A STUDY ON RESIDUAL DISPLACEMENTS OF CONVENTIONAL AND DUAL STRUCTURES LOCATED IN VERY SOFT SOILS

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Abstract
Although damage to structures may be insignificant after an earthquake, the economic impact due to residual displacements (RDs) may be huge. This has recently been highlighted by many research studies. Therefore, this study focuses on the understanding of the parameters affecting RDs in conventional and dual systems (representative of structures equipped with Buckling-Restrained Braces) located in very soft soils. For that purpose, the statistics of the residual displacement demands in conventional and dual SDOF oscillators were investigated when the oscillators were subjected to 220 ground motions recorded on the lakebed zone of Mexico City. The effects of different parameters, such as post-yielding stiffness ratio, period of vibration, strength reduction factor and level of ductility on the responses were studied. It was observed that all these parameters affect the magnitude of the residual displacements. As an example, a positive post-yielding stiffness ratio (*r*) in conventional structures was found to be very effective in reducing the RDs, while negative values of *r* were highly detrimental. On the other hand, for dual systems, it was observed that RDs may be small when the primary part remains elastic. However, if the primary part exhibits inelastic deformations, the expected RDs are increased significantly. In this case, the post-yielding stiffness ratio of the secondary part plays a key role while that of the primary part becomes unimportant. Several recommendations are made to mitigate RDs in conventional and dual structures. The results of this study may be applicable to structures whose response is not significantly influenced by higher modes.

Keywords: Residual displacements, dual systems, very soft soils, lakebed zone of Mexico City, post-yielding stiffness ratio.

1. Introduction

In recent years, there has been a significant increase in the number of papers studying residual displacements (RDs) in structures. The study of RDs is of paramount importance because, even when damage to structures after an earthquake may be insignificant, the cost of repairing them when they have residual displacements may be very high [1-2]. Moreover, structures with large RDs may require demolition even when they have not collapsed [3]. Demolition may be required for several reasons, such as technical, economic and social. The number of examples where demolition has been required due to residual displacements is large, e.g. [4-5].

In order to understand the parameters affecting the RDs in conventional and dual structures located on very soft soils, an extensive parametric study was conducted. The full details of the study are found in ref. [6]- however, a brief summary of the results is offered in this paper.

For convenience, the term conventional structures is used to refer to conventional systems (such as moment resisting frames) whose response can be represented by a bilinear SDOF oscillator; while the term dual structures refers to conventional structures acting in parallel with high energy-dissipating systems, such as buckling-restrained braces (BRBs).

The objective of this study is to estimate the central tendency and dispersion of the residual displacements on single-degree-of-freedom (SDOF) oscillators subjected to earthquake ground motions recorded on very soft soils. These statistical measures are significant in the context of Performance-Based Earthquake Engineering in order to assess the effects of earthquakes with reliability. While the central tendency is represented by the sample mean of the RDs, the dispersion is represented by the coefficient of variation which, in turn, is calculated as the standard deviation divided by the sample mean. The statistical measures are estimated on the basis of the analysis of oscillators for their response to 220 earthquake ground motions recorded in the lakebed zone of Mexico City – where the soils is composed of a clay layer of variable thickness and water content up to 400%. The selected seismic records are listed in ref. [6]. They are characterised by long predominant periods of vibration, long duration, and low frequency contents.

For convenience, all calculations were carried out using the authors' subroutines, which were calibrated using the programme Degtra [7]. Also, it is important to mention that a segment of 40 seconds was added at the end of all the earthquake records in order to determine stabilised responses.

2. Residual displacements on conventional structures

After searching the literature, it has been found that several parameters affect the amplitude of residual displacements in conventional structures. These include:

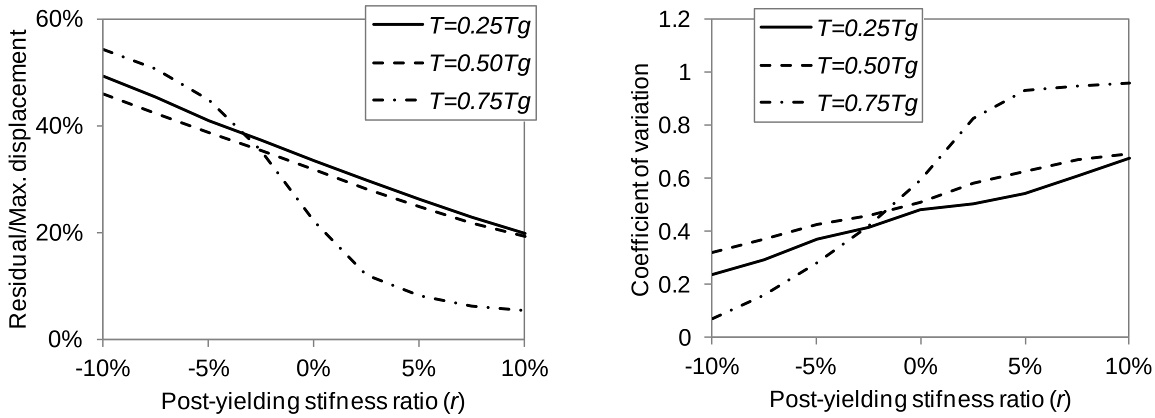
- Post-yielding stiffness ratio (r)
- Period of vibration (T)
- Level of ductility (μ)
- Strength reduction factor (R_y)
- Type of hysteretic response

