



# AN EXERCISE ON THE DERIVATION OF FRAGILITY FUNCTIONS FOR TALL BUILDINGS

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## Abstract

Structural fragility functions are derived for a 20-story reinforced concrete moment-resisting frame building as well as for a 50-story reinforced concrete frame-wall building. Both buildings comply with the provisions of Turkish Seismic Code (2007) and Istanbul Earthquake Resistant Design Guidelines for Tall Buildings (2008). Nonlinear dynamic analyses under the selected acceleration records are performed first. Structural response and ground motion intensity correlations are investigated through regression analysis. Fragility functions representing the probability of being equal to or of exceeding specified thresholds defined on maximum inter-story drift ratio and on maximum floor acceleration are constructed for the considered levels of peak ground velocity and Arias intensity respectively.

**Keywords:** Structural fragility; tall building; vulnerability; dynamic analysis.

## 1. Introduction

As a result of rapid urbanization in highly populated cities with growing economy and restrictions on land use because of environmental and/or man-made factors, on one hand, and due to the advancements in engineering knowledge and services as well as developments in construction technology, on the other hand; tall buildings have become more common worldwide, not only for commercial/business purposes but also as residential buildings.

In an earthquake loss model, it is an essential step to identify building typologies in the building stock and to define their damageability/vulnerability characteristics. Fragility functions for all building typologies including the engineered, high-code tall buildings should be defined. A fragility function is a mathematical expression representing the probability-based relation between the expected response and the performance limits in terms of the cumulative density function of the probability of exceeding of specific damage limit states (or being in certain damage states) for given measures of ground motion severity.

The objective of the study is to address nonlinear structural response and damageability characteristics of tall buildings in relation to characteristics of strong ground motion. This is accomplished by earthquake resistant design of selected study buildings and monitoring global and local damage measures such as drifts, floor accelerations, plastic rotations, shear stresses in shear walls etc., through nonlinear dynamic analyses under the selected acceleration time histories. Ground motion intensity measure-engineering demand parameter correlations are obtained through regression analysis. For the best correlated pairs, i.e. maximum inter-story drift ratios-peak ground velocity and maximum floor acceleration-Arias intensity [1], fragility functions are constructed. Parameters of the log-normal fragility curves, i.e. median and standard deviation, are optimized using the maximum likelihood estimation method.

## 2. Study Buildings

Two reinforced concrete buildings with different heights, structural systems and natural vibration periods are considered in the study:



- Building I: 20-story moment resisting frame building,
- Building II: 50-story frame-wall building.

Building-I is a regular, five-span, planar, 20-story reinforced concrete (RC) moment resisting frame. It has typical story heights of 3 m. and equal span lengths of 5 m. Building-II is a 50-story RC rising on four basement floors, a ground floor and forty five normal floors. First and second, and third and fourth basement floors have equal story heights of 4.0m and 5.5m, respectively. Ground floor is taller than the other stories with a height of 8.0m. Above the ground floor, height of the normal stories is 4.0m. Hence the building is 207m-tall in elevation. The building has a rectangular shape with five and three bays in perpendicular directions. Its general dimensions are 43.0m x 27.0m in plan. Lateral-load-carrying system of the building consists of core wall system with moment resisting frames. Elevation views of the buildings are presented in Fig. 1.

It is assumed that the buildings are located in Şişli-Kağıthane district of Istanbul, approximately 30 km far from the causative main Marmara fault system. The local site conditions correspond to site class C. The buildings were designed in accordance with the provisions of Turkish Seismic Code (2007) [2] and Istanbul Earthquake Resistant Design Guidelines for Tall Buildings (IERDGTB-2008) [3].

The IERDGTB-2008 specifies three levels of earthquake intensities and three levels of building performances. Code-based 5%-damped elastic response spectra for three different earthquake intensity levels are constructed.

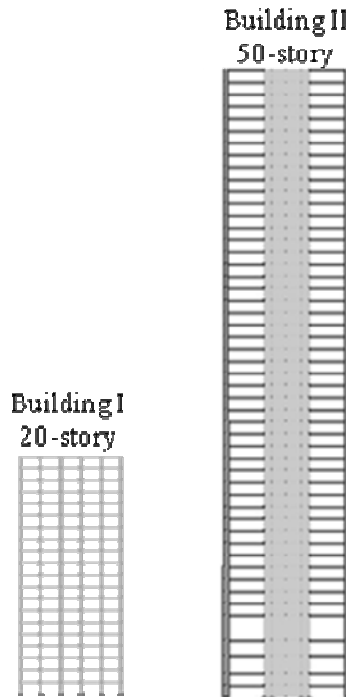


Fig. 1 – Elevation views of the study buildings

For EQ1, EQ2 and EQ3, short- and 1s-period spectral acceleration ( $S_s$  and  $S_1$ ) values for the buildings' location are taken from Appendix A of the IERDGTB-2008:

- EQ1: 50% probability of exceedance in 50 years (72-year return period),  $S_s=0.32s$  and  $S_1=0.15s$ .
- EQ2: 10% probability of exceedance in 50 years (475-year return period),  $S_s=0.64s$  and  $S_1=0.30s$ .
- EQ3: 2% probability of exceedance in 50 years (2,475-year return period),  $S_s=0.94s$  and  $S_1=0.47s$ .

