MAGNITUDE AND DISTANCE SCALING OF ENGINEERING DEMAND PARAMETERS OF MOMENT-RESISTING FRAME STRUCTURES

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Abstract

This paper presents a mathematical formulation that directly estimates the engineering demand parameters (EDPs) of moment-resisting frames based on structural characteristics and earthquake parameters such as magnitude, source-to-site distance and local site conditions. These equations are developed for Maximum Interstory Drift Ratio (MIDR) and Roof Drift Ratio (RDR). One set of generic elastic moment-resisting frames is used to validate the proposed formulation against the results of Response Spectrum Analysis (RSA). The results confirm that the new equations provide a conservative but accurate estimate of the structural response while simplifying the computational process.

Keywords: Probabilistic seismic hazard analysis; ground motion models; EDP hazard curve.
1. Introduction

The goal of Probabilistic Seismic Demand Analysis is the development of the EDP hazard curve $\lambda_{EDP}$, which estimates the mean annual frequency of exceeding a specified threshold of the structural response. This has traditionally been achieved using a two-step approach: First the hazard at the site is estimated based on the seismicity of the area and on the intensity measure IM chosen to represent the ground motions, obtaining the IM hazard curve $\lambda_{IM}$ through Probabilistic Seismic Hazard Analysis PSHA (Cornell (1968) [1], McGuire (1995) [2]), represented by Eq. (1).

$$\lambda_{IM}(im) = \sum_{i=1}^{n} v_i \int P(IM > im|\theta) f_i(\theta) d\theta$$  \hspace{1cm} (1)

Where: $\lambda_{IM}(im)$ represents the mean annual frequency of exceeding the intensity measure value $IM = im$, $n$ is the number of potential earthquake sources at the location of interest, $v_i$ is the average seismic rate of source $i$, $\theta$ represents all the ground motion parameters (magnitude, distance, etc.), and $f_i(\theta)$ is the probability density function of the ground motion parameter combination for earthquake source $i$.

The second step is the definition of the IM-EDP relationship, which is generally developed by fitting a curve to the results of multiple nonlinear seismic analysis (see Baker (2005) [3] for a thorough review of the existing methods). Once the EDP distribution at multiple IM levels is available, it is integrated with the IM hazard curve according to Eq. (2) to obtain $\lambda_{EDP}$.

$$\lambda_{EDP}(edp) = \int_{im} P(EDP > edp|IM = im) d\lambda_{IM}(im)$$  \hspace{1cm} (2)

The accuracy of this approach strongly depends on the chosen IM, which is generally the pseudo-spectral acceleration at the first period of the structure $PSA(T_1)$. In his paper, Luco (2007) [4] gauges the strength of an intensity measure in terms of its efficiency and sufficiency. Several advanced intensity measures have been introduced in recent years to overcome the shortcomings of $PSA$ (see De Biasio (2014) [5], Luco (2007) [4], Tothong (2007) [6] just to name a few), but the simplicity of calculation of $PSA(T_1)$ and its predominance in engineering practice have allowed it to remain the IM of choice to this date, including in the development of the Ground Motion Models as part of the recent NGA-West2 project, Bozorgnia (2014) [7].

This paper summarizes part of the accomplished research in a project that aims at reducing the bias introduced by the selection of an intensity measure by developing a set of Performance Prediction Equations that estimate the EDP distribution based on ground motion parameters alone. These equations can be included in any hazard calculation software to deliver the direct calculation of the EDP hazard curve according to Eq. (3).

$$\lambda_{EDP}(edp) = \sum_{i=1}^{n} v_i \int P(EDP > edp|\theta) f_i(\theta) d\theta$$  \hspace{1cm} (3)

The next section presents the formulation of the first generation of PPEs, which use the existing GMMs to estimate the EDP statistics. Section 4 presents the application of these equations to strike-slip scenarios to show the distribution of the response with several ground motion parameters and to validate the equations against conventional RSA. Additional information about the mathematical formulation and additional results will be available in an upcoming paper.

2. Formulation

As previously mentioned, the goal of this study is to develop and implement an analytical formulation that provides the EDP distribution based on ground motion parameters. The simplest way to achieve this is to use the
existing ground motion models to estimate the IM statistics, which are in turn transformed into EDP statistics using the method summarized in the following. The full mathematical derivation of the EDP statistic equations will be provided in an upcoming paper by the authors.

