THE INCREMENTAL MODAL PUSHOVER ANALYSIS (IMPA): PROPOSAL AND APPLICATION

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Abstract

Existing reinforced concrete frame buildings designed for vertical loads only could suffer severe damage during earthquakes. In recent years, many research activities were undertaken to develop a reliable and practical analysis procedure to identify the safety level of existing structures. The Incremental Dynamic Analysis (IDA) is considered to be one of the most accurate methods to estimate the seismic demand and capacity of structures. However, the executions of many nonlinear response history analyses (NL_RHA) are required to describe the entire range of structural response. The research discussed in this paper deals with the proposal of an efficient Incremental Modal Pushover Analysis (IMPA) to obtain capacity curves by replacing the nonlinear response history analysis of the IDA procedure with Modal Pushover Analysis (MPA). In this work, first, the MPA is examined and extended to three-dimensional asymmetric structures. Than, it is incorporated into the proposed procedure (IMPA) to estimate the structure's seismic response and capacity for given seismic actions. This new procedure, which accounts for higher modes effects, does not require the execution of complex NL-RHA, but only a simpler nonlinear static analysis. Finally, the extended MPA and IMPA were applied to an existing irregular framed building.

Keywords: modal pushover analysis; existing irregular framed building; capacity curve.

1. Introduction

Recently, as a result of seismic events occurrences in Italy, building safety has become a topic of considerable interest. Therefore, the development of fast and reliable analysis procedures determining the safety level of existing structures has become essential.

Incremental dynamic analysis (IDA) is a method for estimating the seismic response and capacity of structures over the entire range of structural responses, from elastic behavior to global dynamic instability. The most accurate way to compute the seismic demands of a structure under a given seismic action is to carry out a nonlinear response history analysis (NL_RHA) on a detailed three-dimensional (3D) mathematical model of the structure. IDA requires the execution of NL_RHA for an ensemble of ground motions, each scaled for various intensity levels, selected to cover a wide range of structural responses, from elastic behavior to global instability. From the results of this computation, it is possible to determine the structural capacities corresponding to the various limit states [1].

However, IDA is onerous for practicing engineers since it requires an intensive computation of many NL_RHAs [2]; recognizing that IDA of practical structures is computationally extremely demanding, even the developers of IDA have devised a simplified, approximate method [3].

Hence, Nonlinear Static Procedures (NSPs) attract the attention of both practicing engineers and the research community since it is more practical and of faster implementation. Different NSPs have been developed and used for their conceptual simplicity, computational attractiveness and capability of providing satisfactory predictions of the building's seismic response: the N2 method [4], the Capacity Spectrum Method (CSM) [5], the Adaptive Capacity Spectrum Method (ACSM) [6], and the Coefficient Method of Displacement Modification [7], etc. Among the current nonlinear static analysis methods, Modal Pushover Analysis (MPA) was developed by Chopra and Goel [8] to take into account the higher mode contributions to the total response; later, Goel and Chopra [9], Chintanapakdee and Chopra [10] reported that MPA yields better results compared to a traditional pushover analysis. However, most of the researches dealing with nonlinear static analysis procedures are limited to planar structures.

Presently a focus on existing buildings, having a remarkable architectural and structural complexity, showed the importance of extending NSP to the entirety of the 3D structure to examine the 3D effects of irregularities, and the relevance of the control point position (both in plan and elevation).

The simplified IDA previously cited (Vamvatsikos and Cornell, 2002), requires to replace NL_RHA with NSP to evaluate seismic demand, for each given seismic intensity level, reducing the computational effort required for the standard IDA: it requires performing several pushover analysis with different load distribution in order to select the most conservative one and, moreover, estimating the elastic stiffness of the SDF system from the IDA curve.

The aim of this paper is to develop the opportunity of an approximate IDA procedure based on MPA realized by Han and Chopra [11], proposing a procedure named Incremental Modal Pushover Analysis (IMPA), for the analysis of complex 3D structures and in particular concrete frame structures. The novelty of this approach is in the evaluation of a multimodal performance point in terms of displacement and base shear and therefore in the evaluation of a capacity curve that can replace the standard capacity curve from pushover, already introduced in previous papers of Bergami et al. [12] [13].

