



## A SIMPLE ESTIMATION METHOD OF THE PROBABILITY DISTRIBUTION OF RESIDUAL DEFORMATION OF QUAY WALLS CONSIDERING VARIATION OF EARTHQUAKE GROUND MOTION

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### Abstract

A major criterion of damage levels of a gravity type quay wall by an earthquake is residual deformation. For example, maximum residual deformations of gravity type quay walls at Kobe port by the 1995 South Hyogo Prefecture Earthquake were about 5 meter although structural members of quay walls were not damaged. Therefore, evaluation of residual deformation is necessary in the design procedure about gravity type quay walls.

However, evaluation of residual deformations of gravity type quay walls cannot be conducted analytically because the deformation is caused by shear deformation of foundation soil layers and so on. Two-dimensional finite element earthquake response analysis considering non-linear characteristics and effects of liquefaction of soil layers can be used for the evaluation of the residual deformation but there is a problem in the computational load for that kind of analysis. At present, introduction of reliability-based design method in view of residual deformation to practical design of gravity type quay walls is very difficult because designers must conduct the analysis many times in order to obtain the probability distribution of residual deformations of the quay wall.

We aim at proposing a simple estimation method of the probability distribution of residual deformations of gravity type quay walls considering the variation of earthquake ground motion. The characteristics of earthquake ground motions are divided into the source effects, the pass effects and the local site effects. We focus on the site effects considering variations in the effects by collecting strong motion record.

Then two-dimensional finite element earthquake response analysis were conducted by using 136 seismic input motions which reflect the variations in the site amplification factors and the site phase characteristics. The probability distribution of the residual deformations of a gravity type quay wall were obtained. The probability distribution of residual deformations was found to be approximated by the logarithmic normal distribution.

Next, the average and the standard deviation of the probability distribution were evaluated for the simple estimation method. Two-dimensional finite element earthquake response analysis were conducted by using 3 input motions: the logarithmic mean of the site amplification factors( $\lambda$ ),  $\lambda$  plus and minus the logarithmic standard deviation( $\zeta$ ). The logarithmic mean and the logarithmic standard deviation were evaluated according to the sum of amplification in the range of 0.5 – 5.0 Hz. The distribution of the residual deformations of the quay wall was simply estimated with the residual deformations by 3 earthquake response analysis.

**Keywords:** *Probability Distribution of Residual Displacement, Variation of Earthquake Ground Motion, Gravity Type Quay Walls*



## 1. Introduction

A major criterion of damage levels of a gravity quay wall by an earthquake is residual deformation. For example, maximum residual deformations of gravity type quay walls at Kobe port by the 1995 South Hyogo Prefecture Earthquake were about 5 meter although structural members of quay walls were not damaged. Therefore, in the earthquake resistant design for gravity type quay walls, the residual deformation is the target of the performance verification. Two types of reference earthquake ground motions specified in ISO23469 are considered in the earthquake resistant design. One is for evaluating serviceability of structures that has a reasonable probability of occurrence during the design working life of quay walls. In Japan, that is called “Level-one earthquake ground motion” with 75 years of return period. The other is for evaluating safety of structures associated with rare events. Because the former type is evaluated by the probabilistic seismic hazard analysis, a probabilistic performance verification method is desirable against serviceability limit state.

The Monte Carlo simulation technique is usually used to calculate a failure probability of a structure. But it requires a large number of trails. The evaluation of the residual deformations of quay walls cannot be conducted analytically because the deformations are caused by shear deformations of foundation soil layers and so on. Two-dimensional finite element earthquake response analysis considering nonlinear characteristics and effects of liquefactions of soil layers must be used for the evaluation of the residual deformations of the quay walls but there is a problem on the computational load for the kind of analysis. At present, it is difficult to introduce to a reliability-based design method in view of the residual deformations of the gravity type quay walls to a practical design because designers must conduct the analysis many times in order to obtain the probability distribution of the residual deformations of the quay walls.