Design basis earthquake response spectra constructed for the values provided above are illustrated in Fig. 3.

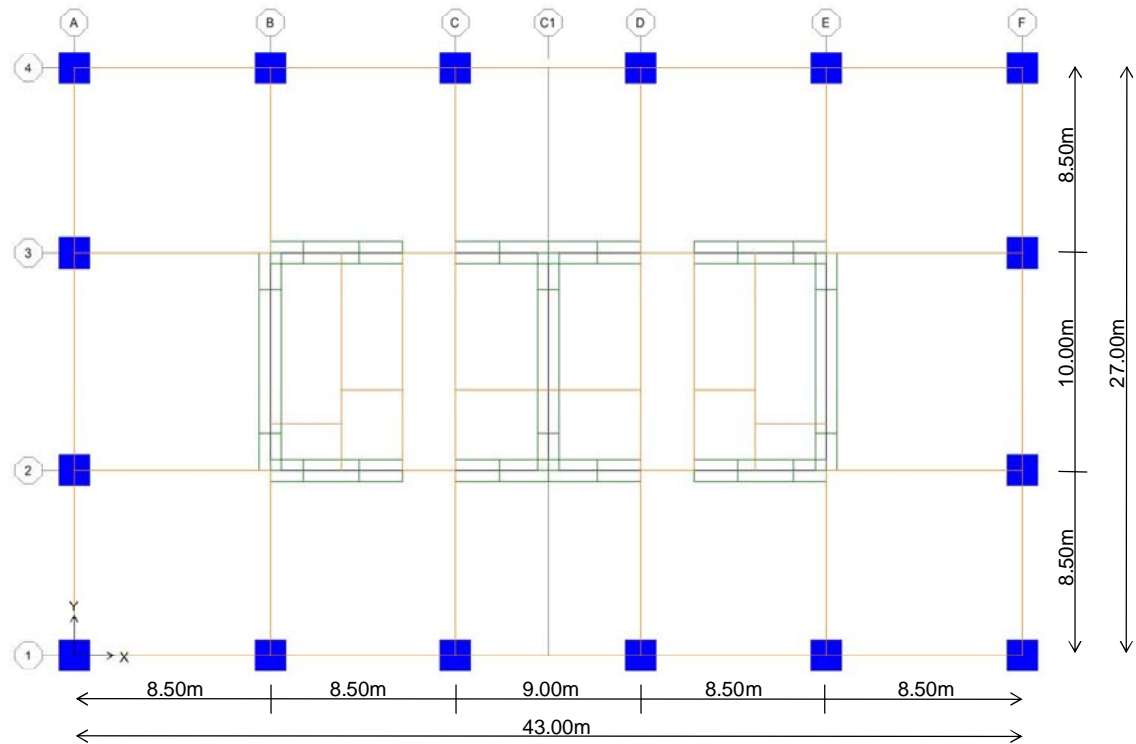


Fig. 2 – Typical floor plan of Building II

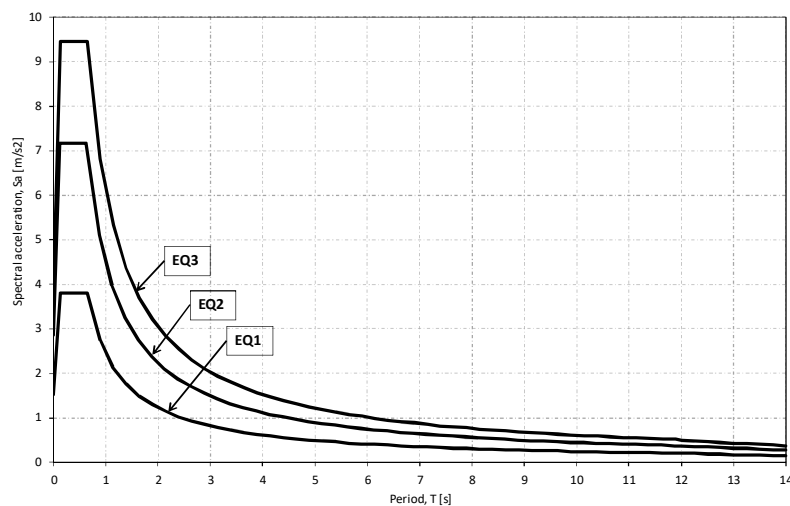


Fig. 3 – 5%-damped elastic response spectra for three levels of design basis earthquake ground motion: EQ1, EQ2 and EQ3

The buildings were modeled as three-dimensional assembly of finite elements using ETABS v9.7.3 software package [4]. Line and shell elements were used for modeling frame elements, i.e. beams and columns, and shear walls and slabs, respectively. In the numerical models, floor diaphragms at each story level were assumed to be infinitely rigid, such that all joints at a floor level move together. Base of the columns and walls at the foundation level was fixed for all the degrees of freedoms and structure-foundation interaction was not taken into account. Design procedure followed is summarized in Table 1. Sectional geometries, characteristic material strengths and free vibration periods for the first three modes are presented in Table 2.