In this paper, the effects of these parameters on the RDs of structures located in soft soils are evaluated. Two additional parameters, which also affect RDS, are studied here, namely:

- Damping ratio (ξ)
- Type of transition from elastic to plastic response

2.1 Effects of post-yielding stiffness ratio and period of vibration

Figure 1 shows the ratios of the residual displacements to the maximum transient demands, hereafter referred as normalised residual displacements, for different post-yielding stiffness ratios (r) and for three oscillators (representative of short-period structures). The oscillators have periods of vibration of T=0.25T_g, 0.50T_g and 0.75T_g, where T_g is the predominant period of vibration of the ground motions. For example, if T_g=2 s, the periods of the oscillators have T=0.5 s, 1 s and 1.5 s, respectively. A damping ratio of 5% and a target ductility demand of 2 were considered in all the calculations. From Figure 1a, it can be observed that both, r and T affect the mean of the normalised RDs. It is also noted that the higher the post-yielding ratio the smaller the normalised RDs especially for the oscillator with T=0.75T_g. From Figure 1b, it is seen that the dispersion tends to increase as r increases. Values higher than 0.6 were easily exceeded.

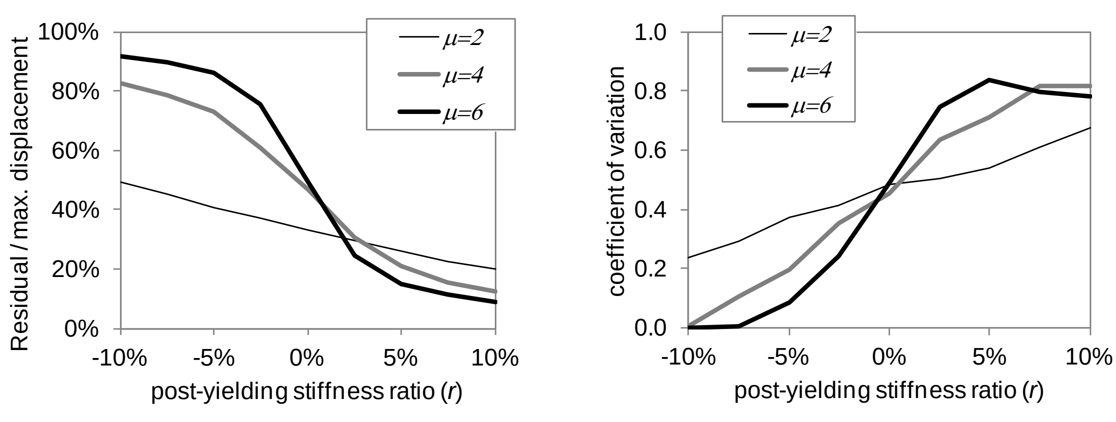


a) b)

Figure 1. Normalised residual displacements of three conventional oscillators: a) mean and b) dispersion

2.2 Effects of level of ductility

Similar to the previous figure, Figure 2 shows the variation of the normalised RDs against r for different levels of ductility (μ) for an oscillator with period of vibration of T=0.25T_g. It is observed that, for positive values of r, the higher the μ the smaller the normalised RDs. On the other hand, for negative r, the higher the μ the higher the RDs. Regarding the dispersion, very high values are seen again (higher than 0.6 in some cases).



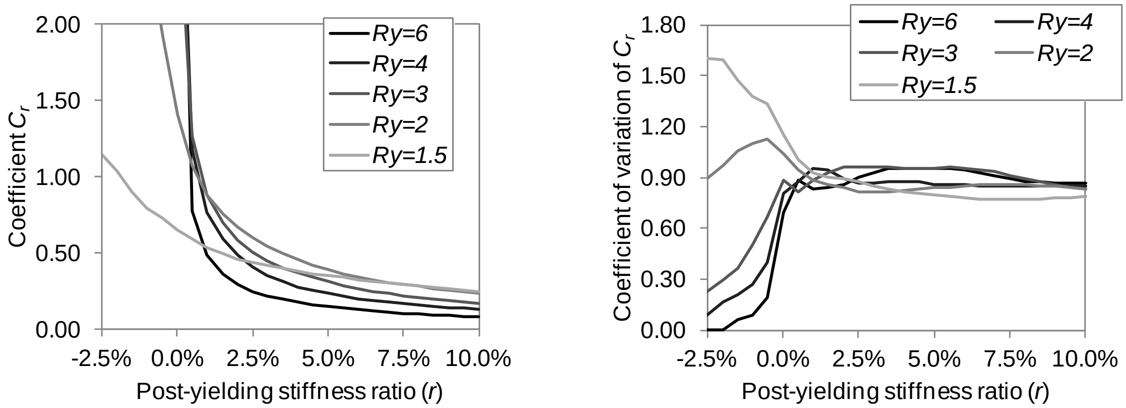
a) b)

Figure 2. Effects of ductility on residual displacements: a) mean and b) dispersion.

