Robust Design of Tuned Mass Dampers for Passive Control of Structures under Earthquake Excitations

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Abstract

Design of Tuned Mass Dampers (TMD's) for passive control of structures subject to earthquake loading is often performed in a deterministic setting, neglecting uncertainties in loading and/or in system parameters. In this paper, it is shown that such designs are not robust w.r.t. the uncertainties present in both loading and system parameters. A methodology is introduced for the optimal design of such systems, yielding TMD configurations which are robust w.r.t. variabilities and uncertainties in loading and system parameters. A robustness measure is introduced in order to compare robust and non-robust solutions. The objective function is written in terms of time-variant system reliability. Monte Carlo simulation is employed as uncertainty quantification solver, and the SQP algorithm is used to solve the constrained optimization problem. A re-start procedure is adopted in order to ensure convergence to a global optimum. A ten-story building is employed as application example. It is shown that the robust formulation leads to structural responses which are less sensitive to variabilities and uncertainties in loading and system parameters.

Keywords: optimal design, robust optimization, optimization under uncertainties, TMD, seismic loading.

1. Introduction

The importance of reducing vibration amplitudes of structures under rare but severe loadings is well recognized. In the last decades, a fast increase in development and application of passive energy dissipation devices, such as viscoelastic dampers, viscous fluid dampers, metallic yield dampers, friction dampers and tuned mass dampers (TMDs) has been observed [1]. One of the main applications of such devices is to reduce the dynamic response of structures subjected to strong earthquake ground-motions.

To achieve the best structural performance, attention has been focused on determining the optimum parameters of TMDs in structures subjected to seismic excitation. Sadek et al. [2] presented a methodology to find optimum parameters of TMDs for single degree of freedom (SDOF) and multiple degree of freedom (MDOF) structures subjected to a number of earthquake excitations. Joshi and Jangid [3] carried out a study to determine the optimum parameters of MTMDs in SDOF structures subjected to a base excitation, which was modelled as a stationary white noise random process. Results showed that the optimally designed MTMD system is more effective than the single TMD. Hadi and Arfiadi [4] discussed the optimum design of single TMDs in MDOF structures subjected to earthquake excitation. The H2 norm was employed as the objective function and Genetic Algorithms were applied to find the best TMD parameters. Chen and Wu [5] presented a study focused on the optimal placement of MTMDs on structures subjected to seismic excitations. Because oscillators are placed one-by-one in sequence, their placement in a structure is selected from a restricted domain. Thus, the final results are indeed suboptimal. Hoang and Warnitchai [6] presented a method that uses a numerical optimizer following a gradient based non-linear programming algorithm to search for optimal parameters of MTMDs. The developed method was applied to optimize MTMDs for a SDOF structure subjected to wide-band excitation. Lee et al. [7] presented an optimal design theory for structures implemented with TMDs. The proposed optimal design theory feasibility was verified by using a SDOF structure with a single TMD, a five-DOF structure with two TMDs, and a ten-DOF structure with a single TMD. Leung et al. [8] employed the particle swarm optimization algorithm to obtain the optimum parameters including the optimum mass ratio, damper damping and tuning frequency of the TMD system attached to a viscously damped single-degree-of-freedom main system, which is subjected to non-stationary excitation. In the sequence, Leung et al. [9] employed again the particle swarm optimization algorithm to obtain the optimum parameters of a TMD system attached to a viscously damped SDOF main system under various combinations of different kinds of excitations. Lin et al. [10] applied a two-stage optimum design procedure for MTMD to reduce structural dynamic responses with the limitation of MTMD's stroke. Shaking table tests of a large-scale three-story building with and without the MTMD under earthquake excitations were conducted. Mohebbi et al. [11] proposed a procedure for designing optimal MTMDs to mitigate the seismic response of MDOF structures. Genetic algorithms were used for solving the optimization problem.

