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SHIELDING BUILDINGS FROM SURFACE WAVES WITH "SEISMIC METASTRUCTURES"

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Abstract

Phononic crystals and metamaterials constitute a broad class of artificially engineered materials able to manipulate the propagation of acoustic/elastic waves at different lengths scale. Filtering and directivity properties of these materials can be controlled and tuned by designing their fundamental building blocks, usually referred as unit cells. Phononic crystals exploit periodicity to induce total wave reflection at selected frequencies. The obtained filtering properties emerge at wavelengths comparable to the periodicity. On the other hand, phononic metamaterials exploit the coupling between propagating waves and local resonators in the fundamental unit cells. As a result, propagation of waves with frequencies around the resonance is inhibited.

More recently the use of phononic crystals and metamaterials has been envisioned for large-scale applications in the field of civil engineering as a mean to shield buildings and infrastructures from natural or man-induced seismic waves. Among these, a solution based on metamaterial concepts and referred as "seismic metastructure" has been recently proposed by some of the authors. The proposed system is realized by an array of buried heavy-cylindrical steel units encased in cylindrical concrete pipes that constitute the metamaterial resonant units. The proper design and arrangement of these resonant units allows filtering longitudinal and shear waves in the typical frequency range of seismic waves (1-10Hz). In fact, analytical/numerical models as well as experimental evidences on a scaled setup proved the shielding capabilities of the proposed seismic isolation system with respect to bulk waves.

Building on the initial results on bulk waves, here we design "seismic metastructure" able to mitigate Rayleigh surface waves. This is of special interest as they are considered the most harmful seismic threat. An analytical model to analyze the propagation of seismic surface waves through a soil engineered with seismic metastructures is developed. The analytical model allows predicting the frequency range within seismic Rayleigh waves are inhibited. A full 3D finite element model is used to numerically validate the prediction of the analytical model. Experimental evidences of the filtering properties of the designed metastructures are obtained on a scaled experimental setup. The excellent agreement of analytical/numerical and experimental results demonstrates the potential of this novel class of seismic isolation devices.

Keywords: Seismic isolation, Seismic metamaterial, Bulk waves, Rayleigh waves, wave steering

1. Introduction

Achieving control on the propagation of acoustic and elastic waves is desirable in different engineering applications, ranging from thermoelectric vibration [1, 2] to sound absorption [3]. A novel approach in this field are phononic crystals and metamaterials; phononic crystals exploit Bragg scattering effects induced by their structural periodicity to create band gaps, i.e. range of frequencies where wave propagation is inhibited in specific or in all spatial directions [4]. These filtering properties arise at wavelengths comparable to the period length of the crystals. In contrast to that, local resonant units are inserted in metamaterials to couple with incident waves and to impede wave propagation at frequencies near resonances. Research on phononic crystals and metamaterials shows the possibility of designing materials for ultrasonic applications, acoustic imaging [5] and acoustic cloaking devices [6, 7]. More recently, similar concepts have been utilized for seismic applications as a mean to protect civil infrastructures from seismic excitation [8-11]. In particular, a first experimental realization of seismic phononic crystals was realized by Brûlé et al. [8] by drilling periodic pattern of meter scale cylindrical holes in sedimentary soil. The tests proved the possibility of reflecting elastic energy by exploiting Bragg scattering effect, achieving large attenuation at a frequency range around 50 Hz. However, this approach would require fairly large structures to function at the harmful seismic spectrum components (1-10 Hz) for which wavelengths are in the order of tens to hundreds meters. Similar periodic structures have been numerically studied to realize composite soil lenses able to reroute seismic waves around buildings [12].

On the contrary, metamaterials do not derive their properties from structural periodicity, and their characteristic dimension can be well below the wavelengths of interest. This property makes metamaterials appealing for civil engineering applications, and in particular for seismic isolation purposes. Sub-wavelength shielding structures based on metamaterial concepts have been proposed to isolate building foundations from incoming seismic bulk waves [13, 14]. In particular, some of the authors of this paper recently proposed the use of arrays of buried resonators to attenuate seismic longitudinal and transversal waves in the low frequency range (1-10 Hz) [14]. However, bulk waves are not the only harmful waves, as the most of the elastic energy released in a seismic event travel along the soil surface in the form of Rayleigh waves. Indeed the proposed array of buried resonators can be equally used to attenuate Rayleigh waves ground shaking with proper design adjustments. The resonators interact with surface waves to redirect their energy to the bulk, away from the surface. Such effects between Rayleigh waves and surface resonators have been recently observed experimentally [15] for surface waves interacting with a forest, where single trees act as surface resonators in a frequency range around 40 Hz. Here we show how engineered resonant structures, originally designed to attenuate longitudinal and transversal seismic waves, interact with surface seismic waves and attenuate them in the frequency range of interest (1-10 Hz).

2. Seismic metastructure: the concept

The proposed seismic isolation system, herein referred as "seismic metastructures", consists of arrays of resonant units buried around buildings or critical infrastructures, designed to reflect/redirect incident seismic waves (Fig. 1a). Each resonant unit consists of a heavy steel cylindrical mass m, suspended by commercial elastic rubber bearings encased in an external concrete cylindrical shell (Fig. 1b). The choice of cylindrical units ensures in plane symmetry of the resonator behavior and thus identical dynamic response for seismic waves coming from different directions. The units can be buried in different lattice arrangements (linear array, triangular lattice, honeycomb lattice etc.) of spacing a to create seismic barriers around critical infrastructure or sensitive buildings.

