A DOUBLE HYPERBOLIC MODEL AND PARAMETER IDENTIFICATION

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Abstract

A new stress-strain model, named "double hyperbolic model (DHP model)" is proposed. This model is composed of two hyperbolic models. The first hyperbolic model is used at strains less than the reference strain or strains at stiffness degradation ratio is 0.5 in which the reference strain is used as a model parameter. The second hyperbolic model is used at strains larger than the reference strain, in which shear strength is used as a model parameter. Two hyperbolic models are connected at the reference strain so that the slope is continuous. It uses only two parameters and can simulate behavior in wide range of strains from very small to large strains.

Accuracy and applicability of the model is examined by using about 500 cyclic shear deformation characteristics test results. Conventional models, the hyperbolic model and the Ramberg-Osgood model, are also examined by the same method.

It is shown that both the hyperbolic and the Ramberg-Osgood models can simulate stress-strain behavior well up to reference strain or strain less than 0.1 %. However, the hyperbolic model underestimate shear stress at large strains and the Ramberg-Osgood model overestimates shear stress at large strains. Therefore, these conventional models may be good in the past situations where input earthquake motion is not very large, but they are not applicable at large strains that are required in the recent Japanese design specifications. On the other hand, the error by the DHP model is much smaller than two conventional models at large strains.

Among three parameters, the reference strain, the maximum damping ratio, and the shear strength, empirical equations are already shown by the authors for the first two parameters. Then statistical approach is made to get an empirical equations for the last parameter, shear strength of the model, is examined. At first it is shown that there is no good correration between reference strain and shear strength as a function with respect to corrected SPT *N*-value, plasticity index, fines content and average grain size, which indicates that small strain behavior and large strain behavior is independent. Then dependency of shear strength ratio (the ration of shear strength to the initial shear modulus) on these parameters are examined. It is found that shear strength ratio increases as the fines contents or the plasticity index increases of average diameter decreases. This indicates that clayey soil shows larger shear stress at large strains compared with the stress at small strains.

Finally, a discussion is made on the shear strength to be used for seismic response analysis of ground and it is encouraged tor future research on this topic.

Keywords: hyperbolic model; reference strain; shear strength; empirical equation

1. Introduction

An Engineer must choose stress-strain models for the seismic response analysis of ground depending on the soil data he has. If he conducts cyclic shear deformation characteristics tests to obtain the strain dependent shear modulus and damping ratio (cyclic shear deformation characteristics in the following), the stress-strain model proposed by the authors [1, 2] gives perfect simulation. On the other hand, if he does not conduct cyclic shear deformation characteristics test, stress-strain parameters are estimated from such parameters as soil type and SPT-*N* value. In these cases, stress-strain models that have a few parameters such as the hyperbolic and the Ramberg-Osgood models are preferable.

The authors collected about 500 sets of data on cyclic shear deformation characteristics and used them to examine the applicability of these stress-strain models [3, 4]. The hyperbolic model is shown to have a tendency to underestimate shear stress and the Ramberg-Osgood model has a tendency to overestimate shear stress at large strains as will be shown later. Therefore applicability of these models is limited at large strains. A new stress-strain model is proposed in this paper, which can simulate soil behavior over a wide range of strains up to the shear strength.

2. Brief review of the previous research

The authors collected about 500 data sets on cyclic shear deformation characteristics test results, which are classified based on the geologic age, depositional environment and soil type as shown in Table 1. They are collected from 95 sites in the Tokyo Metropolitan Area (Kanto district), Japan. Test methods are cyclic triaxial test, cyclic torsion test combined with resonant test using circular solid specimen, and cyclic hollow cylinder torsion test. Initial confining stress σ'_m of each test is shown in Table 1 and Fig. 1 (the legend in Fig. 1 is used in the following figures). The cyclic shear deformation characteristics of all soils are sown in Fig. 2. Here, G denotes secant shear modulus, G_0 denotes initial shear modulus, γ denotes shear strain, and h denotes damping ratio. In the figure, red dashed line indicates clayey soil and blue solid line indicates sandy soil. Although measured shear strains are different in each test, G/G_0 and h are interpolated at strains 1, 2 and 5 in each digit by using a Bezier curve. Recently, large strain behavior is required because design earthquake motion becomes large. On the other hand, applicability of the conventional cyclic shear deformation characteristics test is supposed to be a little larger than 0.1 % [5], but, modulus and damping ratio are frequently measured at strains larger than1 %. Considering these situation, strains from 10^{-6} to 0.01 are used in this study. Therefore there are 13 G/G_0-γ and h-γ data points in each test data.