There are several studies about the adoption of the first-order second moment method for the evaluation of the failure probability of a structure [1]. In this method, the earthquake response analyses are performed only several times, and a simple evaluation of the derivative of the performance function is conducted based on the results, and reliability indices can be evaluated. But those studies did not consider the variation of earthquake ground motions. It is necessary to consider the variation of input ground motions to understand the probability distribution of residual deformations of gravity type quay walls. Earthquake ground motions can be evaluated by source effects, path effects and site effects according to Iwata & Irikura [2]. It is necessary to treat those effects as variables.

This study aims at proposing a simple estimation method of probability distribution of residual deformations of gravity type quay walls considering the variation in earthquake ground motions. We evaluate probabilistic characteristics of site effects by using strong motion records. At first, many earthquake ground motions are generated according to variation in site effects. Secondly, probability distribution of residual deformations of a quay wall is evaluated by conducting earthquake response analyses with all the earthquake motions generated. Finally, we propose a simple estimation method that can evaluate the probability distribution of the residual deformation of a quay wall by performing earthquake response analysis only few times.

## 2. Study method

The variation of earthquake ground motions is considered as the variation of site effects. The site effects can be evaluated with two characteristics: amplification factors and phase characteristics. The amplification factors and the phase characteristics correspond to the Fourier amplitude spectrum and Fourier phase spectrum in frequency domain respectively. We consider variations of site effects both in amplification factors and phase characteristics.

As the target of the study, we chose the port of Sendai-Shiogama in Japan. The earthquake ground motion is estimated by the probabilistic seismic hazard analysis considering source effects, path effects and site effects [3].

The site amplification factor at the strong motion observation station (MYG012) near Sendai-Shiogama port was used for the evaluation of the earthquake ground motion for the port. There is another strong motion observation station (MYG013) near MYG012 as shown in Fig.1. The site amplification factors at both sites were calculated



by spectral inversion technique [4]. Note that MYG012 and MYG013 are the strong motion observation stations of the strong motion seismograph networks called K-NET.

The probability distribution of the site amplification factor at MYG012 is evaluated by the following equation with the seismic records at MYG012 and MYG013 of the same event,

$$G_{MYG012}(f) = G_{MYG013}(f) \frac{O_{i,MYG012} R_{i,MYG012}}{O_{i,MYG013} R_{i,MYG013}} \quad (1)$$

where,  $f$  is the frequency,  $G_j$  is the amplification factor of the site  $j$ ,  $O_{ij}$  is Fourier amplitude spectrum of the seismic motion record of the earthquake  $i$  at the site  $j$ ,  $R_{ij}$  is the epicenter distance of the earthquake  $i$  and the site  $j$ .  $O_{ij}$  is calculated as the square root of the mean of the sum of the squared NS component and the squared EW component.

The necessary conditions of events used in the analysis are as follows: events that occurred after 2004, observed at both sites, the depth of the hypocenter more than 10km and less than 200km, with JMA magnitude greater than 4.0 and less than 7.0 and with sufficient accuracy from 0.2Hz to 10Hz in Fourier amplitude spectrum. As the results, 68 events were conformed to the condition. Calculated amplification factors by Equation 1 are shown in Fig.2. The logarithmic mean of the 68 site amplification factors is shown in the figure as the thick red line and the logarithmic mean plus or minus logarithmic standard deviation is shown as the dotted blue lines. The logarithmic mean is written as  $\lambda$  and the logarithmic standard deviation is written as  $\zeta$ .  $\zeta$  of each frequency is different from one another. The mean of  $\zeta$  in the range of 0.5Hz to 10.0Hz is 0.25.

The deviation of group delay time is considered as phase characteristics. The group delay time is defined as the derivative of Fourier phase spectrum with respect to angular frequency as the following equation,

