New Method to Obtain Ground Velocity and Displacement from Strong Motion Accelerogram Based on Zero Initial Condition

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Abstract:

The displacement and velocity at certain points of a structure is very important in the field of earthquake engineering, structural vibration control and structural health monitoring. However, acceleration is much easily measured than displacement and velocity in experiments. It is, therefore, necessary to integrate acceleration to get velocity and displacement. However, the absence of initial conditions often renders the integration process affected by significant “drift” errors. To attack this problem, a novel integration method is proposed, by which the zero points of velocity and displacement during steady state vibration are sought for and determined by the rule of the conservation of energy. Furthermore, the “drift” caused by the error of sampling spacing and stochastic vibration is removed by detrending the integrated displacement. The proposed method is applied to two recent seismic motion records. The integrated velocity and displacement are compared with known ones, and it shows that the maximum relative error at extremes are less than 2%, which powerfully demonstrates that the proposed integration method is accurate and practical.

Keywords: acceleration integration; reconstruction of velocity; reconstruction of displacement; time-domain integration; zero initial integration

1.Introduction

In recent years, structure collapse and damage caused by natural disasters such as earthquakes and typhoons have made a terrible loss of human life and property. Therefore, the civil engineering structure security problems have drawn greater attention, and structural health monitoring and vibration control have been hot research subjects in the field of civil engineering.

Accurate velocity and displacement of structures during vibration will be beneficial to damage identification and vibration control. In addition, the ground velocity and displacement during earthquake and residual displacement at the end of earthquake are also very important in the field of earthquake engineering. At present, direct measurement methods of structure vibration displacement mainly include precision level, draping method and hydrostatic leveling, etc. However, these methods can be frequently limited by reference point and engineered cost. In recent years, GPS technology is developing rapidly, many scholars apply GPS technology on real-time displacement measurement of structures, but this method is usually affected by environmental factors.

The indirect measurement methods have lower cost and less environmental affection in contrast with direct ones. Yong Xia ,etc[6] treat super-high building as a cantilever beam, they solve displacement and inclination by real-time strain data. By comparison with GPS and inclinometer monitoring data, this method has reliable precision. Yang Xueshan, etc[7] use the QY inclinometer to measure inclination of long-span bridges under dynamic load, and fit the deflection curve according to the measured data. Through laboratory experiments and field tests, it is proved that bridge deflection calculated from QY inclinometer data can meet the engineering requirements precision. Compared with the above two methods of indirect measurement, acceleration sensor measurement method has advantages of widespread application and high technology maturation. This method needs to pre-load acceleration sensors on measured structure to get real-time vibration acceleration data, and velocity and displacement can be solved by integral operation. However, because of the influence of low frequency noise and the absence of initial value, the small errors in velocity and acceleration will accumulate constantly during integration process, and it usually make a obvious “drift”[9] on integrated displacement. Many experts and scholars have done great work on integration method and denoising processing, and achieved abundant research results. David M. Boore, etc[10] pointed out that earthquake acclerograms inevitably contains low frequency and high frequency noise. The high frequency noise need to be filtered by low-pass filter, and the low frequency noise require baseline correction processing. Boore modified earthquake records piecewise by baseline processing, and this method has reliable precision compared with GPS monitoring data. But there is no standard and effective method to identify the section time parameters. Zheng Shuiming, etc[11] applied Boore’s method on Wenchuan Earthquake acclerogram. The low frequency in earthquake records is eliminated by piecewise baseline correction, and the records baseline is roughly back to zero. Chen Weizhen, etc[12] detrend the integration result with noise in order to improve the precision. Li Jianbo, etc[13] detrend the earthquake record with quadratic polynomial which is the fitting curve of the original seismic record to remove the low frequency noise in the case that the spectrum changes little. The above methods still can not accurately determine the boundary conditions of velocity and displacement, therefore the determination of integration initial values has become an important problem which cannot be ignored in this research field.