Table 1 - Summary of the seismic design procedure (Source: IERDGTB-2008)

<b>Design Phases</b>	<b>Phase I-A</b> (preliminary design)	<b>Phase I-B</b> (final design)	<b>Phase II</b> (verification)	<b>Phase III</b> (verification)
<i>Earthquake level</i>	EQ2	EQ2	EQ1	EQ3
<i>Target performance level</i>	Life Safety	Life Safety	Immediate Occupancy	Collapse Prevention
<i>Analysis method</i>	Linear response spectrum	Nonlinear response history	Linear response spectrum	Nonlinear response history
<i>Structural behavior factor</i>	$R \leq 7$	-	$R = 1.5$	-
<i>Component stiffness</i>	Effective rigidity as specified in the code	Effective axial, flexural and shear rigidities from nonlinear section representation	Effective axial, flexural and shear rigidities from nonlinear section representation	Effective axial, flexural and shear rigidities from nonlinear section representation
<i>Strength parameter</i>	Characteristic strength	Expected strength	Expected strength	Expected strength
<i>Limit on inter-storey drift ratio</i>	2%	2.5%	1%	3.5%
<i>Acceptance criteria</i>	Demand to capacity and drift check	Demand to capacity, drift and strain check	Demand to capacity, drift and strain check	Strain and drift check

### 3. Nonlinear Structural Modeling

Analytical modeling of the inelastic response of RC wall systems can be accomplished by using either microscopic finite element models based on a detailed interpretation of the local behavior, or by using phenomenological macroscopic or meso-scale models based on capturing overall behavior with reasonable accuracy. An effective analytical model for analysis and design of most systems should be relatively simple to implement and reasonably accurate in predicting the hysteretic response of RC walls and wall systems. Although microscopic finite element models can provide a refined and detailed definition of the local response, their efficiency, practicality, and reliability are questionable due to complexities involved in developing the model and interpreting the results. Macroscopic models, on the other hand, are practical and efficient, although their application is restricted based on the simplifying assumptions upon which the model is based [5].

Finite element models of the buildings for nonlinear dynamic analysis were realized by OpenSees (Open System for Earthquake Engineering Simulation, McKenna et al., 2002) software package [6]. The analytical models consisted of two-node line elements connected at the nodes representing the joints. The joints and the floor diaphragms at each story level were assumed to be infinitely rigid. ‘beamWithHinges’ elements and ‘nonlinearBeamColumn’ elements of the OpenSees software package were used for modeling the beams and columns and shear walls, respectively. ‘beamWithHinges’ element is based on the iterative flexibility formulation, and considers plasticity to be concentrated over the specified hinge lengths at the element ends. The hinge properties have been defined by assigning to each a previously-defined fiber section. The frame element is assumed to be divided into three parts: two hinges at the ends, and a linear-elastic region in the middle. ‘nonlinearBeamColumn’ elements used for modeling the columns and the walls is based on the iterative force formulation and considers the spread of plasticity along the element. The end moment-rotation relation is obtained by the integration of the section response along the element with the help of integration control points.



Use of a single beam-column element at the wall centroidal axis is a common modeling approach (e.g., FEMA 356, 2000 [7]). In this case, an equivalent column is used to model the properties of the wall, and girders with rigid end zones are connected to the column at each floor level (Fig. 4 –). Based on this approach, a finite element model of the frame on B-axis (see Fig. 2) was elaborated in OpenSees environment (Open System for Earthquake Engineering Simulation) for nonlinear dynamic analysis.

Table 2 - Section and material characteristics of the buildings

	<b>Building-I</b>	<b>Building-II</b>
<b>Columns (cm)</b>	80 by 80 (1 <sup>st</sup> - 4 <sup>th</sup> floors)	
	70 by 70 (5 <sup>th</sup> - 8 <sup>th</sup> floors)	140 by 140 (1 <sup>st</sup> - 9 <sup>th</sup> floors)
	60 by 60 (9 <sup>th</sup> - 12 <sup>th</sup> floors)	120 by 120 (10 <sup>th</sup> - 21 <sup>st</sup> floors)
	50 by 50 (13 <sup>th</sup> - 16 <sup>th</sup> floors)	100 by 100 (22 <sup>nd</sup> - 50 <sup>th</sup> floors)
	40 by 40 (17 <sup>th</sup> - 20 <sup>th</sup> floors)	
<b>Beams (cm)</b>	30 by 60 (all floors)	60 by 60 (all floors)
<b>Structural Wall Thickness (cm)</b>		100 (1 <sup>st</sup> - 8 <sup>th</sup> floors)
	-	80 (9 <sup>th</sup> - 18 <sup>th</sup> floors)
		60 (19 <sup>th</sup> - 50 <sup>th</sup> floors)
<b>Coupling Beams (cm)</b>	-	60 by 100 (all floors)
<b>Concrete <math>f'_c</math> (MPa)</b>	35	50
<b>Steel <math>f_y</math> (MPa)</b>	420	420
<b><math>T_1</math> (s)</b>	2.15	6.61
<b><math>T_2</math> (s)</b>	0.80	4.60
<b><math>T_3</math> (s)</b>	0.47	3.78

Fiber (distributed inelasticity) beam-column models involve subdividing the wall section into concrete and steel fibers. In a fiber model, the cross-section geometry is prescribed, and concrete and steel fibers are individually defined. It is needed to define a constitutive model for the material to establish the section response. The stress-strain relation of Menegotto and Pinto, (1973) [9] was adopted for the steel behavior. The characteristic yield strength of  $f_y=420$  MPa and ultimate strength of  $f_u=550$  MPa with a  $6.64 \times 10^{-3}$  of strain hardening ratio were used. Confined and unconfined concrete models were adopted for the core and cover concretes respectively. Confined concrete model proposed by Mander et al., (1989) [10] was used and strength and strain values are given in Table 3.