The capacity curve obtained defines the relationship between the base shear and top displacement of the building and can be used to evaluate seismic performance: this is a novelty in fact MPA has been developed and applied to determine structural response in terms of displacements and drift distribution. Is opinion of the authors that the evaluation of global forces and displacement is an important information dealing with retrofitting of existing structures, for example with additional energy dissipation devices (Bergami et al., [14] [15]), as long as foundations are usually barely to be characterized and therefore the designer needs to evaluate if the global actions can be supported and if the retrofitting process reduces or not e.g. the base shear.

Therefore IMPA is a proposal of a new approach to be used as standard pushover procedures as commonly used. In the following report, first the MPA is discussed and then, an existing building, presenting both vertical and plan irregularities, was selected as a case study to verify whether the MPA procedure for asymmetric structures could be successful, even in the case of very complex irregular conditions and develop the building's capacity curve through IMPA.

2. Incremental modal pushover analysis (IMPA)

The IMPA procedure to determine the capacity curves uses MPA procedures rather than NL_RHA to estimate seismic demands for each intensity level of earthquake motions. The MPA procedure is described in a convenient step-by-step form (Chopra and Goel, 2002).

The incremental modal pushover analysis (IMPA) proposed is a pushover based procedure that requires the execution of MPA and an evaluation of structural performance within a range of different seismic actions and intensity. Data resulting from MPA application within an identified range of seismic intensity provides all information necessary to estimate the seismic response from different intensity levels. Differently from MPA, this approach is finalized to develop a multimodal capacity curve in terms of base shear and top displacement: the MPA has been developed and used to analyze displacements and drifts distribution. Therefore, dealing with MPA, the evaluation of drifts has to be related with other damage index in order to evaluate the structural performance. With IMPA the author's want to develop a new pushover procedure useful for the same targets of other pushover methods but more suitable for buildings sensitive to higher order modes. For each seismic intensity level, the corresponding Performance Point (P.P.) for the multi-degree-of-freedom (MDOF) system, in terms of roof displacement and corresponding base shear, can be obtained by combining the P.P. determined through the application of many different procedures: in this paper the Capacity Spectrum Method (CSM) has been used but other approaches could be evaluated if considered more suitable. Therefore, according to the procedure selected to be applied for each significant mode: the P.Ps will be combined through the Square Root of the Sum of Squares (SRSS) rule. It is thus possible to obtain a range of multimodal performance points (P.P.mm), each one corresponding to a specific seismic intensity level: CSM (or other approaches as well) are applied using Response Spectrums (RS) for all the intensity level considered (the RS will be scaled up to obtain a range of intensities such as in IDA with the time histories) By connecting all the P.P.mm, a curve can be obtained: this curve is called the "Multimodal Capacity Curve" (MCC). The detailed step-by-step implementation of the IMPA procedure is presented below:

1. Compute the natural frequencies, w_n and modes, φ_n for the linear elastic vibration of the building;

2. Select the ground motions and the RS for a range of intensity levels;

3. For the intensity level *i*, represented by Peak Ground motion Acceleration (PGA), CSM is adopted to find the P.P. for the predominate modes: for the *n*th mode, transform the capacity curve, which is defined in terms of base shear and roof displacement, into a capacity spectrum and transform the RS into an Acceleration Displacement Response spectrum (ADRS) format, and plot them on the same chart. Their intersection is taken as the P.P., as shown in Fig. 1 a). Obtain the corresponding P.P. from the capacity curve, as shown in Fig. 1 b). It is important to note that, for the *n*th mode, if the structure enters a nonlinear plastic stage, then the demand spectrum should be reduced by the spectral reduction factor which depends on the effective viscous damping of structure ξ_{ni}:

$$\xi_{ni} = \xi_0 + k\frac{1}{4\pi}\frac{E_{dni}}{E_{S0ni}} = \xi_0 + k\xi_{eqni} \tag{1}$$

