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SEISMIC VULNERABILITY FUNCTION FOR MASONRY BUILDINGS CONSIDERING THE VARIABILITY OF MATERIALS

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Abstract

This paper proposes a methodology and an equation to determine the vulnerability of masonry structures taking into account the variability of mechanical properties due to lack of supervision during its construction in terms of three important parameters: the type of mortar defined by its resistance, the thickness of joints between bricks and the concrete strength on confinement beams and columns. The proposed expression is based on the statistical treatment of results obtained from carrying out Incremental Dynamic Analysis (IDA) using a set of seismic records for common structures built in the country. With the proposed expression the estimation of expected damage on masonry buildings under seismic actions will be more accurate.

Keywords: Seismic vulnerability, Masonry structural behavior, Risk assessment, Incremental Dynamic Analysis

1. Introduction

Masonry is the most commonly used material to build dwellings in many underdeveloped countries to be economic and to have a good behavior under seismic actions. However, the lack of an adequate structural design and supervision in situ in many of these structures under the responsibility of unqualified personnel brings an important variability on the structural behavior. This makes that the estimation of the expected damage of a masonry structure during the occurrence of an earthquake has an important variation with respect to that reported for public institutions when and earthquake has already occurred. As part of a solution, several researches have been developed for several years, focused on understanding the structural behavior to estimate the damage that could occur under certain intensities of demand [1, 2, 3, and 4]. For example, Esteva [1] is one of the pioneers in estimating the damage of structures associated with the action of a seismic phenomenon. He proposes a set of curves that define the structural vulnerability for different types of systems.

Despite the foregoing and other number of papers that have been developed, not only in Mexico but around the world, there are no studies that take into account through a quantitative parameter the mechanical consequences that represent the variation of the strength and stiffness properties in an actual structure from those considered in its design.

As a first step in defining a solution to the above problem, this paper describes a proposed methodology employed to estimate the seismic vulnerability of masonry structures considering the change in mechanical properties of confined masonry walls, reflected in the decrease of strength design in walls and confinement elements. From this study, an equation to compute in a faster way the seismic vulnerability is proposed.

2. Proposed Methodology

The methodology proposed in this paper to take into account the variation in the properties masonry components to assess the seismic vulnerability follows the next steps: 1) it is necessary to have a structural design of a masonry building regardless of the stage of its construction process (design, construction or operation), from



which the structural behavior of analytical model of the building will be estimated; 2) hence, to have a better representation of actual behavior of the walls, information available from experimental tests is required to obtain the hysteretic parameters to be entered in the analytical masonry walls model; 3) subsequently, the compressive strength and diagonal stress design of the masonry walls is affected by the variation of the design properties of the materials, which are considered to be directly dependent on three main parameters: mortar joint thickness, type of mortar and the quality of concrete in confining elements. The variations in strength of the walls are characterized through a set of analytical models of masonry buildings; 4) for each of the considered analytical models, Incremental Dynamic Analysis (IDA) is performed considering different seismic records that define the seismic environment of the site where the structure is located; 5) from the IDA curves obtained, which characterize the structure is calculated and its vulnerability curve is built; 6) from this set of curves, an equation that provides the best approximation to the variation of the vulnerability curves due to strength uncertainty is defined by employing as base the equation proposed by Miranda [2]. In fig. 1 a graphic representation of the proposed methodology is shown.



Figure 1. Flow diagram of methodology used



For the implementation of the above methodology, the design of a masonry building of 3 levels composed of red brick proposed by Arias [5] is used, which has an area of 51.28 m² and a total height of 7.20 m, as shown in fig. 2a. The structural system is composed by five types of masonry walls: a under opening wall 1.05 m high and 1.12 m long axes (M1); a wall 2.4 m high and 0.74 m long axes (M2); a wall 2.4 m high and 2.93 m long axes; a wall 2.4 m high and 3.88 m long axes (M4); and a wall of dimensions 2.4 m high and 7.16 m long axes (M5), located as shown in the fig. 2b. Aspect ratios (L/H) are: M1 = 0.3, M2 = 0.5, = 1.2 M3, M4 = 1.6 and M5 = 3.0. The nominal dimensions of the red brick annealing are 6 x 12 x 24 cm (height, width and length) and the columns and beams are 12 x 12 cm and 12 x 23 cm, respectively. For both, the confinement steel is 1/8 inch, the steel cross-section is composed of four rods 3/16 inch and the design strength of the concrete is $f'c = 200 \text{ kg/cm}^2$.