Table 3 - Parameters for Mander's confined concrete model

	<b>Building-I</b>	<b>Building-II</b>
<b>Nominal Concrete Compressive Strength (<math>f'_c</math>)</b>	35 MPa	50 MPa
<b>Confined Concrete Strength (<math>f_{pc}</math>)</b>	44.80 MPa	68.75 MPa
<b>Strain at Peak Stress (<math>\epsilon_{pc}</math>)</b>	0.0048	0.0038
<b>Ultimate Strain (<math>\epsilon_{pu}</math>)</b>	0.0195	0.0235

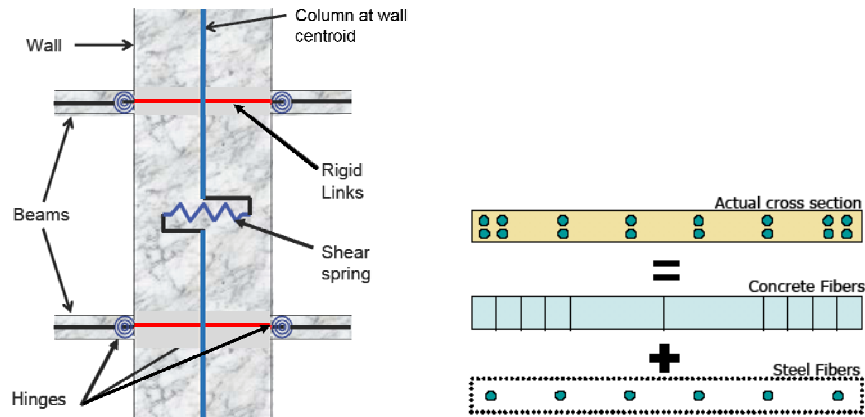


Fig. 4 – Illustration of shear wall modeling by fiber beam-column elements in OpenSees (adopted from [8])

## 4. Dynamic Analyses

### 4.1 Ground motions

The input ground motion database was compiled from PEER website (<http://ngawest2.berkeley.edu/site>). It consists of horizontal components of recordings from 13 earthquakes. The selection is based on the moment magnitude and the Joyner–Boore distance ( $R_{JB}$ ) of the records. For the input ground motion dataset, the magnitudes ( $M_w$ ) and source-to-site distances ( $R_{JB}$ ) range between 6.0 - 7.5 and 0-100 km respectively. The selected recordings came from different soil conditions designated with the NEHRP site classes (B, C, D and E) based on the  $V_{s,30}$  values and faulting mechanisms identified by the rake angle and source mechanism (strike-slip, normal, reverse, reverse oblique and normal oblique). Only accelerograms from strike-slip earthquakes were included in the ground motion library. Accelerograms were only chosen if their moment magnitude, Joyner-Boore distance, site classification and faulting mechanism were known and this reduced the size of the input ground motion dataset. The records were grouped into three sets on the basis of magnitude ranges as follow:  $M_w=6.0-6.49$ ,  $M_w=6.5-6.99$  and  $M_w=7.0-7.5$ . Numbers of records in each sub-set are 56, 104 and 92, respectively, and 252 records in total. Variation of magnitude values with Joyner-Boore distance is presented in Fig. 5.

Both time and frequency domain characteristics of the strong ground motion are used in engineering applications. Parameterization of these characteristics is a useful tool for their incorporation in further studies such as assessment of earthquake hazard or structural response to a particular seismic excitement. The ground motion parameters we have considered are peak ground acceleration (PGA), velocity (PGV), and displacement (PGD), Arias intensity (AI), cumulative absolute velocity (CAV), spectral acceleration/displacement ( $S_a/S_d$ ) and the energy flux (SSV). Energy flux provides a dynamic measure of seismic energy, and can be used to characterize the intensity of ground shaking, as well as the response of structures [11]. Energy flux is defined as the amount of energy transmitted per unit time through a cross-section of a soil or a structural medium. It is equal to kinetic energy multiplied by the propagation velocity of seismic waves.