This paper proposed a novel integration method of measured acceleration to velocity and displacement which is based on the relationship between the maximum and zero of acceleration, velocity and displacement. This method will help to solve the problem of integration initial value absence. Moreover, the "drift" caused by the error of sampling spacing and stochastic vibration is further removed by detrending the integrated displacement, and modify the calculated velocity by subtracting slope of the displacement's linear trend. It is proved that the proposed method has reliable calculation precision for structural response caused by harmonic excitation and earthquake. At last, this method is applied on two typical seismic motion records. The integrated velocity and displacement are compared with given ones, and the comparison results show that the accuracy at extremes can meet with the requirement of practical application.

2.Theoretical Derivation

2.1 The integration method based on zero initial value

According to the energy conservation principle, the structural kinetic energy and potential energy convert continuously in the process of vibration. During the steady state vibration, total energy of structure nearly remains constant. When the structure kinetic energy is zero, the vibration velocity is zero, and the potential energy comes to the maximum. At the same time, the elastic force, the acceleration and displacement come to the maximum, too. When the potential energy is zero, the structure displacement and acceleration are zero. At the same time, the kinetic energy come to the maximum, and the velocity is at the peak.

Based on this principle, if the acceleration a_i is the maximum of steady state vibration at a fixed point of structure, then the corresponding velocity at the same time is zero, namely $v_i=0$. The velocity of next moment can be calculated according to trapezoid rule:

$$v_{i+1} = v_i + \frac{a_i + a_{i+1}}{2} = \frac{a_i + a_{i+1}}{2} \tag{1}$$

$$v_{i+2} = \frac{a_i + 2a_{i+1} + a_{i+2}}{2}\Delta t \tag{2}$$

$$v_{i+3} = \frac{a_i + 2a_{i+1} + 2a_{i+2} + a_{i+3}}{2}\Delta t \tag{3}$$

$$\cdots\cdots$$

$$v_n = \frac{a_i + 2\sum_{j=i+1}^{n-1} a_j + a_n}{2}\Delta t \tag{4}$$

$v_i=0$ can also be treated as the boundary condition to calculate the velocity before i

$$v_i = v_{i-1} + \frac{a_{i-1} + a_i}{2}\Delta t \tag{5}$$

$$v_{i-1} = v_i - \frac{a_{i-1} + a_i}{2}\Delta t = -\frac{a_{i-1} + a_i}{2}\Delta t \tag{6}$$

$$v_{i-2} = v_{i-1} - \frac{a_{i-2} + a_{i-1}}{2}\Delta t = -\frac{a_i + 2a_{i-2} + a_{i-1}}{2}\Delta t \tag{7}$$

… …

$$v_1 = -\frac{a_i + 2\sum_{j=2}^{i-2} a_j + a_1}{2}\Delta t \tag{8}$$

Similarly for the displacement, if v_i is the maximum of steady state vibration, then the corresponding displacement is zero, namely $u_i=0$, and the displacement can be calculated as follows,

$$u_n = \frac{v_i + 2\sum_{j=i+1}^{n-1} v_j + v_n}{2}\Delta t \quad (n > i) \tag{9}$$

$$u_n = -\frac{v_i + 2\sum_{j=n}^{i-2} v_j + v_n}{2}\Delta t \quad (n < i) \tag{10}$$

2.2 Identification of steady state vibration

Searching velocity and displacement of zero accurately is the key to application of the proposed method, and the first step is to identify structural steady state vibration. Structural vibration response includes free vibration which is dependent on initial conditions, concomitant free vibration and forced vibration[14]. Eq (11) is the equation of single degree of freedom system vibration under action of harmonic force.

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = \frac{P}{m}\sin\theta t \tag{11}$$

The general solution of Eq(11) is,

$$\begin{aligned} x(t) &= e^{-\zeta\omega t}\left(\frac{\dot{x}_0 + \zeta\omega x_0}{\omega_\zeta}\sin\omega_\zeta t + x_0\cos\omega_\zeta t\right) \\ &+ Ae^{-\zeta\omega t}\left(\sin\phi\cos\omega_\zeta t + \frac{\varepsilon\sin\phi - \theta\cos\phi}{\omega_\zeta}\sin\omega_\zeta t\right) + A\sin(\theta t - \phi) \end{aligned} \tag{12}$$

$$\omega_\zeta = \omega\sqrt{1-\zeta^2} \tag{13}$$

$$\phi = \arctan\frac{2\zeta\omega\theta}{\omega^2 - \theta^2} \tag{14}$$

Eq (12) shows that when the damped item exp(-ζωt) is less than 10^{-n} (n depends on length of signal), the first two items are close to zero. The third item is forced vibration, and it can be considered that the system's energy remains constant and the system is in steady state vibration phase. Then the above integration method can be applied.