Figure 2. Masonry building analyzed, a) Isometric y b) plan view (Arias, 2005)

2.2 Element modelling

In this article, a fibers model [6] is used to represent the behavior of columns, which reproduces the interaction between bidirectional flexure and axial loading through a set of uniaxial springs. The number of springs is directly dependent on the properties of materials, the section size and distribution of reinforcing steel. The idealized column is formed by a linear element and two sets of multiaxial springs at its ends. The behavior of the springs depends on stress-strain diagrams defined to each material.

The masonry walls was modelled with the "Shear element" of the program CANNY [6], which considers the effects of shear deformations and axial force in the plane. The area of a wall panel is delimited by four nodes located at the corners and the shear capacity and axial force are idealized by springs in the central part of the wall.

The parameters used for the "Shear element", was that given by Cruz [7], who tested seven confined masonry walls to study the effect of resistance to shear force due to change in the aspect ratio (L/H). Besides that, Cruz [7] also performed assaying piles and low walls to obtain the design strengths on compression and diagonal tension.

Four of the walls tested (ME1, ME4, ME6 and ME7) present similarities in terms of type and aspect ratios with the ones employed by Arias [5]. Therefore, it is considered that the results obtained by Cruz [7] reflect considerably approximate the behavior that may occur in the walls employed in the building proposed by Arias [5].

Fig. 3 shows the envelope of the analyzed walls behavior and the response of the same wall obtained from the analytical modeling using behavioral parameters.



Figure 3. Envelope of the tested walls ME1, ME4, ME6 y ME7 and the hysteretic curve used in the wall models.

2.3 Variation in material properties

The parameters that define the variation in design strength (PVS) of structural elements that conform the building are:

- Type of mortar
- Mortar joint thickness
- Quality concrete confinement of beams and columns

Type of mortar

Parral [8] performed an experimental study on piles and low walls, in order to know the differences obtained in the design compression strength (fm *) and design strength to diagonal compression (vm *) in masonry walls using as a variable the mortar types proposed in the Complementary Technical Norms for Design and Construction of Masonry Structures NTCM [7], mortars type I, II and III.

Based on his experimental study, Parral [8] obtains 31.78 kg/cm² of average compression strength, using a mortar type I, when using a mortar type II obtains a resistance of 32.16 kg/cm² and for a type III, 28.77 kg/cm². From the results, it is seen that the compressive strength of the piles and low walls do not have much variation, also those built with type II mortar are more resistant than those built with mortar type I or III, Parral [8] associates that variation in strength specimens to the strength of brick pieces, because of the handmade production of the specimens the same mechanical properties are not retained, therefore, a variation range of 27 to 36 kg/cm² was obtained from the total of assaying.

Parral [8] also obtained average design strengths to diagonal compression of 5.39, 4.60 and 4.01 kg/cm² using mortar type I, II and III respectively.



In order to obtain the factors that represent the strength variation due to mortar (SVM), the strengths obtained from the walls constructed with mortars type II and III are normalized with respect to the one obtained with mortar type I, since type I is recommended by NTCM [9] to join the pieces of masonry. Table 1 shows the normalized SVM obtained for compression strength and a diagonal tension low walls.

Mortar Type	Compression strength factor	Diagonal tension factor
Ι	1	1
II	1.05	0.85
III	0.94	0.75

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Mortar joint thickness

Similarly, Parral [8] calculates the variation in the design strength to compression and diagonal compression low wall and piles based on four joint thickness, 1.5, 2, 3 and 4 cm. The average strength to compression obtained are 29.74, 31.24, 30.66 and 31.96 kg/cm² respectively. From the results appears that, like the results of the evaluation of the compressive strength based on the type of mortar, strength average compression low walls with thicknesses of joint of 2 and 4 cm were higher than that obtained with a thickness of 1.5 cm, due to the above, Parral [8] also associates this variation to the strength of the masonry pieces.

Moreover, Parral [8] reaches design strengths to diagonal compression of 5.25, 4.62, 4.80 and 4.00 kg/cm² for each type of joint thickness, 1.5, 2, 3 and 4 cm respectively. From the results appears that, as joint thickness increases, strength to diagonal compression decreases.

Based on the recommendations indicated in NTCM [9], the thickness of optimal design for handcraft masonry is 1.5 cm, therefore, the normalization of the strength is made for strength low wall made with that joint thickness. Factors strengths variation based on the joint thickness (FVJ) to compression and diagonal stress, are presented in table 2.