2.3 Effects of strength reduction factor

Now, the effect of the strength reduction factor (R_y), defined as the ratio of the strength required to maintain a SDOF oscillator elastic to its yielding strength, is evaluated for the same oscillator with T=0.25T_g. However, for this particular case the residual displacements are not normalised by the peak transient displacements but divided by the elastic spectral displacements. The reason for doing this is that inelastic displacements estimated using constant strength reduction factors tend to be very large in the short-period range [2], which hides the effects of

R_y on residual displacements. Figure 3a shows that for positive values of *r*, the higher the strength reduction factor the smaller are the RDs. On the contrary, for negative *r*, the higher the R_y the higher are the RDs. It is significant to note that values much larger than unity are reached in the negative range, which means that the residual displacements are larger than the elastic spectral displacements. Regarding the dispersion, Figure 3b shows coefficients of variation around 0.90 for positive values of *r*. However, in the negative range two trends are seen: small dispersions for $R_y \geq 2$, and very large dispersions for $R_y < 3$. In the first case, those dispersions are expected because the residual displacements are very large (almost the same as the peak transient displacements) for almost all the earthquake ground motions. For the other case, dispersions are very large due to very high uncertainty in the estimation of RDs for negative values of *r*.

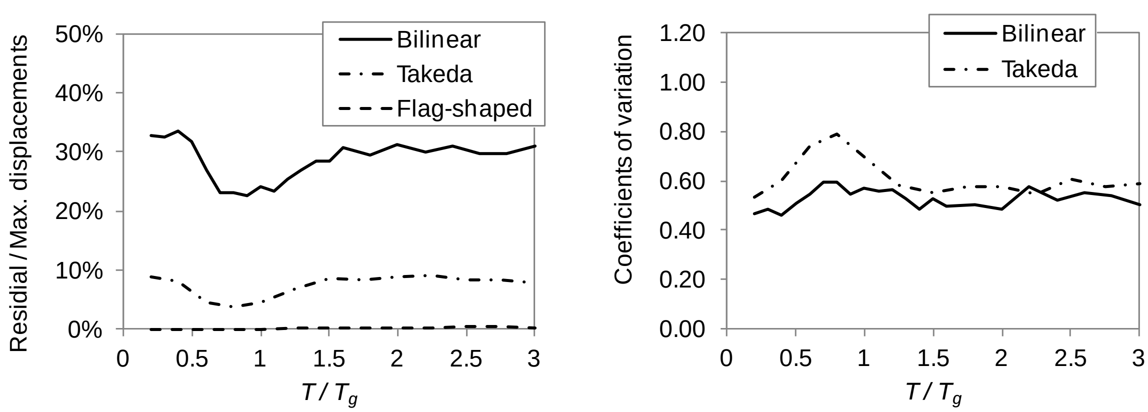


a) b)

Figure 3. Effects on RDs, divided by the elastic spectral displacements, due to R_y: a) mean and b) dispersion.

2.4 Effects of type of hysteretic response

The effects of the type of hysteretic response on normalised RDs are shown in Figure 4 for different periods of vibration. Three hysteresis models are considered, namely: bilinear (representative of steel structure); Takeda [8] (representative of concrete structures); and flag-shaped [9] (representative of structures with self-centring capacity). A damping ratio of 5% and a target ductility of 2 were considered. From the figure, it is clearly observed that the type of hysteretic response affects the normalised RDs significantly. The bilinear model presented the highest RDs, followed by the Takeda model which presented RDs of 10% or less and then the flag-shaped model which presented negligible RDs. Regarding the dispersion, coefficients of variation of around 0.5 and 0.60 are seen for the bilinear and Takeda models, respectively. No dispersion is found for the flag-shaped model because the normalised RDs were negligible under all of the earthquake ground motions.



a) b)

Figure 4. Effects of type of hysteretic response on normalised RDs: a) mean and b) dispersion.

2.5 Effects of damping ratio

The effects of damping ratio on RDs are shown in Figure 5 for different periods of vibration considering elastic-perfectly plastic behaviour (i.e. r=0) and target ductility of 2. It is noted that the mean of the normalised RDs and dispersions are not significantly affected by the damping ratio. However, when the RDs are normalised by the maximum displacements estimated for a constant damping ratio of 5% (Figure 5c), it is observed that the higher the damping ratio the smaller are the RDs. This observation implies that providing supplemental damping to structures may result in the benefit of reducing both, the peak and the residual displacements.