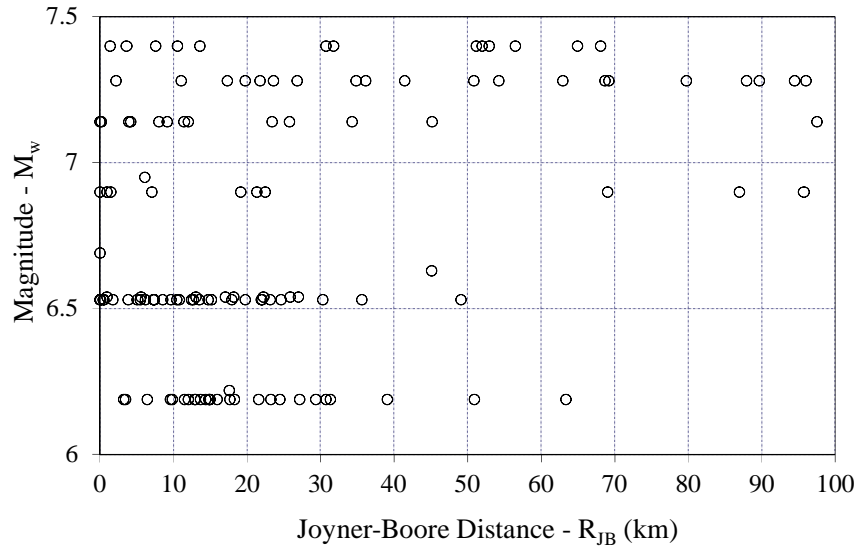


Fig. 5 – Variation of magnitude ( $M_w$ ) values with Joyner-Boore distance

#### 4.2 Nonlinear dynamic response assessment

Nonlinear dynamic analyses of the study buildings were performed under the selected 252 real earthquake recordings. First gravity load analysis as a combination of dead loads and live loads ( $G+0.3Q$ ) was performed. Eigen values were computed through free vibration analysis. Then the buildings were analyzed under the actions of uniform ground motion excitation. P-delta effects were included in the analysis.

Through the OpenSees simulations, acceleration and displacement time histories of a selected joint at each floor in the buildings were monitored. The selected joint was the left-outer beam-column node at the floor levels and the same through the elevation of the building. The recorded acceleration and displacement values at that joint were the relative values with respect to supports of the building. Also, shear force, moment and strain time histories of the shear walls at each floor were tracked. The outputs were post-processed using an ensemble of Matlab [12] scripts and the peak responses over the entire time were obtained. In this study, maximum inter-story drift ratio, maximum floor acceleration, maximum plastic end rotation of beams and maximum shear forces and strains at the critical sections of the shear walls were chosen as engineering demand parameters. Inter-story drift ratio is computed as the difference between horizontal displacements of the adjacent stories divided by the story height. Peak inter-story ratio is the maximum value of the absolute inter-story drift ratios at each story level over the entire time history. Member end rotation is defined as the rotational displacement over the end of the member induced by the earthquake ground motion. Member end rotation is measured in local element coordinates so is the rotational displacement over the end of the member relative to the rotation of the joint at the end of the member. Peak end rotation is the maximum value of the plastic rotations computed at both ends of the selected beams at each floor over the entire time history. Peak floor acceleration is the maximum value of the absolute acceleration of the selected joint at each floor over the entire time history. The maxima of those peak inter-story drift ratios and peak floor accelerations amongst all stories in a building were considered as MIDR and MFA, respectively, and fragility functions were derived in terms of these two engineering demand parameters. For the sake of brevity in this paper, only the peak inter-story drift ratios and the floor accelerations through the elevations of the buildings are presented in Fig. 6 and Fig. 7.