Joint thickness, cm	Compression strengths factor	Diagonal tension factor
1.5	1	1
2	1.05	0.87
3	0.97	0.912
4	1.07	0.762

Table 2-FVJ compression and diagonal tension strengths

Quality concrete confinement in beams and columns

El-Dash and Ramadan [10] performed experimental studies to determine the strength of a series of concrete confined cylinders of 150 mm3 by using as the main variable the type of coarse aggregate: gravel, dolomite and basalt, in addition to this variation, authors consider two types of closed stirrups (soldier and lapped), as well as four separations stirrups (20, 40, 60 and 80 mm), leaving a total of 48 combinations.

To represent the variation of the concrete strength due to the three variables considered, a standard deviation is used and it is obtained by a group strengths that depend on each of the considered variables, such that, for each types of aggregate the standard deviations from basalt, dolomite and gravel are 7.16, 10.04 and 11.29 kg/cm² respectively; for closing soldier stirrup is 9.87 kg/cm² and for the lapped one is 14.30 kg/cm²; and for the



separation of stirrups deviations obtained are 10.63, 6.52, 6.78 and 4.13 kg/cm² for 20, 40, 60 and 80 mm respectively.

Based on the above, it is considered that the least resistance that may occur when using each of these variables is represented by the design strength minus one deviation, ie, considering a design strength of 250 kg/cm^2 , the most unfavorable strengths that may occur for different types of aggregates are 177, 148 and 135 kg/cm² for basalt, dolomite and gravel, respectively. Similarly, the most unfavorable strengths that may occur considering a type of closure are 149 and 104 kg/cm² for lapped and welded respectively. As the above variables, the most unfavorable strengths that may occur considering the separation of stirrups are 142, 184, 181 and 208 kg/cm² for 20, 40 60 and 80 mm respectively.

It is noteworthy that, due to the lacks of experimental data, it is considered that the standard deviations mentioned in the preceding paragraph are constant and can be applied to reduce any strengths value design employed.

In order to consider the worst cases that may occur by varying the type of aggregate, closing stirrup and stirrup separating, it is considered only the lowest value and the mean value obtained from the minimum strengths mentioned above, ie values of 104 and 177 kg/cm² respectively and the strength of the concrete one f'c = 250 kg/cm² is taken as the design. In fig. 4 the graphs of the stress-strain curves of the values that will define the concrete strength change are shown. The behavioral model used to define the compressive strength of concrete is the Hognestad model [11].



Figure 4. Considered resistant stress in beams and columns confinement

Once PVS values that will affect the mechanical properties of design masonry walls depending on the type of mortar (I, II and III), joint thickness (1.5, 2, 3 and 4 mm) and strength of concrete in beams and columns (104, 177 and 250 kg/cm²), all possible combinations that may occur are shown in table 3. Each of these combinations represents a different structural system generated, having a total of 36 structural systems.

		Combination											
Variable		1	2	3	4	5	6	7	8	9	10	11	12
f'c = 250 kg/cm ²	Type of mortar	Ι	Ι	Ι	Ι	Π	II	II	II	III	III	III	III
	Mortar joint thickness	1.5	2	3	4	1.5	2	3	4	1.5	2	3	4
$f'c = 177 \text{ kg/cm}^2$	Type of mortar	Ι	Ι	Ι	Ι	II	II	Π	Π	III	III	III	III
	Mortar joint thickness	1.5	2	3	4	1.5	2	3	4	1.5	2	3	4
$f'c = 104 \text{ kg/cm}^2$	Type of mortar	Ι	Ι	Ι	Ι	II	II	Π	Π	III	III	III	III
	Mortar joint thickness	1.5	2	3	4	1.5	2	3	4	1.5	2	3	4

Table 3 – 36 Structural combinations

2.4 Definition of seismic demand



The seismic demands employed in this study were a set of earthquake records from a single event obtained from The Mexican Base of Strong Earthquakes, MBSE [12]. Each of the records will have a simple filtering and baseline correction for maximum speed. The event used is the one in the state of Guerrero in 1998 registered at different stations in two orthogonal horizontal directions recorded at each station. The stations are: Coyuca, Cerro de Piedra, Las Mesas, El Paraiso, San Marcos, Teacalco, beams and Xaltianguis.

2.5 Analysis of the structure

To perform the analysis of the structure, Incremental Dynamic Analysis (IDA) Vamvatsikos and Cornell [13] was used. This type of analysis is used to estimate more accurately the behavior of a structure under seismic actions. This involves carrying out a set of Nonlinear Dynamic Analysis (NLDA) step by step, where for each analysis, seismic intensity of each record is increased. As result of that, an IDA curve, maximum response curve vs intensity of the earthquake, is obtained. To perform an IDA analysis it is necessary to define three parameters: a scale factor (SF), a measure of intensity of the demand (IM) and a parameter of response (DM).