A simple way of generating IM-EDP relationship is to utilize response spectrum analysis - with complete quadratic combination CQC (Wilson (1981) [8]) - where the maximum SDOF responses for the first \( n \) modes of vibration are combined according to Eq. (4) to estimate the total response, using displacement as the EDP of interest:

\[
\text{Disp}_k = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{Disp}_{i,k} \alpha_{T_i T_j} \text{Disp}_{j,k} = \sqrt{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \Gamma_i \Phi_{i,k} \frac{\text{PSA}_i}{\omega_i^2} \alpha_{T_i T_j} \Gamma_j \Phi_{j,k} \frac{\text{PSA}_j}{\omega_j^2}
\]

Where: \( k \) is the story number, \( i \) is the mode number, 
\[\Gamma_i = \int_{-\infty}^{\infty} \Phi_i(x)dx\] is the modal participation factor of mode \( i \) (from Chopra [9]),
\[\Phi_i(x)dx\] is the pseudo-spectral acceleration at the \( i \)th mode period,
\[\Phi_{i,k} \] is the \( i \)th mode shape of the structure at story \( k \),
\[\omega_i = 2\pi/T_i\] is the circular frequency of mode \( i \),
\[\xi, \alpha\] which is equal to 1.0 when \( i = j \) (Wilson (1981) [8]),
\[\xi\] is the damping ratio, 0.05.

Eq. (4) establishes a multivariate nonlinear relationship \( g(x) \) between the response variable \( \text{Disp}_k \) and the pseudo-spectral acceleration at several periods, which can be approximated using Taylor series, as shown in Eq. (5):

\[
Y \sim g(\bar{x}) + \sum_{i=1}^{n} (x_i - \bar{x}_i) \frac{\partial g(x)}{\partial x_i} \bigg|_{x=\bar{x}} + \frac{1}{2!} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - \bar{x}_i)(x_j - \bar{x}_j) \frac{\partial^2 g(x)}{\partial x_i \partial x_j} \bigg|_{x=\bar{x}} + \cdots
\]

If only the linear term is used, and using the linear properties of the expectation operator, it is possible to derive formulas that provide the statistics of the response variable \( \text{Disp}_k \) based on the statistics of the PSA vector, which are readily available from any existing GMM. It is also reasonably assumed that the distribution of the EDP be lognormal, therefore, the mean and variance of \( \ln \text{Disp}_k \) fully describe its distribution. These values are provided in Eq. (6), where \( A_i \) is a vector representing the structural properties for mode \( i \).

\[
\left\{\begin{align*}
\mu_{\ln \text{Disp}_k} &= \frac{1}{2} \ln \left( \sum_{i=1}^{3} \sum_{j=1}^{3} A_{i,k} A_{j,k} \alpha_{T_i T_j} \mu_{\text{PSA}_i} \mu_{\text{PSA}_j} \right) = \frac{1}{2} \ln \text{CQCA}_k \\
\sigma^2_{\ln \text{Disp}_k} &= \frac{1}{\text{CQCA}_k^2} \sum_{i=1}^{3} \sum_{j=1}^{3} \rho_{\text{PSA}_i} \sigma_{\text{PSA}_i} \sigma_{\text{PSA}_j} A_{i,k} A_{j,k}
\end{align*}\right.
\]
Where: 
\[ A_{i,k} = \frac{\gamma}{\omega_i^2}, \]
\[ AA_{i,k} = A_{i,k} \sum_{i=1}^{3} \mu_{PSA} \alpha_{T_i, T_j}, \]
\[ CQCA_k = \sum_{j=1}^{3} \sum_{j=1}^{3} A_{i,k} A_{j,k} \mu_{PSA} \mu_{PSA}. \]

Similar formulas can be obtained for interstory drift ratio IDR by using \( \Delta \theta_i = (\theta_{i+1} - \theta_i)/h \) instead of \( \theta_i \), and are reported in Eq. (7).

\[
\mu^{\text{in}}_{IDR,k} = \frac{1}{2} \ln CQCB_k \\
\sigma^2_{\text{ln}IDR,k} = \frac{1}{CQCB_k} \sum_{i=1}^{3} \sum_{j=1}^{3} \rho_{PSA} \sigma_{PSA} \sigma_{PSA}, BB_{i,k} BB_{j,k}
\]

Where: 
\[ B_{i,k} = \frac{\gamma}{\omega_i^2}, \]
\[ BB_{i,k} = B_{i,k} \sum_{i=1}^{3} B_{i,k} \mu_{PSA} \alpha_{T_i, T_j}, \]
\[ CQCB_k = \sum_{j=1}^{3} \sum_{j=1}^{3} B_{i,k} B_{j,k} \mu_{PSA} \mu_{PSA}. \]

3. Structural models: Generic elastic moment-resisting frames

The analytical models used in this study were first developed in Drain-2DX by Zareian (2009) [10] as a set of generic elastic moment-resisting frames. Fig.1 shows the general layout of the models. These 2D frames consist of 3 bays and a variable number of stories: 4, 8, 12, and 16. Each building has a set story height of 12 ft (i.e., \( h = 12' \)) and a beam span of 36 ft.