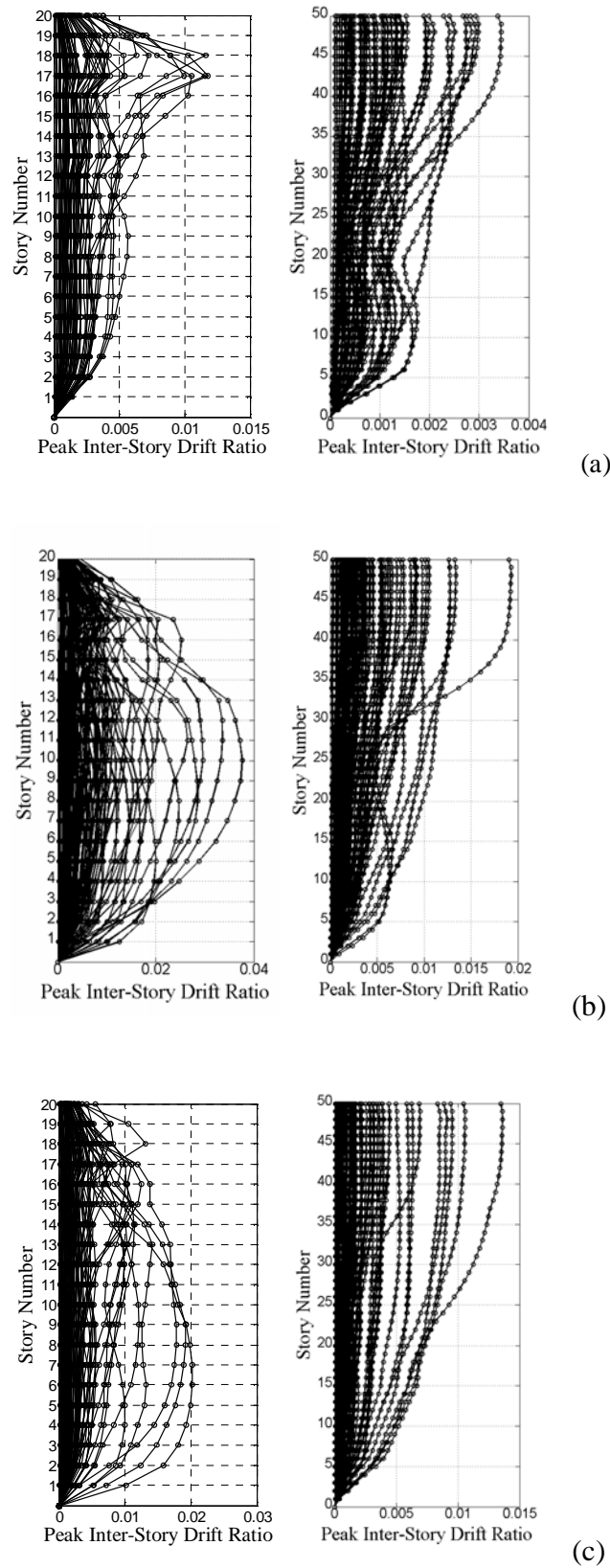


Fig. 6 – Peak inter-story drift ratios under the records of a) Mw=6.0-6.49; b) Mw=6.5-6.99 and c) Mw=7.0-7.5



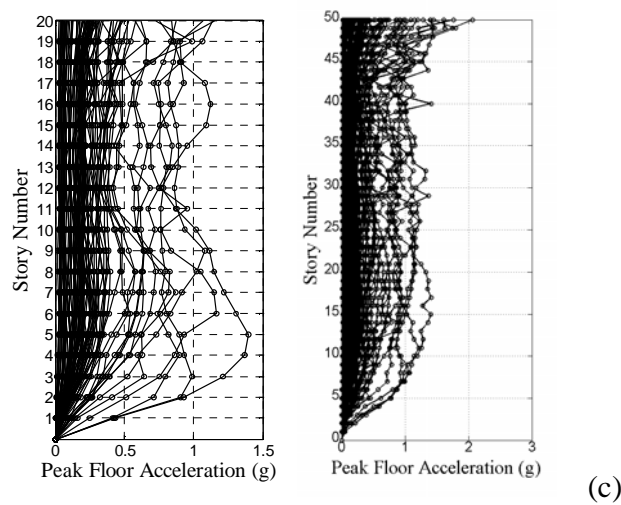
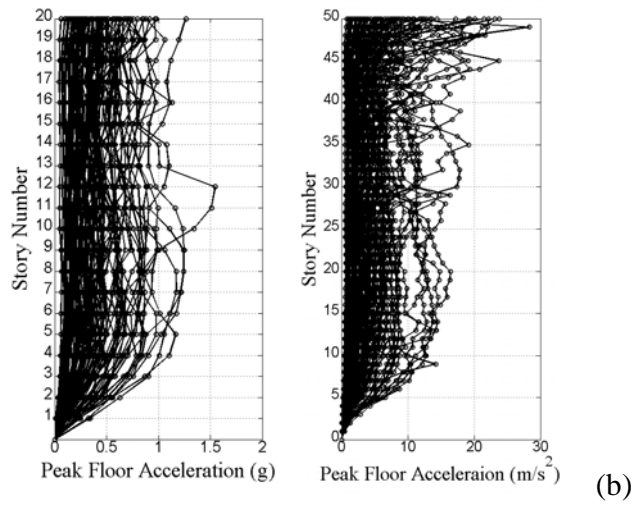
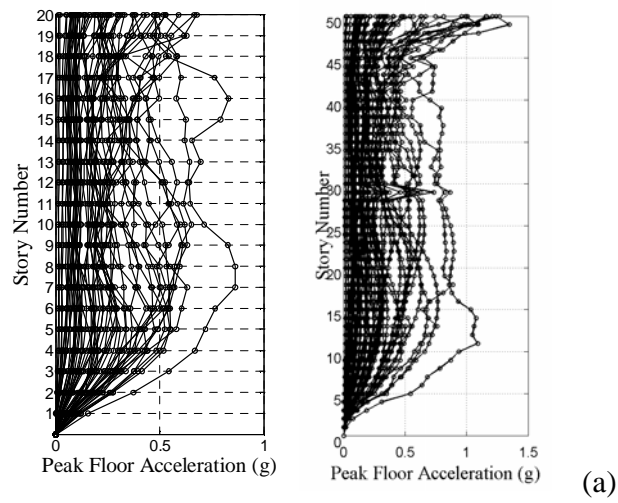


Fig. 7 – Peak floor accelerations under the records of a) Mw=6.0-6.49; b) Mw=6.5-6.99 and c) Mw=7.0-7.5

### 4.3 Structural response-Intensity measure correlations

The properties of a random variable from a finite sample of data can be estimated by statistical inference. In this study, we applied non-parametric statistical inference technique [13]. In non-parametric model, regression was used on the pairs of records' intensity measure values and associated engineering demand parameters obtained from nonlinear dynamic analyses. A linear relationship between the logarithms of the two variables are obtained to estimate of the mean value of engineering demand parameter over the range of ground motion intensity levels covered by the records in the input motion ensemble.