Each resonant unit is characterized by two fundamental modes, i.e. a horizontal mode at frequency f_h and a vertical mode at frequency f_v:

$$f_h = \frac{\sqrt{\frac{2K_H}{m}}}{2\pi} \quad f_v = \frac{\sqrt{\frac{2K_V}{m}}}{2\pi}, \tag{1}$$

where $K_H = \frac{G_b A_b}{H_b}$ and $K_V = \frac{M_b A_b}{H_b}$ approximate the horizontal and vertical stiffness of the elastic bearings, G_b and M_b the shear and constrained moduli of the bearing material, A_b and H_b the area and the height of the bearing.

Elastic bearings are commercially available in different sizes and materials (see for example [16]), such that different masses can be accommodated or different resonant frequencies targeted.

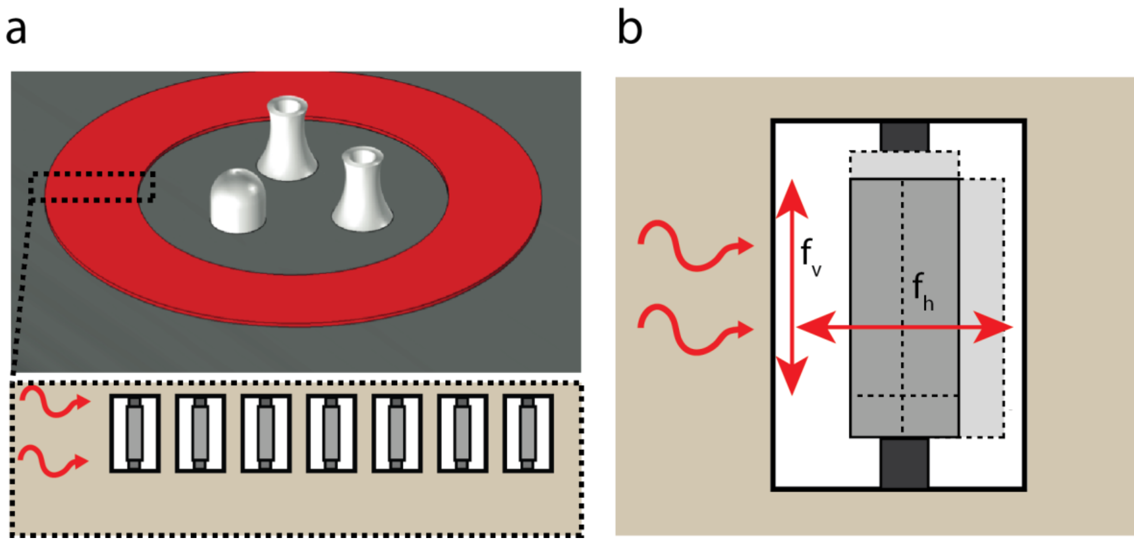

Fig. 1 – a) Schematic of the seismic metastructure. b) Seismic resonant unit (adapted from [14]).

Resonator dimensions and spacing are sub-wavelength in the target seismic frequency range. The proposed seismic resonators are designed to be embedded in soft sedimentary soil (shear velocity $c_{s,soil} = 50 - 500$ m/s) characterized by seismic wavelengths $\lambda = \frac{c}{f}$ of tens to hundreds meters. We investigate the shielding capacity of the proposed seismic metastructure against incoming bulk waves and Rayleigh surface waves. The system can mitigate both bulk waves and Rayleigh waves, exploiting different resonant modes.

3. Interaction with bulk waves

We analyze the interaction between bulk waves (longitudinal and shear) and seismic resonators using a two-dimensional (2D) model and assuming plain strain conditions (see Fig. 2a). Therefore, we restrict our interest to longitudinal and transversal waves propagating in the *x*-*y* plane that excite the horizontal motion of the resonator.

An equivalent 2D model (see Fig 2(b)) is used to simulate the dynamic response of the resonator. In this model, a rigid mass represents the inner cylindrical steel mass while the springs on top and bottom of the resonator are replaced by an effective surrounding medium with Young's modulus:

$$E_{eff} = \frac{f_h^2 r_r{}^2 \rho_r R^2 \left(1 - R^2 + 9\left(1 + R^2\right) ln(R)\right)\pi}{3(1+R^2)} \qquad \text{with} \qquad R = \frac{r_c}{r_r} \tag{2}$$

that ensures the same resonant frequency f_h.

