ON THE ACCURACY OF FRAGILITY ANALYSIS

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Abstract

Current methods for fragility analysis are evaluated in a setting in which seismic hazard is known so that reference fragilities, i.e., fragilities describing structural performance exactly, can be calculated. It is shown that fragilities of single-degree-of-freedom Bouc-Wen system and bilinear system under the postulated hazard obtained by current methods differ significantly from reference fragilities. Moreover, notable differences are found between fragilities obtained by current methods.

Keywords: fragility; conditional spectrum; comparison.

1. Introduction

Fragilities are probabilities that structural systems enter specified damage states for given ground motion which are characterized by intensity measures such as pseudo-spectral acceleration $PS_a(T_1)$ where T_1 is the fundamental period of the structure. They constitute essential tools for performance based seismic design [1].

Current fragilities are commonly constructed by the conditional spectrum-based ground motion selection method (CS) [2, 3] and the random selection method (RS) [4]. Both methods select ground motions in some manner and scale these motions to construct fragilities. The essential difference between them relates to the set of ground motions used for fragility analysis.

Various procedures have been proposed to assess the accuracy of current method for constructing fragilities [5]. Recent studies examine the performance of fragility estimates by using concepts of information theory [6] and benchmark studies based on synthetic ground motions [7].

Due to the scarcity of intense earthquake acceleration records, various stochastic models have been proposed for generating synthetic ground motions [8, 9] for analysis purposes. In this study, the specific barrier model [9, 10] is applied. It provides a complete description of acceleration power spectra of seismic ground motion. It gives the probability law of ground motions given moment magnitude M, source-to-site distance R and other site parameters.

Our objective is to further investigate the accuracy of fragility estimates by comparing the reference fragilities defined in Section 4.1 with approximate fragilities by the CS and RS methods. The reference seismic hazard and approximate seismic hazard are used respectively for the construction of reference and approximate fragilities. In this study, the comparison of ground motion selection procedures is developed in the context of the intensity-based assessment. The ground motion selection algorithm for matching the mean and variance of a target response spectrum is applied for constructing fragility function for the CS method [11]. A random selection procedure is also included in the comparison analysis [4]. A procedure to construct reference fragilities is presented to quantify the differences between all the sets of fragilities.

2. Seismic Hazard

A hypothetical seismic hazard, referred to as the reference seismic hazard, is constructed and used to assess the performance of intensity measures and associated fragilities. The following sections define the reference and approximate seismic hazards.

2.1 Reference seismic hazard

We model the reference seismic hazard by seismic activity matrices and ground motions. The seismic activity matrices are histograms of seismic events plotted against earthquake magnitude M and site-to-source distance R. The law of the seismic ground motion process is given by the specific barrier model [9, 10] and is a Gaussian process with mean zero and second-moment properties depending on (M, R).

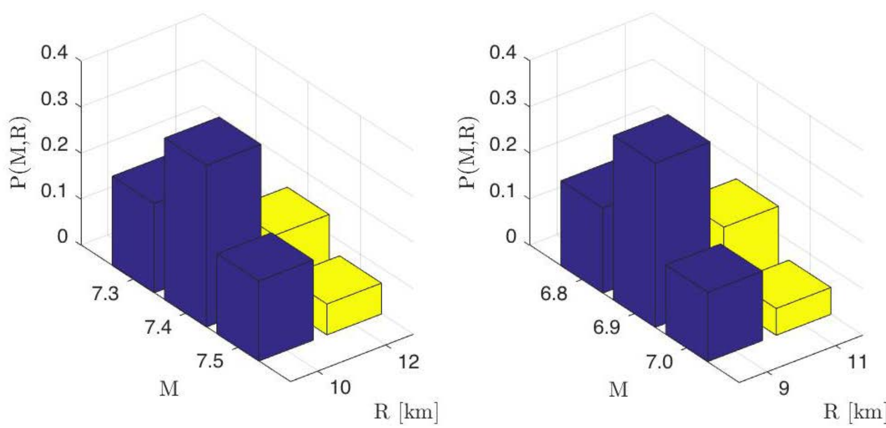

Fig. 1 – Seismic activity matrices for fault I and fault II

2.1.1 Seismic activity matrix

For simplicity, the site is assumed to be affected by two strike-slip line faults, each of which has its own seismic activity matrix as shown in Fig.1. We assume earthquakes occur along each fault with equal probabilities. The moment magnitude M of each earthquake follows Gutenberg-Richter distribution [12]. The rates of occurrence of earthquakes for Fault I and Fault II are 0.05 and 0.02, respectively. The shear-wave velocity in the top 30 m of the soil is 520 m/s. This site is an extension of the model in [7].