Regression was applied first in order to see effects/correlations of magnitude, site class and source-to-site distance on7with structural response parameters, we applied the non-parametric regression technique. It has been observed from the analyses results that it is difficult to propose a direct correlation between the structural responses and ground motion characteristics in terms of earthquake magnitude and soil conditions. However the analyses results allowed us to develop the functions for the variation of the structural responses with source-to-site distance as shown in Fig. 8. It is observed while the spectral acceleration/displacement at the first fundamental vibration period of building is a well correlating intensity measure with the structural response (e.g. displacement, drift) for Building-I, it is not for Building-II at all. Energy flux, which is defined as the amount of seismic energy transmitted to the structure and can be computed as the sum of the squared ground motion velocities, appears a well correlating ground motion intensity measure not only with global response parameters (i.e. displacement, drift) but also with local response parameters (i.e. shear force and moment in the walls) for Building-II. Peak ground acceleration and Arias Intensity are found to be representative intensity measures for the assessment of floor accelerations in both structural systems. Regarding the plastic rotations in beams and columns as well as the strains in the walls, the best correlations are obtained with the peak ground velocity.

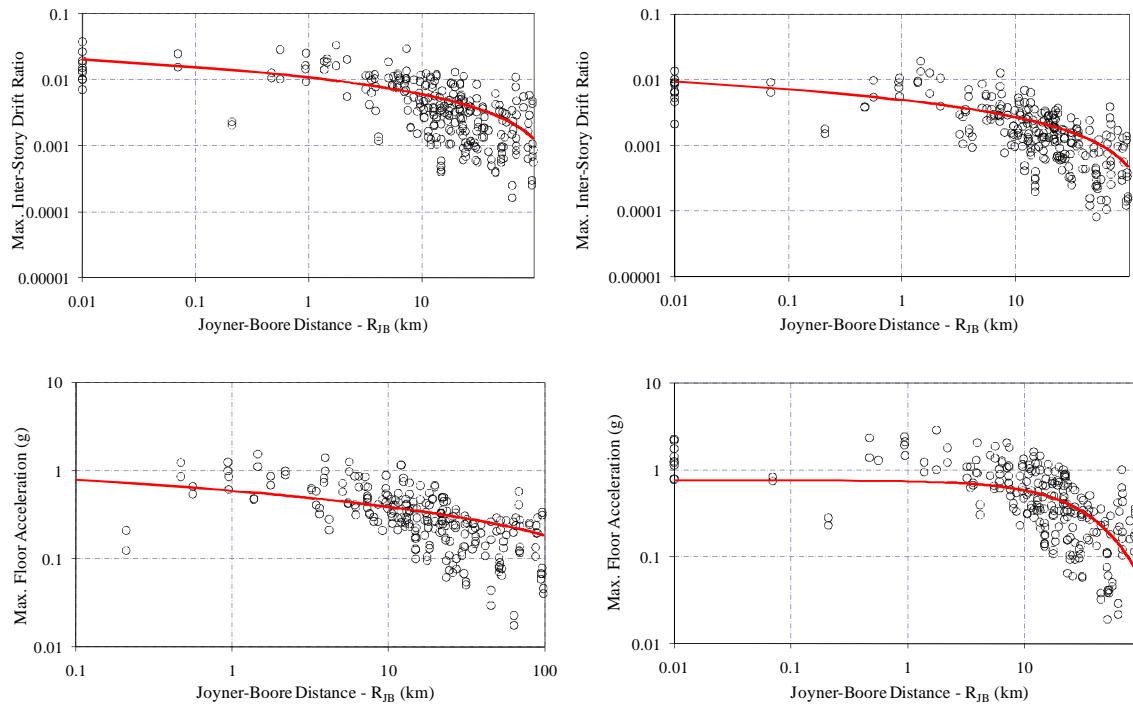


Fig. 8 – Variation of maximum inter-story drift ratios and maximum floor acceleration wrt source-to-site distance and fitted regression lines: Building I (left) and Building II (right)

## 5. Derivation of Fragility Functions

Fragility functions were obtained by statistical processing, i.e. log-normal curve fitting, of the pairs of structural response and ground motion intensity parameters. Two sets of fragility curves were developed for being equal to



or exceeding specified threshold levels on maximum inter-story drift ratios (MIDR) and maximum floor accelerations (MFA). Ground motion intensities were represented by peak ground velocity (PGV) and Arias intensity (AI) as of the best correlated intensity measures with each of structural response parameters respectively.

Parameters of the log-normally distributed fragility functions, i.e. median and standard deviation, can be computed by maximum likelihood estimation (MLE) method originally given by Shinozuka et al. [13]. In this process, it is checked whether the threshold on the MIDR or MFA is either surpassed or not. If the threshold is exceeded, a '1' outcome value is given to the corresponding intensity measure, i.e. PGV or AI. Similarly, if the threshold is not attained, a '0' value is attached to the IM. This results in a plot of points either lying on the axis of abscissas at a value of 1 or at a value of 0 for a range of IM. A best fitted log-normal curve is then optimized by MLE, thus obtaining the fragility curve representing the probability of exceeding considered level of structural response. Derived fragility curves are presented in Fig. 9 and Fig. 10.