The applicability of the frequently used the hyperbolic model the Ramberg-Osgood mode (R-O model):

$$\text{Hyperbolic model: } \tau = \frac{G_0\gamma}{1+\gamma/\gamma_r} \tag{1}$$

$$\text{Ramberg-Osgood model: } \gamma = \frac{\tau}{G_0}\left\{1+\alpha\left(\frac{\tau}{\tau_f}\right)^{\beta-1}\right\} \tag{2}$$

is examined in Figs 3 and 4. Here, τ denotes shear stress, γ_r denotes reference strain τ_f denotes shear strength, and α and β are parameters. The hyperbolic model uses one parameter and the Ramberg-Osgood model uses two independent parameters to express nonlinear characteristics in addition to the elastic modulus G_0.

The reference strain in the hyperbolic model is evaluated at a strain where G/G_0=0.5 in Fig. 3. Agreement at small strain is good, but shear stress is underestimated at large strains in almost all data. Two parameters of the Ramberg-Osgood model are evaluated so that test and model agrees at G/G_0=0.5 and 0.8 in order to get good agreement at relatively small strains in Fig. 4; shear stresses at large strains is overestimated in this model. From these observations, large strain behavior is difficult to simulate by the conventional stress-strain models although behavior at small strain simulated well. In our previous study (Yoshida and Wakamatsu, 2012), several methods are examined to calculate values of parameters and evaluate disagreement or error of these models. Here error of i-th test data E_i is calculated by

Table 1. Classification of cyclic shear deformation characteristics test data

Geologic age		Depositional environment	Soil Type	Geologic category	Number of data σ'_m	I_p	F_c	D_{50}
Man-made		Fill	Clayey soil	1-Bc	10	7	10	10
			Sandy soil	2-Bs	14	2	14	14
Holocene	Upper	Aeolian	Sandy soil	3-As	2	0	0	0
		Marine	Sandy soil	4-As	13	0	13	13
			Gravel	5-Ag	0	0	0	0
		Brackish-water	Clayey soil	6-Ac	12	10	12	12
			Sandy soil	7-As	15	3	15	15
		Fluvial	Peat	8-Ap	3	2	2	2
			Clayey soil	9-Ac	30	29	30	25
			Sandy soil	10-As	28	0	28	28
			Gravel	11-Ag	0	0	0	0
	Lower	Marine	Clayey soil	12-Ac	119	99	113	113
			Sandy soil	13-As	20	3	18	18
		Fluvial	Gravel	14-Ag	0	0	0	0
Pleistocene	Upper	Marine Brackish-water	Clayey soil	15-Ac	17	15	17	17
			Sandy soil	16-As	5	1	5	5
	Upper	Volcanic ash fall	Loam	17-Lm	15	14	14	14
			Clayey soil	18-Dc	6	5	6	6
			Sandy soil	19-Ds	0	0	0	0
			Gravel	20-Dg	0	0	0	0
	Middle	Volcanic ash fall	Loam	21-Lm	0	0	0	0
			Clayey soil	22-Dc	76	53	71	70
			Sandy soil	23-Ds	92	1	74	74
			Gravel	24-Dg	1	0	1	1
Pliocene-Pleistocene	Lower Pleistocene Upper Pliocene	Marine	Mudstone	25-Dc	4	4	4	4
			Sandy-gravel rock	26-Dsg	0	0	0	0