2.1.2 Ground motion

We use the specific barrier model to generate synthetic ground motions. The model gives the probability law of seismic ground motions given the moment magnitude M, the source-to-site distance R and other site parameters. It assumes that the ground motion process $A(t)$ is a zero-mean, non-stationary, Gaussian process defined by

$$A(t) = h(t)Z(t), \quad 0 \leq t \leq t_f \tag{1}$$

where t_f is the duration of the simulated ground motion defined by the specific barrier model, and

$$h(t) = e_1 t^{e_2} \exp(-e_3 t) \tag{2}$$

is a deterministic modulation function with constant parameters e_1, e_2 and e_3 and $Z(t)$ is a zero-mean, stationary, Gaussian process with one-sided power spectral density $g(\nu; M, R)$ depending on the moment magnitude M and the source-to-site distance R. Fig.2 shows the spectral density function of $Z(t)$ and the sample of the seismic ground motion process $A(t)$ for $(M = 7.3, R = 10\ km)$ at this site defined in Section 2.1.1.

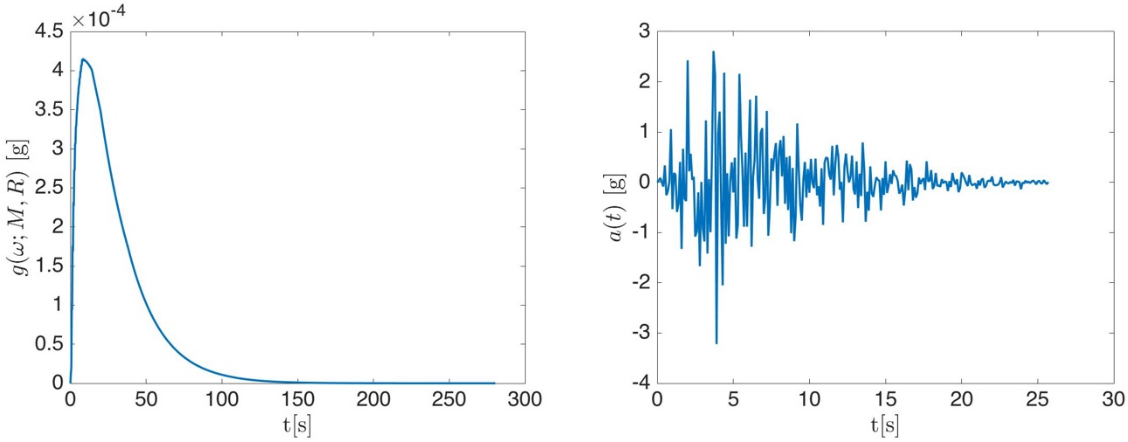

Fig. 2 – Spectral density of $Z(t)$ and the sample of the ground motion process $A(t)$ for $(M = 7.3, R = 10\ km)$

2.2 Approximate seismic hazard

The approximate seismic hazard is described by a table of samples of the ground motion process $A(t)$ defined in the previous section. This definition mimics practice in the sense that the information on the seismic hazard is limited to a finite set of ground motion records. The essential difference from practice is that in our setting the probability law of the ground motion is known. Following practice, the table of synthetic ground motions is used to construct intensity measures and two types of fragilities.

In practice, a ground motion prediction model is needed to describe the intensity measure $PS_a(T)$ as a function of moment magnitude M, source-to-site distance R and period T. Also a correlation model is needed to describe correlation between spectral acceleration values at multiple periods. The current prediction and correlation models are not consistent with the ground motions in this study. To make the ground motion prediction model consistent with the ground motions in our study, data-consistent prediction and correlation models are developed.