Despite the importance of the studies mentioned above, it is today widely acknowledged that deterministic optimization is not robust with respect to the uncertainties which affect the performance of engineering facilities. Deterministic Design Optimization (DDO) does not explicitly address the uncertainties inherently present in structural, especially environmental loads, material strengths, and load effect and member strength models. Reliability-based design optimization (RBDO) has emerged as an alternative to model the safety-under-uncertainty part of the problem. However, both deterministic and reliability-based design optimizations suffer from the same drawback: these methods allow optimum structures to be designed, but by considering conservative safety margins with respect to failure modes. Such methods are not appropriate for the optimal design of control structures such as TMD and MTMD. DDO is not appropriate because it is not robust with respect to the uncertainties. RBDO is not appropriate because control needs to be designed at the precise point (design point), and not within a conservative safety margin with respect to the design point. In fact, one can say that control is not control unless it is robust with respect to the uncertainties described above. Specifically, structural control should be made robust with respect to the uncertainties in the loading, mass, damping and stiffness, in the inherent variability of materials properties, imperfect modelling of loads, load effects, system response, and so on [12-19]. Thus, in the design and optimization of TMD passive control systems, it is of paramount importance to take uncertainties into account.

Within this context, this paper deals with the robust optimal design of TMD systems for structures subjected to seismic loading. In Section 2 the structural model is presented. Time-variant reliability analysis of oscillators is presented in Section 3. The robust optimization problem and a robustness measures are introduced in Section 4. Numerical examples are presented in Section 5, in order to illustrate the importance of taking into account uncertainties in the optimum design of TMD systems. For the sake of simplicity and to focus on the formulation of the robust design optimization problem, the numerical analysis section deals with one TMD only. It should be remarked, however, that the extension to MTMD is straightforward. The paper is finished with a Summary in Section 6.

2. Structural model

The differential equation that governs the motion of multi-degree-of-freedom systems, with *one* TMD located on the top floor (Figure 1) and subjected to earthquake ground motions may be written as:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=-\mathbf{M}\mathbf{B}\ddot{\vec{y}}(t) \tag{1}$$

in which **M**, **C** and **K** represent the $(n+1)\times(n+1)$ structural mass, inherent damping and stiffness matrices, respectively, and n is the number of degrees of freedom of the model. $\mathbf{x}(t)$ is the $(n+1)$-dimensional relative displacement vector with respect to the base and a dot over a symbol indicates differentiation with respect to time. **B** is a $(n+1)\times d$ matrix of ground motion influence coefficients, i.e., this matrix contains the cosine directors of the angles formed between the base motion and the associated displacement direction with the considered degree of freedom. d is the number of considered ground motions (directions). $\ddot{\vec{y}}(t)$ is a d-dimensional vector representing the seismic excitation, *i.e.*, the ground motion or base acceleration. For design purposes, assume that all relevant structural parameters (e.g. nominal values of mass, damping and stiffness) can be assembled in a parameter vector **p**. In the TMD design optimization process, the design variables are considered to be its stiffness and damping, which are then grouped in the design vector **d** (further details are given in Section 4).

3. Time-variant reliability of oscillators

The classical reliability problem for an oscillator is depicted in Figure 2. During an excitation event of specified duration t_E, a specific, scalar displacement response of the oscillator $x(t)$ (e.g., relative displacement between floors, displacement of top floor, etc.), should not exceed $\pm b$, where b is a given barrier which corresponds to a critical response level. In this paper, the displacement of top floor is chosen as the relevant displacement response. The critical response may represent permanent damage like cracking of concrete, or it may represent ultimate failure due to loss-of-equilibrium, and is further addressed latter in this paper. For a given barrier level b and excitation duration t_E, the failure probability is given by the classical Poisson model [20]:

$$P_f(b,t_E)=1-\exp\left(-2\int_0^{t_E} v_x^{+}(b,t)dt\right) \tag{2}$$

where v_x^{+} is the up-crossing rate. For stationary excitation and for a time-invariant barrier $b(t)=b$, the crossing rate is also constant and $\int_0^{t_E} v_x^{+}(b,t)dt = v_x^{+}(b)t_E$. For a linear system excited by a Gaussian process, the response is Gaussian and the crossing rate can be evaluated as:

$$v_x^{+}(b)=\frac{\sigma_{\dot{x}}}{\sigma_x}\frac{1}{2\pi}\exp\left(-\frac{b}{2\sigma_x^{2}}\right) \tag{3}$$

where σ_x and $\sigma_{\dot{x}}$ are the standard deviation of the displacement and of the velocity response, respectively.

