RELIABILITY-BASED DESIGN OF COMPLEX STRUCTURAL SYSTEMS UNDER EARTHQUAKE EXCITATION

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Abstract

This work presents an efficient strategy for dealing with reliability-based design problems of complex structural systems under earthquake excitation. The excitation is modeled as a non-stationary stochastic process which combines a point-source model with a velocity pulse model. The solution of this class of problems is computationally very demanding due to the large number of structural analyses required during the design process. A model reduction technique combined with an appropriate optimization scheme is proposed to carry out the design process efficiently in a reduced space of generalized coordinates. In particular, a method based on substructure coupling technique for dynamic analysis is implemented to define a reduced-order model for the structural system. The re-assembling of the reduced-order model matrices due to changes in the values of the design variables are avoided during the optimization process. The effectiveness of the proposed methodology is demonstrated with one application problem consisting in the reliability-based design of a bridge structural model.

Keywords: Advanced simulation techniques; High dimensional reliability analysis; First excursion probability; Model reduction techniques; Reliability-based design optimization.

1. Introduction

Structural optimal design via deterministic mathematical programming techniques has been widely accepted as a viable tool for engineering design. However, in most structural engineering applications response predictions are based on models involving uncertain parameters. This is due to a lack of information about the value of system parameters external to the structure such as environmental loads or internal such as system behavior. Under uncertain conditions the field of reliability-based optimization provides a realistic and rational framework for structural optimization which explicitly accounts for the uncertainties [1,2]. In the present work, structural design problems involving finite element models under stochastic earthquake loading are considered. The design problem is formulated as the minimization of an objective function subject to multiple design requirements including standard and reliability constraints. The probability that any response of interest exceeds in magnitude some specified threshold level within a given time duration is used to characterize the system reliability. This probability is commonly known as the first excursion probability [3]. The corresponding reliability problem is expressed in terms of a multidimensional probability integral involving a large number of uncertain parameters. The solution of reliability-based design problems involving finite element models under stochastic excitation requires a large number of finite element analyses to be perform during the design process. These analyses correspond to finite element re-analyses over the design space (required by the optimizer), and over the uncertain parameter space (required for reliability estimation). Consequently, the computational demands depend highly on the number of finite element analyses and the time taken for performing an individual finite element analysis. Thus, the computational demands in solving reliability-based design problems of complex structural systems may be large or even excessive. In order to cope with these difficulties a model reduction technique combined with an appropriate optimization scheme is proposed to carry out the design process efficiently in a reduced space of generalized coordinates. The goal is to reduce the time consuming operations involved in the re-analyses and dynamic responses of medium/large finite element models. Specifically, a model reduction technique based on substructure coupling for dynamic analysis is considered in the present implementation [4]. The proposed method corresponds to a generalization of substructure coupling applicable to a class of linear and nonlinear systems. The technique includes dividing the linear components of the structural system into a number of substructures obtaining reduced-order models of the substructures, and then assembling a reduced-order model for the entire structure.

The organization of this work is as follows. The formulation of the reliability-based design problem is presented in Section 2. Next, the characterization of the structural systems of interest is considered in Section 3. Implementation issues such as reliability estimation, optimization strategy and model reduction are discussed in Sections 4, and 5. The integration of the model reduction technique into the design process is discussed in Section 6. The effectiveness of the proposed strategy is demonstrated in Section 7 by the reliability-based design of a bridge structural model. The paper closes with some final remarks.

2. Design Formulation

The reliability-based design problem is characterized in terms of the following constrained non-linear optimization problem

$$\begin{aligned} \mathrm{Min}_{\theta} \quad & \mathrm{C}(\boldsymbol{\theta}) \\ s.t \quad & \mathrm{g}_i(\boldsymbol{\theta}) \leq 0 \qquad\qquad , i = 1, \ldots, n_c \\ & P_{F_i}(\boldsymbol{\theta}) - P_{F_i}^* \leq 0 \quad , i = 1, \ldots, n_r \\ & \boldsymbol{\theta} \in \boldsymbol{\Theta} \end{aligned} \tag{1}$$