SF is a positive scalar value for which the earthquake record is scaled, thus, giving, a "new" accelerogram of greater or lesser intensity than the original. A SF value of 1 denotes the natural scale of the earthquake record

The IM characterizes the intensity of the seismic demand; in this paper the intensity values used in the IDA applications were 0.1g, 0.5g, 0.9g, 1g, 1.2g, 1.7g and 1.5g 2g, where g = 9.805 m / s2.

The DM is a positive value that represents the response of a structural model due to the action of a seismic action. In this work the maximum interstory drift index is chosen as the parameter of structural response, as it correlates well with structural and non-structural damage [2].

2.6 IDA curves

The curve that represents the expected structural behavior is given by the geometric mean of all structural responses obtained for a specific acceleration. In fig. 5 a set of IDA curves obtained for direction X and Y from the model with the design mechanical properties is shown.



Figure 5. IDA Curves obtained for X and Y

From the previous figure, may be observed that in the X direction, the level of drift is significant for intensities lower to 1g, otherwise in Y direction. This difference is associated to the lower length to its height of the walls that are located in the X direction, therefore, the area to resist shear force is also smaller. Furthermore, due to its area/height ratio, those walls act more to flexural than shear. Otherwise, in Y direction the ME4 and ME7 walls



are located, and those walls are longer than the X direction walls, therefore, have a greater ability to withstand shear forces.

3 Vulnerability functions

To represent mathematically structural vulnerability, in this article the expression proposed by Miranda [2] is used, which proposes a way to analytically estimate the vulnerability functions based on the maximum interstory drift index that generates the structure during the occurrence of a seismic phenomenon. Such expression is the following:

$$E(\beta \mid \gamma_i) = 1 - \exp\left[\ln 0.5^{\left(\frac{\gamma_i}{\gamma_{50}}\right)^{\rho}}\right]$$
(1)

where $E(\beta|\gamma_i)$ is the expected value of the β damage due to a level of structural response γ_i generated by a seismic intensity; γ_{50} is the intensity of structural behavior which represents 50% of damage (for this article, interstory drift index); ρ is a parameter that depends on the type of structural arrangement, materials and construction details that determine the capacity of deformation.

In this article, the interstory drift index that represents 50% of damage of the structure is obtained from the IDA curves. From the curves results, it was observed that the most significant differences presented were for the strength of concrete parameter, so that an interstory drift index average for each of the three types of concrete strengths were estimated; fc = 104, 177 and 250 kg/cm². The γ_{50} values to be used to obtain the vulnerability curves for each of the structural models are shown in Table 4.

Table 4 – Interstory drift index that represents 50% of damage

	f'c = 104 kg/cm2	f'c = 177 kg/cm2	fc = 250 kg/cm2
Y50	0.002	0.0022	0.003

Based on the values presented in table 4 and considering a concrete strength of fc = 250 kg/cm2, the vulnerability curve is obtained from the model that represents to the structure with the mechanical properties of design (VCD), and it is shown in fig. 6. It may observed that after an intensity of 0.5g, a considerable increase of structural damage occurs, this is due to the fragile failure that occurs in the masonry walls after reaching the cracking drift.





Figure 6. IDA Vulnerability curve from design model

Similarly, in fig. 7 it is shown a comparison between the vulnerability curves that represent each of the 36 combinations of structural strength of the system studied with the VCD. It is noted that some of the curves have an estimate of damage minor than the structural design model in periods up to 1.5 s, this trend is associated to combinations of values SVM (Table 1) and FVJ (Table 2) obtained from the tests performed by Parral (2007), where some strengths presented are upper to the design and consequently, improve the strength of the structure.



Figure 7. Set of vulnerability curves obtained

3.1 Proposed vulnerability function that considers the variation in strength

To determine the vulnerability of any masonry structure due to the change in mechanical properties considered in the design in a straightforward manner, ec. 2 is proposed, in which vulnerability is in terms of a value α f that relate, the variation (in percentage) of estimated damage between vulnerability curves. This paper considers three levels of variation in the masonry and the confining elements: "High", "Medium" and "Low".

$$E\left(\beta_{\text{mod}}|I\right) = E\left(\beta_{\text{design}}|I\right)(1+\alpha_f)$$
(1)

where $E(\beta_{mod}|I)$ is the expected damage of a structure associated with an intensity of demand I, $E(\beta_{design}|I)$ is the expected damage of a structure with design features associated with an intensity of demand I, affected by a parameter that depends on the level of variation α_f .