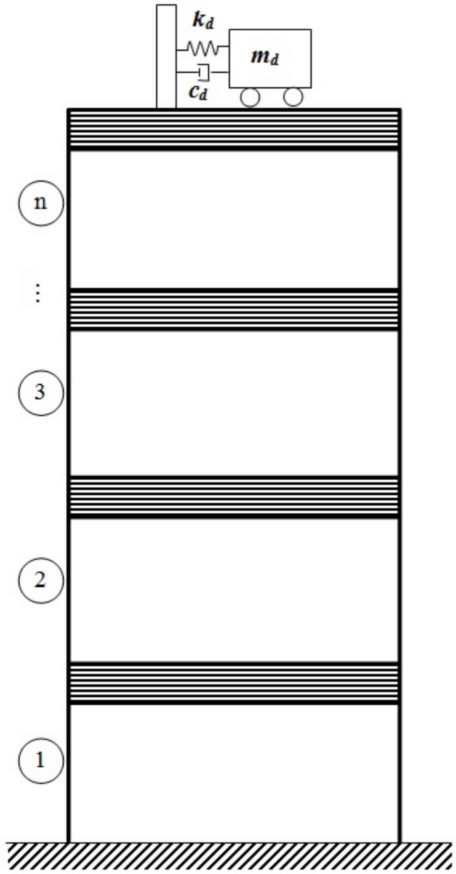

Fig. 1 – n-DOF building with single TMD on the top floor.

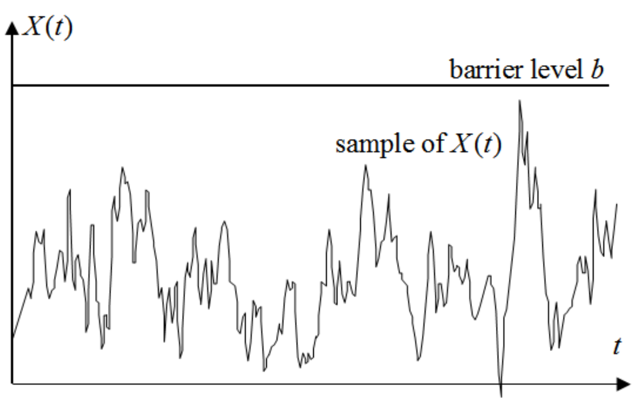

Fig. 2 – Classical time-variant reliability problem.

Many types of environmental structural loading can be described by the arrival of an unknown number of events (winds, storms, sea waves, earthquakes. By modeling the arrival of events as a Poisson process, and for a design life t_D, the failure probability becomes:

$$P_f\left(b,t_D\right)=\sum_{i=1}^{\infty}P_f\left(b,t_E\mid i\right)p_i\left(t_D\right) \tag{4}$$

where $P_f(b, t_E \mid i)$ is the conditional probability of failure, given the occurrence of exactly i events during the design life, and $p_i(t_D)$ is the probability of having exactly *i* events, given by the Poisson distribution:

$$p_i(t_D) = \frac{(\nu t_D)^i \exp(-\nu t_D)}{i!} \tag{5}$$

In Eq. (5), ν is the arrival rate of events. It can be shown that the conditional failure probability, assuming independence, is given by [20]:

$$P_f(b, t_E \mid i) = 1 - \left(1 - P_f(b, t_E)\right)^i \tag{6}$$

For convenience, failure probabilities can be converted into reliability index (β) by means of the classical First Order Reliability result:

$$\beta = -\Phi^{-1}\left(1 - P_f(b, t_D)\right) \tag{7}$$

where Φ is the standard Gaussian cumulative distribution function. Clearly, the reliability index is a function of the vector of design parameters as well as the vector of structural parameters **p**. Hence, once can write $\beta = \beta(\mathbf{d}, \mathbf{p})$.

