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# ESTIMATION OF UNIFORM VULNERABILITY SPECTRA FOR SEISMIC DESIGN OF STRUCTURES

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## *Abstract*

A methodology for the estimation of uniform vulnerability spectra (UVS) is proposed. The methodology includes two main stages: On the one hand, the zone seismicity definition in terms of exceedance rate of magnitudes, and on the other hand the computation of the exceedance probability of strength demand given an expected damage. The proposed methodology is the following: (1) computation of the exceedance rate of magnitudes, (2) characterization the seismic hazard in terms of ground motion records according to the considered magnitudes in the above step, (3) computation of damage spectra associated to several strength values for each ground motion record, (4) statistical and probabilistic treatment of the structural response in terms of damage index for each magnitude, strength value and structural period, (5) obtention of the expected lateral strength corresponding to a damage level given a structural period and a magnitude, (6) computation of lateral strength exceedance probability given a damage level for each structural period and magnitude (7) computation of exceedance rate of lateral strength given a damage level and a structural period (8) construction of the UVS by selecting, in several exceedance rate of lateral strength curves associated to several structural periods, the lateral strength value that has the same given exceedance rate. Finally is presented an example of proposed methodology applied to a hypothetic single seismic source.

*Keywords: spectra, vulnerability, structural design, structural damage.*

## 1. Introduction

Past earthquakes (Michoacán 1985, Loma Prieta 1989, Northridge 1994, Kobe 1995, Ecuador 2016) have demonstrated that traditional design philosophies do not always provides the expected security levels. Because of this, several researchers have made efforts to introduce explicitly the structural behavior on the seismic design through of design methodologies [1, 2, 3, 4]. In addition, performance-based design spectra as a useful tool for seismic design of structures have been developed, this type of spectrums provide the resistance associated to an objective performance. For example, Sewel and Cornell [5] presented an algorithm based on modify the response spectra through reduction factors which depend, among other parameters, of the structural period and the non-linear deformation level. Collins *et al.*, [6] obtained uniform hazard spectra using synthetic ground motions records, then, associating the structural displacements with several seismic yield coefficients, they proposed an empirical equation to estimate the annual exceedance probability of ductility level as a function of the seismic yield coefficient. Mendoza *et al*. [7] presented a methodology to compute uniform reliability design spectra through Monte Carlo simulations. Ghosh and Collins [8] established the bases to use uniform energy spectra in the structural design. Avelar *et al*. [9] developed a methodology to obtain performance-based design spectra associated to a ductility level. Niño *et al*. [10] presented a methodology to obtain performance-based design spectra of uniform hazard associated to an objective damage index, which are useful in performance-based design philosophy. Rivera and Ruiz [11] formulated a procedure to compute uniform reliability spectra for structural systems with energy dissipaters, they showed that the hysteretic curves do not depend on structural velocity or vibration period but the relative displacement between the energy dissipater edges. Luco *et al*. [12] indicated that uncertainties in the structural characteristics cause to the failure probability do not be necessarily equal to the uniform hazard in the zone, they proposed to adjust the existent seismic hazard maps to obtain uniform failure probability maps. Niño and Ayala [13] developed a process to include variability in the structural characteristics in order to compute the lateral strength associated to a performance-based design spectra. Baker [14] proposed to use conditional mean spectrums (CMS), as CMS provides a response spectra conditioned to an objective spectral acceleration. Besides CMS is an efficient tool to select seismic records useful in subsequent dynamic analysis. Lin *et al*. [15] demonstrated that use intensity-based design spectra might cause overestimations in the structural response. Loth and Baker [16] showed that the conventional uniform hazard spectra are typically conservative to estimate the structural response in terms of any structural parameter on multimodal structures.

As can be seen, efforts to considerer structural characteristics to determinate the design forces associate to a performance level, have been advancing in the last years. However, it should be noted that most of the aforementioned methodologies are useful to compute spectrums associated to only one performance level, considering the exceedance probability associated to a damage state. Such spectrums provide the lateral strength needed to reach or exceed a single damage state (fragility). In this paper, the proposed spectrums consider the occurrence probability of all damage states associated to a same seismic demand, obtaining in the process the mean damage state (vulnerability), so that an Uniform Vulnerability Spectra (UVS) provides the necessary design resistance so that, on average an structure presents an objective damage level each certain period of time.