where $\boldsymbol{\theta}$ $(\theta_i, i = 1, \ldots, n_d)$ is the vector of design variables with side constraints $\boldsymbol{\Theta}$, $\mathrm{C}(\boldsymbol{\theta})$ is the objective function, $\mathrm{g}_i(\boldsymbol{\theta}) \leq 0$ is a standard constraint, and $P_{F_i}(\boldsymbol{\theta}) - P_{F_i}^* \leq 0$ is a reliability constraint where $P_{F_i}(\boldsymbol{\theta})$ is a failure probability function and $P_{F_i}^*$ is the target failure probability. The standard constraints are related to general design requirements such as geometric conditions, material cost components, availability of materials, etc. On the other hand, the reliability constraints are associated with design specifications characterized through the use of reliability measures given in terms of failure probabilities with respect to specific failure criteria. For structural systems under stochastic excitation the probability that design conditions are satisfied within a particular period T provides a useful reliability measure [4]. Such measure is referred as the first excursion probability and quantifies the plausibility of the occurrence of unacceptable behavior (failure) of the structural system. In this context, a failure event $F_i(\boldsymbol{\theta}, \boldsymbol{z})$ can be defined as $F_i(\boldsymbol{\theta}, \boldsymbol{z}) = d_i(\boldsymbol{\theta}, \boldsymbol{z}) > 1$ where d_i is the normalized demand function defined as

$$d_i(\boldsymbol{\theta}, \boldsymbol{z}) = \max_{j=1,\dots,l} \max_{t \in [0,\mathrm{T}]} \frac{\left| r_j^i(t, \boldsymbol{\theta}, \boldsymbol{z}) \right|}{r_j^{i^*}} \tag{2}$$

where $r_j^i(t, \boldsymbol{\theta}, \boldsymbol{z})$ are the responses functions associated with the failure event F_i, $r_j^{i^*}$ are the acceptable response levels and $\boldsymbol{z} \in \boldsymbol{\Omega}_{\boldsymbol{z}}$ is the vector of uncertain variables involved in the definition of the stochastic excitation. The vector $\boldsymbol{z}$ is characterized by a probability density function $p(\boldsymbol{z})$ which indicates the relative plausibility of the possible values of the uncertain parameters $\boldsymbol{z} \in \boldsymbol{\Omega}_{\boldsymbol{z}}$. Note that the responses $r_j^i(t, \boldsymbol{\theta}, \boldsymbol{z})$ are functions of time (due to the dynamic nature of the excitation), the design vector $\boldsymbol{\theta}$, and the random vector $\boldsymbol{z}$. These response functions are obtained from the solution of the equation of motion that governs the structural system. Finally the probability of failure evaluated at the design $\boldsymbol{\theta}$ is formally defined as

$$P_{F_i}(\boldsymbol{\theta}) = P\left[\max_{j=1,\dots,l} \max_{t \in [0,\mathrm{T}]} \frac{\left| r_j^i(t, \boldsymbol{\theta}, \boldsymbol{z}) \right|}{r_j^{i^*}} > 1 \right] \tag{3}$$

where $P[\cdot]$ is the probability that the expression in parenthesis is true. Equivalently, the failure probability function evaluated at the design vector $\boldsymbol{\theta}$ can be written in terms of the multidimensional probability integral

$$P_{F_i}(\boldsymbol{\theta}) = \int_{(d_i(\boldsymbol{\theta},\boldsymbol{z})>1)} p(\boldsymbol{z})\, d\boldsymbol{z} \tag{4}$$

where all terms have been previously defined.

3. Structural Model

A general type of nonlinear structural systems can be cast into the following equation of motion

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{k}\big(\mathbf{u}(t), \dot{\mathbf{u}}(t), \boldsymbol{\tau}(t)\big) + \mathbf{f}(t) \tag{5}$$

where $\mathbf{u}(t)$ denotes the displacement vector, $\dot{\mathbf{u}}(t)$ the velocity vector, $\ddot{\mathbf{u}}(t)$ the acceleration vector, $\mathbf{k}\big(\mathbf{u}(t), \dot{\mathbf{u}}(t), \boldsymbol{\tau}(t)\big)$ the vector of non-linear restoring forces, $\boldsymbol{\tau}(t)$ the vector of a set of variables which describes the state of the nonlinear components, and $\mathbf{f}(t)$ the external force vector. The matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ describe the mass, damping, and stiffness, respectively. Note that some of the matrices and vectors involved in the equation of motion depend on the vector of design variables $\boldsymbol{\theta}$ and/or the uncertain system parameters $\boldsymbol{z}$ and therefore the solution is also a function of these quantities. The explicit dependence of the response on these quantities is not shown here for simplicity in notation. The evolution of the set of variables $\boldsymbol{\tau}(t)$ is described by the first-order non-linear differential equation

$$\dot{\boldsymbol{\tau}}(t) = \boldsymbol{\kappa}(\mathbf{u}(t), \dot{\mathbf{u}}(t), \boldsymbol{\tau}(t)) \tag{6}$$

where $\boldsymbol{\kappa}$ represents a nonlinear vector function. This particular characterization of the nonlinear components allows to model different types of nonlinearities including hysteresis and degradation [5,6]. The equation of motion (5) together with equation (6) that describes the evolution of the set of variables $\boldsymbol{\tau}(t)$ constitute a system of coupled non-linear differential equations for $\mathbf{u}(t)$ and $\boldsymbol{\tau}(t)$. The previous formulation is particular well suited for cases where most of the components of the structural system remain linear and only a small part behaves in a nonlinear manner.