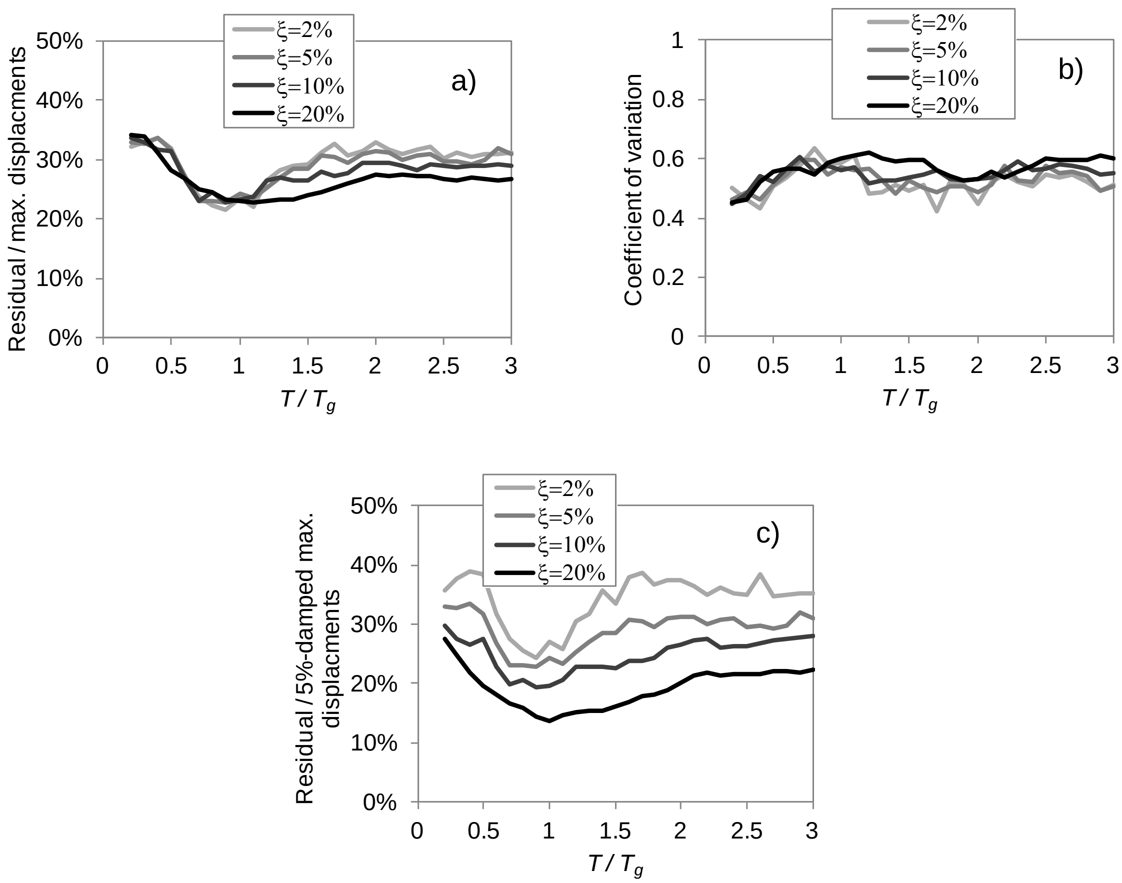
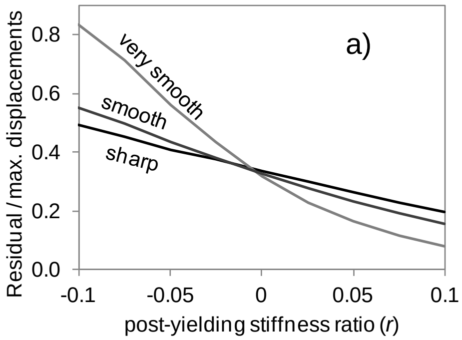


Figure 5. Effects of damping ratio on RDs: a) mean of normalised RDs, b) dispersion, and c) mean of RDs divided by 5%-damped maximum displacements.

2.6 Effects of type of transition from elastic to plastic response

Since most of the structural elements in a structure yield at different levels of displacement, the transition from elastic to plastic response is often not sharp but smooth. Therefore, it is of interest to assess the effects of the type of transition on RDs. In this regard, an oscillator with period of vibration of T=0.25T_g, damping ratio of ξ=5% and target ductility of μ=2 was subjected to the same 220 ground motions described previously for different values of post-yielding stiffness ratio. Three types of transition were considered, namely sharp, smooth and very smooth, which correspond to transition exponents of 100, 7 and 3, respectively, in the Bouc-Wen model [10]. From Figure 6a, it is seen that the type of transition is only significant for $r \neq 0$. Moreover, in the positive range the smoother the transition the smaller are the normalised RDs. The opposite is true in the negative range. The dispersion reaches very high values, which increase with the post-yielding stiffness ratio.



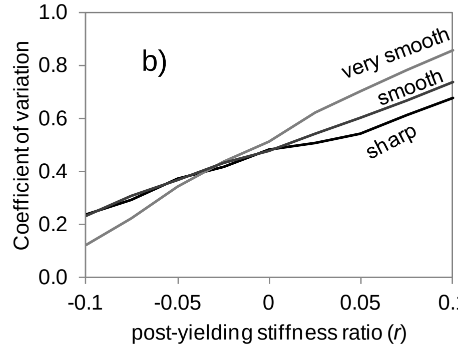
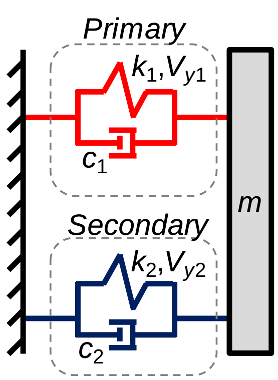


Figure 6. Effects of type of transition on normalised RDs: a) mean and b) dispersion.