where ξ_{ni} is the effective damping for *n*th mode, ξ_0 is the inherent damping of the elastic structure, about 5% for reinforced concrete structures; E_{dni} is the energy dissipated in an ideal hysteretic cycle, which corresponds to the area enclosed by the hysteresis loop; E_{s0ni} is the maximum strain energy dissipated by the structure corresponding to the area of the hatched triangle. The term *k* is the damping modification factor that is an adjustment factor to approximately account for changes in hysteretic behavior in reinforced concrete
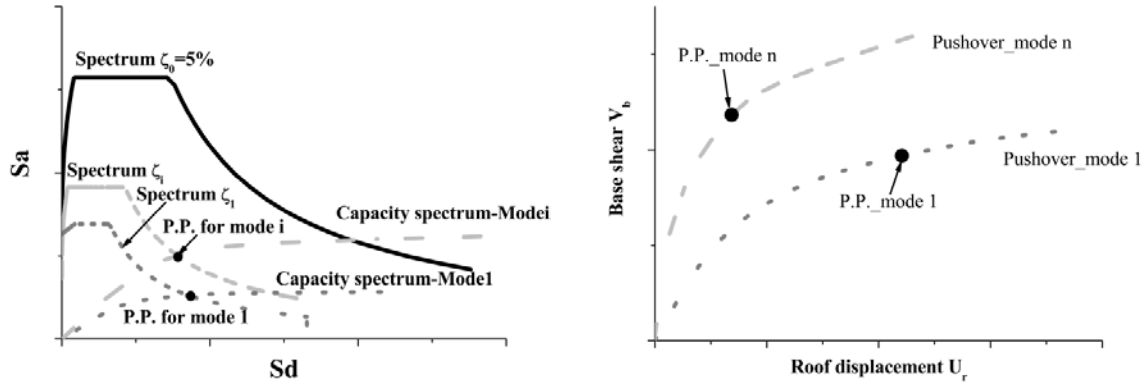
structures.

Fig. 1 – Evaluation of the performance points (P.P.) for each capacity curve that belongs to the pushover analysis with the selected load distributions: proportional to Mode1..Mode n. a) for each capacity curve the P.P. is determined via C.S.M. b) P.P. can be plotted in the V-U plane

4. Determine multimodal performance point (P.P.mm) in terms of multimodal base shear V_{bmmi} and multimodal roof displacement u_{rmmi}.

The value of the roof displacement of the selected control point is determined, for each level of earthquake intensity considered, by combining the modal displacements of the control point u_{rni} using the SRSS rule.

$$u_{rmmi} = ((\sum_n u_{rni}^2)^{1/2}) \tag{2}$$

Instead for the considered earthquake intensity level, to derive the base shear, the procedure adopted follows:

a) **if the structure remains elastic** the value of the base shear of the structure is determined using the same procedure;

$$V_{bmmi,el} = ((\sum_n V_{bni}^2)^{1/2}) \tag{3}$$

b) **if the structure enters the inelastic range** a different procedure is used:
   step 1) the total value of the plastic hinge rotation, θ_{cb} at column end of the first level is estimated as the SRSS combination of the values θ_{cbi} obtained with the pushovers with each modal load distribution.
   step 2) the corresponding bending moments in the columns are estimated through the relevant moment-rotation diagram at the value of the plastic hinge rotation calculated from the SRSS combination
   step 3) shears in the columns are calculated using the corrected bending moments, and the base shear is calculated as the sum of the column shears.

$$V_{bmmi,y} = V(\theta_{cb}); \ \theta_{cb} = (\sum_n \theta_{cbi}^2)^{1/2} \tag{4}$$

5. Repeat steps 2–4 for as many intensity levels needed to form the IMPA curve, as shown in Fig. 2 where P.P. are the performance points obtained with a lateral load distribution proportional to modal shape 1,..,n.; according to MPA approach the multimodal performance point (P.P.mm) can be obtained to trace the multimodal capacity curve (MCC).
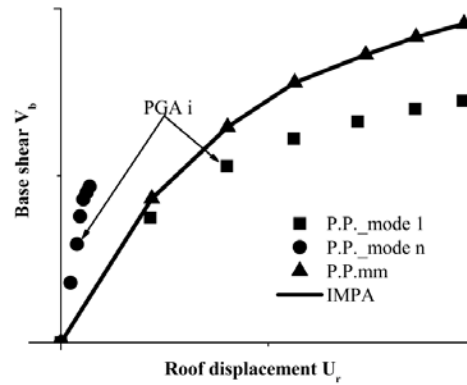
Fig. 2 – Construction multimodal capacity curve (MCC) from the IMPA procedure. By applying the SRSS rule with the P.P. obtained with each load distribution (Mode1..Mode n) and for each intensity level (the response spectrum is scaled from lower to higher intensity levels) the MCC) can be obtained.