As previously pointed out the external force vector $\mathbf{f}(t)$ is modeled as a non-stationary stochastic process. Depending of the application under consideration and the available information different methodologies can be used for generating samples of these types of processes. Such methodologies include techniques based on filtered Gaussian white noise processes, stochastic processes compatible with power spectral densities, point source-based models, subevents source-based models, record-based models, etc. [3,7,8,9,10,11,12]. A common aspect of the aforementioned methodologies is that the generation of the corresponding stochastic processes samples involves in general a large number of random variables, e.g. of the order of hundreds or thousands. Therefore the evaluation of the failure probability function for a given design (Eq. 4) constitutes a high-dimensional problem which is extremely demanding from a numerical point of view.

4. Reliability Estimation and Optimization Scheme

The reliability constraints of the nonlinear constrained optimization problem (1) are defined in terms of first excursion probability functions. These reliability measures are given in terms of high-dimensional integrals. The difficulty in estimating these quantities favors the application of simulation techniques to cope with the probability integrals. It is important to note that each sample, in the context of simulation, implies the solution of the set of non-linear differential equations (5) and (6) that characterizes the structural model (a dynamic finite element analysis). Therefore an efficient simulation technique is required in the context of the present formulation. A general applicable method named subset simulation is adopted here [13]. In this advanced simulation technique the failure probabilities are expressed as a product of conditional probabilities of some chosen intermediate failure events, the evaluation of which only requires simulation of more frequent events. The intermediate failure events are chosen adaptively using information from simulated samples so that they correspond to some specified values of conditional failure probabilities. Therefore, a rare event simulation problem is converted into a sequence of more frequent event simulation problems. The method uses a Markov chain Monte Carlo method based on the Metropolis algorithm for sampling from the conditional probabilities. This is the most widely applicable simulation technique because it is not based on any geometrical assumption about the topology of the failure domain. In fact, validation calculations have shown that subset simulation can be applied efficiently to a wide range of dynamical systems including general linear and non-linear systems [14,15]. In addition, subset simulation is very-well suited for parallel implementation in a computer cluster. This feature allows a very efficient numerical implementation of this advanced simulation technique. For a detailed description of subset simulation, from the theoretical and practical point on view, the reader is referred to [13].

On the other hand, the solution of the reliability-based optimization problem defined in Eq. (1) can be obtained in principle by a number of techniques such as standard deterministic optimization schemes or stochastic search algorithms [16,17]. In particular, a class of interior point algorithms based on the solution of the first-order optimality conditions is implemented here [18]. The scheme has proved to be quite effective for a wide range of applications in the context of deterministic and stochastic optimization problems. The details of the aforementioned algorithm can be found in [19].

5. Substructure Coupling Technique

The solution of the reliability-based optimization problem (1) is computationally very demanding due to the large number of dynamic analyses required during the design process. In fact the reliability estimation at each design requires the evaluation of the system response at a large number of samples in the uncertain parameter space (of the order of hundreds or thousands). In addition, the iterative nature of the optimization strategy may impose additional computational demands. Consequently, the computational cost may become excessive when the computational time for performing a dynamic analysis is significant. To cope with this difficulty, a model reduction technique is considered in the present formulation. In particular, a method based on substructure coupling technique for dynamic analysis is implemented in order to define a reduced-order model for the structural system [4,20]. The general idea of the methodology is to divide the linear components of the structural system into a number of linear substructures obtaining reduced-order models and then assembling a reduced-order model for the entire structural system. More specifically, after the division of the structure linear components into substructures, the model reduction technique involves two basic steps: definition of sets of substructure modes; and coupling of the substructure-modes models to form a reduced-order system model. The dimension of the reduced-order model can be substantially smaller (one or more orders of magnitude) than the dimension of the original unreduced model. Details of the procedure can be found in the previous references.