## 2. Proposed Formulation

The proposed model is based on the Probabilistic Seismic Hazard Assessment (PSHA) equation [17, 18, 19]. PSHA provides the exceedance rate of seismic intensity through the computation of the occurrence probability of an earthquake with magnitude $M$ and the conditional probability of exceedance of an intensity value $A$, given a magnitude $M$ and a distance $R$.

The exceedance rate of design strength associated to an expected damage value, can be explained as how often a seismic demand exceeds a design resistance associated to a damage level $\beta$, if occurs an earthquake with magnitude $M$ from a distance $R$.

Based on the aforementioned, the exceedance rate of lateral strength can be computed as the sum of all seismic sources and magnitudes for the product of the mean number of earthquakes with magnitude, $M$, of the $i$-th seismic source by the exceedance probability of a strength demand $s$, given that an earthquake of magnitude, $M$, from a distance, $R$, has occurred producing a damage, $\beta$, on a structure with vibration period, $T$. The proposed expression is presented in the Eq. (1):

$$\nu(s) = \sum_{i=1}^{N} \int_{Mo}^{Mu} -\frac{\partial \lambda(M)}{\partial M} \cdot \Pr\left(S \geq s \mid M, T, DI\right) dM \tag{1}$$

where $\nu(s)$ is the exceedance rate of the lateral strength, $\frac{\partial \lambda(M)}{\partial M}$ is the annual mean number of earthquakes with magnitude, $M$, that the seismic source produce and Pr ($S \geq s \mid M, R, \beta, T$) is the probability of a strength demand exceed a design strength, $s$, given a magnitude, $M$, an expected damage, $DI$, and a structural period, $T$, the design strength, $s$, given in terms of spectral acceleration ($Sa$), may be computed as:

$$s = \frac{R_y}{m} \tag{2}$$

where $R_y$ is the lateral strength and $m$ is the mass of the system.

## 3. Proposed Methodology

The proposed methodology to estimate UVS is the following:

1. Compute the exceedance rate of magnitudes $\lambda$ ($M$).
2. Represent the site seismicity through ground motion records, if do not exist enough real seismic records for this purpose, using synthetic seismic records is recommended.
3. Compute damage spectra for all seismic record associated to a single magnitude, these spectra must be computed for a collection of yielding strength values.
4. Select a structural period in a damage spectra collection associated to a magnitude and yield strength, then fit a probability function to the damage index behavior for such period.
5. Obtain the exceedance probability of damage index, given a structural period, magnitude and yield strength.
6. Compute the probability of design strength value be equal to a strength demand given a damage level
7. Compute the expected value and standard deviation of lateral strength given a damage acceptance.
8. Compute the exceedance probability of a design strength given an accepted damage value.
9. Compute, through of the Eq. (1), the exceedance rate of design strength given a damage level.

   Steps 4 to11 must be repeated for several structural periods.

10. Obtain UVS by selecting the lateral strength associated to a same exceedance rate for several structural periods

## 4. Case of study: Uniform vulnerability spectra associated to the seismic source “Guerrero Gap” for the lake zone in Mexico City

Mexico City has grown exponentially, particularly in the last century, achieving an important urban development. It is founded over three different soil types (firm, transition and soft soil), where seismic waves from far seismic sources might be amplified or attenuated. Over the soft soil region the Communications and Transports Ministry (SCT, for its acronym in Spanish) seismic station is located. This station is known as the most representative place in terms of dynamic amplifications on alluvial valleys, such place shows the negative effects caused by the site effects on the Mexico City lake zone.

The Mexico City seismic hazard is defined by three seismic source types: subduction, intermediate depth and continentals. In Mexico, subduction earthquakes are the most frequent, and they have caused plenty of fatalities and economic losses [20]. One of seismic zones that has drawn attention of seismologist and seismic engineers is the Guerrero Gap (shown in the Fig.1), because it has not generated an important earthquake ($M$ > 7.0) since 1911 [21], this suggest the null stress liberation might result in a large earthquake.