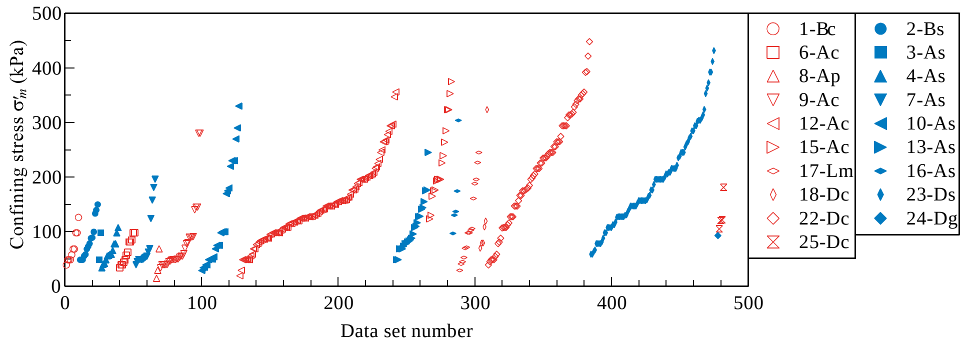


Fig. 1 – Distribution of initial effective mean stress σ'_m

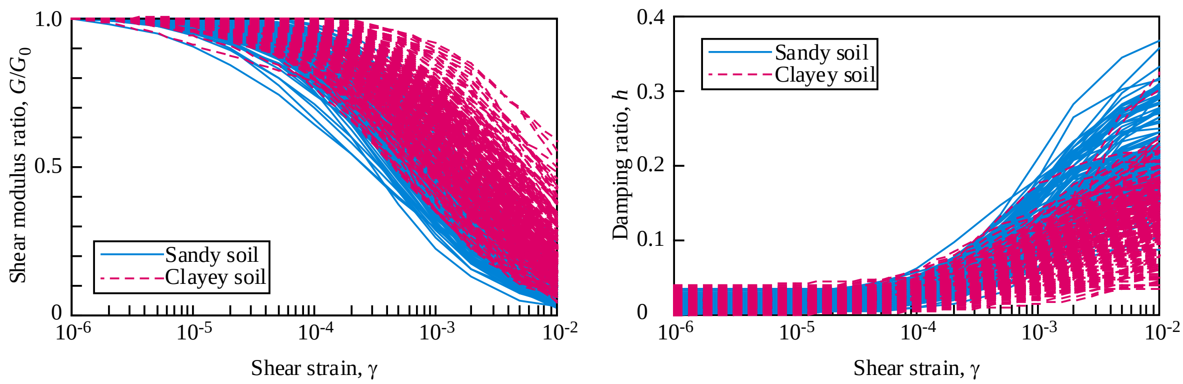


Fig. 2 – Cyclic shear deformation characteristics of all test specimens

$$E_i = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\left(\frac{\tau}{G_0}\right)_{k,\text{test}} - \left(\frac{\tau}{G_0}\right)_{k,\text{model}}\right)^2} = \frac{1}{G_0}\sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\tau_{k,\text{test}} - \tau_{k,\text{model}}\right)^2} \tag{3}$$

where N denotes number of data and is 13 as explained before, and subscripts "test" and "model" denote test data and model. Error can be smaller than the cases shown in Figs. 3 and 4 if parameters another set of parameters, but general tendency is the same.

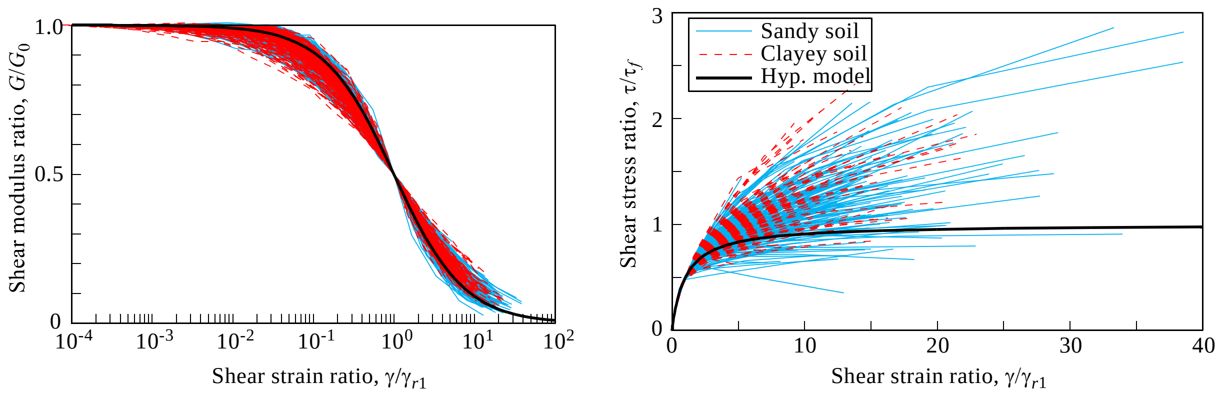


Fig. 3 – Applicability of hyperbolic model

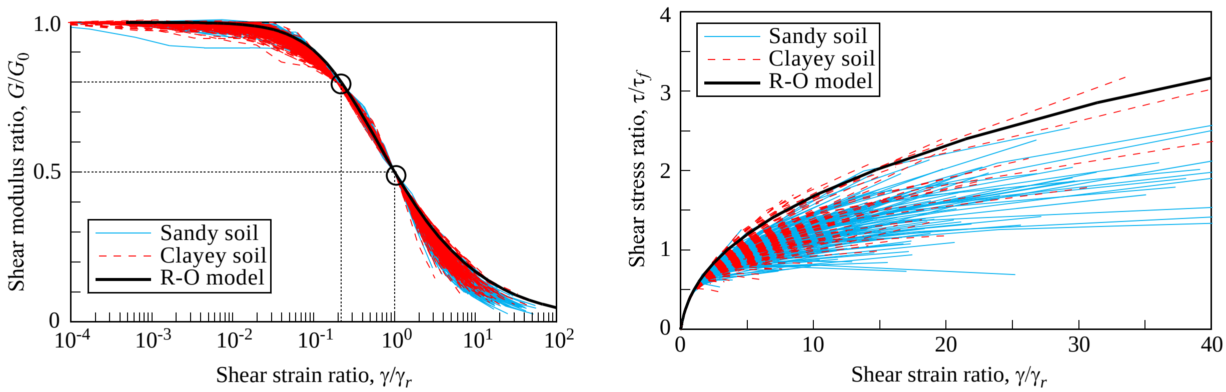


Fig. 4 – Applicability of Ramberg-Osgood model

3. A double hyperbolic model

The model proposed in this paper is composed of two hyperbolic models, and is named a double hyperbolic model (DHP model). Two equations are