4. Robust optimization problem

This paper aims at pursuing the TMD design optimization of a given structure subjected to seismic loading, taking into account the uncertainties present in the structural system and its loading. That is, an approach for the robust design optimization of TMD is presented. In order to take into account these uncertainties, some parameters of the structure are modeled as random variables and the seismic loading is modeled as a random process.

One classical objective function in the deterministic design of TMD is the minimization of some mean square response of the structure. However, it has been reported in the literature that such an objective function does not necessarily correspond to the optimal design in terms of reliability [21, 22]. Hence, the probability of failure of the structure is chosen as the objective function to be minimized in this optimization problem. Such an objective function has been adopted, for instance, in the works by Taflanidis et al. [15] and Taflanidis [17], to name just a few. This probability of failure is given by Eq. (4) and the design variables are the stiffness (k_d) and damping constant (c_d) of the TMD. These two design variables are grouped into the vector $\mathbf{d} = [\mathbf{k}_d \ \mathbf{c}_d]$ for notation convenience. As commented in the previous section, the reliability index is chosen as the reliability measure. Since the higher its value is, the lower the probability of failure is, the objective function is given by $-\beta(\mathbf{d},\mathbf{p})$. Thus, the optimization problem can be posed as:

$$\begin{aligned} \text{Find: } & \mathbf{d} \\ \text{which minimizes: } & -\beta(\mathbf{d}, \mathbf{p}) \\ \text{subject to: } & d_i^{min} \le d_i \le d_i^{max}, \quad j = 1, 2 \end{aligned} \tag{8}$$

This formulation leads to an unconstrained optimization problem, where the feasible design set is limited only by the bounds of the design variables. In this context, several optimization algorithms can be used to solve this problem, e.g. gradient-based algorithm, Nelder-Mead algorithm, among others. In the numerical analysis section, a sequential quadratic programming (SQP) algorithm, coupled with a restart procedure, is employed in order to ensure the convergence to a global optimum. This coupling has been tested in several papers and has successfully solved complex optimization problems [23-25]. For the interested reader, a detailed explanation of the optimization algorithm is given in these references.

Optimal TMD parameters found by the above formulation are intrinsically more robust than TMD parameters found by any equivalent deterministic formulation. However, it is important to demonstrate that indeed results obtained by the robust optimization are less sensitive to changes in the estimated structural parameters **p**: mass, stiffness and damping of the structural elements and of the TMD. Hence, a perturbation is introduced in vector $\mathbf{p}_{\Delta}$, such as to verify what would be the actual reliability index of the structure if parameter values where not the nominal values used in designing the TMD. Then, the perturbed reliability index β_{Δ} can be compared with the original, and a measure of robustness is obtained: the smaller the relative difference, the more robust is the design:

$$\text{robustness}: \ \frac{\beta}{\left|\beta-\beta_{\Delta}\right|} \tag{9}$$

The robustness measure proposed in Eq. (9) in employed in the next section to demonstrate that results of the optimal design of TMD considering uncertainties in structural and TMD parameters are more robust than optimal TMD designs obtained by considering only uncertain loading.

5. Numerical examples

In this section, an example optimization problem is solved: robust design of TMD of a structure subjected to seismic excitation.

5.1 Problem definition

A classical example, previously studied by several researchers [4, 7, 11], was selected from the literature. The structure is a ten-story shear frame as illustrated in Figure 1. In the top floor of this structure a TMD is to be installed. The mass of the TMD (m_d) is chosen a priori to be 3% of the total mass of the structure. The parameters to be determined by the designer are mean values of the stiffness (k_d) and damping coefficient (c_d) of such a TMD. Two variants of the example are solved: a) only the seismic excitation is considered random; b) the seismic excitation is random, but uncertainties are also considered in the mass, stiffness and damping coefficient of the structure and the TMD. These are then modeled as random variables whose characteristics are presented in Table 1. Table 1 also shows the perturbations considered in order to evaluate the robustness of the solutions, following Eq. (9). These perturbations are applied to the random variables of the structure. The critical combination of perturbations is given by a reduction of stiffness and damping, and an increase of the mass (this explains the signs on the perturbations presented in Table 1).