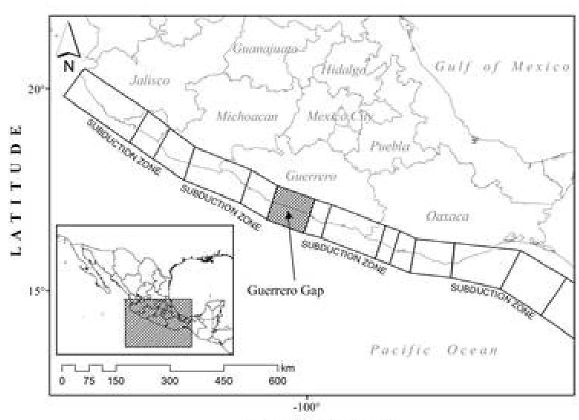


Fig. 1 – Mexican subduction zone [22] and the Guerrero Gap (highlighted area)

In this example for illustrative purposes, the computation of UVS is associated to a single seismic source (Guerrero Gap) and is applicable to a single place (SCT seismic station site), however it is necessary to explain that a complete study of seismic hazard must consider all the seismic sources that affect an interest area.

Step 1: Exceedance rate of magnitudes

To compute the magnitude exceedance rate, the earthquakes were delimited to the characteristic earthquakes with magnitudes from 7.2 to 8.2, due to earthquakes with these magnitudes have historically caused the largest damage on Mexico City. To compute the exceedance rate of magnitudes the characteristic earthquake model [23] was used (Eq. 3).

$$\lambda\left(M\right)=\lambda_0\frac{1-\Phi\left(\dfrac{M-E(M\,|\,T00)}{\sigma_M}\right)}{\left[\Phi\left(\dfrac{M_u-E(M\,|\,T00)}{\sigma_M}\right)-\Phi\left(\dfrac{M_o-E(M\,|\,T00)}{\sigma_M}\right)\right]} \tag{3}$$

The exceedance rate of magnitudes was computed using the following values [9]: *T00* = 80 years, $M_o$ =7.0 *Mu* = 8.4, *F* = 0.0, *D* = 7.5, $\sigma_M$ = 0.27 y $T_o$ = 39.7 years. Fig. (2) shows the exceedance rate curve corresponding to the Guerrero Gap characteristic earthquakes.

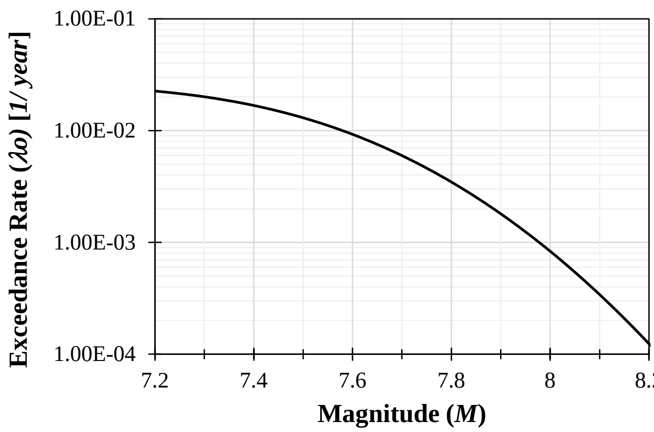


Fig. 2 – Exceedance rate of magnitude for the Guerrero Gap source

Step 2: Seismicity at the interest zone

Due to the considered magnitudes correspond to unusual earthquakes, it was necessary to use synthetic ground motion; a total of 11,000 synthetic ground motion records were used to describe the seismicity at the SCT site. These seismic records were computed by Niño *et al*. [13] through empirical Green function approach using two corner frequencies and a two stages summation scheme. The considered seismic magnitudes were from 7.2 to 8.2 with an increase in 0.1, resulting 1,000 seismic records per magnitude. The objective of this step is to characterize the intensity levels at the study site.