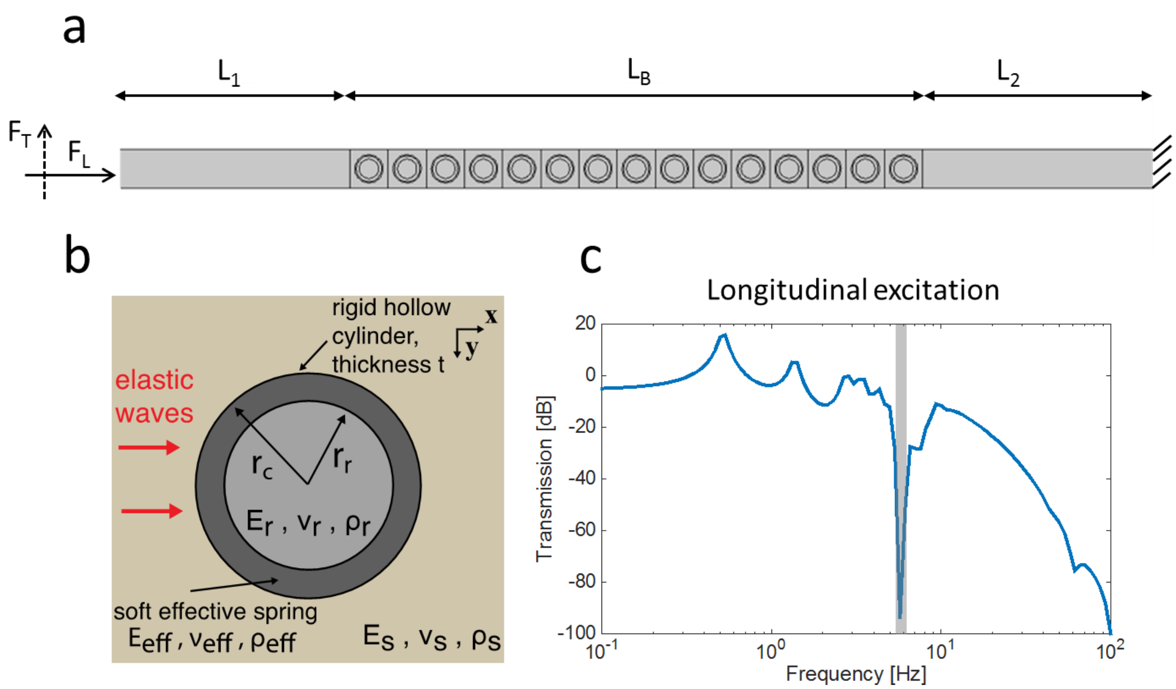

Fig. 2 – a) 2D FEM model of an array of 15 seismic resonators. b) Equivalent 2D resonator model (adapted from [14]). c) Transmission spectrum for longitudinal waves.

The bulk waves attenuation by an array of seismic resonators is analyzed by means of finite element (FEM) simulations using Comsol Multiphysics©. We model the transmission of longitudinal and shear waves through an array of 15 resonators (Fig. 2(a)) using the above described 2D equivalent resonator model.

For the numerical example a soft sedimentary soil with Young's modulus $E_{soil} = 20\ MPa$, poisson's ratio of 0.3 density $\rho_{soil} = 1300\frac{kg}{m^3}$ is used; moreover a small isotropic loss factor $\eta_s = 0.03$ is used to account for soil energy dissipation. For this soil a resonator with a steel mass of radius $r_s = 0.225\ m$ and height $h_s = 1.8\ m$ is designed, with a total mass of approximately $2000\ kg$. Elastic bearing connections are designed to obtain an horizontal resonant frequency $f_h = 5\ Hz$. The external concrete shell has a radius $r_c = 0.6\ m$ and the spacing along the arrays is $d = 0.9$ m, well below the seismic transversal and longitudinal wavelengths in the frequency range around the resonant frequency. The FEM model consists of a homogeneous inlet soil stripe of length L_1, followed by a seismic barrier of length $L_b = n_{res} * d$ and a by an outlet soil stripe of length L_2, (Fig 2.a), for a total length $L = 70\ m$. The system is excited with harmonic longitudinal (transverse) forces F_L (F_T) at the left boundary of the stripe. The output is measured as the reaction force F_R in x (y) direction on the fixed right end side of the stripe model. We calculate transmission function of the system T as the ratio between reaction force and input excitation for longitudinal and transverse force, respectively:

$$T_{L(T)} = \frac{F_{R,x(y)}}{F_{L(T)}} \tag{3}$$

Figure 2c shows the transmission of longitudinal waves incident on an array of 15 seismic resonators. The resonators are extremely effective in attenuating excitation around the resonant frequency, leading to a transmission dip higher than -80 dB. However, the wave attenuation is confined to a very narrow frequency range, as already observed in other elastic metamaterial devices [5]. The narrow frequency range represents a major drawback in the context of seismic engineering, where both structures and earthquake excitation are characterized by a rather broadband spectrum. As such, to attenuate waves in a broader frequency spectrum, the concept of rainbow trapping, i.e. array of units resonating at different frequencies, can be exploited. This concept, proposed earlier in optical [17] and acoustic systems [18], is based on the idea of splitting propagating waves into a spatial spectrum. The principle can be translated to seismic excitations, such that the different frequency components of the incident wave are attenuated by different spatial regions of the barrier. As a result, a broad attenuation frequency range can be achieved. For the proposed system, the tuning of the single unit resonant frequency is obtained by changing the dimensions, and so the stiffness, of the bearing connectors. In particular, an array of resonators with resonances logarithmically spaced between 4 to 7 Hz is studied. To this aim an identical FEM stripe model with seismic barriers composed by different number ($N_{res} = 5 - 40$) of resonant units is analyzed. In Fig. 3 we show the transmission for seismic barrier composed of $N_{res} = 15$ and $N_{res} = 35$ resonators. The transmission of the longitudinal excitation (Fig. 3a) for the case of 15 logarithmically spaced resonators shows lower attenuation (-40 dB) as compared to the single frequency case. However, as desired a much wider frequency spectrum is covered with the use of the rainbow trapping array. As far as the transverse excitation is concerned, a higher attenuation is observed (-60 dB for 15 resonators and more than 100 dB for 35 resonators) as compared to the longitudinal case (see Fig. 3b). Such difference can be explained by the generally higher damping of shear waves.