$$\tau = \frac{G_0\gamma}{1+\gamma/\gamma_r} \qquad (\gamma \le \gamma_r) \tag{4}$$

$$\tau = \frac{G_0(\gamma-\gamma_0)}{A+B(\gamma-\gamma_0)} \qquad (\gamma > \gamma_r) \tag{5}$$

where A, B and γ_0 are parameters. These parameters are determined under the following conditions.

1) Two models have the same values at $\gamma=\gamma_r$ and the slope at $\gamma=\gamma_r$ is continuous.
2) Shear strength is τ_f.

Then Eq. (5) yields

$$\tau = G_0\gamma_r \frac{k\gamma/\gamma_r + k - 1}{4k - 3 + \gamma/\gamma_r} \qquad (\gamma > \gamma_r) \tag{6}$$

Here, the parameter $k = \tau_f/(\gamma_r G_0)$ is called shear strength ratio in the following because $G_0\gamma_r$ is the shear strength parameter of the first (small strain) hyperbolic model. Shear strength ratio takes the value larger than 0.5. The conventional hyperbolic model is obtained by setting k=1. Fig. 5 shows change of the cyclic shear deformation characteristics depending on k value.

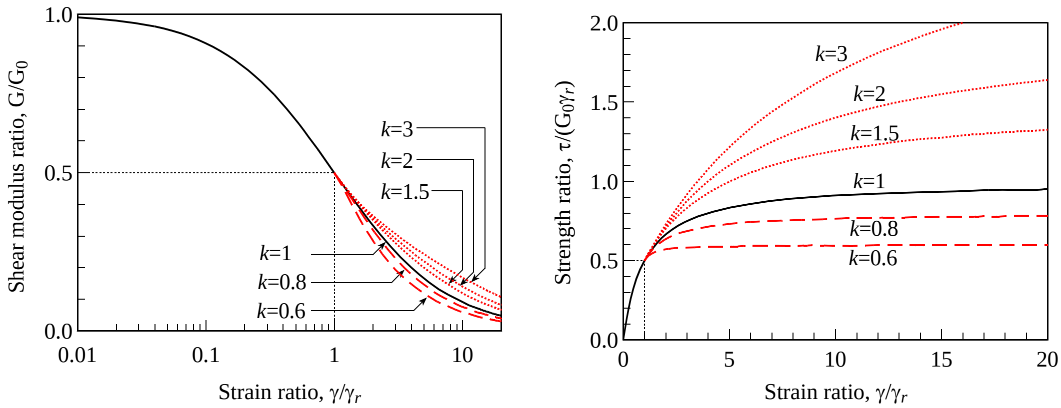


Fig. 5 – Parametric study of double hyperbolic model

4. Parameters and accuracy

Among the two parameters used in the double hyperbolic, the reference strain is already studied in detail in our previous study (Yoshida and Wakamatsu, 2013). Therefore, values of shear strength ratio k are studied in this paper. They are evaluated by two different methods, i.e., least square method and the nonlinear method.

Equation (6) is re-written as

$$\gamma_r + \frac{\tau}{G_0}\left(\frac{\gamma}{\gamma_r} - 3\right) = k\left(\gamma_r + \gamma - 4\frac{\tau}{G_0}\right) \tag{7}$$

This equation indicates that relationships between "$\gamma_r + \tau/G_0(\gamma/\gamma_r - 3)$" and "$\gamma_r + \gamma - 4\tau/G_0$" are linear. Then the shear strength ratio is obtained by the least square method. Strains larger than the reference strain is used to evaluate shear strength ratio.

An iterative procedure is used to make the error in Eq. (3) minimum by changing k in the second method, which is called a nonlinear method in the following because this process is equivalent to solving a nonlinear equation. In our previous study on the hyperbolic model, reference strains obtained both by the least square method and by the nonlinear method are almost identical. Therefore, in the nonlinear method, all the data is used to evaluate the error because the shear strength ratio obtained is expected to be same as the one by the least square method if strains larger than the reference strain are used.