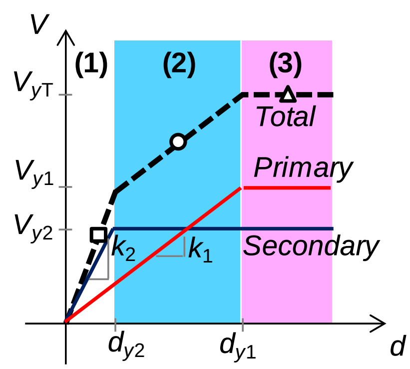
3. Residual displacement on dual structures

Dual systems are composed of two substructures working in parallel, namely: a primary part and a secondary part (Figure 7). While the primary part (representing the main structural resisting system, such as moment resisting frames) has a stiffness, k_1, a yielding displacement, d_{y1}, yielding load capacity, V_{y1} and damping coefficient, c_1; the secondary part (representing an energy dissipative system such as BRBs) has a stiffness, k_2, a yielding displacement, d_{y2}, yielding load capacity, V_{y2} and damping coefficient, c_2. The combined load capacity of the dual system is represented by the dashed-dark line in Figure 7b, which is the summation of the two lower curves.

Since the interactions between the composing parts of dual systems generate differences in the response demands [6], it is of interest to study the effects of these interactions on the residual displacements. It can be anticipated that the residual displacements would be highly affected by the magnitude of the maximum displacement demands on the dual system. In other words, if the maximum displacement demand is located within zone 1 (square dot in Figure 7b), the residual displacements are expected to be null because both parts of the system have sufficient restoring load capacity to return the structure to zero displacement position. On the other hand, if the maximum displacement demand is located within zone 2 (circular dot in Figure 7b), residual displacements are expected because the secondary part present plastic deformation. Finally, if the maximum displacement demand is located within zone 3 (triangular dot in Figure 7b), the residual displacements might be large because both parts present plastic deformation.



a) Parts of a dual SDOF system



b) Strength-displacement curve

Figure 7. A dual SDOF oscillator and its capacity curves.

3.1 Effects of the ductility of the secondary part

Once the capacity curves of the primary and secondary parts of a dual system are given, their ductility demands are given by:

$$\mu_1 = \frac{d}{d_{y1}} \tag{1a}$$

$$\mu_2 = \frac{d}{d_{y2}} \tag{1b}$$

where d is the maximum transient displacement and d_{y1} and d_{y2} are defined in Figure 7b. Therefore, the ratio of ductilities is given by:

$$\frac{\mu_2}{\mu_1} = \frac{d_{y1}}{d_{y2}} \tag{2}$$

It should be noted that, once the capacity curves of the parts are given, the ratio of ductilities is determined and remains constant, so that any increase of the ductility demand on the secondary part is proportional to the increase in the ductility demand on the primary part.

Now, in order to evaluate the effects of μ_2 on the residual displacements, a dual SDOF oscillator is subjected to the same 220 ground motions described earlier. The oscillator has a period of vibration of T=0.25T_g and a total damping ratio of ξ=5%. Elastic-perfectly plastic behaviour is assumed in both parts of the system. A yielding displacement ratio of d_{y1}/d_{y2}=8 is considered in so that μ_2/μ_1=8 (see Eq. 2). Figure 8 shows the normalised RDs for target ductilities of the secondary part between 2 and 7. It is noted that μ_1<1 due to the μ_2/μ_1=8 constrain. It is observed that the larger the ductility of the secondary part, the smaller are the normalised RDs. This is in agreement with the observations of Figure 1, where conventional oscillators with positive post-yielding stiffness ratios presented smaller normalised RDs as r increased. Regarding the dispersion, the coefficient of variation remained almost constant with very high values around 0.80.

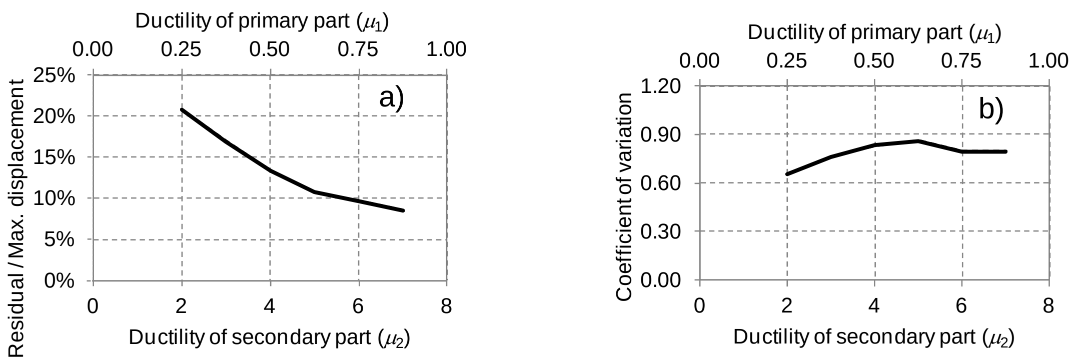


Figure 8. Effects of μ_2 on normalised residual displacements: a) mean and b) dispersion.