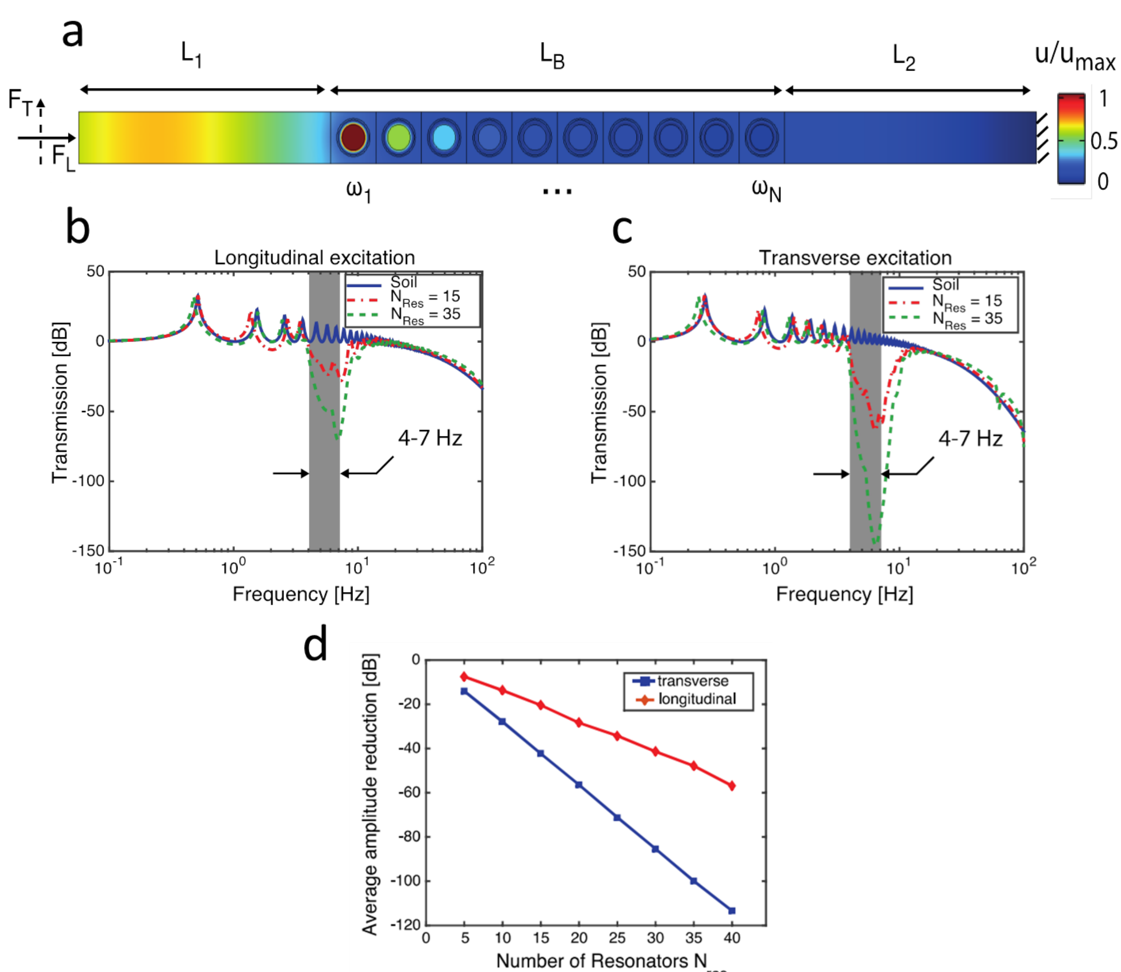

Fig. 3 a) Schematic of the rainbow trapping array using the 2-D equivalent model. b) Transmission spectrum for longitudinal waves for varying number of resonators (0, 15 and 35). c) Transmission spectrum for transverse excitations. d) Average attenuation as a function of the number of resonators in the array (adapted from [14]).

The relation between the averaged amplitude reduction between 4 and 7 Hz and the total number of resonators is shown in Fig. 3d; such relation can be used as guideline for seismic barrier design.