Shear strength ratios obtained by two methods are shown in Fig. 6. Values of k scatter widely up to 9. Many of them are larger than 1.0, which agrees with the fact that hyperbolic model shown in Fig. 3 underestimates shear stress at large strains.

Shape of distributions in Fig. 6 is similar, which can also be confirmed from the comparison of shear strength ratio in Fig. 7. Many points lie on the 1:1 line. In the same manner, Errors by two methods are compared in Fig. 8, which also lie on the 1:1 line although errors by the nonlinear methods is a little smaller than those by the least square method. These observations indicate that agreement at large strains is important to make the error small.

Fig. 9 shows errors E_i of all data for three models. Here model parameters are evaluated to make the error minimum by using the iterative nonlinear methods in all models. Generally speaking, errors on the Ramberg-Osgood model is the largest and those by the DHP model is the smallest. In order to see accuracy in detail, error by the DHP model is compared with other models in Fig. 10.

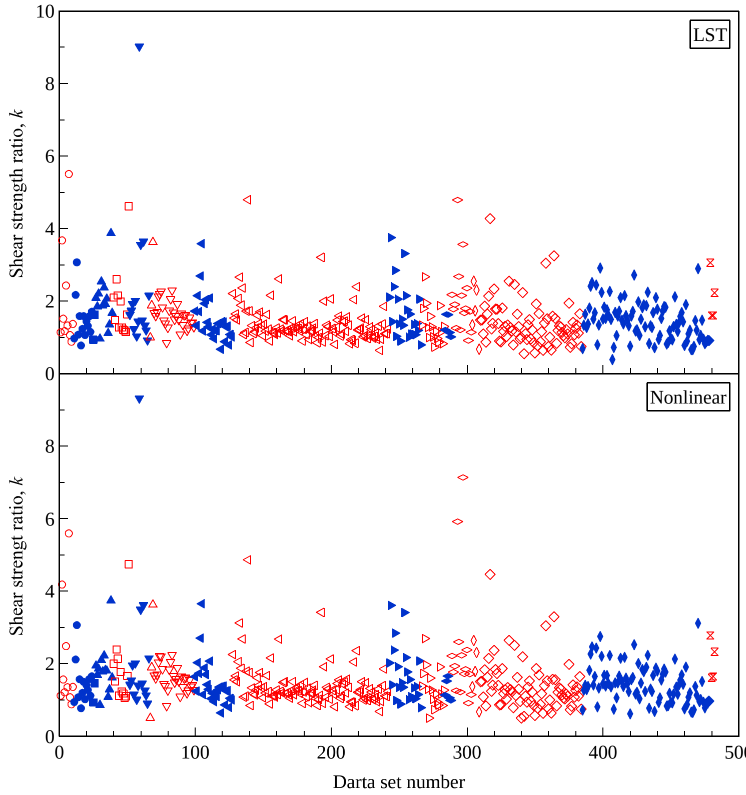
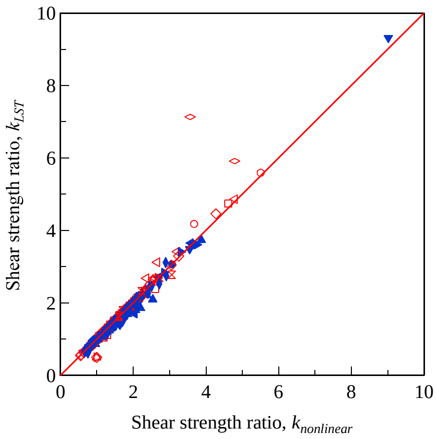