3. Structural systems

The structures considered for the illustrative purpose are single-degree-of-freedom Bouc-Wen and bilinear system. Let $X(t)$ be the relative displacement of a single-degree-of-freedom system subjected to the seismic ground acceleration $A(t)$. For the Bouc-Wen [13] and bilinear systems [14], $X(t)$ satisfies the following equations respectively

$$\text{Bouc-Wen: } \ddot{X}(t) + 2\zeta\omega_0\dot{X}(t) + \omega_0^2(\rho X(t) + (1 - \rho W(t))) = -A(t) \tag{3}$$

$$\dot{W}(t) = a\dot{X}(t) - b|\dot{X}(t)||W(t)|^{n-1}W(t) - c\dot{X}(t)|W(t)|^n$$

$$\text{Bilinear: } \ddot{X}(t) + 2\zeta\omega_0\dot{X}(t) + \omega_0^2\phi(X, x_m, \alpha) = -A(t) \tag{4}$$

Where ω_0 is natural frequency, ζ is damping ratio, x_m is yield displacement, α is post-yield hardening ratio, and ρ, a, b, c, n are system parameters. $W(t)$ defines the hysteretic displacement for Bouc-Wen system and $\omega_0^2\phi(X, x_m, \alpha)$ describes the hysteretic restoring force for bilinear system. Numerical results are shown for ω_0 = 2π rad/s, ζ = 0.02, x_m = 0.01, α = 0.05, ρ = 0.1, a = 0.1, b = 0.5, c = 5 and n = 1. The hysteretic restoring force of the single-degree-of-freedom Bouc-Wen and bilinear systems are plotted for a sample of $A(t)$ as shown in Fig.3.

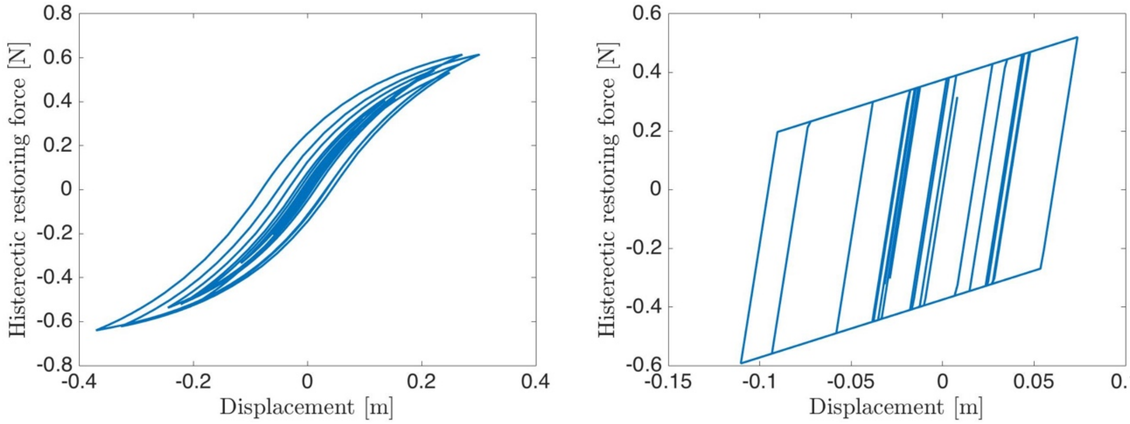

Fig. 3 – Dynamic response for the Bouc-Wen and bilinear systems subjected to a sample of $A(t)$

The engineering demand parameter of interest is the maximum relative displacement of the structure defined by

$$\tilde{x}_{i,\tau} = \max_{0\leqslant t\leqslant\tau}(|x_i(t)|) \tag{5}$$

where $x_i(t)$ is the relative displacement of the system subjected to a ground motion sample $A_i(t)$. The intensity measure IM is the pseudo-spectral acceleration at the fundamental period T_1 of the structure given by

$$PS_{a,i}(T_1) = \omega_0^2 \max_{0\leqslant t\leqslant\tau}(|x_i(t)|) \tag{6}$$

4. Fragility analysis

Seismic fragility is a function of scalar ground motion intensity measure. Various methods have been proposed for constructing fragility functions. The reference fragilities are developed by using the law of the ground motion process as defined in Section 2.1.2. Unscaled synthetic ground motions are used as inputs to the systems to avoid any change of the probability law of random functions [15]. The approximate fragilities are developed by using the table of samples of ground motion process as defined in Section 2.2. For the CS method, different sets of ground motions are selected and scaled for each intensity level for construction of fragilities. For the RS method, a distinct set of ground motions are scaled to each intensity level to construct fragilities. The procedure to construct the reference fragilities is presented in Section 4.1. The procedures to construct approximate fragilities by the CS and RS methods are summarized in Section 4.2.

4.1 Reference fragility

The construction of the reference fragilities involves the following steps.

Step.1 For each pair of (M, R) in the seismic activity matrix, n_{gm} ground motions are simulated from the specific barrier model [9,10].

