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ANALYTICAL CONTENT VULNERABILITY ASSESSMENT METHODOLOGY FOR EARTHQUAKE CATASTROPHE MODELS

K. Rokneddin⁽¹⁾, A. Safiey⁽²⁾, M. Javanbarg⁽³⁾, V. Manghnani⁽⁴⁾, W. Pang⁽⁵⁾

- (1) Senior Research Scientist, Cat Management & Analytics Center of Excellence, AIG, keivan.rokneddin@aig.com
- (2) Graduate Research Assistant, Glenn Department of Civil Engineering, Clemson University, asafiey@g.clemson.edu
- (3) Manager, Cat Management & Analytics Center of Excellence, AIG and Adjunct Faculty, Glenn Department of Civil Engineering, Clemson University, mohammad.javanbarg@aig.com
- ⁽⁴⁾ Head of Analytics and Exposure, Cat Management & Analytics Center of Excellence, AIG, vijay.manghnani@aig.com
- (5) Associate Professor, Glenn Department of Civil Engineering, Clemson University, wpang@clemson.edu

Abstract

The scarcity of detailed claims data for building contents (Coverage C) from historical earthquake events poses a significant challenge for property insurance catastrophe models to reliably estimate the losses associated to building contents. To develop content vulnerability functions empirically, one would need to have access to data from a multitude of historical events; however, loss disaggregation by coverage is rarely reported even when claims data become available from recent significant events such as Maule (2010) and Tohoku (2011). While damage to the building structure (Coverage A) can be estimated analytically using simulation-based fragility functions to amend sparse historical observations, the adoption of analytical approaches for other coverages is limited in the current generation of catastrophe models. In the absence of analytical methods, content loss estimation often relies on a combination of expert opinion and abstract reasoning on top of precious-little available data which is often limited to residential properties. In this paper, the authors employ FEMA P-58's component-based methodology to develop a framework for simulation-based derivation of content vulnerability functions. Following a review of published literature and the types of content components in FEMA P-58's PACT library, the authors present the simulation-driven vulnerability function for a four-story office building in Los Angeles, and compare the results against respective functions for office buildings from commercial models. Moreover, this paper discusses the need for new content component types in offices and professional service occupancy. Through this study, the authors demonstrate the possibility of improving content loss estimates in catastrophe models by adopting approaches similar to those involved in the development of structural vulnerability functions.

Keywords: Catastrophe Models, Insurance, Contents Vulnerability, Performance-Based Earthquake Engineering, Component-Based, FEMA P-58



1. Introduction

Property and Casualty (PC) insurers generally rely on a suite of catastrophe models (Cat models) to stochastically evaluate the risk to their portfolio from known hazards such as earthquake and windstorms. Since their adoption in the insurance industry at the end of 1980s, Cat models have significantly enhanced the practice of managing the risk from natural hazards to the insurers' exposure [1]. For the earthquake hazard, the statistical framework in Cat models consists of three modules: 1) the hazard module to evaluate the ground motion intensity from a set of simulated seismic events, 2) the vulnerability module to simulate the normalized loss (severity) for a given level of ground motion intensity, and 3) the financial module to incorporate the policy terms and determine the retained and ceded risks. A similar framework is also employed in HAZUS [2].

Significant progress was made in the field of performance-based earthquake engineering in the early 2000s to account for uncertainties in hazard and severity evaluations [3–6]. Nowadays, the literature includes enhanced consideration of uncertainties for earthquake occurrence rates [7], ground motion prediction equations [8,9], and structural response [10–13]; and consideration of correlated building responses in a portfolio of buildings [14,15]. However, the commercially available Cat models leave out significant amounts of uncertainties in hazard and vulnerability modules and make simplifying assumptions to achieve more favorable computation times. A drawback of commercial models for PC insurers is in their transparency which makes it difficult for users to understand the modeling assumptions and make adjustments. A few open source Cat modeling platforms, including the GEM Foundation's OpenQuake [16], have emerged in recent years to address this issue by publishing all modeling details, including the considered sources of uncertainties. These platforms enable the users to make changes to any of the three primary modules and still use the platform's computational engine to evaluate the updated losses.