Fig. 6 – Shear strength ratios obtained by least square and nonlinear methods



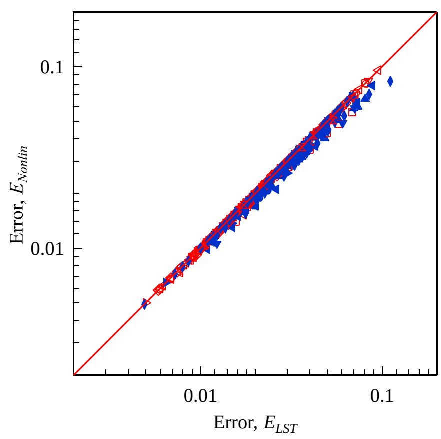


Fig. 7 – Comparison of shear strength ratio

Fig. 8 – Comparison of error

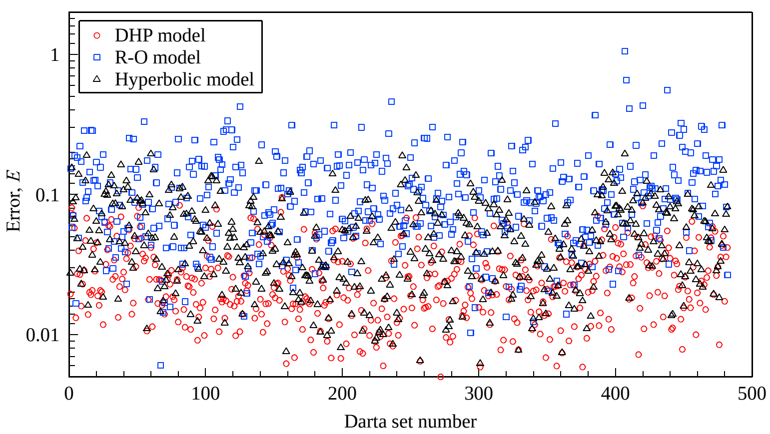
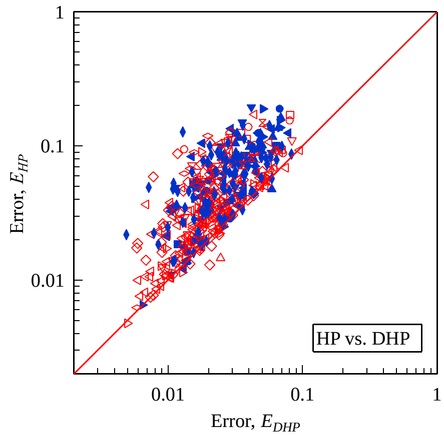
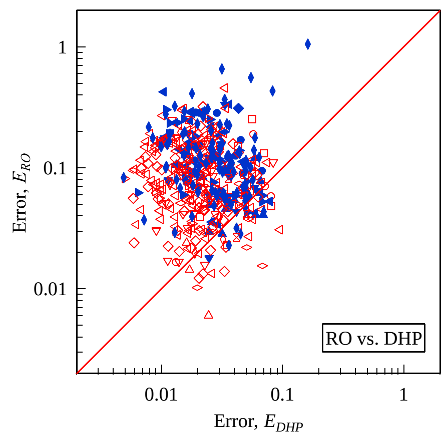


Fig. 9 – Minimized error of each model

Maximum error of the DHP model is about 0.1, and that of the hyperbolic model is about 0.2. Average error by the hyperbolic model is several times larger than that of DHP mode. On the other hand, that of the Ramberg-Osgood model is much larger up to 1.05.



(a) Hyperbolic model



(b) Ramberg-Osgood model

Fig. 10 – Comparison of errors with other models

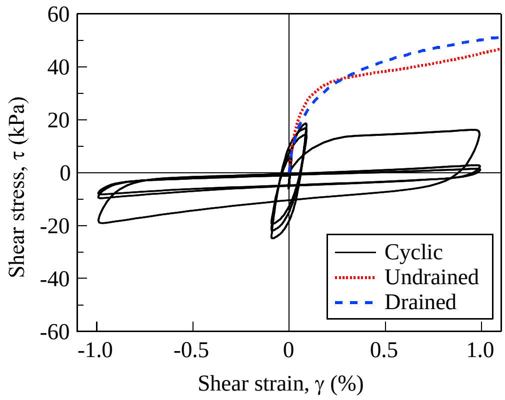


Fig. 11 – Stress-strain curves under static and cyclic loading

5. Empirical equations

It is shown that the proposed DHP model shows much small error compared with past conventionally used models such as the hyperbolic and the Ramberg-Osgood model although it uses only two parameters for nonlinear behavior, i.e., reference strain γ_r and shear stress ration k. This model also uses maximum damping ratio h_{max} shown by Hardin and Drnevich [6].

$$h = h_{max}(1 - G/G_0) \tag{8}$$

Among these three parameters, empirical equations for γ_r and h_{max} are already examined by the authors [4]. They are summarized in Tables 2 and 3 for clayey soil and sandy soil, respectively. More equations are shown in the original paper based on detailed classification, but only equations based on rough classifications are shown in the table because it it generally difficult to make detailed classification based on geological age and depositional conditions shown in Table 1. Here I_p denotes plasticity index, σ_m is initial effective mean stress in kPa, and D_{50} is average diameter in mm.

Then empirical equations for the shear strength ratio is a final problem. Fig. 12 shows relationships between shear stress ratio k and various parameters such as corrected SPT N-value N_1, fines content F_c, plasticity index I_p, and average diameter D_{50}. It is seen that no good correration for all parameters. It indicates that there is no relationships between reference strain and shear strength.

Therefore, as a next stage, shear stress itself is used to evaluate from various parameters. Fig. 13 shows the relationships between shear stress and the same parameters used in Fig. 11. Here, since shear deformation characteristics is shown as G/G_0-γ relationships. Therefore, shear strength is also normalized by G_0, and is also called shear stress ratio, but there is no confution between k and τ_f/G_0.because k and τ_f/G_0.are always written in this paper.