3.2 Effects of ductility of the primary part

In order to assess the effects of the ductility of the primary part on the normalised RDs, the same oscillator as in the previous section is analysed considering a ratio of μ_2/μ_1=4 and target ductilities of the primary part between 0.5 and 3. Figure 9a shows that: 1) when the maximum transient displacement demands are located within zone 2 (i.e. $\mu_1 \leq 1$), the normalised RDs tend to decrease as μ_1 increases; and 2) when the maximum transient displacement demands are located within zone 3 (i.e. μ_1>1), the normalised RDs increase dramatically as μ_1 increases. Therefore, when designing dual systems, a good recommendation is to maintain the maximum displacement demands within zone 2 in order to avoid very large RDs. Regarding the dispersion, the coefficient of variation is very high, with more of the values higher than 0.50.

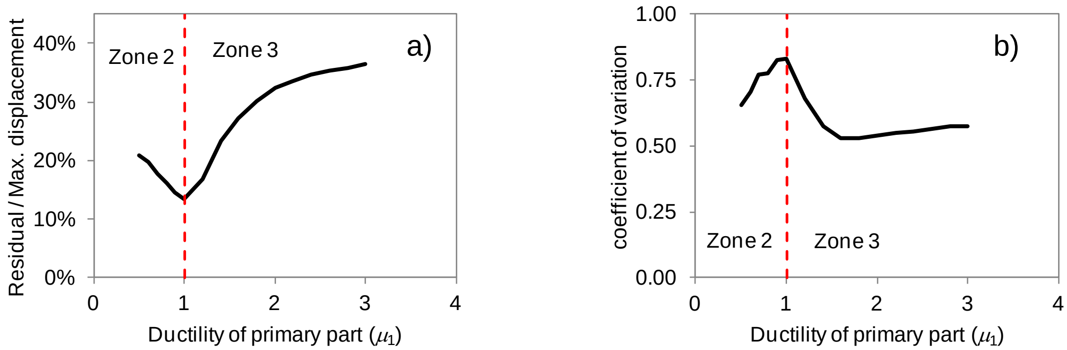


Figure 9. Effects of μ_1 on normalised residual displacements: a) mean and b) dispersion.

3.3 Effects of positive post-yielding stiffness ratio of the parts

The normalised RDs of the previous section, which correspond to elastic-perfectly plastic behaviour in both parts of the dual system (i.e. zero post-yielding stiffness ratios, r_1=r_2=0), are now compared to two cases where either the primary part or the secondary part present positive post-yielding stiffness ratio (i.e. Case 1: r_1=5% and r_2=0 or Case 2: r_1=0 and r_2=5%). The results are presented in Figure 10, where it can be seen that the effect of a positive r_1 on the normalised RDs is negligible (see Case 1). On the other hand, it can be observed that the effect of a positive r_2 is very significant and, at the same time, beneficial in reducing the mean of the normalised RDs (see Case 2).

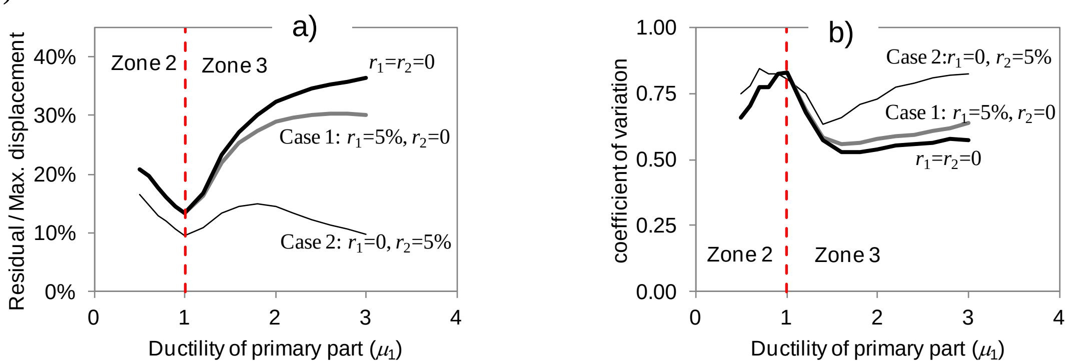


Figure 10. Effects of positive post-yielding stiffness ratios on normalised RDs: a) mean and b) dispersion.

3.4 Effects of negative post-yielding stiffness ratio of the parts

Now, in order to assess the effects on RDs of negative post-yielding stiffness ratio in one of the parts of the dual system, the following two cases are analysed: Case 1 for r_1= -5% and r_2=0; and Case 2 for r_1=0 and r_2= -5%. Figure 11 compares the results against those of elastic-perfectly plastic behaviour (i.e. r_1=r_2=0). From the figure, it can be observed that a negative r_1 does not have a significant effect on the normalised RDs. However, a negative r_2 is highly detrimental because the mean of the normalised RDs are increased dramatically. Therefore, when designing dual systems with maximum displacement demands located within zone 3 (as defined previously in Figure 7b), the post-yielding stiffness ratio of the secondary part must not be negative. In other words, r_2>0 shall be satisfied.