3. Case study

3.1 Building description

The building used as a case study is an existing nine story RC framed building located in Italy, designed for gravity loads only and built in the 1970s. Details of this building are available in Bergami et al., however the building consists of a ground floor, an eight-story elevation and a roof terrace: the plan and elevation layout of existing building are shown in Fig.3.
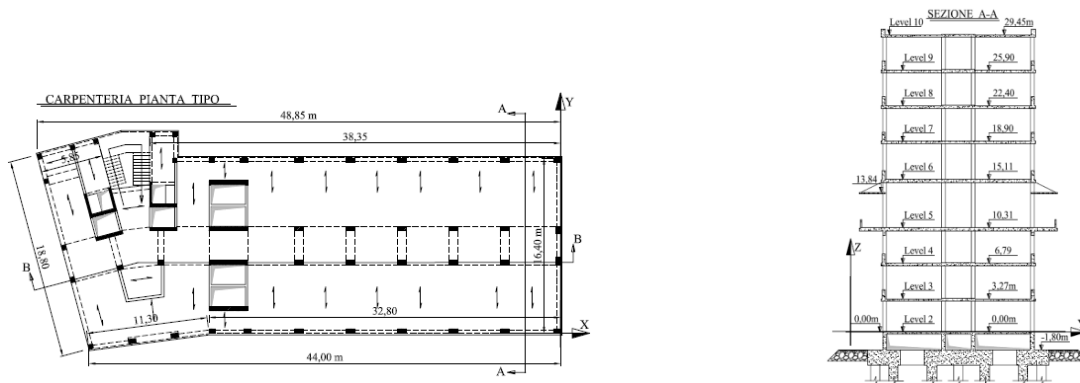

Fig. 3 – Layout of the existing building, a) plan b) section A

From a structural point of view, the plan is an irregular polygon where the resistant elements are distributed unevenly: the concentration on one side of shear walls and the one way beam orientation cause a strong irregularity.

3.1 Seismic input

In this study, the seismic action is defined using both the elastic response spectrum according to NTC'08 and a set of 7 natural time histories. In both cases, seismic action is described by two orthogonal components assumed as being independent and represented by the same response spectrum or by time history; the vertical component of the seismic action has been ignored.

Seismic action details are available in Bergami et al., nevertheless in agreement with NTC'08, seismic action has been defined according to the site and return period detected: the return period depends on the limit state and the category of the existing building. The life safety (SLV) limit state has been adopted.

According to the elastic response spectrum, a set of 7 natural time histories are defined using Rexel software [16]. In Fig. 4 the elastic response spectrum defined by NTC'08 is shown with the response spectrum of each time history record (RS record a-b-c-4-d-e-l).
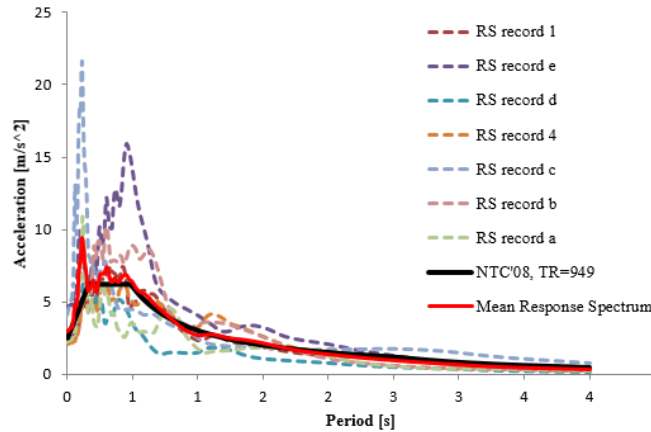

Fig. 4 – Response spectra of the code-compliant set of Time Histories: RS record a..RS record l are the selected ground motion records, NTC'08 is the response spectra according to the Italian technical code for a returning period of T_R=949 years.

4. Results

Modal analysis is employed to identify the dynamic behavior of the existing structure and investigate the relevance of higher modes (Fig. 5). In this paper only the Y direction will be discussed; along Y the first, fourth and seventh modes exhibit more than 78% of the participation mass and therefore these modes will be considered in the IMPA.
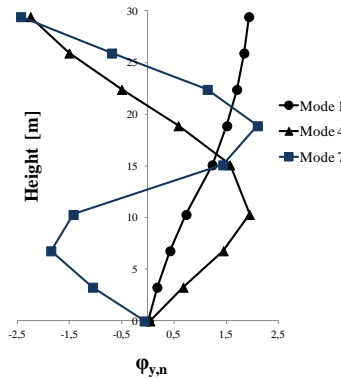