Step 3: Damage spectra computation

The damage spectra allows to know the response of a single degree freedom system family (SDOF) in terms of damage index (*DI*) [24]. In this work, the selected damage index was the proposed by Teran and Jirsa [25], due to it presents a good agreement with the ground motions in Mexico City lake zone [26], furthermore, its parameters are already calibrated for the Mexico City soft soil. The Teran and Jirsa damage index expression is defined as follows:

$$DI = \frac{NE_{H\mu}\left(2-b\right)}{r\left(2\mu_u - 1\right)} \tag{4}$$

where $NE_{H\mu}$ is the normalized hysteretic energy dissipated in each hysteretic cycle, $\mu_u$ is the ductile capacity of the structural system under monotonic load action, *a* and *b* are hysteretic stability parameters.

The normalized hysteretic energy $NE_{H\mu}$ is computed as follows:

$$NE_{H\mu} = \frac{E_{H\mu}}{\delta_y F_y} \tag{5}$$

where $E_{H\mu}$ is the dissipated hysteretic energy, $\delta y$ is the yield displacement and the $F_y$ is the yield strength.

The parameter values depend of the seismic excitation and the hysteretic model used which must be suitable for the structural system type will be analyzed; in the case of this work, a bilinear hysteretic model was used [27].

Damage spectra for all seismic records were computed. Lateral strengths were selected in a range of spectral acceleration from 10 to 2000 gals with an increase in 10 gals, the maximum and minimum lateral strength were selected as a function of the maximum and minimum spectral acceleration observed in all response spectra. Fig.3 shows some damage spectra obtained for five arbitrary earthquakes and two strength values.

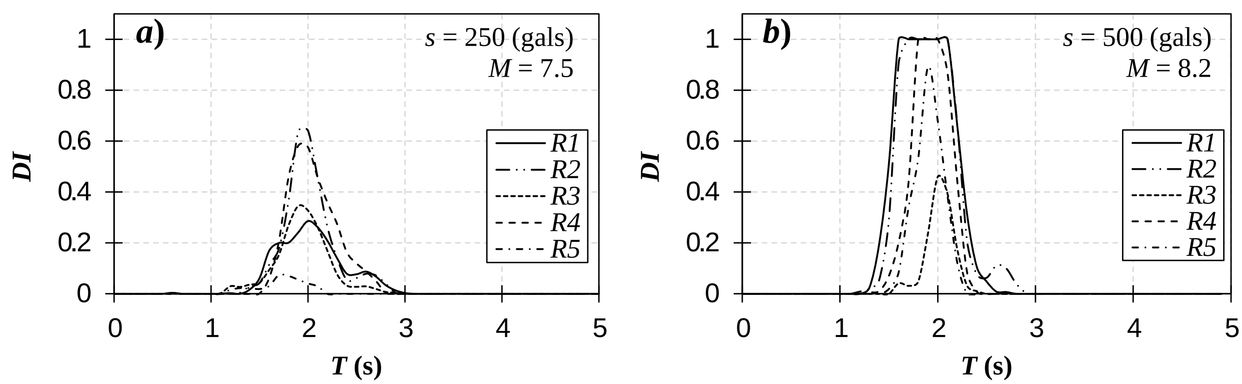


Fig. 3 – Damage spectra associated to five seismic records (*R1*, *R2*, *R3*, *R4*, *R5*) and lateral strength of *a*) 250 gals and *b*) 500 gals, for 7.5 and 8.2 magnitudes correspondingly

Step 4: Fit a probability function to the damage index behavior

Within a damage spectra family associated to a magnitude and a lateral strength, all damage indexes corresponding to a single vibration period were selected, in order to fit a probability density function to the occurrence of damage index behavior.

For illustrative purposes Fig. 4 shows four relative frequency histograms associated to a structural period and an earthquake magnitude for several lateral strength values. Fig 4 shows the maximum frequency values that are closer to a damage index equal to one, correspond to lower lateral strength values, while that the maximum frequency values that are closer to a damage index equal to zero, correspond to a larger lateral strength.

A probability distribution function was fitted to the observed data; in this case the random variable is represent by the damage index. To represent the damage index behavior the Beta distribution function was proposed (Eq. 6) due to it is delimited among values from zero to one, as the case of the employed damage index.

$$f(DI) = \frac{1}{B} DI^{a-1}\left(1 - DI\right)^{b-1} \tag{6}$$

where $B$ is the Beta function, $DI$ is the damage index, $a$ and $b$ are Beta distribution control parameters computed in terms of damage values.