$$T_{gr}(\omega) = \frac{d\phi(\omega)}{d\omega} \quad (2)$$

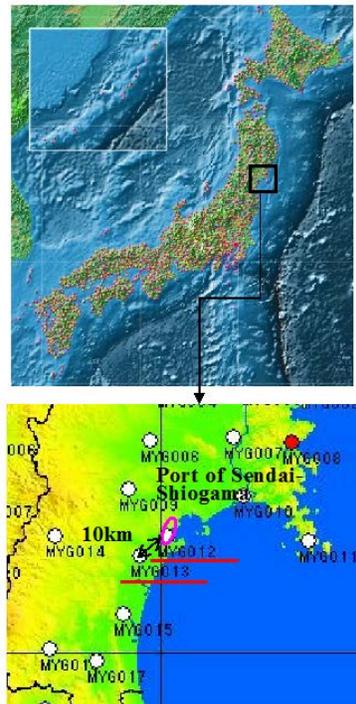


Fig. 1 – Positional relations of the strong motion observation sites

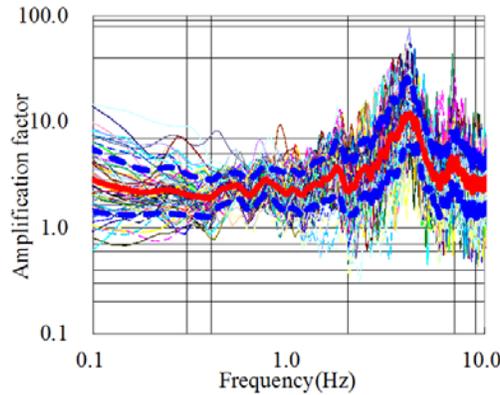


Fig. 2 – Variation in site amplification factor

where  $\varphi$  is Fourier phase spectrum,  $\omega$  is the angular frequency [5]. The group delay times for earthquake response analysis are generated by the following equation,

$$T_{grn}(\omega) = (1 + r)T_{gr0}(\omega) \quad (3)$$

where  $T_{grn}$  is the group delay time for the  $n$  th analysis,  $r$  is the random variable with normal distribution whose mean is zero and standard deviation is 0.3. The group delay time of the Level-one earthquake ground motion at the port of Sendai-Shiogama is used as  $T_{gr0}$  as shown in Fig.3. Fig.4 shows the ground motion calculated with the mean of the site amplification factors shown in Fig.2 and the mean of the group delay time shown in Fig.3. Two group delay times are generated for each single site amplification factor. By coupling the group delay times and the site amplification factors, 136 input motions are generated.

The target of this study is a gravity type quay wall which consists of a reinforced concrete caisson on a rubble foundation. The FEM model of the quay wall is shown in Fig.5. Computer program FLIP (Finite Element Analysis Program for Liquefaction Process) is used for the analysis, which can consider the liquefaction of soils and is often used in seismic design for port structures in Japan [6]. The soil parameters are shown in Table 1. The replaced sand under the caisson and the reclaimed sand behind the caisson are liquefiable. Residual