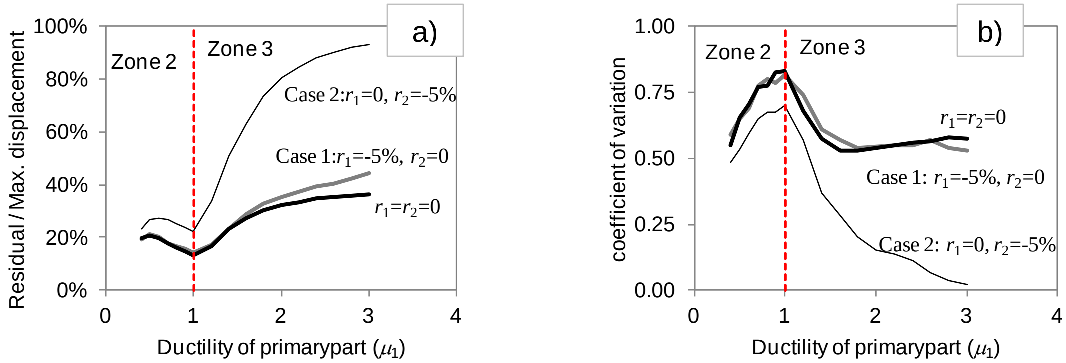


Figure 11. Effects of negative post-yielding stiffness ratios on normalised RDs: a) mean and b) dispersion.

3.5 Effects of type of hysteretic response of the primary part

Another interesting analysis is the evaluation of the residual displacements in dual systems when a dissipative system is combined with different types of hysteretic responses. For that purpose, the three hysteretic models of section 2.6 (i.e. bilinear, Takeda and flag-shaped) are considered here for the primary part of the dual system, while bilinear model is kept for the secondary part because this represents well the behaviour of dissipative systems such as BRBs. It is important to highlight that the effects of the type of hysteretic response of the primary part is only meaningful when the maximum displacement demands are located within zone 3 of the capacity curve (as defined in Figure 7b) – otherwise, the response of the dual system would be exactly the same irrespective of the type of hysteretic response. The results of the analysis are shown in Figure 12 for periods of vibration between $0.1T_g$ and $3T_g$. From the figure, it can be seen that, while the bilinear model presented the largest normalised RDs, the flag-shaped model presented the smallest ones. The Takeda model presented values somewhere in the middle. Regarding the dispersion, the coefficients of variation are very large; values around 0.5 and 0.7 are observed for the bilinear and Takeda models, respectively. The dispersion of the flag-shaped model is not shown in the figure because most of the mean RDs are very small, leading to unrealistic values of dispersion.

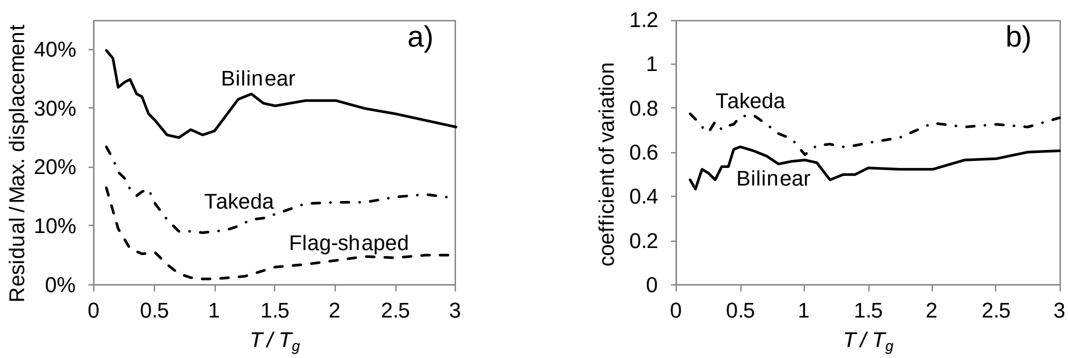


Figure 12. Effects on normalised residual displacements of the type of hysteretic response of the primary part: a) mean and b) dispersion.

4. On the reliability and the value of the results

Since high dispersions of the estimated data were observed in sections 2 and 3 (i.e. the coefficients of variation were often higher than 0.6), confidence intervals of the means and coefficients of variation are estimated in this section in order to assess the reliability of the resulting data. In the pursuit of simplicity, only two representative cases are selected and analysed in this section. Both cases correspond to the bilinear behaviour of conventional and dual structures, as shown previously in Figures 4 and 12, respectively. Figures 13 and 14 show the 95%

confidence intervals for the conventional and dual structures. From the figures it is seen that the estimates of the mean and dispersions are reliable.