To determine the optimum TMD parameters, a stationary earthquake excitation is assumed, which can be modeled as a white noise signal with constant spectral density, S_0, filtered through the Kanai–Tajimi spectrum. The PSD function is given by:

$$s(\omega)=S_0\left[\frac{\omega_g^4+4\omega_g^2\xi_g^2\omega^2}{(\omega^2-\omega_g^2)^2+4\omega_g^2\xi_g^2\omega^2}\right], \quad S_0=\frac{0.03\xi_g}{\pi\omega_g(4\xi_g^2+1)} \tag{10}$$

Table 1 – Statistical information about the random variables of the structure.

		Mean Value	C.o.V. [%]	Perturbations considered in robustness analysis
Stiffness (k_i) [N/m]	Story (i)	650.0×10^6	15.0	-5%
	TMD	k_d	15.0	-
Mass (m_i) [kg]	Story (i)	360.0×10^3	10.0	+3%
	TMD	108.0×10^3	10.0	-
Damping (c_i) [Ns/m]	Story (i)	620×10^6	25.0	-8%
	TMD	c_d	25.0	-

where ξ_g and ω_g are the ground damping and frequency, respectively. In case b) of the analysis, filter parameters are also considered as random variables. Their mean values were adopted as $\xi_g = 0.6$, $\omega_g = 37.3$ rad/s [11] and peak ground acceleration (PGA) = 0.20g. These random variables are considered independent, Gaussian and their coefficients of variation were set to 20%. Similar procedures were adopted in Taflanidis et al. [15]. It should be remarked that for more realistic situations a non-stationary excitation should be employed. This procedure is detailed, for instance, in [14].

The upper bound and the lower bound value of the stiffness (k_d) and damping coefficient (c_d) of the TMD are 0 - 4000 kN/m and 0 – 1000kNs/m, respectively. In all cases, the Newmark method ($\gamma = 1/2$ and $\beta = 1/4$) was employed to solve the dynamic problem. The time step adopted was $\Delta t = 0.02$s. The mechanical model was calibrated and it is in accordance with the results provided in the literature.

5.2 Convergence of σ_x and $\sigma_{\dot{x}}$

Since MCS is employed to evaluate the standard deviations σ_x and $\sigma_{\dot{x}}$ required in Eq. (3) to approximate the probability of failure of the structure, a proper sample size needs to be selected. Thus, a study of the convergence of these standard deviations is shown in this section. Results are shown in Figure 3 and 4. One may see from these figures that around 250 samples are required to stabilize the convergence curves. Thus, a sample size of 250 is employed herein to evaluate σ_x and $\sigma_{\dot{x}}$.

5.3 Minimization of the probability of failure

The optimization problem presented in Sections 4 and 5.1 is solved in this section. A SQP algorithm from the MATLAB optimization toolbox, coupled with a restart procedure, is employed for this purpose. Three variants of each analysis are presented, by varying parameter b in Eq. (2): (i) $b= h/300$, (i) $b=h/400$ and (iii) $b=h/500$, where h is the total height of the shear frame. Results of the analyses considering only excitation uncertainty are shown in Table 2. Results of the robust optimal design, i.e. considering uncertainty in excitation and structural parameters, are presented in Table 3. It should be remarked that the reliability indexes presented in these tables were evaluated considering uncertainty in the excitation and in the structural parameters, in order to allow for comparisons between the different analyses. For instance, optimal values of k_d and c_d in Table 2 were obtained solving the optimization problem given by Eq.(8) and considering uncertainty in the excitation only; whereas reliability indexes shown in this table were evaluated also considering uncertainties in structural parameters in order to allow for direct comparison of the reliability indexes.