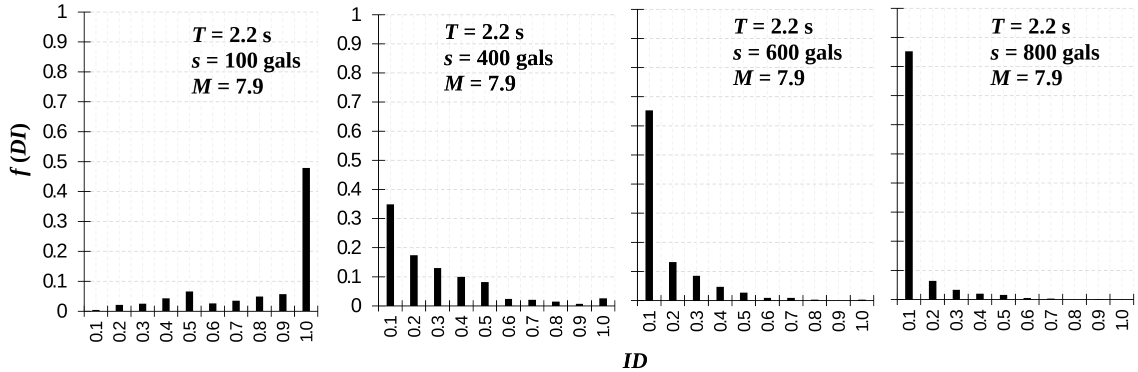


Fig. 4 –Relative frequencies distribution of damage index for a structural period, a magnitude and some lateral yield strength

To verify the fit to Beta distribution, Kolmogorov-Smirnov goodness of fit test was carried out.

Step 5: Exceedance probability of a damage value

The exceedance probability of damage value given a magnitude, a structural period and a lateral strength was obtained through Eq. (7).

$$\Pr\left(di \geq DI \mid M, T, s\right) = 1 - \frac{1}{B}\int_{0}^{di} DI^{a-1}\left(1-DI\right)^{b-1} dDI \tag{7}$$

where Pr ($di \geq DI \mid M, T, s$) is the probability that an expected damage $DI$ be exceed given the occurrence of an earthquake with magnitude, $M$, affecting a structure with vibration period, $T$, and a design strength, $s$.

In this case of the damage characterization, it was represented by a damage index, however, any quantitative or qualitative damage characterization might be used, associating in any case, a quantitative relationship between the employed damage index and another damage measure (e.g. losses, damage states, etc.).

For illustrative purposes, Fig. 5 shows five exceedance probability of damage index curves associated to five lateral strength values.

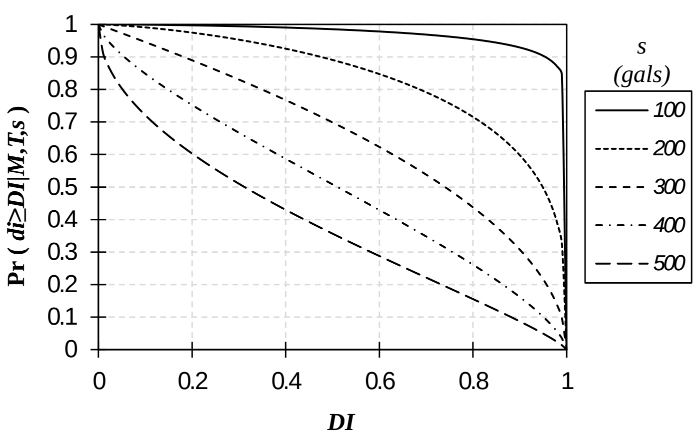


Fig. 5 –Exceedance probability of damage index for five arbitrary values of lateral yield strength ($T$ = 2.2 s, $M$ = 8.2)

In Fig.5 shows we can see the exceedance probability of a damage index equal to zero is equal to one, this is logical because a damage index equal to zero means the structure does not have any damage, in other words, this damage state will always be exceed. On the other hand the exceedance probability of a damage index equal to one is zero, this is logical, due to a damage index equal to one means the structure has collapsed, so it is impossible to admit more damage. In the same figure, it can see that for a same damage index, the exceedance probability increases while the lateral strength decreases, for instance, for a damage index equal to 0.4, the exceedance probability has values of 0.99, 0.91, 0.77, 0.59 and 0.42 depending of the structure lateral strength.