6. Optimal Design Based On Reduced-Order Model

The previous model reduction technique is quite general in the sense that dividing the structure into substructures and reducing the number of physical coordinates to a much smaller number of generalized coordinates certainly alleviates part of the computational effort. However, the generation of the reduced-order model at each design implies the computation of the so-called normal modes associated with each substructure [4]. This procedure can be computationally very expensive due to the substantial computational overhead that arises at substructure level. In order to make the model reduction technique more efficient a particular parametrization scheme in terms of the design variables is considered in the present formulation. Specifically, it is assumed that the stiffness and mass matrix of each substructure depend on only one of the design variables. Such dependency can be linear or nonlinear. Of course, different substructures may depend on different design variables. It should be pointed out that the previous parametrization is often encountered in a number of practical applications. By using this parametrization scheme it can be shown that the reduced-order matrices (mass, stiffness and damping) can be written explicitly in terms of the design variables. In other words, the reduced order matrices can be expressed in terms of a set of matrices which are independent of the values of the vector of design variables $\boldsymbol{\theta}$ and a set of parametrization functions [20,21]. To save computational time these matrices are computed and assembled once for a reference model. Therefore there is no need to compute these matrices during the iterations of the design process due to changes in the value of the design variables. This feature results in substantial computational savings. In addition, the formulation guarantees that the reduced order model is based on the exact substructure modes for all values of the design variables. For the more general case, that is, when the substructure matrices depend on more than one design variable the reduced-order model matrices should be re-assembled for new values of the vector of design variables. This repeated computation, however, is usually confined to a small number of substructures in many practical applications. So, even in the more general case a significant saving may still arise. Finally, it is noted that parallelization techniques are also possible at the model level. In fact, the definition of all substructure matrices in generalized coordinates can be carried out in parallel, reducing the computational time of the proposed implementation even further.

7. Application Problem

To evaluate the effectiveness of the proposed methodology the reliability-based design of the bridge structural model shown in Figure 1 is considered. The bridge is curved in plan and has a total length of 119 m. It has 5 spans of lengths equal to 24.0m, 20.0m, 23.0m, 25.0m, and 27.0m, respectively, and four piers of 8m height that support the girder monolithically. Each pier is founded on an array of four piles of 35 m height. The piers and piles are modelled as column elements of circular cross-section with 1.6 m and 0.6 m diameter, respectively. The deck cross section is a box girder which is modelled by beam and shell elements. It rests on each abutment through two rubber bearings that consist of layers of rubber and steel plates, with the rubber being vulcanized to the steel plates. The rubber bearings are characterized by the external diameter D_r, internal diameter D_i and the total height of rubber H_r. The force-displacement characteristics of the rubber bearings are modelled by a biaxial hysteretic behavior [22]. An analytical model based on a series of experimental tests conducted for real-sized rubber bearings is used in the present application. For a detailed description of the analytical model that describes the nonlinear behavior of the bearings the reader is referred to [22,23]. The interaction between the piles and the soil is modelled by a series of translational springs along the height of the piles with stiffnesses varying from 11200 T/m at the base to 0.0 T/m at the surface. The following values of the material properties of the concrete structure are considered. The Young's modulus is taken to be E= 2.0 x 10^{10} N/m^2, the Poisson ratio ν=0.2, and mass density ρ=2500kg/m^3. Finally, a 3% of critical damping is added to the model. The selected finite element model for the bridge structure has 10.068 degrees of freedom.