![Generic Moment-Resisting Frame Diagram](image)

Fig. 1 – Schematic view of the generic moment-resisting frame, with the geometry on the left and the analytical model on the right (from Zareian (2009) [10])

For each number of stories, two values of the first mode period of the structure \( T_i \) were considered: \( T_1 = 0.1 \) NS or \( T_1 = 0.2 \) NS, where NS is the number of stories. Damping was set at 5% for the first and third modes.

Three modes of stiffness distributions along the height were considered in the original report. In the “Shear” models the beam stiffness at each story varies proportionately with the story shear experienced by the building when subjected to the NEHRP lateral load pattern. This assures that the building deflected shape is linear under the same load. The “Uniform” case represents buildings with constant beam stiffness along the height, and the “Intermediate” case, as the name suggests, represents the average of the previous two at each story. Fig.2 schematically represents the variation in stiffness along the height for the three distributions. It is assumed that variation along the height of beam strength follows the stiffness distribution, based on design considerations.
All beams in the same floor have equal moment of inertia and strength. The same applies to columns, and all columns in each story have the same moment of inertia as the beams in the floor above. To avoid undesirable collapse mechanisms, capacity design is implemented in the generic frames, using stronger columns than beams. All columns in the same story have equal strength as well as stiffness.

Even though the original models were designed with concentrated plasticity at the element ends, only their elastic behavior described by their modal properties is considered as part of this study. Additional information about the models can be found in the original paper, Zareian (2009) [10].

4. EDP distribution with ground motion parameters

This section presents the distribution of the structural response of the aforementioned generic buildings when subjected to several strike-slip earthquake scenarios. Only the results of the “Shear” type buildings are presented, other buildings show similar results.

The earthquake scenarios are represented by their median response spectra and their dispersions, calculated using the Campbell-Bozorgnia 2014 horizontal Ground Motion Model [11] and characteristic fault geometry considerations: Consistently with the original paper, each moment magnitude is assumed to be caused by a characteristic fault width and depth, whose values are reported in Table 1.

Table 1 – Characteristic fault geometry for strike-slip scenarios, adapted from CB14 [11]

<table>
<thead>
<tr>
<th>$M$</th>
<th>$Z_{BOT}$ [km]</th>
<th>$W$ [km]</th>
<th>$Z_{TOR}$ [km]</th>
<th>$Z_{HYP}$ [km]</th>
</tr>
</thead>
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<tr>
<td>3.5</td>
<td>15</td>
<td>0.5119</td>
<td>7.1449</td>
<td>7.5626</td>
</tr>
<tr>
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<td>15</td>
<td>1.6572</td>
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<td>15</td>
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<td>4.2887</td>
<td>7.2779</td>
</tr>
<tr>
<td>6.5</td>
<td>15</td>
<td>14.1259</td>
<td>0.8741</td>
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<td>10.2267</td>
</tr>
<tr>
<td>8.0</td>
<td>15</td>
<td>15</td>
<td>0.0</td>
<td>10.2267</td>
</tr>
</tbody>
</table>

Five values of shear wave velocity $V_{s30}$ are considered: 255, 360, 525, 760, and 1070 m/sec, representative of NEHRP site classes D, CD, C, BC, and B respectively, as recommended in the original paper. These strike-slip scenarios are all characterized by a 90-degree dip angle, the $Z_{2.5}$ depth was calculated from $V_{s30}$ using Eq. (33) in CB14, and $A_{1100}$ was calculated as the PGA value corresponding to a $V_{s30} = 1100$ m/sec.

The goal of this section is to validate the linearized equations developed in Section 2 by comparing the structural response calculated using these formulas with the result of response spectrum analysis RSA using the
CQC combination, introduced in Eq. (4). To provide statistical robustness to the calculations, 1,000,000 Monte Carlo simulations of $PSA(T_1)$, $PSA(T_2)$, and $PSA(T_3)$ were developed for each earthquake scenario that are consistent with the intensity measure distribution. The correlation coefficient $\rho_{lnPSA_i lnPSA_j}$ formulas developed in Baker (2008) [12] were used in the calculations, given the lack of updated equations.