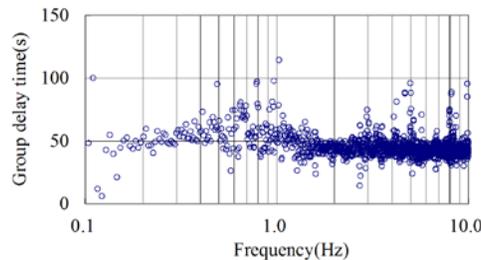


Fig. 3 – Mean of the group delay time

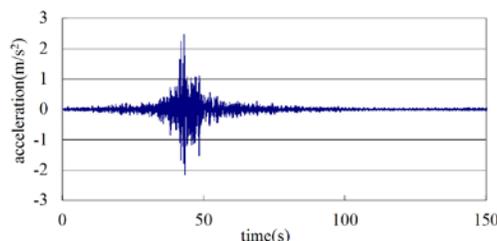


Fig. 4 – Earthquake ground motion with mean site effects

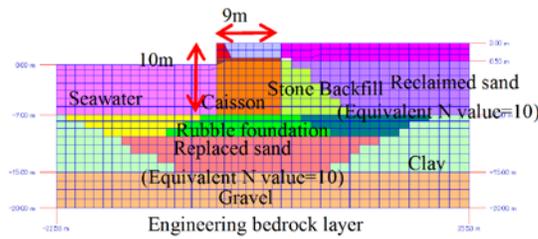


Fig. 5 – FEM model of a gravity type quay wall

Table 1 – Soil parameter values

	Wet unit weight (t/m <sup>3</sup> )	Reference effective confining pressure kN/m <sup>2</sup>	Reference initial shear modulus kN/m <sup>2</sup>	Reference initial volume modulus kN/m <sup>2</sup>	Cohesion kN/m <sup>2</sup>	Angle of internal friction °	Porosity	Maximum damping ratio
Reclaimed sand(Over ground water level)	1.84	98	84900	221000	—	40	0.45	0.24
Reclaimed sand(Under ground water level)	2.04	98	84900	221000	—	40	0.45	0.24
Clay	1.7	143	74970	195500	—	30	0.55	0.2
Replaced sand	2.04	98	84900	221000	—	40	0.45	0.24
Rubble foundation	2.04	98	180000	469400	20	35	0.45	0.24
Stone Backfill	2.04	98	103200	269200	20	35	0.45	0.24
Gravel	2.04	98	152000	396000	0	42	0.45	0.24
Base viscous boundary	Wet unit weight=1.81t/m <sup>3</sup> , Vs=330m/s, Vp=1890m/s							

deformations are calculated by earthquake response analyses with inputted 136 ground motions.

### 3. Study results

The result of the earthquake response analysis using the earthquake ground motion with the mean of the site effects is shown in Fig.6, as an example. The figure shows the residual deformation of the gravity type quay wall and the peak excess pore water pressure ratio, which ranges from 0.0 to 1.0 according to the liquefaction level. It is obvious that the liquefaction occur in the replaced sand and the reclaimed sand.

The histogram of the residual deformations is shown in Fig.7, which are the results of earthquake response analysis with 136 earthquake ground motions considering variation in the site effects. The mean of the residual deformations is 0.09m and the standard deviation is 0.06m. The solid line in the figure shows the frequency distribution regarding the distribution of the residual deformations as the logarithmic normal distribution. The cumulative probability is shown in Fig.8 and Fig.9. Fig.9 is an enlarged view of Fig.8. Note that the relative cumulative frequency is the cumulative probability of the 136 results of earthquake response analysis. It is important to estimate the upper tail of the distribution of the residual deformations. The logarithmic normal distribution gives larger residual deformation than the normal distribution in the range of the cumulative probability higher than 0.93 and it also gives larger residual deformation than the relative cumulative frequency in the range of the cumulative probability higher than 0.96. Therefore, the logarithmic normal distribution gives the residual deformation on the conservative side and highly applicable to the practical design.

We try to estimate the cumulative probability of the logarithmic normal distribution shown in Fig.8 and Fig.9 by conducting earthquake response analysis only several times. Assuming the distribution of both the site amplification factor and the residual deformation as logarithmic normal distribution, as a simple estimation method, the earthquake response analysis are conducted seven times using ground motions of  $\lambda$ ,  $\lambda$  plus and minus  $\zeta$ ,  $\lambda$  plus and minus 0.75 times  $\zeta$  and  $\lambda$  plus and minus 0.5 times  $\zeta$  about the site amplification factors. Fig.10 shows the comparison of the cumulative probability with the results by the simple estimation method. The simple estimation method gives very large residual deformation than others in the range of the cumulative probability higher than about 0.8. Therefore, the simple estimation method is not applicable to estimate the distribution of the residual deformations.