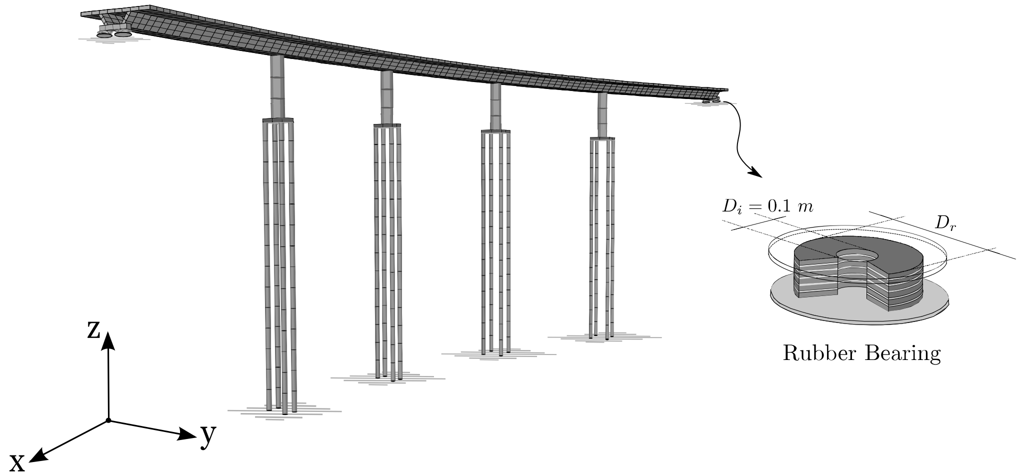

Figure1. Finite element model of bridge structure

The bridge structure is subjected to ground acceleration in a direction defined at 25^o with respect to the x axis. The ground acceleration is modelled as a non-stationary stochastic process. In particular, a stochastic point-source model characterized by a series of seismicity parameters such as the moment magnitude and rupture distance is considered in the present implementation [8,9]. The model is a simple, yet a powerful means for simulating ground motions with high and low frequency components. The methodology, which was initially developed for generating synthetic ground motions, has been reinterpreted to form a stochastic model for ground excitation [24]. The input for the stochastic excitation model involves a white noise sequence and a series of seismological parameters as previously pointed out. Details of the entire procedure can be found in [8,23]. The duration of the excitation is equal to T=30s with a sampling interval equal to Δt=0.01s. Based on the characterization of the point source model it can be shown that the generation of the stochastic ground motion samples involves more than 3.000 random variables for the duration and sampling interval considered. Thus, the vector of uncertain parameters $\boldsymbol{z}$ involved in the problem has more than 3.000 components. For illustration purposes, Figure 2 shows a synthetic excitation sample generated by the stochastic point-source model. It is emphasized that the proposed reliability-based design strategy is not restricted in any way to this particular excitation model. In fact, alternative ground motion models can be used as well. For the dynamic analysis a homemade finite element code was implemented.

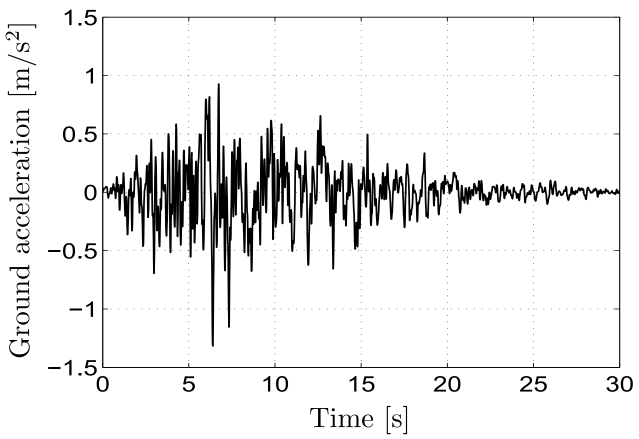

Figure 2. Ground acceleration time history sample

7.1 Design Problem

The reliability-based design problem is defined in terms of the optimization problem

$$\begin{aligned} \mathrm{Min}_{\theta} \quad & \mathrm{C}(\boldsymbol{\theta}) \\ & P_{F_i}(\boldsymbol{\theta}) \leq P_{F_i}^* \quad , i = 1, 2 \\ & \boldsymbol{\theta} \in \boldsymbol{\Theta} \end{aligned} \tag{7}$$

where $\boldsymbol{\theta}$ ($\theta_i, i = 1, \ldots, 6$) is the vector of design variables, $\mathrm{C}(\boldsymbol{\theta})$ is the cost function which is assumed to be proportional to the total volume of rubber in the bearings and to the total volume of the pier elements, and $P_{F_i}^* = 10^{-4}, i = 1,2$ are the corresponding target probabilities. The design variables include the diameter of the piers circular cross section, the external diameter of the bearings and the total height of rubber of the bearings. Design variables$\theta_1, \theta_2, \theta_3$, and θ_4 are related to the diameter of the circular cross section of the four piers, while design variables θ_5 and θ_6 are associated with the external diameter and total height of rubber of the bearings located at the abutments. The relationship between the design variables and the actual structural parameters is given by $D_{pi} = \theta_i \overline{D}_{pi}, \mathrm{i} = 1,2,3,4$, $D_r = \theta_5 \overline{D}_r$ and $H_r = \theta_6 \overline{H}_r$, where $D_{pi}, i = 1,2,3,4$ are the diameters of the circular cross section of the piers, D_r is the external diameter of the bearings, H_r is the total height of rubber of the bearings, and $\overline{D}_{pi}$ (1.6m), $\overline{D}_r$ (0.8m), and $\overline{H}_r$ (0.17m) are the corresponding nominal values of the structural parameters. The side constraints for the design variables are given by: $0.75 \leq \theta_i \leq 1.25, i = 1,2,3,4$; $0.75 \leq \theta_5 \leq 1.25$, and $0.88 \leq \theta_6 \leq 1.47$. Failure, that is unacceptable performance, is defined in terms of the relative displacement of piers and the relative displacement of the rubber bearings. Thus, the corresponding failure probability functions are given by