Fig. 5 – (right) Modal shapes: main three modes in terms of participating mass along Y. (left) Evaluation of the P.P. with CSM: in the plot there is the elastic Response spectrum $\zeta = 5\%$ (P.P. for Mode 4 and Mode 7 are in the elastic range) and the Response spectrum reduced according to a damping of $\zeta = 23\%$ (23% is the equivalent viscous damping at the P.P. for Mode 1)

Fig. 6a shows the performance point (P.P.) obtained with the CSM for each one of the capacity curves obtained applying a pushover analysis with a load distribution proportional to the three modal shapes considered. The demand spectrum used has obtained according to NTC'08 (PGA=0.25g). The structure enters in the nonlinear state for the first mode, and linear elastic state for fourth and seventh mode. According to CSM, when structure enters nonlinear plastic stage, and the spectral reduction factor depends on the effective viscous damping of equivalent Single Degree of Freedom (SDOF) system ξ_i.

By repeating this procedure for other intensity levels the Multimodal Capacity Curve (MCC) has been obtained. By combining the responses considering a range of intensity levels through SRSS rule, both for roof displacements and base shear are determined. Responses are obtained applying CSM on capacity curves obtained considering each modal shape and the response spectrum is scaled from PGA=0.05g to PGA=0.30g so for each intensity level an SRSS combined P.P. (P.P.mm) is obtained. Connecting this sequence of P.P. the multimodal capacity curve (MCC) can be defined (Fig.6b).
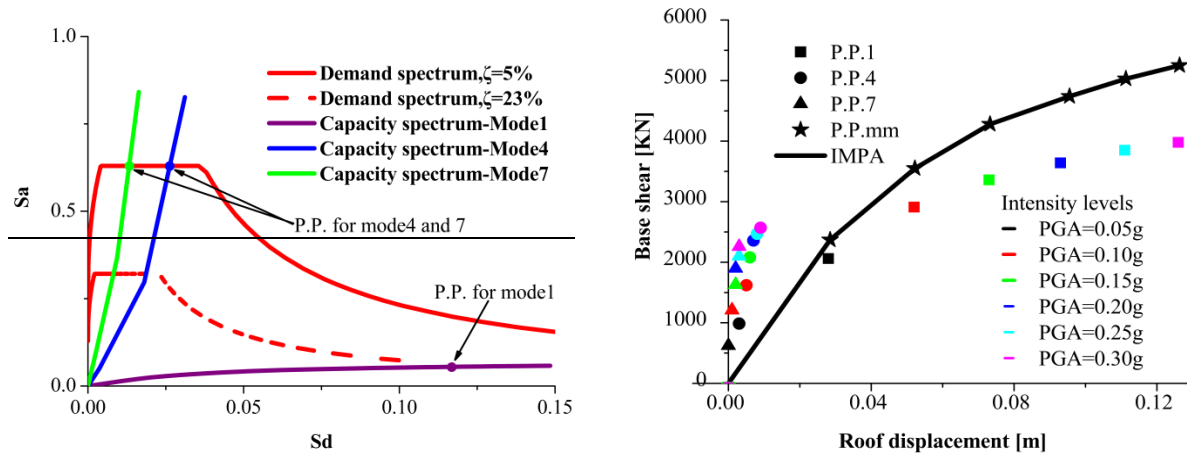

Fig. 6 – a) Evaluation of the P.P. with CSM: in the plot there is the elastic Response spectrum ζ = 5% (P.P. for Mode 4 and Mode 7 are in the elastic range) and the Response spectrum reduced according to a damping of ζ = 23% (23% is the equivalent viscous damping at the P.P. for Mode 1), b) - Construction of MCC from the IMPA procedure. The P.P.mm is obtained by applying SRSS rule with the P.P. obtained from single mode pushover (Mode 1, 4, 7)