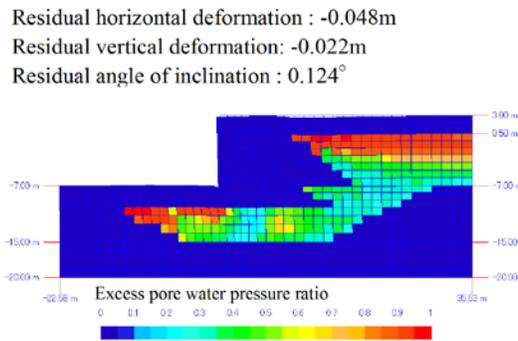


Fig. 6 – An example of the earthquake response analysis

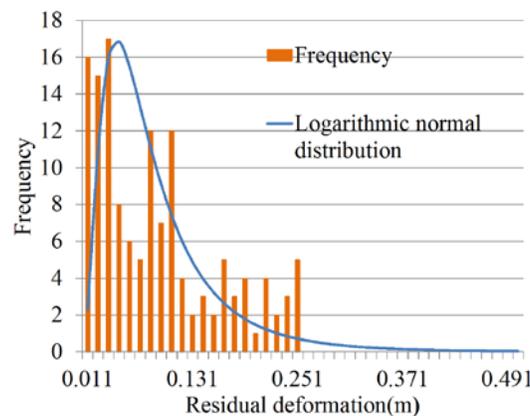


Fig. 7 – Histogram of the residual deformation distribution

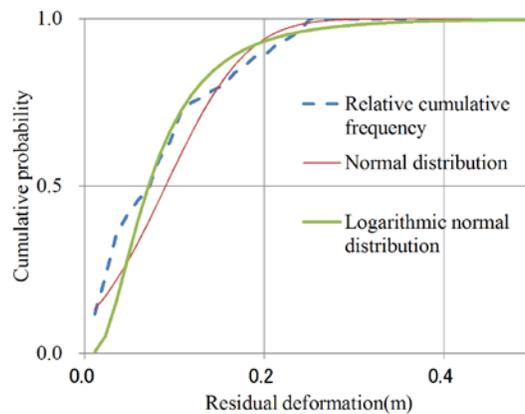


Fig. 8 – Comparison of cumulative probability of residual deformation

The reason of the estimation error by the simple estimation method is thought to be derived from the modeling method of the variation of the site amplification factor. Each site amplification factor in Fig.2 have large fluctuation for each frequency band and is larger than  $\lambda$  at a certain frequency band and smaller at another frequency band. On the other hand, modeled site amplification factors such as  $\lambda$  plus  $\zeta$  have larger site amplification factors than  $\lambda$  at every frequency. Therefore, we overestimated or underestimated the residual deformations by using earthquake ground motions with those site amplification factors.

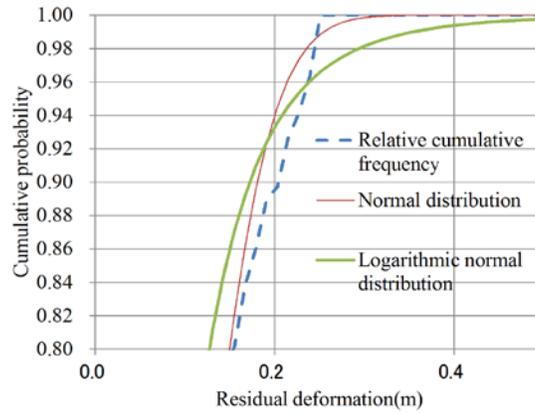


Fig. 9 – Comparison of cumulative probability of residual deformation (an enlarged view of Fig.8)

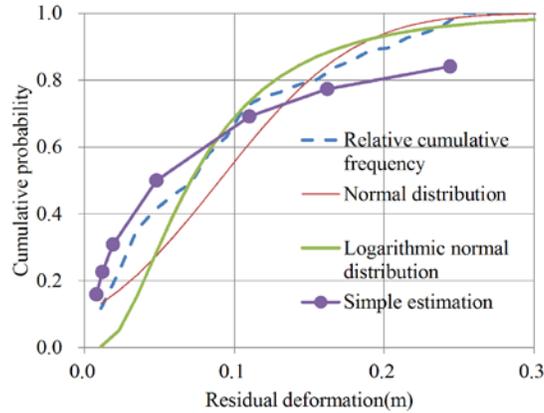


Fig. 10 – Comparison of cumulative probability of residual deformation with simple estimation method

The magnitude of Fourier amplitude spectrum in the frequency range from 0.5Hz to 5.0Hz mainly contributes to a residual deformation of a gravity type quay wall. So we consider the sum of a site amplification factor with the range from 0.5Hz to 5.0Hz as the index of magnitude of the site amplification factor. Fig.11 shows the relationship between the residual deformations and the sum of site amplification factors with range from 0.5Hz to 5.0Hz. The correlation between the sum of the site amplification factors and the residual deformations is high.

We apply the following equation as an index of the magnitude of the site amplification factor.

$$x' = \frac{amp - \lambda_{amp136}}{\zeta_{amp136}} \quad (4)$$

where  $amp$  is the sum of each site amplification factors with the range from 0.5Hz to 5.0Hz,  $\lambda_{amp136}$  and  $\zeta_{amp136}$  are the logarithmic mean and the logarithmic standard deviation of the sum of the site amplification factors of 136 earthquake ground motions respectively. The relationship between the index of the site amplification factors and the residual deformations are shown in Fig.12.