4. Interaction with surface waves

Seismic Rayleigh waves are non-dispersive wave solutions that travel along the earth surface causing the ground to shake in an elliptical motion. Rayleigh waves travel at a velocity slightly lower than bulk shear waves ($c_R \approx 0.9 c_{s,soil}$). As surface solutions, Rayleigh waves show a decay of the displacement amplitude along the soil depth with the wave front displacement confined within a depth $d_R = 1.5\,\lambda_R$. Their surface elliptical motion excites both the vertical and horizontal mode of the resonators. Nonetheless, the excitation further couples with the rotational mode of the cylindrical mass. This mode has a frequency that can be approximate as:

$$f_r = \sqrt{\frac{K_r}{I}} \tag{4}$$

where $K_r = \frac{K_H h_r^2}{2}$ is the rotational stiffness, h_r the height of the resonator mass and I the rotational inertia of the mass. To capture the interaction between Rayleigh excitation and seismic resonators, a 2D (plane x-z) analytical model is used. The model is adapted from an analytical model originally developed to study the interaction between surface acoustic waves and contact resonances at the microscale [19]. The soil is modelled as a semi-infinite elastic space (z<0) and the buried resonators as 3-degree of freedom systems with translational mass m and a rotational inertia I. The mechanical resonators are directly attached to the surface with the elastic translational springs K_V, K_H and the rotational spring K_R (Fig. 4a), assuming that their dimension along the depth is negligible with respect to the wave front depth. This assumption effectively transforms the effect of the resonant motion of the resonator into a set of dynamic boundary conditions for the semi-infinite elastic space. The 2D analytical model is used to extract the dispersion relation of the soil-resonator system, i.e. the frequency-wavenumber ($f - k$) relation describing the full set of wave solutions that can propagate through the soil resonator system. The dispersion relation is obtained following the typical approach for Rayleigh waves [20] and replacing the standard free boundary conditions of the semi-infinite elastic medium with the normal $\sigma_{zz,res}$ and tangential stresses $\sigma_{xz,res}$ exerted by the resonators on the surface soil:

$$\begin{gathered} \nabla^2 \Phi = \frac{1}{c_{L,soil}^2}\frac{\partial^2 \Phi}{\partial t^2} \qquad \nabla^2 H_y = \frac{1}{c_{S,soil}^2}\frac{\partial^2 H_y}{\partial t^2} \\ \sigma_{zz}(0) = \sigma_{zz,res} \quad \sigma_{xz}(0) = \sigma_{xz,res} \end{gathered} \tag{5}$$

Eq. 5 describes the set of wave equations and boundary conditions needed to extract the dispersion curve.

In Eq. 5 $\Phi = Ae^{kz\sqrt{1-\frac{c^2}{c_{L,soil}^2}}} e^{ik(ct-x)}$, $H_y = Be^{kz\sqrt{1-\frac{c^2}{c_{S,soil}^2}}} e^{ik(ct-x)}$ are respectively the dilatation and distortional wave potentials and c the phase velocity of the surface wave solution. The normal and tangential resonator stresses are calculated assuming the resonator excited by the base displacements $u_{z,0} = \frac{\partial \Phi}{\partial z} + \frac{\partial H_y}{\partial x}\Big|_{z=0}$, $u_{x,0} = \frac{\partial \Phi}{\partial x} - \frac{\partial H_y}{\partial z}\Big|_{z=0}$ induced by the wave motion.

For the numerical example, we calculate the dispersion curve for an array of seismic resonator with a mass m = 6700 kg ($h_r = 1.7\,m, r_r = 0.4\,m$) and targeting horizontal, rotational and vertical resonances $f_h = 2.6\,Hz$, $f_r = 4.6\,Hz$ $f_v = 4.9\,Hz$, respectively. The resonators are embedded in a soft sedimentary soil, $c_{S,soil} = 121\,\frac{m}{s}$ with a triangular arrangement of lattice constant $a = 1.3$ m.

The dispersion relation for such soil-resonator system is shown in Fig. 4b. The solid red line describes the surface wave solutions supported by soil-resonator medium at the different frequencies. Around the resonant frequencies the solutions are strongly dispersive (the phase velocity c varies with frequencies) and the branches are flattened. As a result some frequency ranges emerge in which surface solutions are not supported and elastic energy is converted into the soil substrate in the form of shear bulk waves. Two surface band gaps around the horizontal (i.e. 2.6-2.7 Hz) and the vertical resonance (4.9-7.5 Hz) can be identified. However, the horizontal resonance is only able to open a narrow frequency gap, negligible for engineering purpose. On the contrary, the vertical mode of the resonators opens a significant band gap for surface waves. Therefore, an analytical model that neglects the translational and rotational motion of the resonators can be used to obtain a conservative closed expression for the surface band gap edges as:

$$f^- = f_v \qquad f^+ = f_v(\beta + \sqrt{\beta^2 + 1}) \quad \text{where } \beta = \frac{m f_v \pi}{A \rho c_S}\sqrt{1 - \frac{c_S^2}{c_L^2}} \tag{6}$$

The non-dimensional parameter β can be used to guide the design of the resonators to target different frequencies as well as for different (softer-harder) soils. For the resonator size and geometry considered, we obtain $\beta = 0.37, f^- = 4.9\ Hz, f^+ = 7.1\ Hz$.