Fig. 7 shows the capacity curves obtained with a standard pushover (load profile proportional to Mode 1: Push-Mode 1) or performing IMPA (connecting the P.P.mm) and NL_RHA (in the plot named IDA-Umax-Vmax); we underline that maximum displacements and maximum base shear from a NL_RHA are not contemporary, therefore this curve can be considered the upper bond of the capacity curve (any other pushover based curve will be lower).
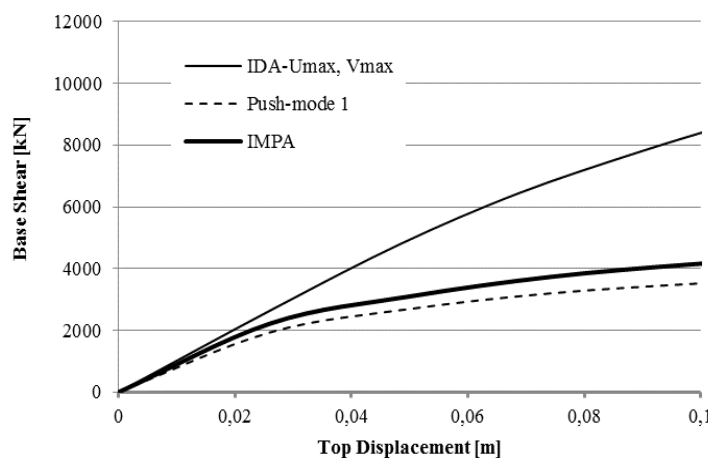

Fig. 7 - Capacity curves obtained from different methods: the standard pushover analysis along Y (for the predominate mode), IMPA method and IDA method.

Comparing IMPA to standard single mode pushover curve we can observe an increase of base shear that makes the capacity curve from IMPA stiffer than Push-Mode 1, and closer to the IDA curve.

The capacity curves are almost the same, it is also another indication that the pure translation along this direction. When the structure enters into inelastic state, IMPA would underestimate the base shear compared to IDA.

If we compare IMPA and IDA procedures in terms of Intensity Measure (IM), as a Peak Ground Acceleration (PGA) or Spectral Acceleration at the structure's first mode period ($S_a(T_1, 5\%)$), and Damage Measure (DM), as maximum interstory drift or maximum roof displacement, the incremental curves are evidently closer.

Fig. 8a shows the incremental curves obtained with performing IMPA (connecting the P.P.mm in terms of top displacement and PGA) and NL_RHA (in the plot named IDA-Umax, PGA). IDA curve and IMPA curve have almost the same values. Fig. 8b shows, in the same way, the incremental curves obtained with performing IMPA (connecting the P.P.mm in terms of maximum interstory drift and S_a) and NL_RHA (in the plot named IDA-Drift$_{max}$, S_a). The incremental curves, IDA and IMPA, are very close: IMPA method overestimates slightly maximum interstory drift values.
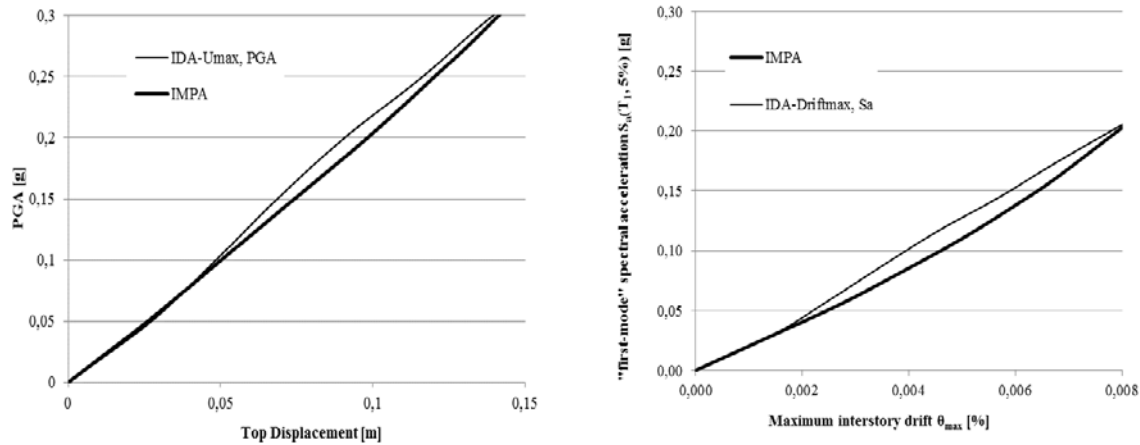

Fig. 8 - Incremental curves obtained from two methods: IMPA method and IDA method, a) in terms of Peak Ground Acceleration (PGA) and maximum top displacement b) in terms of "first mode" spectral acceleration $S_a(T_1, 5\%)$) and maximum interstory drift.