It is clear that the sum of the site amplification factors of 136 earthquake ground motions and the residual deformation have positive correlation as shown in Fig.11 and the sum of the site amplification factors of modeled earthquake ground motions used by the simple estimation method and the residual deformation have also positive correlation as shown in Fig.12. Therefore, there is a possibility that the probability distribution of 136 residual deformations is to be estimated by the sum of the site amplification factors as an index.

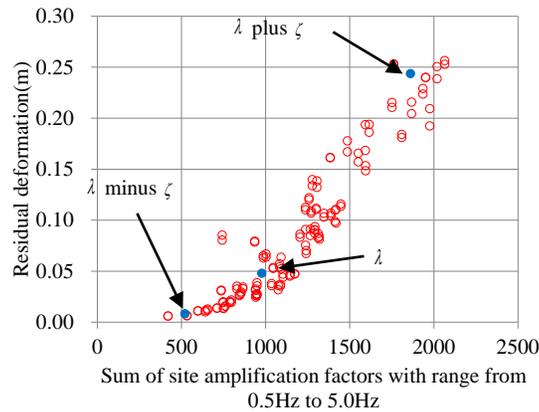


Fig. 11 – Relationship between the site amplitude factor and the residual deformation

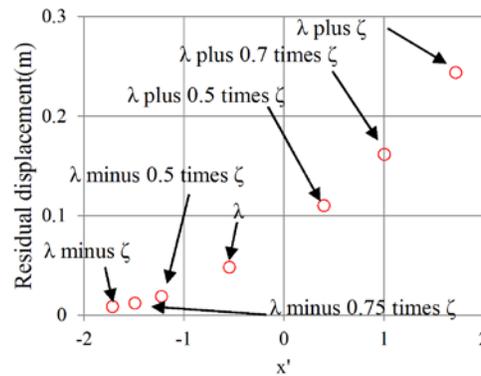


Fig. 12 – Relationship between the site amplitude factor and the residual deformation

Assuming the logarithmic normal distribution, the probability distribution of the residual deformations can be estimated by using the least squared approximation. The three cases which are  $\lambda$ ,  $\lambda$  plus and minus  $\zeta$  about the site amplification factors are extracted from Fig.12 and shown in Fig.13. The regression equation by the least squares approximation using the three points is also shown in Fig.13. According to Eq. (4), if  $amp$  is equal to  $\lambda_{amp136}$ ,  $x'$  is equal to 0, and if  $amp$  is equal to  $\lambda_{amp136}$  plus  $\zeta_{amp136}$ ,  $x'$  is equal to 1 in Fig.12. So the mean and the standard deviation of the residual deformations are calculated as 0.11m and 0.071m respectively by using the regression equation as shown in Fig.13. The means and the standard deviations of the residual deformations which are calculated by the same method with other three points ( $x$  is equal to 0.75 and 0.5) are shown in Table 2. The case which  $x$  is equal to 1.0 is the best to approximate the residual deformation among the three cases shown in Table 2. The approximation of the frequency distribution of the residual deformations is compared with the result of earthquake response analysis as shown in Fig.14, and the cumulative probability is shown in Fig.15 and Fig.16. Because the 3-point approximation gives the residual deformation on the conservative side than the logarithmic normal distribution as shown in Fig.15 and Fig.16, the 3-point approximation is thought to be more applicable to the practical design.

#### 4. Conclusions

We proposed the simple estimation method of probability distribution of residual deformations of gravity type quay walls considering variation of earthquake ground motions. 136 input earthquake ground motions were generated considering variations of site amplification factors and those of site phase characteristics. Residual deformations of a gravity type quay wall were calculated by the earthquake response analysis with the generated 136 earthquake ground motions.