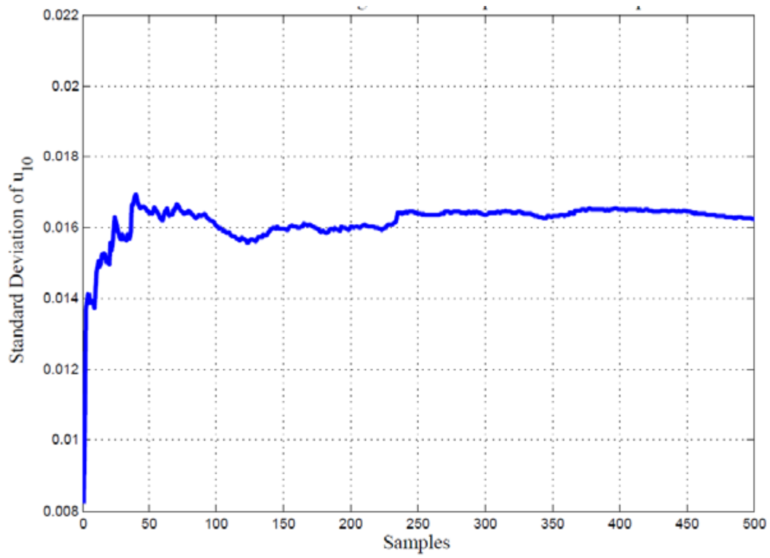

Fig. 3 – Convergence of the standard deviation of top floor displacement (σ_x).

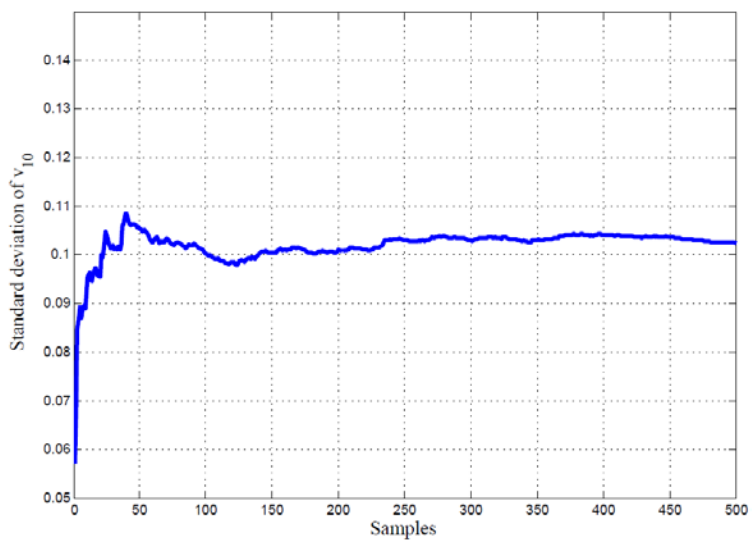

Fig. 4 – Convergence of the standard deviation of top top floor velocity ($\sigma_{\dot{x}}$).

In Tables 2 and 3 one observes that reliability index of uncontrolled structures ($\beta_{\text{uncontroled}}$) is significantly smaller than the reliability index of the TMD-controlled structures (β). This result shows the effectiveness of the TMD in reducing the probabilities of failure of the frame structures under seismic loading.

The reliability indexes of the TMD controlled structures (β), obtained by considering uncertainty in the excitation only (Table 2), are smaller than the β's obtained by the robust optimization (Table 3). This result is a first indication that the optimization discussed herein, considering uncertainties in structural parameters, leads to more robust control by the TMD. To further demonstrate this result, the robustness measure introduced in Eq. (9), and evaluated for the perturbations presented in Table 1, is also shown in Tables 2 and 3. One can see that the robust design discussed herein, considering uncertainties in structural parameters, leads to larger values of the robustness measure. Hence, structural control obtained by the robust optimal design of TMD becomes less sensitive to fluctuations or uncertainties in structural and TMD parameters.

Table 2 – Results of the TMD optimization considering only excitation uncertainty.