$$\begin{aligned} P_{F_1}(\theta) &= P\left[\max_{t\in[0,T]} \frac{|\mathrm{u_b}(t,\boldsymbol{\theta},\boldsymbol{z})|}{0.10\mathrm{m}}\right] > 1 \\ P_{F_2}(\theta) &= P\left[\max_{t\in[0,T]} \frac{|\delta(t,\boldsymbol{\theta},\boldsymbol{z})|}{0.07\mathrm{m}}\right] > 1 \end{aligned} \tag{8}$$

where $\mathrm{u_b}(t,\boldsymbol{\theta},\boldsymbol{z})$ represents the maximum relative displacement between the deck girder and the base of the rubber bearings at each abutment (in the x or y direction), and $\delta(t,\boldsymbol{\theta},\boldsymbol{z})$ denotes the maximum relative displacement between the top of the piers and their connection with the pile foundations (in the x or y direction). It is noted that the estimation of the failure probability functions for a given design $\boldsymbol{\theta}$ represents a high-dimensional reliability problem. In fact, as previously pointed out, 3.000 random variables are involved in the corresponding probability integral (Eq. 4). Regarding the excitation, the same set of samples generated by the stochastic excitation model is used throughout the design process (common random number stream approach).

7.2 Definition of Reduced-Order Finite Element Model

Considering the previous design formulation the bridge structure is divided into a number of substructures. The division is guided by a parametrization scheme so that the substructure matrices for each one of the introduced substructures depend on only one of the design variables. In particular, the structural model is subdivided into six linear substructures and two nonlinear substructures as shown in Fig. 3. Substructure S_1 is composed by the pile elements, substructures S_2, S_3, S_4, and S_5 include the different pier elements, and substructure S_6 corresponds to the deck girder. Finally, substructures S_7 and S_8 are the nonlinear substructures composed by the rubber bearings located at the left and right abutments, respectively. With this subdivision substructures S_1 and S_6 do not depend on the design variables, while substructures S_2, S_3, S_4, and S_5 depend on the design variables $\theta_1, \theta_2, \theta_3$, and θ_4 respectively, and design variables θ_5 and θ_6 are associated with the nonlinear substructures S_7 and S_8.

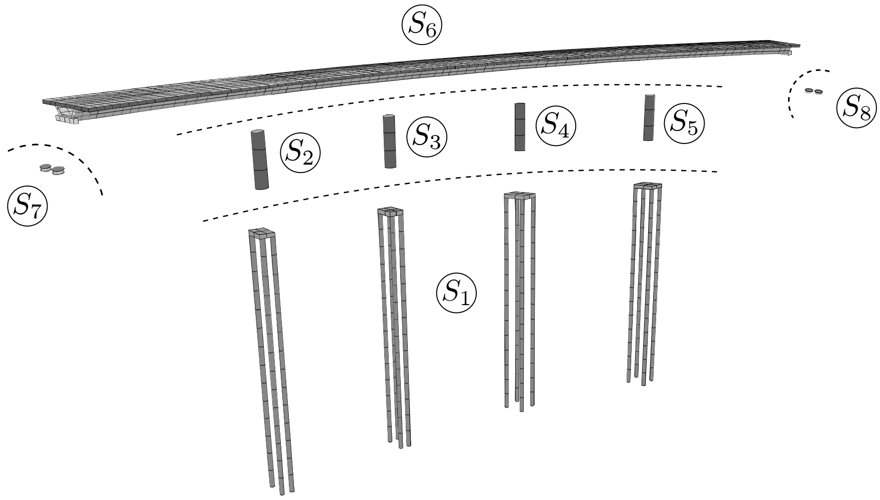

Figure 3. Substructures of the finite element model

Based on the previous substructures the reduced order model is characterized as follows. Ten generalized coordinates are retained for substructure S_1, two for each one of substructures S_2, S_3, S_4, and S_5 and ten for substructure S_6. Validation calculations show that this reduced order model is adequate in the context of the present application. In fact, the error between the modal frequencies using the full nominal reference finite element model and the modal frequencies computed using the reduced-order model falls below 0.5% for the lowest six modes. The corresponding matrix of MAC-values (modal assurance criterion) between the first six modal vectors computed from the unreduced finite element model and from the reduced-order model is shown in terms of a 3-D representation in Figure 4. It is observed that the values at the diagonal terms are close to one and almost zero at the off-diagonal terms. Thus, the modal vectors of both models are consistent. The comparison with the lowest 6 modes is based on the fact that the contribution of the higher order modes (higher than the 6th mode) in the dynamic response of the model is negligible.