While open source platforms have increased transparency, the insurance companies still face the challenge of replacing the default values and assumptions, particularly for hazard and vulnerability modules, with proper alternatives to form their own view of the risk. In this paper, the authors focus on one such challenge which emerges in the vulnerability module, i.e., the estimation of normalized losses given the value of spectral acceleration at the base of a structure. The normalized loss is defined as the incurred loss to a building normalized by its total replacement value, and is a random variable which takes values in [0, 1]. For any given seismic event, the damage ratio depends on the geometric, material, and structural characteristics of the building as well as the level of intensity measure (such as the spectral acceleration) which is caused by the event at the building's location. From a structural engineering point of view, the performance of a structure can be estimated as a function of the input ground motion using either numerical simulations (e.g., FEM analysis) or physical model testing (e.g., shake table tests). In addition, the Cat model developers generally have access to a dataset of damage indices gathered from post-event surveying of damaged buildings in historical earthquakes. Using damage index or claims data, the model developers calibrate the results from the numerical simulations and present them in the form of vulnerability (damage) functions, establishing a relationship between the mean damage ratio and the intensity measure. However, structural analysis and historical damage data are often only useful to evaluate the damage to the structural system, and cannot directly assess the losses caused by damage to non-structural systems (e.g., elevators, roofs) or building contents (e.g., computers, furniture), nor can be used to estimate restoration times. Those losses can be as significant as losses to the building itself (Coverage A) if not larger for many events [17]. Since the Cat model developers have very little detailed historical loss information about content losses (Coverage C), their content loss assessments have traditionally been subjective. Model developers' assumptions for content damage functions have ranged a set percentage of building damage ratios for every scenario to heuristic relationships combining the building damage ratio and hazard intensity level.

The framework for component-based loss assessment methods, published by FEMA in P-58 [18], presents an opportunity to improve the content loss assessment methodology. FEMA P-58 presents a platform in which all components in the building (structural, non-structural, and content) are specifically modeled with their customized fragility and consequence functions [18]. Therefore, and for particular building archetypes where the content components either are known or can be reasonably assumed, P-58 provides a path for analytical development of content vulnerability functions. The authors have used the P-58 framework to analytically derive



the content vulnerability function for one of the most frequent commercial building archetype for commercial insurers: the office and technical services occupancy. Section 2 reviews the literature on content damage estimation methods, followed by the description of the component-based methodology in Section 3. Section 4 presents the details of the case study for which the content vulnerability function and comparisons against those in commercial packages are provided in Section 5. Finally, the authors discuss the need for new damage states to be defined for content components in FEMA P-58's library.

2. Content Fragility and Vulnerability in the Literature

Damage to the content objects in commercial and industrial environments can be widespread, as demonstrated in post-event surveys of the Christchurch earthquake in 2010 [19]. Commercial occupancies such as office buildings incurred losses associated with overturning shelves, broken sprinkler pipes, and damaged furniture, among others. Industrial facilities also experienced major damage to heavy machinery and equipment. Two photos of documented damage are shown in Fig. 1.



Fig. 1. Examples of damage to building contents: a) commercial environment (NIWA [20]), and b) displacement of heavy equipment in an industrial facility [19].

Non-structural or content components have been often modeled as rigid bodies susceptible to rocking or overturning [21,22]. Garcia and Soong [23,24] proposed the sliding mode of failure, and modeled the content components as either unrestrained or anchored rigid bodies. They developed fragility functions for sliding bodies against horizontal peak base acceleration. Moreover, researchers who modeled content objects as rigid bodies classified the failure modes into displacement-sensitive and acceleration-sensitive. An object may be prone to failure due to displacement if there is another object nearby which can have a negative impact on the former on contact. An acceleration-sensitive mode of failure occurs when collision between an object and its environment depends on the level of acceleration. In order to improve the damage estimates to content and non-structural components, Jaimes et al. [25] developed a relationship between peak ground acceleration and peak ground velocity. Clearly, combinations of these failure modes may exist in a given environment and accordingly, developing a comprehensive model is a challenge without having extensive knowledge on the types of objects and their environment.

Other researchers continued expanded the sliding block model and applied it to other types of contents (e.g., lab equipment [26]). Reinoso et al. [17,22] expanded the rocking body model of content objects to develop fragility functions for a variety of shelf objects in a residential setting. Although rocking and sliding body models can simulate the behavior of many shelf objects, they do not generally extend to objects such as furniture, fine arts, or valuable papers which have received less attention in the literature.



GEM Foundation has also investigated the content loss estimates for its OpenQuake platform, and considered more component types [27]. We have adopted GEM's fragility functions for this study and made minor adjustments to their consequence functions, as explained in Section 4. All considered content objects in GEM's document are sensitive to peak floor acceleration.

3. Methodology

We propose a framework, based on P-58's methodology, to develop content and building vulnerability functions. This framework is developed for use with non-linear time-history analysis in the form of IDA [5], but can modified to accommodate P-58's simplified method based on capacity spectrum analysis or other similar methods. Similar P-58 based frameworks have been proposed by other researchers [28], albeit using different numerical methods. This framework requires fragility and consequence functions for every considered component in the building, whether obtained from P-58's PACT companion library or the literature. The PACT library includes damage states, fragilities, and consequences for the most generic types of components in supported occupancies. However, user input is often required for non-structural and content components, a limitation which restricts straightforward application.