The structural behavior obtained from two methods is showed on Fig. 9, the story mechanism of deformation is the same of all PGA's values. However, IMPA method overestimates maximum interstory drift for the story levels lower and underestimates them for the story levels higher.
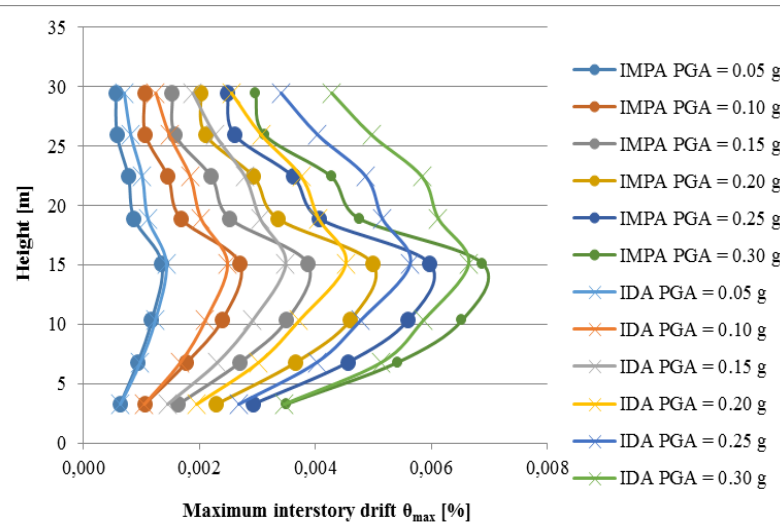
Fig. 9 - Structural behavior obtained from IMPA method and IDA method.

5. Conclusion

This work proposes a non linear static procedure to evaluate the seismic capacity of buildings that are sensitive to higher modes. The procedure, named Incremental Modal Pushover Analysis (IMPA), is proposed as an alternative to the non linear Incremental Dynamic Analysis (IDA).

It is well known that IDA implies the execution of many non linear response histories that entail complex and intense computational activity (in this case we used a PC-Intel ® Core ™ i7-4770 CPU @ 3.40 GHz for 52 hours of analysis): non linear dynamic analysis (NL_RHA) requires the preliminary definition of a set of time histories and the execution of the analysis, does not give an immediate and univocal interpretation of results. In fact the IDA curve, which represents the structure's non linear response to each of the selected and scaled time histories, can be realized with different approaches: maximum displacement and maximum base shear, maximum displacement and corresponding base shear, or maximum base shear and corresponding displacement, etc. Therefore, considering that the non linear dynamic analyses are of difficult execution, a simpler approach such as IMPA may constitute a valid alternative, especially for professional use.

IMPA is performed based on the well known modal pushover analysis (MPA) (Chopra and Goel, 2002) but its objective is to obtain a capacity curve (the capacity curve is commonly obtained with a standard pushover analysis) also considering the effect of higher modes: this approach widens applicability range of the non linear static analysis to include irregular and high rise buildings.

In IMPA the MPA is used to estimate the seismic demand and capacity of structures over the entire range of structural responses: the demand curve (the response spectrum) is scaled from lower to higher intensity values starting from the definition of a design response spectrum. Using the capacity spectrum method (CSM), for each single mode, and then the combination rule SRSS, a multimodal performance point (P.P.$_{mm}$) can be defined for each intensity level: the multimodal capacity curve (MCC) is the conjunction of all the multimodal performance points obtained.

The approach was tested by applying it to an existing irregular mid-rise building, therefore IMPA is suggested to predict the capacity of structures and, in particular, in case of structures sensitive to higher modes. Finally, comparing results in terms of base shear-top displacement between IDA, standard pushover and IMPA, the effectiveness of IMPA has been demonstrated: the multimodal capacity curve obtained with IMPA results closer to the IDA curve. IMPA's effectiveness is sensitive to the PGA (the distance between IMPA and IDA increases with PGA). Both IMPA and IDA curves show a hardening behaviour, with IDA resulting stiffer in the plastic range, while the pushover curve is mostly elasto-plastic. However, considering exact values resulting from IDA

method, for the pure translation direction, standard pushover underestimates base shear with an error of 28%, the IMPA underestimates base shear with an error of 13%.

Therefore this procedure can be considered a valid tool for professional use to estimate the capacity of structures even for non regular and high rise structures.

6. Acknowledgements

7. References

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