinear dynamic analyses and IDAM [3]

Table 5 illustrates that the parameter of constant dispersion ( $\beta_{cost}$ ) increases when the number of records increases for both structures and for both demand parameters ( $D_{roof}$  and  $\delta_{max}$ ). Lastly, with regard to dispersion, it is useful to perform some evaluations. By calculating the ratio between the parameter of constant dispersion obtained for both structures with and without dampers in the three cases, it comes out that the dispersion of the structure with dampers is about 80% of the dispersion of the structure without dampers (Table 6). By calculating the ratio between the coefficients of the regression lines  $\beta=a+bS_a$  obtained for the structures with and without dampers in the three cases, one gets the relation which allows to obtain the dispersion of the structure with dampers, knowing that for the structure without dampers (Table 6). Given the previous observations on the trends of dispersion, the most reliable correlations seem to be those obtained for Case 3, characterized by a greater number of records (180).

Table 5 –  $\beta_{cost}$  with the increasing of the number of records for  $D_{roof}$  and  $\delta_{max}$

	Structure with dampers			Structure without dampers	
	104 records	170 records	180 records	104 records	180 records
$\beta_{cost}(D_{roof})$	0.42236	0.4523	0.4696	0.5252	0.5651
$\beta_{cost}(\delta_{max})$	0.42239	0.4595	0.4803	0.5629	0.6381

Table 6 –  $\beta_{cost}$ , ratio between the parameters of constant dispersion, obtained for both structures with and without dampers in the three cases;  $\beta_{reg}$ , ratio between the coefficients of the regression lines obtained for the structure with and without dampers in the three cases

	$\beta_{cost,dam}/\beta_{cost,withoutdam}$				$\alpha=a_{dam}/a_{withoutdam} \quad \lambda=b_{dam}/b_{withoutdam}$		
	Case 1	Case 2	Case 3		Case 1	Case 2	Case 3
$D_{roof}$	0.8612	0.8042	0.831	$\alpha$	1.714	0.943	0.566
				$\lambda$	1.034	1.009	1.198
$\delta_{max}$	0.8162	0.7503	0.7518	$\alpha$	-1.828	-1.082	0.613
				$\lambda$	0.836	0.810	1.024

#### 4.2. Evaluation of the annual failure probability

The second step of this research is the study of the simplified formula proposed by the 2000 SAC/FEMA method to assess the annual probability of exceeding a given performance level (Eq. (3)). The following assumptions have been considered in this study: consideration of the near collapse limit state; adoption of different criteria for approximating the hazard curve: interpolation on the whole range of examined return periods (criterion (a),  $T_R=30, 50, 101, 201, 475, 664, 975, 1950$  and  $2475$  years); interpolation on a small range close to the return



period of 975 years, which is associated in the code to the near collapse limit state (criterion (b)  $T_R=475, 664, 975, 1950$  and  $2475$  years). Moreover, interpolations of the first and second order have been performed for both the criteria identifying thus four hazard curves. The approximation of the second order has been determined according to the method proposed by Vamvatsikos [8]. The expressions of the obtained hazard curves are reported in Table 7. Lastly, the determination of the capacity values for the near collapse limit state and for the two considered demand parameters has been made according to two criteria described in the following. In particular, two collapse conditions are defined, one based on the ultimate roof displacement ( $D_{roof,u}$ ), the other on the ultimate interstorey drift ( $\delta_{max,u}$ ).  $D_{roof,u}$  has been determined through a pushover analysis, under a modal pattern of lateral load, as the roof displacement when the first plastic hinge in a column reaches the collapse rotation. This value is  $D_{roof,u}=0.145$  m.  $\delta_{max,u}$ , to be compared with the demand evaluated in terms of maximum drift along the height, has been determined through a pushover analysis as the maximum drift along the height when the first plastic hinge in a column reaches the collapse rotation. Since in the pushover analysis a column sway mechanism at the fourth storey has been observed, the mentioned drift corresponds to the ultimate drift of the fourth storey, equal to 2.5375%. It should be noticed also that during the nonlinear dynamic analyses, in all the cases in which the collapse has been reached, a mechanism at the third or fourth storey has been observed.