Table 2 Empirical equations for clayey soil

	γ_r					h_{max}				
	$\log\gamma_r=a\sigma_m\times10^{-4}+b$		$\log\gamma_r=a\sigma_m\times10^{-4}+bIp\times10^{-3}+c$			$h_{max}=a\sigma_m\times10^{-5}+b$		$h_{max}=a\sigma_m\times10^{-5}+bI_p\times10^{-4}+c$		
	a	*b*	*a*	*b*	*c*	*a*	*b*	*a*	*b*	*c*
Pleistocene	7.26	-2.77	10.5	6.70	-3.14	2.35	0.170	3.26	-4.70	0.189
Holocine	4.91	-2.77	5.79	7.66	-3.11	4.75	0.175	5.65	-3.54	0.186
I_p>30	4.49	-2.65	7.18	6.38	-3.05	5.82	0.162	4.12	-4.03	0.188
I_p<=30	10.3	-2.96	11.7	13.2	-3.33	-3.78	0.194	0.102	-0.985	0.188
Clayey soil	7.74	-2.80	8.86	7.33	-3.14	0.951	0.177	3.06	-4.28	0.191

Table 3 Empirical equations for sandy soil

	γ_r					h_{max}				
	$\log\gamma_r=a\sigma_m\times10^{-4}+b$		$\log\gamma_2=a\sigma_m\times10^{-4}+b\log D_{50}+c$			$h_{max}=a\sigma_m\times10^{-5}+b$		$h_{max}=a\sigma_m\times10^{-5}+b\log D_{50}\times10^{-2}+c$		
	a	*b*	*a*	*b*	*c*	*a*	*b*	*a*	*b*	*c*
Pleistocene	9.96	-3.20	11.2	-0.176	-3.35	2.36	0.223	-0.0143	-7.45	0.179
Holocene	6.52	-3.13	8.08	-0.210	-3.30	-5.87	0.211	-4.88	-1.27	0.200
Sandy soil	8.64	-3.17	9.69	-0.216	-3.34	4.64	0.211	6.97	-4.62	0.175

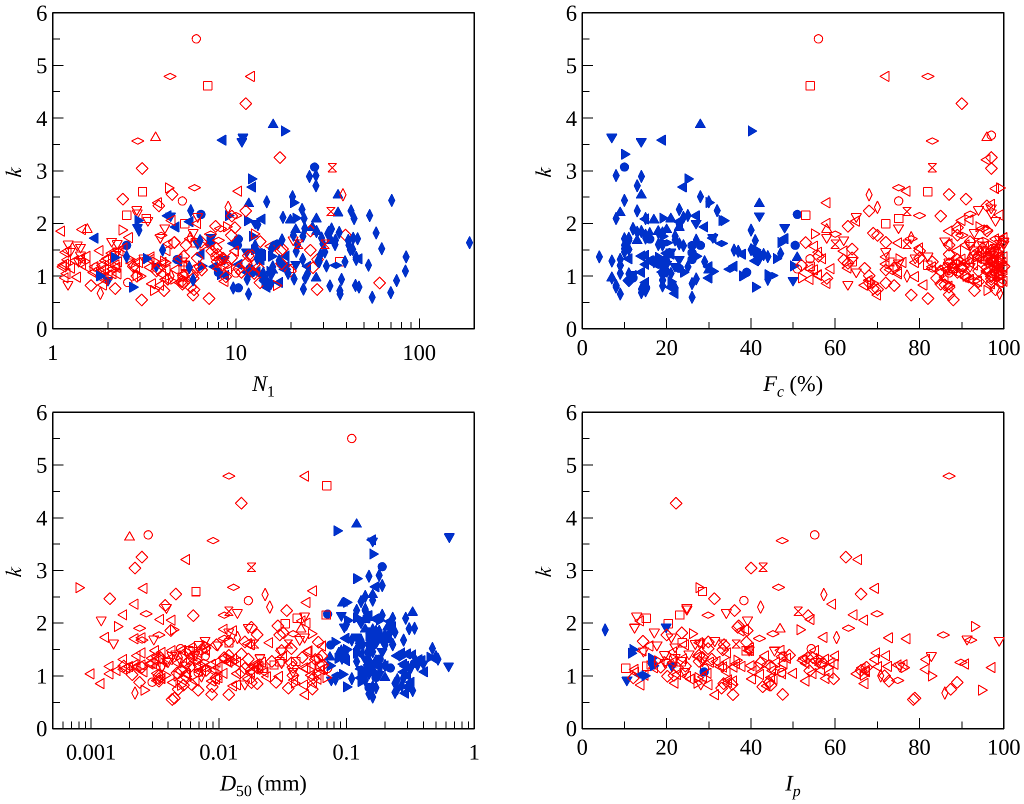


Fig. 12 – shear stress ratio k vs. various parameters.