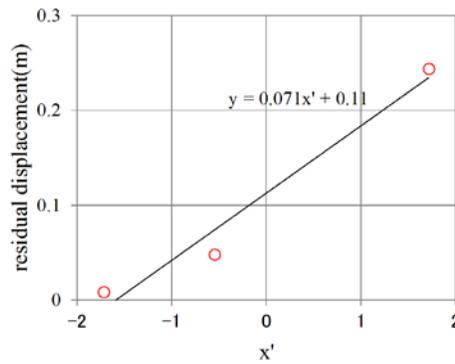


Fig. 13 – 3-point approximation of residual deformation with sum of site amplification factors as index

Table 2 – Estimation of residual deformation by 3-point approximation

Site amplification factors $\lambda$ plus or minus $x$ times $\zeta$	Residual deformation(m)	
	mean	standard deviation
0.50	0.085	0.057
0.75	0.095	0.061
1.00	0.113	0.071

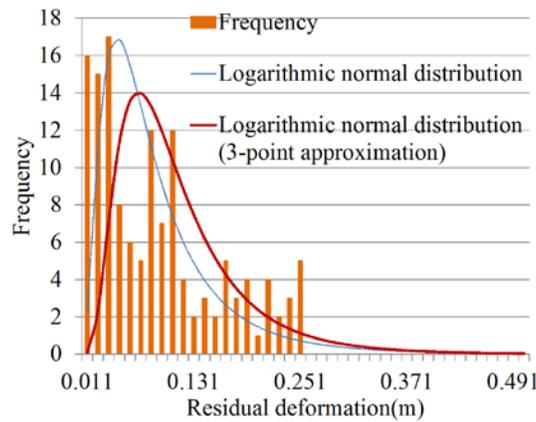


Fig. 14 – Frequency distribution of residual deformation (3-point approximation:  $\lambda$ ,  $\lambda$  plus and minus  $\zeta$ )

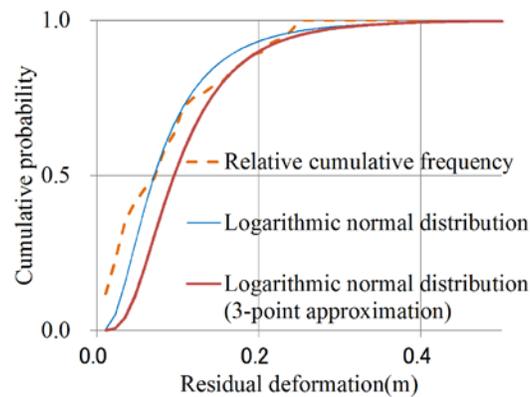


Fig. 15 – Comparison of cumulative distribution of residual deformation (3-point approximation:  $\lambda$ ,  $\lambda$  plus and minus  $\zeta$ )

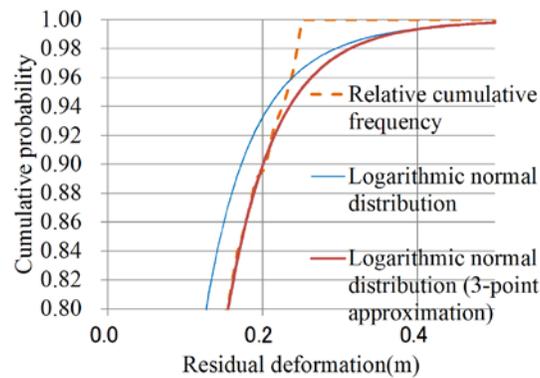


Fig. 16 – Comparison of cumulative distribution of residual deformation (an enlarged view of Fig.15)

We tried to estimate the cumulative probability of the logarithmic normal distribution by conducting earthquake response analysis only several times. Assuming the distribution of the site amplification factor as the logarithmic normal distribution, the earthquake response analysis were conducted for seven modeled ground motions in cases of  $\lambda$ ,  $\lambda$  plus  $\zeta$  and minus 1, 0.75, 0.5 times about the site amplification factors. We applied the sum of the site amplification factors with the range from 0.5Hz to 5.0Hz as an index of magnitude of the site amplification factors and examined the relationship between the index and the residual deformations. We next proposed a simple estimation method of the distribution of residual deformations by the least squares approximation using the results of three cases, i.e.,  $\lambda$ ,  $\lambda$  plus and minus  $\zeta$  of the site amplification factors.

As the result, it was shown that the distribution of the residual deformations can be estimated with the residual deformations by 3 earthquake response analyses.

As a future problem, it is necessary to verify the applicability of the proposed method to other sites. In addition, as we only considered as the variation in earthquake ground motions in this study, it is necessary to consider other the variations such as those in soil properties.

## 5. References

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