Step.2 Nonlinear response history analyses are performed for the system of interest with all unscaled synthetic ground motions. The maximum relative displacements $\tilde{X}_\tau$ are plotted with respect to corresponding $PS_a(T_1)$ as shown in Fig.4.

Step.3 The range of $PS_a(T_1)$ is partitioned into equal subintervals. For each subinterval, the center value of each subinterval is denoted as the intensity level. The fragility $P(X > x_{cr}|M = m, R = r)$ can be estimated by the fraction of failures in each subinterval for a selected displacement threshold x_{cr}.

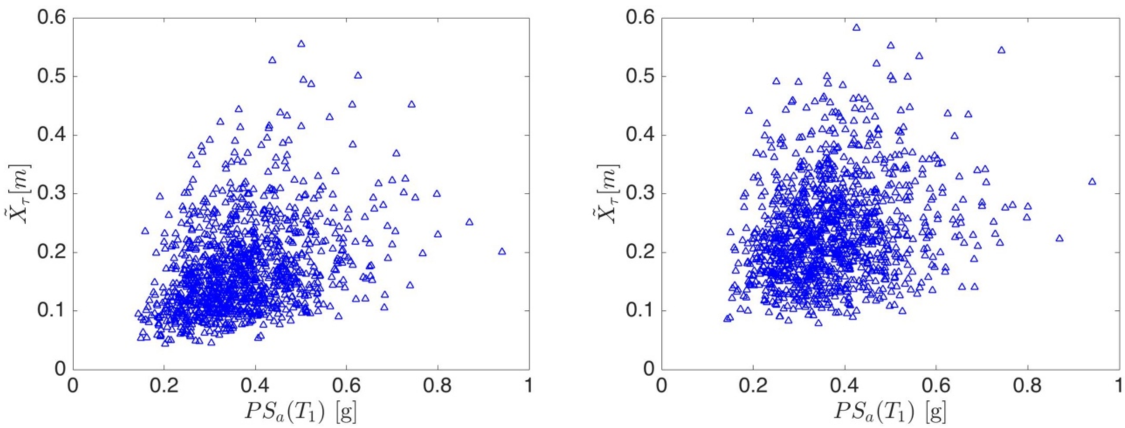

Fig. 4 – $\tilde{X}_{\tau}$ vs. $PS_a(T_1)$ for sdof Bouc-Wen and bilinear system

Step.4 The fragilities for different intensity levels can be fitted to a lognormal distribution by maximum likelihood method.

Step.5 The reference fragility can be obtained by combining fragility curves for all pairs of (M, R)

$$P(X > x_{cr}) = \sum_{i=1}^{n} p(M = m, R = r)P(X > x_{cr}|M = m, R = r) \quad (7)$$

where n is the number of pairs of (M, R), $p(M = m, R = r)$ is the probability of occurrence of earthquakes for $(M = m, R = r)$.

4.2 Approximate fragility

4.2.1 Conditional spectrum-based selection method

A set of ground motions is selected for each intensity level by matching the mean and variance of the target response spectrum. The procedure for ground motion selection based on this method [2, 11, 16] can be summarized as follows:

Step.1: The associated M, R and $\epsilon(T)$ can be obtained from deaggregation given a target intensity level. The deaggregation information represents the distribution of $(M, R, \epsilon(T))$ for combinations causing the exceedance of a certain intensity level. $\epsilon(T)$ is defined as the number of standard deviations by which an observed logarithmic spectral acceleration $\ln PS_a(T)$ differs from the mean of $\ln PS_a(T)$ from a ground motion prediction model.

The conditional distribution of moment magnitude M is calculated by

$$P(M = m|IM > x) = \frac{\lambda(IM>x, M=m)}{\lambda(IM>x)} \quad (8)$$

$$\lambda(IM > x, M = m) = \sum_{i=1}^{n_{sources}} \lambda_i \sum_{k=1}^{n_R} \mathbb{P}\,(IM > x|m, r_k)_i P(M_i = m)P(R_i = r_k) \quad (9)$$

where IM represents intensity measure, $n_{sources}$ is the total number of seismic sources, λ_i is the rate of occurrence of earthquakes greater than the lower limit of moment magnitude of interest. $P(IM > x|m, r_k)_i$ is computed by

$$P(IM > x|m, r) = 1 - \Phi\left(\frac{\ln x - \overline{\ln IM}}{\sigma_{\ln IM}}\right) \quad (10)$$

where the term $\overline{\ln IM}$ and $\sigma_{\ln IM}$ are the outputs of the ground motion prediction model. The conditional distribution of source-to-site distance $P(R = r|IM > x)$ can be obtained in the same manner. Then the conditional mean $\overline{m}$ and $\overline{r}$ can be obtained, where $\overline{m} = E[M = m|IM > x]$ and $\overline{r} = E[R = r|IM > x]$.