The EDPs chosen to quantify the structural response are the Roof Drift Ratio ($RDR$) and the Maximum Interstory Drift Ratio ($MIDR$), where the height of each story is set to 12 ft. All results are presented in percentage points. Similar formulas and plots for base shear $V_b$ were developed as part of the study, and are available upon request.

Fig.3 and Fig.4 show the distance attenuation of $MIDR$ and $RDR$ respectively for an 8-story structure with a first mode period $T_1$ of 1.6 sec located on sites with $V_{S30}$ equal to 360 m/sec and 1070 m/sec. The median results calculated using the linearized equations are represented with a solid line while the dots are the medians of the Monte Carlo simulations. The different colors represent four values of magnitude: $M = 4.5$ is in red, $M = 5.5$ is in blue, $M = 6.5$ is in black, and $M = 7.5$ is in green. Consistently with the characteristic fault geometry assumption, the results are truncated for values of $R_{RRP}$ lower than the depth to the top of the fault $Z_{TOR}$.

![Fig. 3 – Distribution of median MIDR with distance $R_{RRP}$ for scenarios with $V_{S30} = 360$ m/sec and 1070 m/sec for an 8-story building with $T_1 = 1.6$ sec](image)

![Fig. 4 – Distribution of median RDR with distance $R_{RRP}$ for scenarios with $V_{S30} = 360$ m/sec and 1070 m/sec for an 8-story building with $T_1 = 1.6$ sec](image)
The figures show the distance attenuation and the magnitude scaling of the results. Moreover, as expected, the structural response is generally slightly lower on stiff soil with high $V_{S30}$ than on softer soil. The figures also show that the Monte Carlo simulations consistently fall below the solid lines, suggesting that the linearized equations offer a conservative estimate of the response. Furthermore, the small distance between the two guarantees that there is no bias when using the simplified approach.

Fig. 5 offers a different perspective on the same results. The MIDR and RDR are here plotted against magnitude for four values of $R_{rup}$: $R_{rup} = 5$ km is in red, $R_{rup} = 10$ km is in blue, $R_{rup} = 30$ km is in black, and $R_{rup} = 100$ km is in green. The structure is an 8-story building with a first mode period of 1.6 sec located on a site with $V_{S30} = 360$ m/sec. As in the previous figures, the linearized results are represented by solid lines, with interpolation of the results between magnitude values, and individual dots are used for the Monte Carlo simulations. It is easy to appreciate that the previous conclusions still apply, and the linearized equations appear to be a conservative but accurate estimate of the results.

Fig. 6 presents the distribution of $\sigma_{lnMIDR}$ for a 12-story structure with $T_1 = 1.2$ sec located on soft (left) and stiff (right) soil. The results show a low dependence of the dispersion on distance and shear wave velocity, especially for more flexible structures. Magnitude appears to be the only important parameter, with slightly higher dispersion in scenarios with magnitudes in the lower range. These results are consistent with the mathematical formulation of the underlying GMM [11].

![Fig. 5](image1.png)  ![Fig. 6](image2.png)

**Fig. 5** – Distribution of median MIDR and RDR with magnitude for scenarios with $V_{S30} = 360$ m/sec for an 8-story building with $T_1 = 1.6$ sec

**Fig. 6** – Distribution of $\sigma_{lnMIDR}$ with magnitude for scenarios with $V_{S30} = 255$ m/sec and 1070 m/sec for a 12-story building with $T_1 = 1.2$ sec
5. Conclusions

This paper introduces a simplified mathematical formulation to determine the EDP distribution based on IM statistics. Paired with any existing ground motion model, these linearized equations can be included in any hazard software to provide a direct calculation of the EDP hazard curve. This paper reported the final equations for floor displacement $\text{Dis}_{pk}$ and interstory drift ratio $\text{IDR}_k$, but the reader can find additional information on the upcoming companion paper by the same authors.

A set of generic moment-resisting frames developed by Zareian (2009) [10] was used to determine the distribution of roof drift ratio ($\text{RDR}$) and maximum interstory drift ratio ($\text{MIDR}$), using Campbell-Bozorgnia 2014 [11] as the underlying ground motion model.

The distribution of the structural response with the ground motion parameters was validated against the results of Response Spectrum Analysis (RSA) using 1,000,000 Monte Carlo simulations. The results show that the linearized equations provide a conservative and accurate estimate of the structural response.

6. Acknowledgements

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7. References