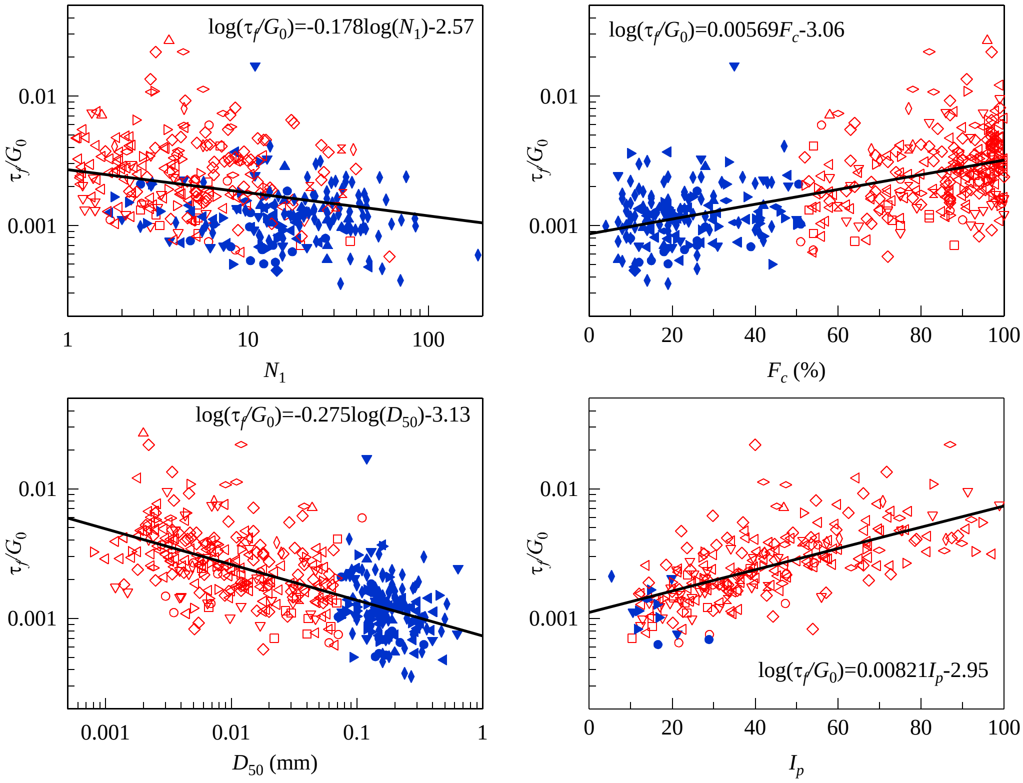


Fig. 13 – shear stress ratio τ_f/G_0 vs. various parameters.

It is seen that shear stress ratio τ_f/G_0.correlate to all parameters It increases as I_p and F_c increases. It indicates that clayey soil shows higher shear strength, which agrees with past experience. On the other hand, it decreases D_{50} and N_1 increases. Relationships between D_{50} are same tendencies above.

Empirical equations obtained by applying the least square method is shown in Fig. 13. Since vertical axis is a log-axis, correration cannot be a good, but this kind of scattering is frequently observed.

6. Concluding remarks

A new stress-strain model, named a double hyperbolic (DHP) model, is proposed. This model uses only two parameters in expressing the nonlinear behavior, which are the reference strain (strain at which G/G_0=0.5) and the shear strength (shear stress ratio is calculated from the shear modulus and the reference strain). Simulation of about 500 cyclic shear deformation characteristics shows that error of this model is much smaller than those by the conventional stress-strain models (hyperbolic model and Ramberg-Osgood model). The mechanical meaning of the parameters is very clear, which is another advantage of this model.

Here, it is noted that shear strength use in evaluating k is not the shear strength obtained by a monotonic loading test nor calculated from the Mohr-Coulomb criteria. Fig. 11 show test results of monotonic and cyclic loading test results; result of the cyclic test is a part of the cyclic shear deformation characteristics test and monotonic test is carried under drained and undrained conditions [7]. Stresses at large strains by the cyclic loading test are much smaller than those by the monotonic loading test because of the excess porewater pressure generation. Therefore, the relevant evaluation of the shear strength for the DHP model remains a problem for the future study.

It is shown that shear strength does not relate well with reference strain. It relates various parameters such as N_1, I_p, F_c, and D_{50}. However, accuracy cannot be good and a future research is necessary. This is not the requirement of the double hyperbolic model but also for all other models.

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