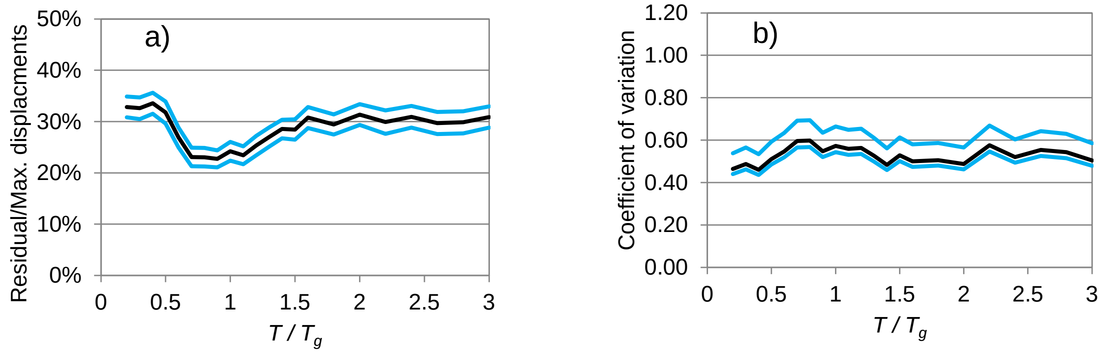


Figure 13. Confidence intervals for conventional structures: a) mean and b) dispersion.

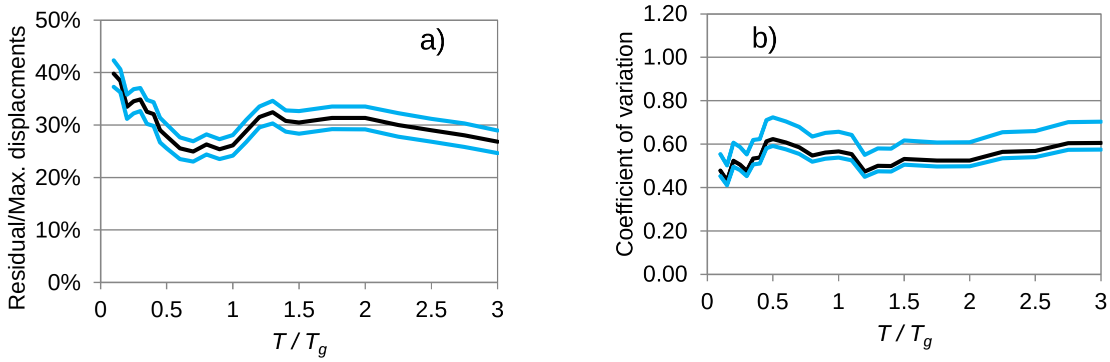


Figure 14. Confidence intervals for dual structures: a) mean and b) dispersion.

5. Recommendations

After analysing the factors affecting the demands of residual displacements on conventional and dual SODF structures located in very soft soils, some recommendations are made in order to mitigate them:

In conventional systems:

- Provide a high post-yielding stiffness ratio (say) r > 5% or 10%.
- Provide self-centring capacity.
- If these recommendations are not feasible, the following recommendations may be followed:
    - Design for reduced levels of ductility.
    - Design for reduced strength reduction factors.
    - Provide supplemental damping, which reduces both residual and peak displacements.
    - Use reinforced concrete structures rather than steel structures.
    - Avoid negative values of r because they increase RDs dramatically.

In dual systems:

- Design for a maximum displacement demands within zone 2, i.e. $\mu_1 \leq 1$.
- If the maximum displacement demands reach zone 3 (i.e. $\mu_1 > 1$), provide a post-yielding stiffness ratio of the secondary part r_2 > 5% or 10%.
- If the previous suggestions are not feasible, provide self-centring capacity to the primary part.
- If the previous recommendations are not feasible, the following recommendations may be followed:
    - Preferably, use a primary part made of reinforced concrete.
    - Provide supplemental damping.
    - Do not use negative values of post-yielding stiffness ratio of the secondary part (r_2).

6. Conclusions

Statistics of the residual displacement (RDs) demands, normalised by the peak transient displacement demands, were determined for conventional and dual SDOF structures located on very soft soils. Using a relatively large set of 220 earthquake ground motions, several analyses were conducted and the results are presented in this paper. It was observed that there are several parameters affecting the magnitude of the normalised residual displacements. As an example, a positive post-yielding stiffness ratio (*r*) in the conventional structures is very effective in reducing the normalised RDs, while negative values of *r* are highly detrimental.

Regarding dual systems, it was noted that normalised RDs are small when the maximum displacement demands are located within zone 2 (as defined in Figure 7b) and very large when the maximum displacement demands are in zone 3. For the latter case, it was observed that the post-yielding stiffness ratio of the secondary part (r_2) is key to constrain RDs, i.e. while a positive value of r_2 reduces RDs significantly, a negative r_2 is highly detrimental. Also, it was seen that by combining an energy dissipation system (e.g. BRBs) with a primary structure having self-centring capacity may be very efficient because the RDs are small.

Several recommendations were made for conventional and dual systems in order to mitigate the residual displacements. While some recommendations are very effective others may not be so. In conventional structures, the most effective ways to reduce RDs are providing: 1) a high value of *r* in conventional oscillators or 2) self-centring capacity. In dual systems, the most effective recommendations are: 1) controlling the displacement demand so that the primary part remains elastic; 2) providing $r_2 > 5\%$; or 3) providing self-centring capacity to the primary substructure.

Finally, in order to account for the uncertainty associated to the estimations of RDs, the dispersion was determined by means of the coefficient of variation. Very high values were seen in all the cases studied, reaching values higher than 0.6. Therefore, these high dispersion values must be considered when assessing residual displacements.

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