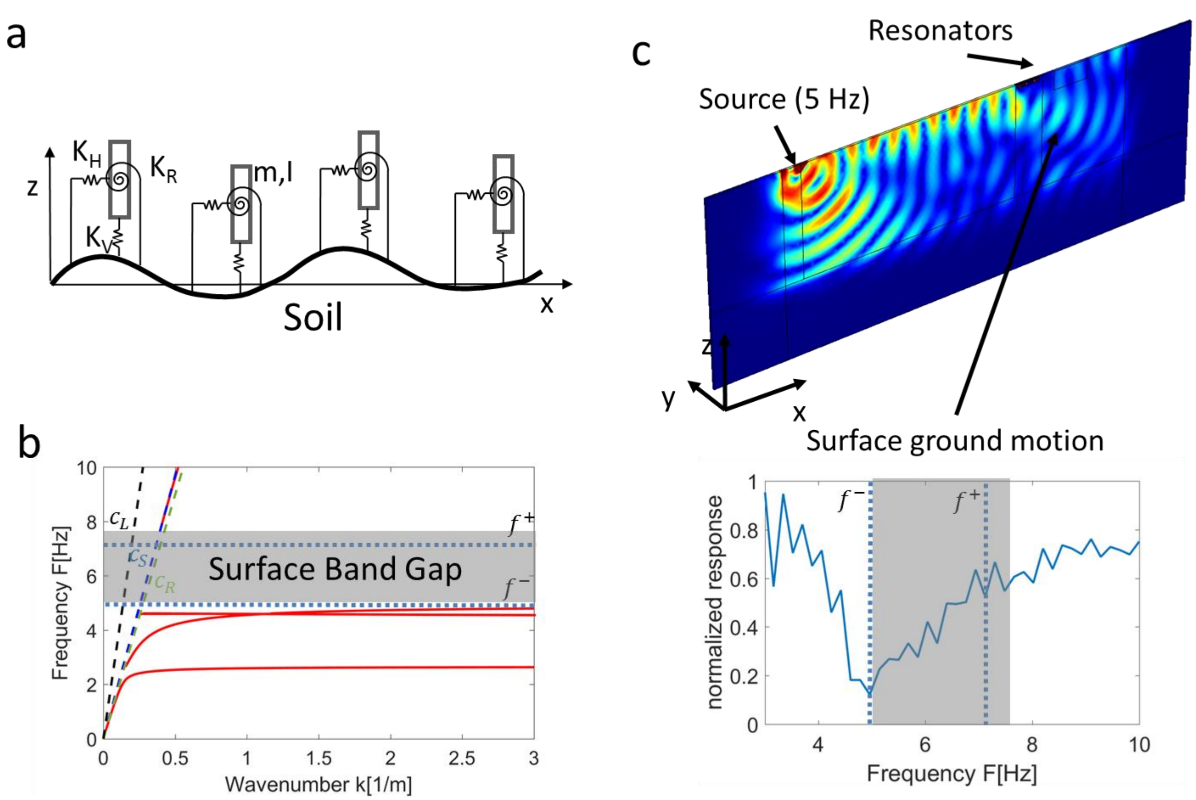

Fig. 4. a) Schematic of the 2D analytical model. b) Dispersion relation of the soil-resonator system. c) FEM simulations of a representative stripe of soil engineered with 12 lines of resonators and driven by harmonic surface excitation: surface to bulk wave conversion and surface response.

FEM simulations are used to verify the analytical predictions and the mechanism behind the existence of surface wave band gaps. To this aim we used a 3D stripe model of soil engineered with an array of 12 resonators and driven with a harmonic surface excitation between 2-10 Hz. In the 3D model periodic boundary conditions along the *y*-direction and Perfectly Matched Layers (PML) at the bottom and side boundaries of the model are used to avoid wave reflections (Fig. 4c). The amplitude of surface ground motion after the seismic barrier, normalized with respect to its value at 2 Hz, is shown in Fig 4c. It is shown a clear drop within the predicted surface bandgap, with an average amplitude reduction of the 60% at the center of the band gap (at 6Hz). The surface to bulk wave conversion mechanism responsible of the ground motion attenuation is shown in the full displacement field in Fig. 4c.

To verify the validity of the analytical and numerical findings, we designed a tabletop experimental setup with 30 metal resonators arranged in triangular lattice (a_{scal} =17 mm), embedded in a polyester resin matrix (Fig. 5a). The scaled experiment is characterized by the dimensionless parameter $\beta_{exp} = 0.37$, which matches the real scale scenario modelled in the numerical example.