The variability of the annual probability of failure is then examined. With regard to the first order approximation of the hazard curve, the results for  $D_{roof,u}$  are reported in Table 8, which shows the values of probability together with the corresponding values of hazard and dispersion. It can be noticed that: under the same conditions of structure and dispersion, the values of annual probability of failure vary considerably depending on the hazard curves (a) or (b); under the same hazard curves (a) or (b), a variation of the dispersion seems to have a great influence on the final value of the annual probability of failure, also in comparison with a variation of the hazard value; as for the parameter of constant dispersion, a much greater annual probability of failure has been obtained for the structure without dampers than for the structure with dampers; this does not always occur when the parameter of variable dispersion is used, as for Case 2, hazard curve (b). The same observations can be made for the collapse defined by  $\delta_{max,u}$  (not shown) with the additional observation that, when the variable dispersion parameter ( $\beta_{reg}$ ) is used, the annual probability of failure is much greater for the structure with dampers than for the structure without dampers. Consequently, the results are in line with the expectations mainly when the parameter of constant dispersion ( $\beta_{cost}$ ) is used. Table 8 shows also the values of collapse probability obtained for Case 3 using the median curves of  $D_{roof}$  estimated with the direct assessment method [3] and the dispersion values ( $\beta_{cost}$ ) or expressions ( $\beta_{reg}$ ) from nonlinear dynamic analyses. The values of probability with IDAM are larger than those obtained from nonlinear dynamic analyses due to the conservative estimates obtained for the median curves of  $D_{roof}$  and to the different trends of such curves.

As for the second order approximation of the hazard curve, for both the demand parameters and for the same structure, there is a smaller difference among the values of  $H(S_{a,1}^{NC})$  determined with different intervals for interpolating the hazard curve (criterion (a) or (b)) than with the first order approximation. This is shown in Table 9 for  $D_{roof,u}$  and for the Case 3. As a consequence, considering the second order approximation of the hazard curve, the variation, for the same structure and dispersion, of the annual failure probability with the number of values of  $T_R$  is reduced if compared with the same variation obtained with the first order approximation. Moreover, considering the second order approximation of the hazard curve, a lower influence of the dispersion on the annual failure probability has been obtained than with the first order approximation. Finally it is possible to observe that, considering the second order approximation and using both the dispersion parameters, the results are in line with the expectations, i.e. the annual failure probability for the structure without dampers is always much greater than that obtained for the structure without dampers.

Table 7 – Approximation of the first and second order of the hazard curve, obtained with criterion (a) and (b)

	Hazard curve criterion (a)	Hazard curve criterion (b)
1st order approximation	$H(S_a^{NC}) = 3 \cdot 10^{-5} (S_{a,1}^{NC})^{-2.827}$	$H(S_a^{NC}) = 7 \cdot 10^{-6} (S_a^{NC})^{-4.03}$
2nd order approximation	$H(S_a^{NC}) = 2.62 \cdot 10^{-6} \cdot e^{(-0.878 \ln^2 S_a(T_1) - 5.923 \ln S_a(T_1))}$	$H(S_a^{NC}) = 3.04 \cdot 10^{-7} \cdot e^{(-2.18 \ln^2 S_a(T_1) - 9.312 \ln S_a(T_1))}$



Table 8 – Annual failure probability for the first order approximation of the hazard curve with criterion (a) and (b) for collapse defined by  $D_{roof,u}$

$D_{roof,u}=0.145$ m									
		Case 1		Case 2		Case 3		Case 3 (IDAM)	
		With dampers 170	Without dampers 104	With dampers 104	Without dampers 104	With dampers 180	Without dampers 180	With dampers 180	Without dampers 180
$H(S_{a,1}^{NC})$	a)	$8.8 \cdot 10^{-5}$	$1.2 \cdot 10^{-4}$	$1.2 \cdot 10^{-5}$	$1.2 \cdot 10^{-4}$	$1.2 \cdot 10^{-4}$	$4.8 \cdot 10^{-4}$	$2.6 \cdot 10^{-4}$	$2.65 \cdot 10^{-4}$
	b)	$3.3 \cdot 10^{-5}$	$4.8 \cdot 10^{-5}$	$1.8 \cdot 10^{-6}$	$4.8 \cdot 10^{-5}$	$4.9 \cdot 10^{-5}$	$3.6 \cdot 10^{-4}$	$1.53 \cdot 10^{-4}$	$1.57 \cdot 10^{-4}$
$\beta_{regr}$		0.6043	0.4686	0.6319	0.4686	0.6714	0.6486	0.585	0.6545
$P_{F,NC}$	a)	$5.4 \cdot 10^{-4}$	$7.1 \cdot 10^{-4}$	$3.7 \cdot 10^{-4}$	$7.1 \cdot 10^{-4}$	$7.9 \cdot 10^{-4}$	$2.2 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$5.55 \cdot 10^{-3}$
	b)	$1.3 \cdot 10^{-3}$	$1.9 \cdot 10^{-3}$	$2.0 \cdot 10^{-3}$	$1.9 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$8.2 \cdot 10^{-3}$	$5.55 \cdot 10^{-3}$	$5.43 \cdot 10^{-2}$
$\beta_{cost}$		0.4523	0.5252	0.4223	0.5252	0.4696	0.5651	0.4696	0.5651
$P_{F,NC}$	a)	$2.8 \cdot 10^{-4}$	$9.9 \cdot 10^{-3}$	$7.3 \cdot 10^{-5}$	$9.9 \cdot 10^{-4}$	$3.5 \cdot 10^{-4}$	$1.6 \cdot 10^{-3}$	$8.2 \cdot 10^{-4}$	$3.23 \cdot 10^{-3}$
	b)	$3.4 \cdot 10^{-4}$	$3.8 \cdot 10^{-3}$	$7.7 \cdot 10^{-5}$	$3.8 \cdot 10^{-3}$	$4.4 \cdot 10^{-4}$	$4.3 \cdot 10^{-3}$	$1.56 \cdot 10^{-3}$	$1.81 \cdot 10^{-2}$