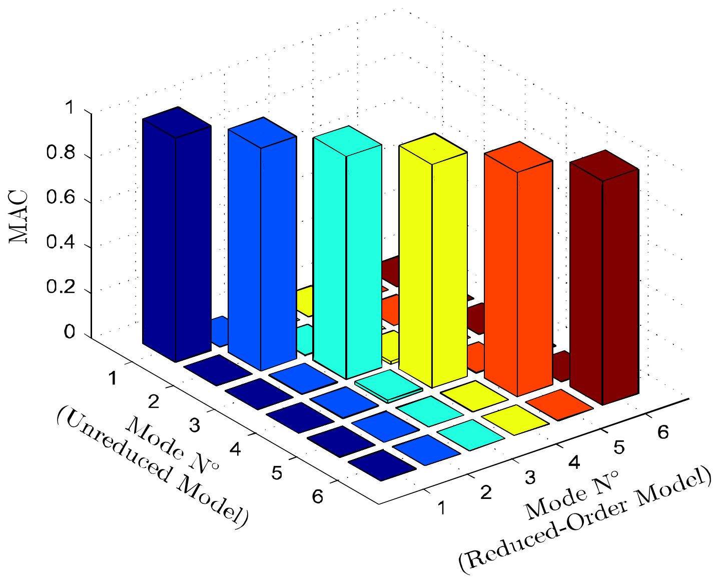

Figure 4. MAC-values between the mode shapes computed from the unreduced finite element model and from the reduced-order model

In summary, a total of 28 generalized coordinates are retained for the six linear substructures. On the other hand, the number of interface degrees of freedom (between the substructures) is equal to 60 in this case. The dimension of the resulting reduced-order model represents a 99% reduction with respect to the dimension of the unreduced model. Thus, a drastic reduction in the number of degrees of freedom is obtained with respect to the original unreduced finite element model. Validation calculations show that the reduced-order model and the full finite element model are equivalent in the context of this design problem. Therefore, the design process of the bridge structural model is carried out by using the reduced-order model. It is important to note that the calibration and definition of the reduced-order model is done off-line, before the design procedure takes place.

7.3 Numerical Results

Taking advantage of the reduced-order model one particular design scenario is investigated in detail in order to get insight into the reliability and general performance of the bridge structure under consideration. Specifically the interaction between bridge structural components and rubber bearing parameters is studied. To this end the design space in terms of the diameter of the circular cross sections of the piers and the external diameter of the rubber bearings is constructed. The design variables associated with the diameters of the circular cross sections of the piers are linked to one design variable $\theta_{pier} = \theta_1 = \theta_2 = \theta_3 = \theta_4$, while the design variables θ_5 associated with the external diameter of the rubber bearings is denoted as $\theta_{bearing}$. Design variable related to the total height of rubber of the bearings is kept constant and equal to its lower bound value, i.e., $\theta_6 = 0.88$ (H_r=0.15 m). Figure 5 shows some objective contours and iso-probability curves as well as the final design. The design space is shown in terms of the actual values of the diameter of the circular cross sections of the piers D_p ($D_p = \theta_{pier}\, \overline{D}_p$) and the external diameter of the rubber bearings D_r ($D_r = \theta_{bearing}\, \overline{D}_r$). From the figure it is observed that the probability of failure event F_2 decreases as the diameter of the circular cross sections of the piers increase. In this case the piers become stiffer and therefore the relative displacements between the top of the piers and their connection with the piles foundations decrease. It is also seen that the failure event F_2 is controlled by the diameter of the circular cross sections of the piers for values of this quantity close to its lower bound, i.e. D_p < 1.45m. In this range of values the iso-probability curves are almost perpendicular. So, the effect of the external diameter of the isolator is negligible. In other words, the flexibility of the pier elements controls the relative displacements between the top of the piers and their connection with the piles foundations, as expected. Contrarily, for values of this quantity close to its upper bound, i.e. D_p > 1.70m a strong interaction between the diameter of the circular cross sections of the piers and the external diameter of the rubber bearings is observed. Thus, for rigid pier elements the relative displacements between the top of the piers and their connection with the piles foundations is controlled by both design variables, that is, D_p and D_r. In fact, the iso-probability curves indicate that for example an increase in the diameter of the circular cross sections of the piers is compensated by a decrease in the external diameter of the bearings. In other words, for such combination of the design variables D_p and D_r the probability of failure remains invariant. On the other hand it seen that the failure event F_1 is mainly controlled by the external diameter of the rubber bearings. Actually, the iso-probability curves associated with failure event F_1 show a relatively weak interaction between the diameter of the circular cross sections of the piers and the external diameter of the rubber bearings. The probability of this event decreases as the external diameter of the isolators increases. The final design for this scenario is given by D_p=1.64m and D_r= 0.67m (point B in the figure) where both reliability constraints are active.