Step 6: Strength probability matrix

Considering the design strength value as a discrete variable, the strength probability matrix can be computed. This matrix allows to know the probability of a strength demand be equal to a lateral strength associated to a damage index. The computation of the strength matrix probability was performing through Eq. (8):

$$\Pr(s = S_i \mid DI) = \Pr(di \geq DI \mid S_i) - \Pr(di \geq DI \mid S_{i+1}) \tag{8}$$

Eq. (7) can be understood as the probability of a design strength, $s$, be equal to the ***i***-th seismic strength demand, $S_i$, necessary to get an accepted damage level, $DI$. This probability is computed as the difference between the exceedance probability of an expected damage index given a lateral strength, $S_i$, and the exceedance probability of the same damage index given a lateral strength $S_{i+1}$. In this case, $S_{i+1}$ is larger than $S_i$.

Step 7: Expected value of lateral strength given an expected damage

The expected value and the standard deviation of strength given a damage value were computed using the Eq. (9) and the Eq. (10):

$$E\left[s \mid DI\right] = \sum_{i=1}^{n} \Pr(s = S_i \mid DI) \cdot (S_i) \tag{9}$$

$$\sigma\left[s \mid DI\right] = \sqrt{\sum_{i=1}^{n} \left\{ \left\{ (S_i) - E[s] \right\}^2 \cdot \Pr(s = S_i \mid DI) \right\}} \tag{10}$$

where $E\ [s|DI]$ is the expected lateral strength as given an accepted damage value $DI$, and $\sigma[s|DI]$ is the standard deviation between possible values of the lateral strength for a same damage level.

For illustrative purposes Fig.6 shows the first two lateral strength statistical moments variation, as a damage index function. The Fig 6 shows that in extremes values where the expected damage is total or null, the standard deviation is zero, this means the estimation of lateral strength for these damage states is successful. In the case of intermediate values, the variation level will depend of the selected damage value.

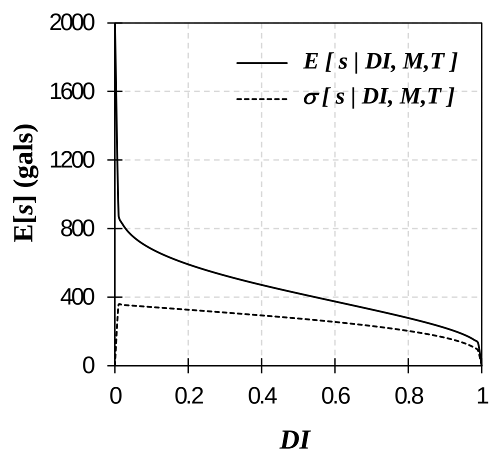


Fig. 6 –First two lateral strength statistical moments variation associated to a damage index, a structural period and an earthquake magnitude ($T$ = 2.2, $M$ = 8.2)

Step 8: Lateral strength exceedance probability given an expected damage

By selecting an accepted damage level on the previous curve (step 9) and the corresponding statistical moments, it is possible to compute the exceedance probability of lateral strength given the selected damage level. Generally the probability distribution that best fits to the lateral strength demand is the lognormal distribution [28, 29] with parameters $E_{ln}$ [$s$] and $\sigma_{ln}$ [$s$]. (Eq. (10)):

$$\Pr\left(S \geq s \mid M, T, DI\right) = 1 - \int_0^S \frac{1}{s \cdot \sigma_{\ln(s)} \sqrt{2\pi}} e^{\frac{-\left(\ln(s) - \mu_{\ln(s)}\right)}{2\sigma_{\ln(s)}}} ds \tag{10}$$

where Pr (S ≥ $s$| $M,DI,T$) is the probability that a design strength, $s$, be exceed by a strength demand, $S$, given a magnitude, $M$, a structural period, $T$, and was accepted a damage value $DI$.