Table 9 – Annual failure probability for different hazard curve approximations and for collapse defined by  $D_{roof,u}$

$D_{roof,u} = 0.145$ m, Case 3				
With dampers $S_{a,1}^{NC}=0.6144$ , $a=0.2421$ , $b=1.0523$ , $\beta_c=0.275$				
	a) I order	a) II order	b) I order	b) II order
$H(S_{a,1}^{NC})$	$1.2 \cdot 10^{-4}$	$3.8 \cdot 10^{-5}$	$4.9 \cdot 10^{-5}$	$1.7 \cdot 10^{-5}$
$\beta_{regr}=0.6717$				
$P_{F,NC}$	$7.9 \cdot 10^{-4}$	$7.19 \cdot 10^{-4}$	$2.37 \cdot 10^{-3}$	$4.99 \cdot 10^{-4}$
$\beta_{cost}=0.4696$				
$P_{F,NC}$	$3.5 \cdot 10^{-4}$	$2.49 \cdot 10^{-4}$	$4.37 \cdot 10^{-4}$	$2.20 \cdot 10^{-4}$
Without dampers $S_{a,1}^{NC}=0.3763$ , $a=0.44$ , $b=1.1357$ , $\beta_c=0.275$				
	a) I order	a) II order	b) I order	b) II order
$H(S_{a,1}^{NC})$	$4.8 \cdot 10^{-4}$	$3.7 \cdot 10^{-4}$	$3.6 \cdot 10^{-4}$	$3.4 \cdot 10^{-4}$
$\beta_{regr}=0.6486$				
$P_{F,NC}$	$2.2 \cdot 10^{-3}$	$2.4 \cdot 10^{-3}$	$8.2 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$
$\beta_{cost}=0.5651$				
$P_{F,NC}$	$1.6 \cdot 10^{-3}$	$1.8 \cdot 10^{-3}$	$4.3 \cdot 10^{-3}$	$1.2 \cdot 10^{-3}$



## 5. Conclusions

Several investigations have been performed on the application, through nonlinear dynamic analyses, of the simplified SAC/FEMA approach for the probabilistic seismic assessment of RC structures without and with viscous dampers. In the following the main results are summarized. As expected, the median values obtained for the structure without dampers are greater than those obtained for the structure with dampers regardless of the number of record considered. Regarding the application of the direct assessment method as an alternative to nonlinear dynamic analyses, the estimates provided by the method for the median curves of  $D_{roof}$  are consistent with the results of nonlinear dynamic analyses and can be applied for the probabilistic assessment.

With reference to the dispersion parameter  $\beta_{reg}$  it is possible to notice that: it increases with seismic intensity; it depends on the results of dynamic nonlinear analyses, and in particular on the number of records; the expected trend, i.e. greater dispersion for the structure without dampers than for the structure with dampers, has been determined only for the higher number of records (180 records). Regarding the dispersion parameter  $\beta_{cost}$ , we can note that: the expected trend has been obtained also for the lower number of records;  $\beta_{cost}$  increases with the number of seismic events both for the structure with and without dampers. Moreover, relations between the expressions of  $\beta_{cost}$  and  $\beta_{reg}$  for the structure without and with dampers have been derived with the purpose to obtain the dispersion for the structure with dampers by knowing that for the structure without dampers.

With regard to the simplified formula to determine the annual probability of failure  $P_{F,NC}$ , it is particularly sensitive to variations of hazard curve approximation and dispersion. It is possible to note that for the hazard curves determined with the first order approximations, different values of  $P_{F,NC}$  have been obtained by changing the interval in which the hazard curve is interpolated. Moreover, the simplified formula for  $P_{F,NC}$  is particularly affected by the dispersion.  $P_{F,NC}$  values always in line with the expectations have been obtained only using the parameter of dispersion  $\beta_{cost}$ . Considering the second order approximation of the hazard curve, the variation of the annual failure probability when changing the interval for interpolating the hazard curve is reduced if compared with the same variation obtained with the first order approximation. With the mentioned second order approximation it is also reduced the influence of the values of dispersion and the  $P_{F,NC}$  is always greater for the structure without dampers than for the structure with dampers. Therefore, it is possible to observe that it is better to use the parameter of constant dispersion  $\beta_{cost}$  when we consider the first order approximation for the hazard curve. Otherwise, using the second order approximation allows to obtain values of probability less sensitive to the interval for interpolating the hazard curve and to the type of dispersion  $\beta_{reg}$  or  $\beta_{cost}$ .

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