Step.2: The conditional mean and standard deviation of the response spectrum is given by

$$\mu_{\ln PS_a(T;\overline{m},\overline{r})|\ln PS_a(T_1;\overline{m},\overline{r})=x} = \mu_{\ln PS_a(T;\overline{m},\overline{r})} + \rho(T,T_1)\epsilon(T_1)\sigma_{\ln PS_a(T;\overline{m},\overline{r})} \quad (11)$$

$$\sigma_{\ln PS_a(T;\overline{m},\overline{r})|\ln PS_a(T_1;\overline{m},\overline{r})=x} = \sigma_{\ln PS_a(T;\overline{m},\overline{r})}\sqrt{1-\rho^2(T,T_1)} \quad (12)$$

where $\ln PS_a(T;M,R)$ is a non-stationary Gaussian process with mean $\mu_{\ln PS_a(T;M,R)}$ and variance $\sigma^2_{\ln PS_a(T;M,R)}$ obtained from a ground motion prediction model with correlation function $\rho(T,T_1)$. The ground motion prediction model and correlation model are constructed consistent using ground motions in this study following standard procedures.

Step.3: A greedy optimization technique [11] is applied to select the set of ground motions which has the best match to the target response spectrum for the target intensity level.

4.2.2 Random selection method

The ground motions selection procedure described in [4] is referred to as the random selection method. In [4], 11 ground motions are selected and amplitude scaled to the given intensity level to construct fragility curve for nonlinear response history analysis of buildings. The random selection method can be summarized as follows:

Step.1 A single set of n_{rand} ground accelerations $\{A_i(t)\}, i = 1,2,\ldots,n_{rand}$ are randomly selected.

Step.2 The selected set of ground motions is scaled to all intensity levels. The response $X_i(t;IM)$ of the systems subjected to the scaled ground acceleration can be calculated. The maximum relative displacement can be obtained for each ground motion from Eq. (5) for the selected values of intensity measure.

Step.3 Fragility for each intensity level is equal to the number of ground motions that satisfies $\tilde{X}_\tau(IM)$ larger than the critical value x_{cr} divided by number of all selected ground motions n_{rand}.

5. Numerical results

Site information is given in Section 2.1. A set of synthetic ground motions are generated according to the seismic activity matrices in Section 2.1.1. The systems of interest are defined by Eq. (3) and Eq. (4) in Section 3.

5.1 Range of $PS_a(T_1)$

Let $A_i(t)$ denote the strong motion part of the ground motion process. The spectral density of displacement process $X_i(t)$ of a linear oscillator with parameter (ω_0,ζ) is

$$g_i(\nu;\omega_0,\zeta) = |h(\omega_0,\zeta)|^2 g_{Ai}(\nu) \quad (13)$$

where $h(\omega_0,\zeta)$ is the transient frequency response function, $g_{Ai}(\nu)$ is the spectral density of the input $A_i(t)$. The mean crossing rate [17] is defined in Eq. (14)

$$\nu_i(y) = \frac{1}{2\pi}\frac{\dot{\sigma}_i}{\sigma_i}\exp\{-\frac{1}{2}(\frac{y}{\sigma_i})^2\} \quad (14)$$

where $\sigma_i^2 = var[X_i(t)] = \int g_i(\nu;\omega_0,\zeta)d\nu, \dot{\sigma}_i^2 = var[\dot{X}_i(t)] = \int \nu^2 g_i(\nu;\omega_0,\zeta)d\nu$.

The probability of the maximum relative displacement larger than a certain critical value x_{cr} can be calculated as follows

$$P(\tilde{X}_\tau > x_{cr}) \approx \sum_i p_i [1 - \exp\{-2v_i(y)\tau\}] \quad (15)$$

where p_i be the probability of earthquake for cell i. Since $PS_a(T_1) = \omega_0^2 S_d(T_1)$, the result can be transferred into representation by $PS_a(T_1)$. As shown in Fig.5, intensity levels larger than 0.9g are extremely unlikely at our site.