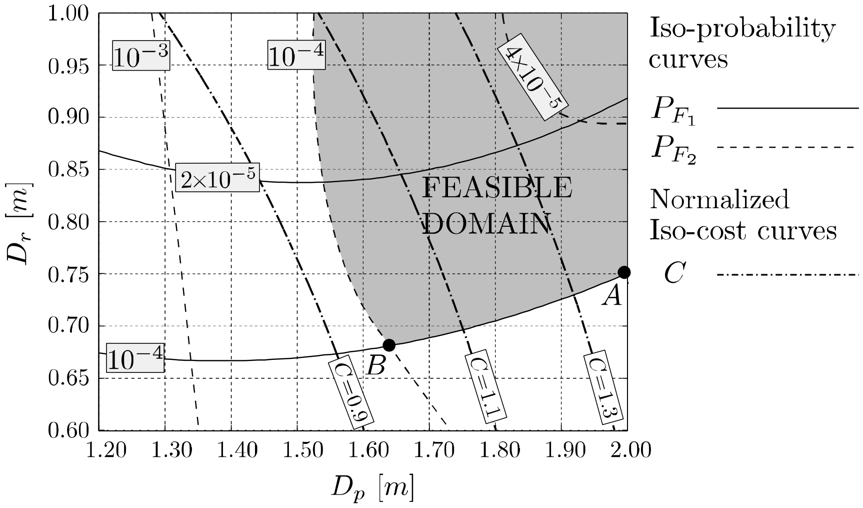

Figure 5. Design space in terms of the circular cross sections of the piers and the external diameter of the rubber bearings

The results shown in Figure 5 can also be used to demonstrate the benefits of designing the isolators and the bridge structure simultaneously. For example, if the design process involves only the isolators and the diameter of the circular cross sections of the piers is kept constant at their upper bound values (D_P= 2.0m), the optimal design is given by D_r = 0.75m (point A in the figure) with a corresponding normalized cost equal to C=1.4. On the other hand, if the diameter of the circular cross sections of the piers is also considered as design variable the final design moves from point A to point B, with a decrease of the normalized cost in about 30%. Thus, taking into account the interaction between the design variables associated with the bridge structure and the isolators during the design process is quite beneficial in terms of the cost of the final design. It is clear that the above observations and remarks give a valuable insight into the complex interaction of the design variables on the performance and reliability of the bridge structural system.

7.4 Computational Cost

Table 1 shows the on-line computational costs involved in the assemblage of the finite element model and the computation of the dynamic response for a given design considering the full finite element model and the reduced-order model. These operations and procedures are performed at each iteration of the design process. It is seen that the time difference by using the full finite element model and the reduced-order model is quite significant. Actually the ratio between these times is almost 100. The off-line computational cost, that is the cost of calculations related to the definition of the reduced-order model which is performed once during the design process, corresponds to approximately two full analyses (finite element model generation and dynamic response) of the unreduced model in this case. Considering this cost an overall speedup value of more than 10 is obtained by the proposed methodology in solving this particular design problem. In this context the speedup is the ratio of the execution time by using the unreduced model and the execution time by using the reduced-order model. The reduction in computational effort is achieved without compromising the accuracy of the final design. Finally, it is noted that once the reduced-order model has been defined several design scenarios in terms of different objective functions, reliability constraints, side constraints, etc. can be explored and solved efficiently. So, higher speedup values (up to two orders of magnitudes) can be obtained for the overall design process of the bridge structure.

Table 1. On-line computational costs for a given design

Task Description	Full FE model Time (s)	Reduced model Time (s)
FE model generation	52.9	0.013
Modal analysis	0.74	0.052
Numerical Integration	1.7	0.51
Sum of different tasks	55.34	0.575

8. Conclusions

A general strategy for dealing with a class of reliability-based design problems of finite element models under stochastic earthquake loading has been presented. It consists in the integration of a model reduction technique with an appropriate optimization scheme. The design process is carried out in a reduced space of generalized coordinates. In particular, a model reduction technique based on substructure coupling technique for dynamic analysis is considered in the present implementation. The reduction technique, which is applied to the linear components of the structural systems, produces highly accurate models with relatively few substructure modes. The numerical results demonstrate that the computational effort involved during the design process is reduced significantly with respect to the process considering the unreduced finite element model. In fact, good speedup values were obtained. On the other hand, the reduction in computational effort is achieved without compromising the accuracy of the design process. Based on the results of this study it is concluded that the proposed approach is potentially an effective tool for solving a class of reliability-based design problems involving complex structural systems under stochastic excitation such as ground motions.

9. References

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