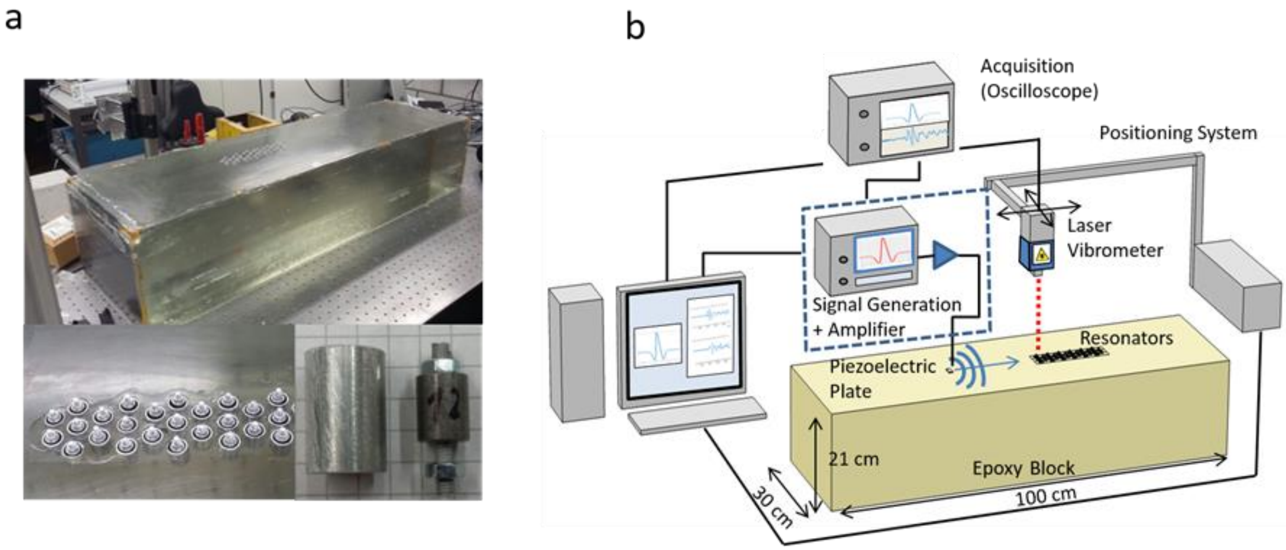

Fig. 5 a) Resin block, resonators arrangement, scaled resonator. b) Schematic of the tabletop experimental setup.

To this aim we built a block of 1 m x 0.3 m x 0.21 m of polyester resin; this material is commonly used in geophysical experiments [21, 22] to realize small scale experimental setups. The density of the resin matrix is 1100 kg/m^3, the shear and longitudinal wave speeds are $c_{s,res} = 1175\, m/s$, $c_{L,res} = 2554\, m/s$, respectively, and the measured Rayleigh wave speed $c_{R,res} = 1100\, m/s$. Each resonator embedded in the block consists of a heavy steel cylinder with radius 4 mm and height 10 mm for an overall mass $m_{scal} = 4\, g$; the mass is encased in an aluminum outer shell with diameter 12 mm and thickness 1 mm and mounted on an aluminum set screw that provides the vertical stiffness to the resonator. Such stiffness can be tuned by changing the free length of the setscrew. We adjust the screw length to fix the vertical frequency $f_{v,scal} = 11.5\, kHz$.

Rayleigh waves are excited using a piezoelectric transducer bonded to the resin block and driven by a signal generator. We studied the propagation of transient waves generated with a Ricker pulse centered at 11.5 kHz. We recorded the normal component of the velocity along the symmetry line of the block passing through the array of resonators using a laser-doppler vibrometer. We obtained the 1D seismograph of Fig. 6a where two distinct events can be identified: a first direct Rayleigh wavetrain ($c_R = 1100\, m/s$) R1 and a second Rayleigh wave front RB2 reflected from the boundaries. The direct Rayleigh wavetrain activates the vertical resonances of the buried resonators and strongly decays passing through the resonator chain. For the experimental setup, Eq. (6) predicts a surface band gap in the frequency range between 11.5 kHz and 16.5 kHz. The comparison between the signal recorded before and inside the chain of resonators (Fig. 6b) shows that the attenuation frequency range is consistent with this prediction.

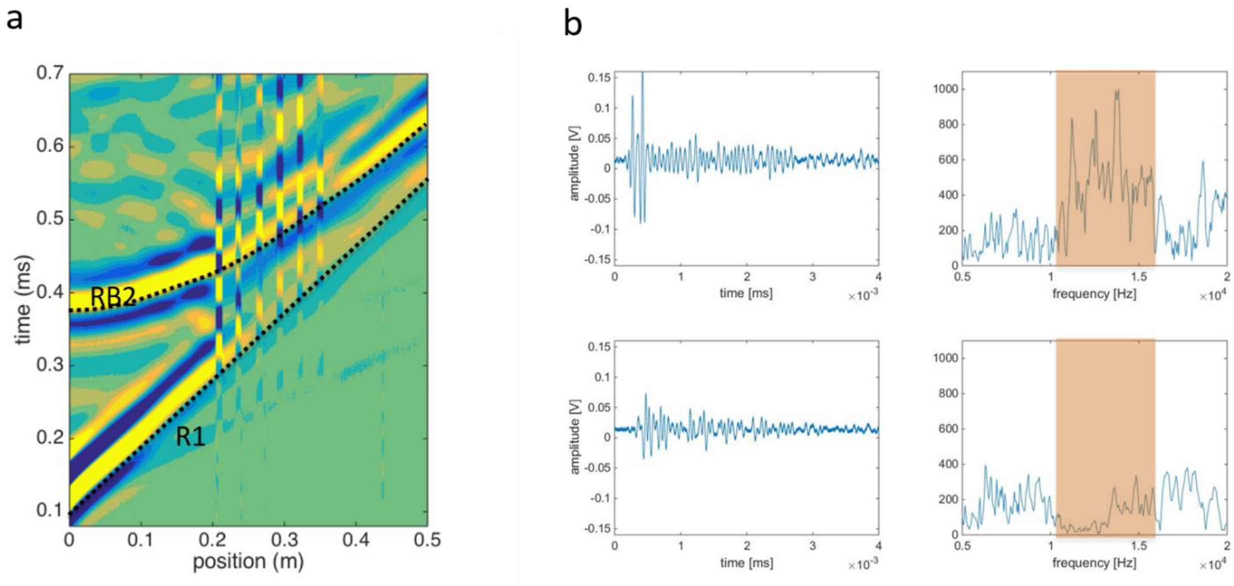

Fig. 6. - a) Acquired seismograph along the symmetry line of the resonators. b) Recorded signal before and inside the resonator array.