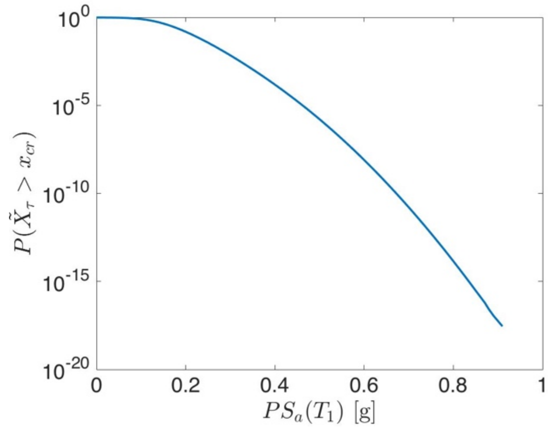

Fig. 5 – Probability of crossing

5.2 Comparisons

The approximate fragilities constructed by the CS and RS methods are compared with the reference fragilities as shown in Fig.6. The reference fragility curve is constructed following the procedure presented in Section 4.1. For the CS method, different sets of 40 ground motions are selected for each intensity level to construct the approximate fragility curve. For the RS method, a single set of ground motions is scaled to all intensity levels to construct the approximate fragility curve. Fragilities are fitted into lognormal distribution using the maximum likelihood method.

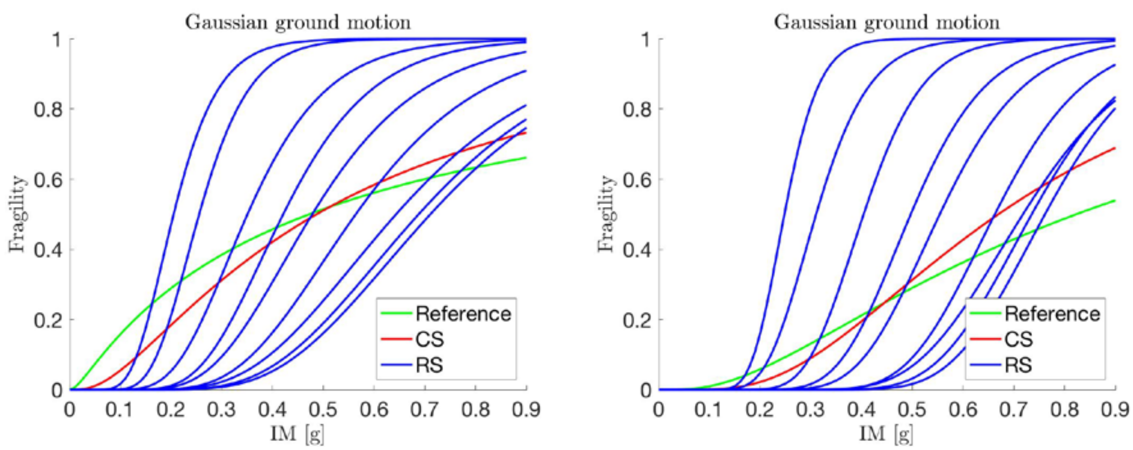

Fig. 6 – Fragility comparison for the Bouc-Wen system and the bilinear system

The range of intensity level is specified in Section 5.1. The intensity levels chosen for the construction of fragility curve are [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9] (g). 9 sets of 40 ground motions are selected respectively for the 9 intensity levels. For the RS method, a family of approximate fragility curves is developed with each curve constructed using one set of ground motions selected by the CS method. Since the set of ground motions selected by the CS method can be regarded as one possible combination of ground motions from random selection, this set of ground motions is scaled to all intensity levels to construct an approximate fragility curve by the RS method.

The plots in Fig.6 show the approximate fragility curves differ significantly from the reference fragility curve. For the Bouc-Wen and bilinear systems, approximate fragility curve by CS method tends to be more conservative than the reference fragility curve at larger intensity level, and less conservative at smaller intensity level. For the approximate fragility curves by the RS method, different sets of ground motions produce significantly different fragilities. The set of ground motions selected by the CS method for larger intensity level tend to produce less conservative fragility curve when used to construct fragility curve by the RS method.

6. Conclusions

Current methods for fragility analysis have been evaluated in the framework of a postulated seismic hazard, which allows the construction of error-free fragilities, referred to as the reference fragilities.

Differences and similarities between current and reference fragilities have been explored in the context of single-degree-of-freedom systems with Bouc-Wen and bilinear hysteresis. It was found that (1) some of the current fragilities are sensitive to the particular set of ground motions selected for analysis, (2) current fragilities differ significantly from the reference fragilities.

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