For illustrative purposes Fig.7 shows several exceedance probability curves, it shows how the exceedance curves decay quickly if the magnitude is lower.

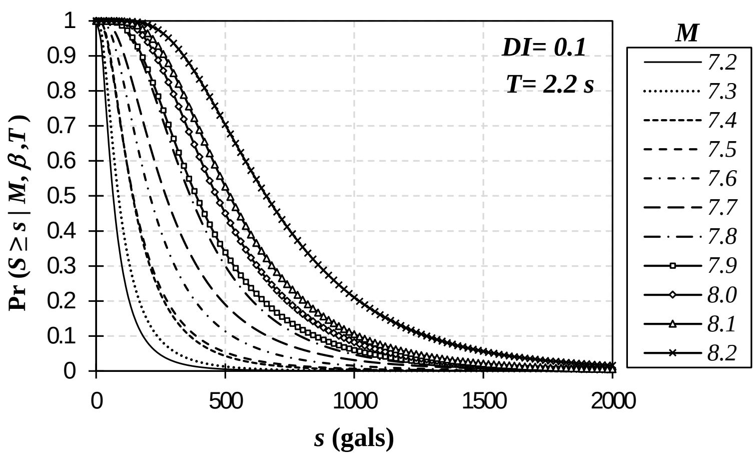


Fig. 7 –Exceedance probability of lateral strength given a damage level and a structural period associated to several magnitude values

Step 9: Exceedance rate of design strength

Once that the exceedance rate of magnitudes and the exceedance probability of lateral strength associated to a damage value have been computed, it is possible to compute the exceedance rate of lateral strength associated to a damage level by using the Eq. (1). However due to the application example is focused to a single seismic source, the Eq. (1) was delimited to this condition, resulting in Eq. (11):

$$\nu(s) = \int_{Mo}^{Mu} -\frac{\partial \lambda(M)}{\partial M} \cdot \Pr\left(S \geq s \mid M, T, DI\right) dM \tag{11}$$

where $\nu(s)$ is the exceedance rate of lateral strength corresponding to a single seismic source, $M_o$ and $M_u$ are the minimum and maximum magnitude considered in that seismic source.

Fig.8 shows several exceedance rate of design strength curves associated to five damages values, it is possible to see the exceedance rate associated to a design strength, decreases as the expected damage increase.

It is necessary to emphasize that the exceedance rate curves computed until this moment, correspond only to one structural period, therefore must be calculated several curves corresponding to all the possible structural periods in the UVS.

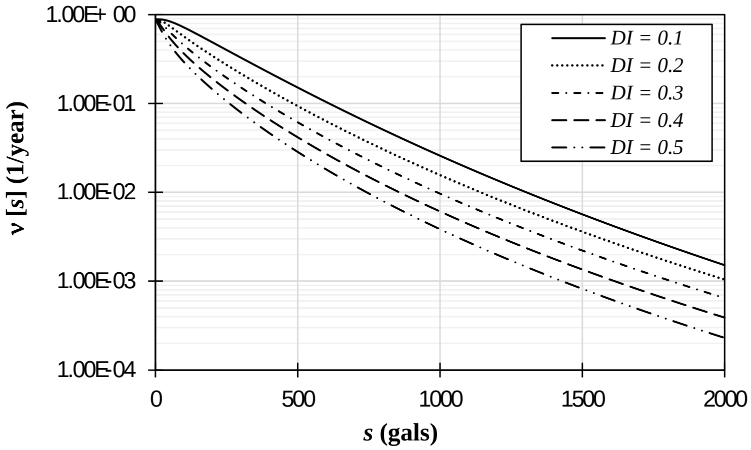


Fig. 8 –Exceedance rate of lateral strength curves for several arbitrary damages levels, all of them are associated to a vibration period of 2.2 s

Step 10: Obtention of uniform vulnerability spectra

To obtain a UVS, the process showed in steps 4 to 10 must be repeat until include all the structural periods considered on the UVS. The selected damage in the step 8 must be the same in all cases so that the resultant UVS will correspond to an only damage level.

When the exceedance rate curves corresponding to all periods included on the UVS have been computed, it is possible to obtain a complete spectra, by relating the strength demands associated to a same exceedance rate for several structural periods. Finally, the resultant strength demands (in terms of spectral acceleration) are presented on a graphic. Fig 9 shows several UVS, computed for several expected damages and several return periods (*RT*).