Obtaining values α_f

In order to estimate the value of α_f for different intensity levels used in this work, it is assigned an expression to the curve that defines each of levels of variation considered. To illustrate this, in fig. 8a it is done the comparison between the VCD to an intensity of demand for 1g with a curve representing a "high" strength variation of any element; in fig. 8b the point (gray) representing the difference of the above comparison, as well as a black point that is obtained of propose an expression that fits comparison values shown.



Figure 8. Example of a proposal for values α_f , a) Comparison between vulnerability curves for Sa=1g and b) Curve that represents the difference between the two curves of vulnerability for Sa=1g

It is considered that the vulnerability curve will represent a variation "Low" in the confining elements if it is obtained from the average of the curves based on the design strength of concrete, in this case $fc = 250 \text{ kg/cm}^2$. Likewise, the curve will represent a variation "Medium" if it is used the vulnerability curve obtained from the average of the curves considering a concrete strength of $fc = 177 \text{ kg/cm}^2$. Just as the previous two curves, the curve representing a variation "High" is obtained from the average of the curves where the strength used in concrete is f 'c = 104 kg/cm².

To consider the variation in the material of the wall, the curve of vulnerability that represents a variation "Low" is derived from the average of the four curves formed by mortar type I and II in combination with thicknesses of 1.5 and 2 cm and are based on a strength of concrete $fc = 250 \text{ kg/cm}^2$; to consider a variation of "High", the average curves are based on a concrete strength of 104 kg fc = / cm2 and which are formed by a type III mortar and board thicknesses 2, 3 and 4 cm, and a type II mortar and a joint thickness of 4 cm; also, to consider a "Medium" variation, the average of those whose behavior is between the previous eight curves and are based on a concrete strength of $fc = 177 \text{ kg/cm}^2$ in this case, it is made up of curves that have a type I and mortar joint thicknesses 3 and 4 cm, a type II mortar with a joint thickness of 3 cm and a type III mortar with joint thickness of 1.5 cm.

Table 5 shows the proposed equations and different constants values to determine α_f , with which may affect vulnerability curves of masonry structures obtained of considering the design characteristics to reflect the levels of strength variation.

		Masonry walls					
	Variation	Low	Medium	High			
		$\alpha_f = a + b\cos(cx + d)$	$\alpha_f = a + b\cos(cx + d)$	$\alpha_f = a + b\cos(cx + d)$			
	Low	a= -0.039 b= 0.024	a= 0.003 b= 0.047	a= 0.036 b= 0.045			
Beam and		c= 6.048 d= -0.730	c= 4.245 d= -4.859	c= 5.061 d= -5.79			
column		$\alpha_f = ab^{\left(\frac{1}{x}\right)}x^c$	$\alpha_f = ab^{\left(\frac{1}{x}\right)}x^c$	$\alpha_f = ab^{\left(\frac{1}{x}\right)}x^c$			
	Medium	a= 0.098 b= 1.344	a= 0.157 b= 1.212	a= 0.183 b= 1.233			
		c= -1.695	c= -1.578	c= -1.398			

Table 5 – Equations to determine α_f factor



It is noteworthy that, based on the results obtained in this article, it is made the consideration that the structural damage occurs only after an intensity of 0.2g, therefore expressions are valid for an intensity of demand higher than this.

In fig. 9 are shown in summary, the set of vulnerability curves that represent the variation in resistance, "Low", "Medium" and "High" that may occur in the already constructed elements that conform the wall of confined masonry (beams and columns of confinement and masonry walls) and therefore, of the overall structure.



Figure 9. Curves of vulnerability for the three levels of variation in beams and columns of confinement and masonry walls

4 Conclusions

In this paper a methodology is proposed to modify vulnerability curves that considers the design features of confined masonry structure based on three levels of variation; "Low", "Medium" and "High". This was done by including a parameter α_f which is in function of the variation in resistance of the masonry walls and the confinement elements. From the results it is observed that the estimate of a masonry structure during the occurrence of an earthquake, physical damage may be almost 30% higher than initially estimated, due to a "high" variation of the elements that conform the wall masonry. On the other hand, if there is a variation "Low" in the elements of the confined masonry wall, the physical damage estimated may be up to 5% lower than initially estimated. Based on the obtained results an expression is obtained.

With this article, it is expected to establish the basis to include, in an explicit way, the variation in the estimate structural damage due to earthquakes into account the variation in strength of materials already constructed compared with those established in the design.



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