Case	k_d (MN/m)	c_d (MNs/m)	$\beta_{uncontroled}$	β	β_Δ	robustness
(i) $b = h/300$	3.69	0.146	2.23	5.23	4.51	7.26
(ii) $b = h/400$	3.69	0.146	0.48	2.95	2.26	4.28
(iii) $b = h/500$	3.69	0.147	*fail*	1.09	0.25	1.30

Table 3 – Results of the robust TMD optimization.

Case	k_d (MN/m)	c_d (MNs/m)	$\beta_{uncontroled}$	β	β_Δ	robustness
(i) $b = h/300$	3.29	0.161	2.23	5.33	4.72	8.74
(ii) $b = h/400$	3.29	0.161	0.48	3.05	2.46	5.17
(iii) $b = h/500$	3.29	0.161	*fail*	1.20	0.50	1.71

The optimization algorithm converged to the same optimum design independently of the starting point chosen by the restart procedure. An approximation of the objective function of case (iii), for the more general case considering uncertain load and structural parameters, is given in Figure 5. One can see from this figure that the objective function is fairly smooth and without local minima.

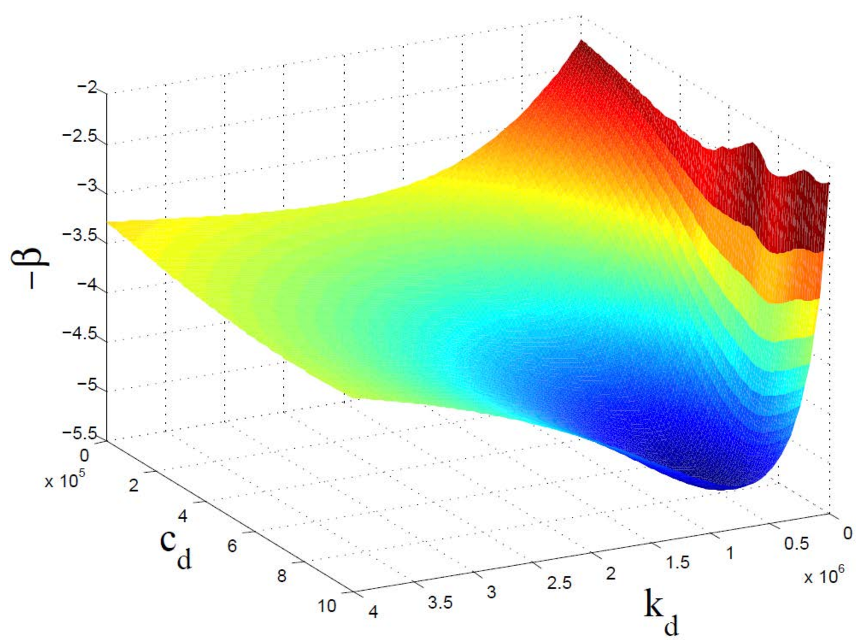

Fig. 5 – Objective function in the design space for the robust optimal design of a TMD (case (iii)).

6. Concluding remarks

This paper dealt with the robust optimum design of TMD systems for structures subject to seismic loading. First, the basic concepts of uncertainty quantification were presented focusing on the evaluation of the probability of failure of a given structure subject to seismic loading. It was detailed that the computation of the standard deviation of the stochastic process that define the displacement and velocity of a given degree of freedom of the structure and the definition of a threshold level for it were necessary to evaluate this probability of failure. Here, the MCS was employed to obtain these standard deviations, however, it may be remarked that other methods may be employed. Then, the robust design of the TMD was posed having as the objective function the minimization of the structural probability of failure. The design variables were chosen to be the stiffness and the damping of the TMD. It led to an optimization problem in which the only constraints were the bounds of the design variables. A SQP based algorithm, coupled with a restart procedure, was suggested as the optimizer to ensure the convergence to a global optimum. Then, a numerical example was presented to illustrate the importance of taking into account uncertainties in the optimum design of TMD systems for structures subject to seismic loading. This was achieved by introducing perturbations, which create a distance between parameter values considered in the design and parameter values "observed" in the actual structure. From the results of the numerical analysis section, it was shown that the structural control obtained by the robust optimal design of TMD becomes less sensitive to fluctuations or uncertainties in structural parameters.

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