5. Conclusions

We demonstrated theoretically and on a scaled experimental setup the possibility of attenuating seismic bulk waves and Rayleigh waves by arrays of buried resonators. Seismic bulk waves can be attenuated exploiting the horizontal resonance of the resonators. By creating arrays with distributed resonance frequencies, exploiting the concept of ‘‘rainbow traps’’, a broader frequency spectrum can be achieved. When seismic Rayleigh waves interact with the resonators, the vertical resonance opens up band gaps and surface waves are redirected into the soil substrate. A close analytical formulation is derived for estimating the surface band gap limits. Finite element simulations and experimental evidences proved the analytical predictions as well as the underlining mechanism for surface wave attenuation.

6. References

[1] Maldovan M (2013): Sound and heat revolutions in phononics. *Nature*, **503** (7475), 209–17.

[2] Maldovan M (2015): Phonon wave interference and thermal bandgap materials. *Nat. Mater.*, **14** (7), 667–674.

[3] Sagartzazu X, Hervella-Nieto L, Pagalday JM (2008): Review in sound absorbing materials. *Arch. Comput. Methods Eng.*, **15** (3), 311–342.

[4] Kushwaha MS, Halevi P, Dobrzynski L, Djafarirouhani B (1993): Acoustic Band-Structure of Periodic Elastic Composites. *Phys. Rev. Lett.*, **71** (13), 2022–2025.

[5] Sheng P, Zhang XX, Liu Z, Chan C T (2003): Locally resonant sonic materials. *Physica B: Condensed Matter*, **338** (4), 201–205.

[6] Stenger N, Wilhelm M, Wegener M (2012): Experiments on elastic cloaking in thin plates. *Phys. Rev. Lett.*, **108** (1).

[7] Farhat M, Guenneau S, Enoch S (2009): Ultrabroadband elastic cloaking in thin plates. *Phys. Rev. Lett.*, **103** (2).

[8] Brûlé S, Javelaud E H, Enoch S, Guenneau S (2013): Experiments on seismic metamaterials: Molding surface waves. *Phys. Rev. Lett.*, **112** (13).

[9] Kim S, Das M P (2013): Artificial Seismic Shadow Zone by Acoustic Metamaterials. *Mod. Phys. Lett. B*, **27** (20), 1–4.

[10] Shi Z, Cheng Z, Xiong C (2010): A New Seismic Isolation Method by Using a Periodic Foundation. *Earth Sp.* 2586–2594.

[11] Shi Z, Cheng Z, Xiang H (2014): Seismic isolation foundations with effective attenuation zones. *Soil Dyn. Earthq. Eng.*, **57**, 143–151.

[12] Colombi A, Guenneau S, Roux P, Richard C (2015): Transformation seismology: composite soil lenses for steering surface elastic Rayleigh waves. *Nat. Publ. Gr.*, **9**,1–19.

[13] Finocchio G, Casablanca O, Ricciardi G, Alibrandi U, Garescì F, Chiappini M, Azzerboni B (2014): Seismic metamaterials based on isochronous mechanical oscillators. *Appl. Phys. Lett.*, **104** (19).

[14] Krödel S, Thomé N, Daraio C (2015): Wide band-gap seismic metastructures. *Extrem. Mech. Lett.*, **4**, 111–117.

[15] Colombi A, Roux P, Guenneau S, Gueguen P, Craster RV (2015): Forests as a natural seismic metamaterial : Rayleigh wave bandgaps induced by local resonances. *Nat. Sci. Reports*., 1–7.

[16] http://www.bridgestone.com/products/diversified/antiseismic_rubber/, “Seismic Isolator for Buildings | Bridgestone Corporation,” 2014.

[17] Tsakmakidis KL, Boardman A D, Hess (2007): “Trapped rainbow” storage of light in metamaterials. *Nature*, **450** (7168), 397–401.

[18] Zhu J, Chen Y, Zhu X, Garcia-Vidal FJ,. Yin X, Zhang W, Zhang X (2013): Acoustic rainbow trapping. *Sci. Rep.*, **3** (1728), 1–6.

[19] Boechler N, Eliason JK, Kumar A, Maznev AA,. Nelson K A, Fang N (2013): Interaction of a contact resonance of microspheres with surface acoustic waves. *Phys. Rev. Lett.*, **111** (3).

[20] Graff KF (1975): *Wave motion in elastic solids*.Courier Corporation.

[21] Bretaudeau F, Leparoux D, Durand O, Abraham O (2011): Small-scale modeling of onshore seismic experiment: A tool to validate numerical modeling and seismic imaging methods. *Geophysics*, **76** (5), 101–112.

[22] Bretaudeau F, Brossier R., D. Leparoux D, Abraham O, Virieux J (2013): 2D elastic full-waveform imaging of the near-surface: Application to synthetic and physical modelling data sets. *Near Surf. Geophys.*, **11** (3), 307–316.