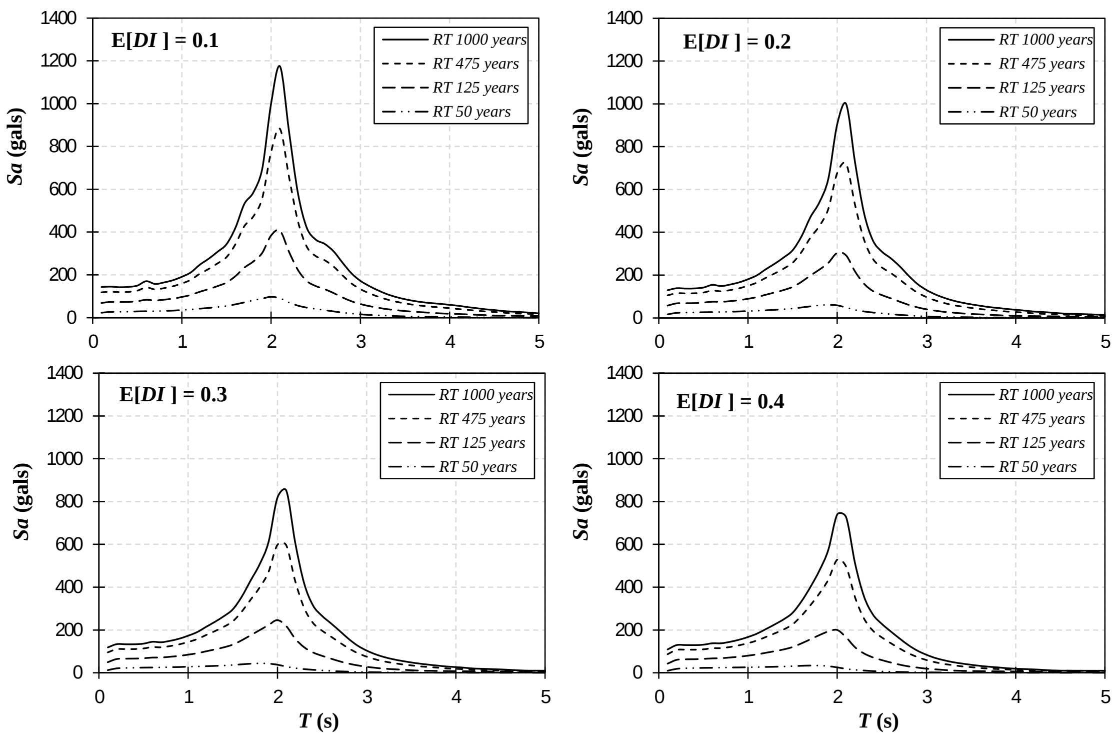


Fig. 9 –UVS to 5% damping, associated to expected damage levels of 10, 20, 30 and 40 percent, they were computed for return periods of 1000, 475, 125 and 50 years.

In the same figure, it is observed that the return period or the expected damage level selection, affects significantly the spectral ordinates, as a result, if the expected damage for the same exceedance rate increases, the spectral ordinates will decrease and vice versa, on the other hand, if the return period is larger, the spectral ordinates will be larger too.

In this example UVS for the characteristic earthquakes of a single seismic source (Guerrero Gap) that affects Mexico City soft ground zone was obtained. The proposed spectrums provide the seismic demands (in terms of spectral acceleration) associated to a selected damage level for a formal design process.

This information could be useful because allows to the designer or the building owner to take better decision about the building security including life safety and financial topics. It is expected that in the future, spectra as shown in this work, can be used to help in the design process.

## 5. Conclusions

The proposed spectra provides a rational way to assess the expected damage at the beginning of design process, this allows to take better decisions about the use of financial and time resources.

On the other hand, when damage is associated to monetary losses, it is possible to compute economic losses associated to a reparation cost, in this way, a curve of exceedance rate of loss can be computed, such representation allows to the decision maker creates a better orientation of economical and human resources.

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## 7. Copyrights



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