In addition, for other forms of disturbing forces this method can also be used to identify steady state vibration. Periodic disturbing force can be transformed into a finite number of sine functions with different frequencies via Fourier series; random vibration can be approximated by a finite number of sine functions via discrete Fourier transform.

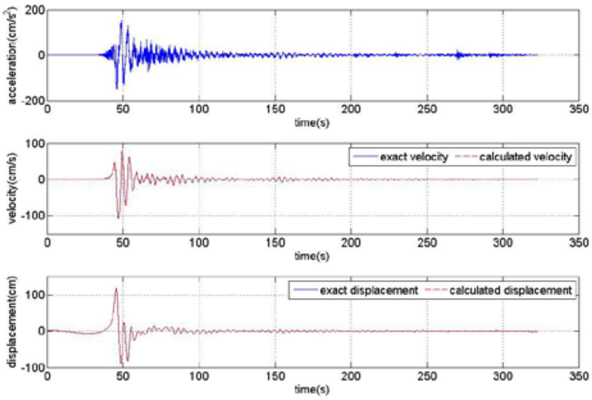

Fig. 1 – Calculation results of east-west(N016.HNE.NC.01)

Table 1 – Error analysis

	Velocity (cm/s)	Displacement (cm)
Accurate Value	-107.3200	116.9500
Calculated Value	-107.3170	116.9771
Relative error at extremes（%）	0.0028	0.0232

2.3 Correction of zero errors

There is always some errors in identification of zero points affected by sample interval and random vibration. For example, the v_i is determined as zero based on above method, but the true value of v_i is v_0. The calculated velocity always differs by v_0 from the true velocity because of properties of trapezoid rule.

And for the calculated displacement,

$$u = \int_0^t (v_t + v_0)dt = u_t + v_0 t \quad (15)$$

Eq (15) shows that the velocity error causes a linear trend in displacement calculation, it is necessary to detrend[11] the calculated displacement. In addition, the slope of linear trend is velocity error v_0 according to Eq (15), and subtract it from calculated velocity to get accurate one. One other thing to note is that this method should be applied on signal whose frequency is more than 2Hz. Because the detrend correction can be regarded as a high-pass filter in frequency domain, and it can not guarantee accuracy at low frequencies.

3. Seismic wave reconstruction calculation

The ground velocity and displacement peaks during earthquake are very important to structure damage identification and design. In this section, the validity of the new method is demonstrated by triaxial velocities and displacements reconstruction of two recent typical earthquakes. Then the calculated data will be compared with the given to analyze precision.

3.1 2015 Nepal Earthquake

The April 2015 Nepal earthquake (also known as the Gorkha earthquake) killed over 8,000 people and injured more than 21,000. It occurred at 11:56 Nepal Standard Time on 25 April, with a magnitude of 7.8Mw or 8.1Ms. Its epicenter was east of Lamjung District, and its hypocenter was at a depth of approximately 8.2 km. It was the worst natural disaster to strike Nepal since the 1934 Nepal–Bihar.

The triaxial earthquake records and related information in this paper are downloaded from Center for Engineering Strong Motion Data (http://www.strongmotioncenter.org/), and the corresponding velocity and displacement time-history curves are also downloaded for comparison with calculated data and error analysis. The station of this earthquake record is Kanti Path, Kathmandu, Nepal.