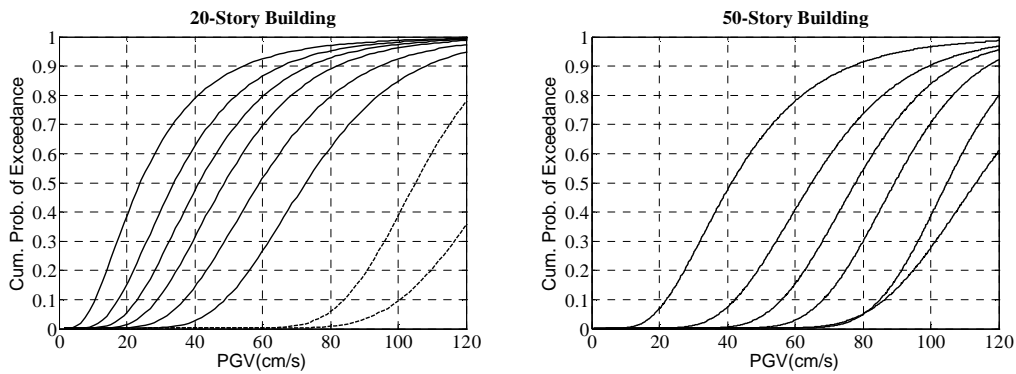


Fig. 9 – Fragility curves for maximum inter-storey drift ratios (MIDR) in terms of peak ground velocity (PGV). Each curve represents a discrete damage state associated to a threshold on maximum inter-story drift ratio. Threshold values: 0.002, 0.004, 0.006, 0.008, 0.01 and 0.012 (dashed curves for 0.02 and 0.03).

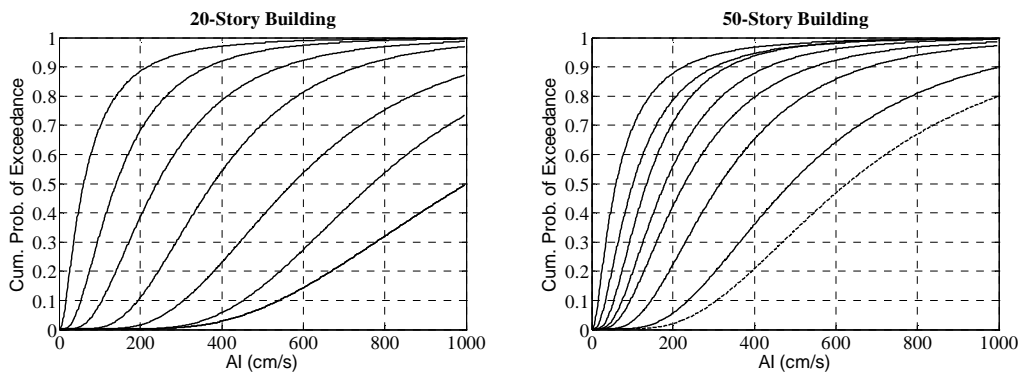


Fig. 10 – Fragility curves for maximum floor accelerations (MFA) in terms of Arias Intensity (AI). Each curve represents a discrete damage state associated to a threshold on maximum floor acceleration. Threshold values: 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 and 1.5 g (dashed curve for 2.0g).

## 6. Discussion and Concluding Remarks

Nonlinear dynamic analyses of a 20-story moment resisting frame building and of a 50-story frame-wall building are performed under a set of 252 real earthquake acceleration time series. The selected accelerograms are grouped on the basis of magnitude and distance and, are further processed to compute ground motion intensity measures such as peak ground motion values, spectral values, Arias Intensity, cumulative absolute velocity and energy flux. Nonlinear structural modeling is achieved with OpenSees software package and global and local



damage measures such as inter-story drifts, floor accelerations, plastic rotations in beams/columns, shear forces in walls are monitored. Regression on the results of nonlinear dynamic analyses is used to identify structural response-ground motion intensity correlations. Two sets of fragility functions are derived from the best correlations, i.e. maximum inter-story drift ratio-peak ground velocity and maximum floor acceleration-Arias intensity.

Regarding the nonlinear behavior modeling, it is deemed that the use of fiber beam-column elements in structural modeling is adequate for the evaluation of global system response in the present study. On the other hand, inelastic response of structural walls subjected to horizontal loads is dominated by large tensile strains and fixed end rotation due to bond slip effects, associated with shifting of the neutral axis. This feature might not be directly modeled by a beam-column element model. Multiple vertical-line-element model (MVLEM) is able to capture those response characteristics (e.g., shifting of the neutral axis, and the effect of a fluctuating axial force on strength and stiffness). However, MVLEM for nonlinear dynamic analysis is not currently available in OpenSees. Although working with commercial software (e.g. Perform 3D [15]) might be an alternative, it requires considerable amount of time when working with hundreds of accelerograms since output processing should be done manually analysis by analysis.

Fragility functions for all building typologies including the engineered, high-code tall buildings should be defined in an earthquake loss model whereas they currently do not exist for tall buildings in Turkey. The study provides fragility curves for certain structural systems with a possible extension to include other structural types in future studies as well as considering further nonlinear modeling approaches.

## 7. Acknowledgements

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