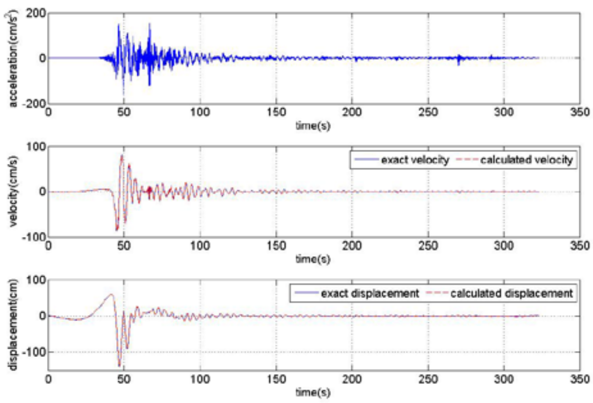

Fig. 2 – Calculation results of north-south(N016.HNN.NC.01)

Fig. 1 is east-west earthquake record and the values calculated by new integration method. Table 1 shows that relative error of velocity is less than 0.003%, and the error of displacement is less than 0.03 mm at the position of extreme compared with the given one.

Fig. 2 is north-south earthquake record and the calculated data. Table 2 is error analysis of extremes, the relative errors of velocity and displacement at peaks are both less than 1%.

Table 2 – Error analysis

	Velocity (cm/s)	Displacement (cm)
Accurate Value	-86.0480	-139.0400
Calculated Value	-86.0498	-138.7256
Relative error at extremes（%）	0.0021	0.2261

Fig. 3 is calculated values and given ones of vertical direction. Table 2 shows the error of displacement at extreme is less than 0.2mm, and relative error of velocity at extreme is about 0.0032%.

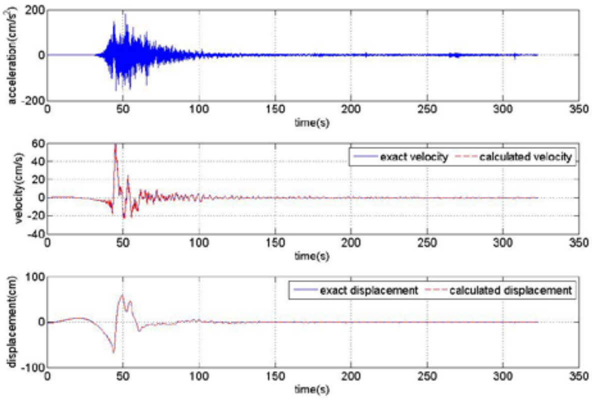

Fig. 3 – Calculation results of vertical direction(N016.HNZ.NC.01)

Table 3 – Error analysis

	Velocity (cm/s)	Displacement (cm)
Accurate Value	58.7930	-66.9430
Calculated Value	58.7949	-67.1410
Relative Error at extremes（%）	0.0032	0.2957

The triaxial time-history curves of the given and the calculated match well in Fig 1 to 3. And the method has reliable accuracy at extremes which are the most important to engineering application.

3.2 Ferndale Earthquake

Ferndale Earthquake occurred at 5:17 of 9 Mar 2014. Earthquake magnitude is 6.9. It was felt across northern Mexico and southern Canada. The earthquake epicenter coordinates are north latitude 40.8° and west longitude 125.0° . Data of this paper is from station Ferndale，Fire Station(ID:1023)，and the epicentral distance is 73.2 km.

Fig. 4 is 90° earthquake records and the calculated values of above method. The error analysis in Table 4 shows the relative errors of velocity and displacement at peaks are below 0.1%.

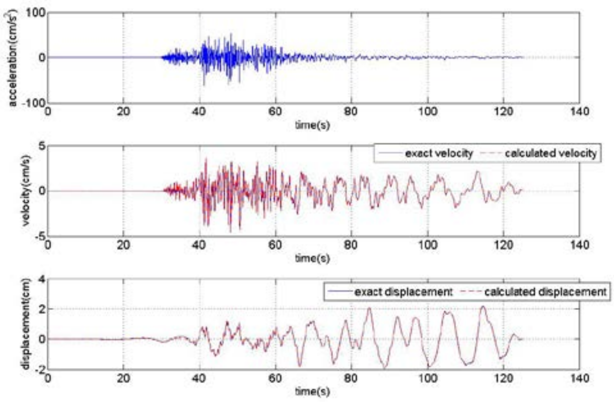

Fig. 4 – Calculation results of 90°(1023a)

Table 4 – Error analysis

	Velocity (cm/s)	Displacement (cm)
Accurate value	-4.5591	2.1980
Calculated value	-4.5590	2.2124
Relative error at extreme（%）	0.0030	0.6553

Fig. 5 is 0° Ferndale Earthquake records and the calculated values. The error analysis in Table 5 shows the relative errors of velocity at the peak is below 0.1%. The error of the maximum displacement is less than 0.2mm.

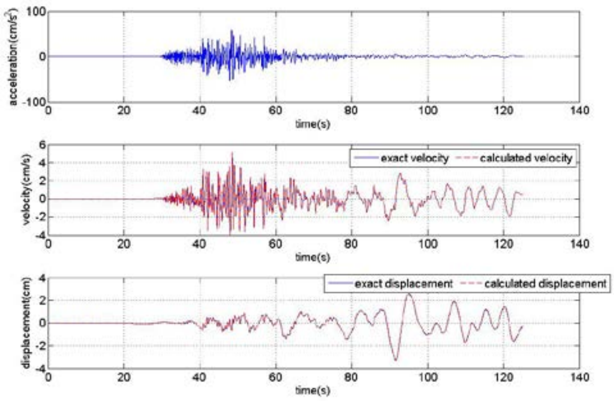

Fig. 4 – Calculation results of 0°(1023b)

Table 5 – Error analysis

	Velocity (cm/s)	Displacement (cm)
Accurate value	5.0750	-3.2991
Calculated value	5.0755	-3.2743
Relative error at extremes（%）	0.0089	0.7520

Fig. 6 is time-history curves comparison in vertical direction. Table 6 shows the relative error of velocity extreme is about 0.024%, and the difference at peak of the two displacement data is less than 0.3mm.

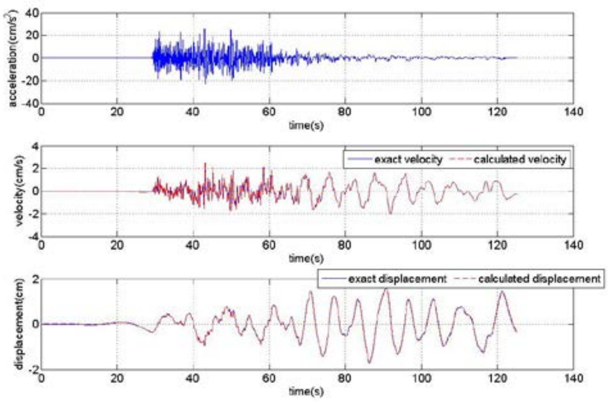

Fig. 7 – Calculation results of vertical direction(1023c)

Table 7 – Error analysis

	Velocity (cm/s)	Displacement (cm)
Accurate Value	2.4715	-1.7031
Calculated Value	2.4709	-1.7306
Relative Error at extremes（%）	0.0242	1.6153

In this section, the new integration method is applied on Ferndale Earthquake to calculate ground velocity and displacement. The error analysis indicates that this method has reliable precision at extreme points.

4. Conclusion and summary

In this paper, a new integration method which is based on zero initial condition is proposed, and the analysis results demonstrate that it can reconstruct the response of structure under simple harmonic and stochastic vibration from measured acceleration. This method has three obvious innovation points as follows:

(1)Zero points of velocity and displacement are searched in the phase of steady state vibration based on principle of conservation of energy, it effectively solves the problem of the integral boundary values' absence;

(2)The integrated displacement is detrended to remove the "drift" caused by the error of sampling spacing and stochastic vibration. And subtract the slope of the linear trend from velocity to improve the calculation accuracy;

(3)This method adopts trapezoidal rule with no accumulative error and high computational efficiency.

The proposed method is applied to calculate ground velocity and displacement from filtered earthquake acceleration records. The calculated data shows that the relative errors at extreme values are all below 2% which can be satisfied with engineering accuracy. These above numeric simulation tests have powerfully demonstrated that this new method is accurate and has a high value of practical application.

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