The framework of component-based loss assessments is presented in Fig. 2. Our methodology uses incremental dynamic analyses (IDAs) for non-linear time-history simulation of two dimensional models of buildings. Upon completion of the numerical simulations, the framework uses realizations of the demand in terms of an engineering demand parameter (e.g., peak floor acceleration or residual displacement) and examines two building-level damage states, collapse and reparability. We assume total loss for the building structure (Coverage A) in case the building response exceeds either of those damage states. If the building survives, damage to each structural, non-structural, and content component is estimated using the component's fragility and consequence functions. For content loss estimation, the framework does not check against building reparability, as it does not induce damage to content components.

$$E[L_i \mid IM = im] = \sum_k E[L_i \mid D_i = d_i^k, IM = im] \cdot P[D_i = d_i^k \mid IM = im]$$
(1)

where L_i is loss to Component i and d_i^k is the damage state k defined for Component i. The first term on the right hand side of the equation shows the mean value of the consequence function for a given damage state and intensity while the second term denotes the fragility function. The mean value of the total loss from all components is then simply evaluated by adding all expected component losses as in Eq. (2).

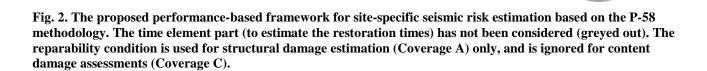
$$E[L | IM = im] = \sum_{i=1}^{N} E[L_i | IM = im]$$
(2)

Finally, the mean damage ratio (MDR) per intensity level is evaluated from Eq. (2) in the form of Eq. (3).

$$MDR = E[DR \mid IM = im] = \frac{E[L \mid IM = im]}{TRV}$$
(3)

Repair Time = Replacement Time Condition Tag=Unsafe

Finish



Repair Time = Replacement

Condition Tag = Unsafe

Calculate Non-collapse Casualties

4. Case Study

Calculate Repair Time Determine Condition Tag

We consider a four-story office building in Los Angeles, California to demonstrate the methodology and develop content vulnerability functions. The building has a Reinforced Concrete distributed moment resisting frame and has a contemporary seismic design. See [29] for structural details.

Given the particular occupancy, one may find the following major types of content components: office furniture and fixtures, EDP/computer equipment, server racks, hanging artwork, valuable papers, and kitchen items. Whether components are secured or anchored in an office environment depends on the details of the business continuity management plans for the resilience of work environment, and can vary significantly across different environments. Regardless of the state of component anchorage, FEMA's accompanying PACT library lacks the details for damage states and/or the consequence functions for the majority of these items. Therefore, the authors adopted the values from a case study published by Porter et al. [27] which investigated the content types in the business school building of the Colorado State University. Table 1 demonstrates the types of objects considered in this study as well as the fragility and consequence functions. Porter et al. presented the fragility functions against the horizontal peak floor acceleration as the intensity measure. We adopted the same type of objects in our study since they resemble the typical items found in an office environment.

We estimated the total value of content objects to be close to 800,000 USD for this particular office building in Los Angeles. For the consequence functions, we use a normal distribution with the mean and coefficient of variation values shown in Table 1.

The peak floor accelerations for this study are estimated following an IDA analysis on a two dimensional model of the building using the 22 ground motion records from FEMA P-695 [30]. The considered ground motions are recorded far-field time-histories with a greater than 10km distance from the fault, and do not include



the vertical component of the ground motion. The non-linear time-history analyses were performed in OpenSEES [31] on Clemson's distributed computer cluster (Palmetto).

Table 1. Fragility and Consequence functions for the considered content components (adopted from Porter et al. [27]). The fragility functions are based on horizontal peak floor accelerations.

Objects	Fragility Function		Mean Value	C.O.V.
	Median	β	Mean value	C.O. v.
Computers	0.4	0.5	316,700	0.2
Furniture	9.9	0.5	307,400	0.2
Bookshelves	0.25	0.5	166,500	0.2
Hanging artwork	0.25	0.5	8,400	0.2
Fragile items	0.25	0.5	1,000	0.2
SUM			800,000	

5. Results and Discussions

Fig. 3 presents the developed component-based content vulnerability functions in CDF form (probability of non-exceedance $P[DR < l \mid IM = im]$) for ten spectral acceleration levels (0.1g - 1.0g). The demonstrated spectral accelerations are at the structure's natural period of vibration of 0.94 seconds. For the assumed acceleration-based failure damage states (Table 1), the damage ratio does not exceed 0.5 for spectral accelerations up to 1g. Based on Table 1 values, furniture objects are almost invulnerable to horizontal floor accelerations. In reality, however, furniture and other content components are susceptible to other modes of failure, such as falling heavy or sharp objects or water from sprinkler pipe leakage. Therefore, the CDFs shown in Fig. 2 are likely to be a lower bound for the actual loss ratios, particularly given the high relative value of furniture in Table 1.

Fig. 4 presents the mean damage ratio (mean of the distributions from Fig. 3) for each level of spectral acceleration, as in Eq. (3). Also presented are the mean vulnerability functions from two commercial Cat models. Since the assumed natural period of vibrations for four-story RC office buildings in Los Angeles is different in the Cat models, we have transformed all spectral acceleration periods to 0.5 seconds using USGS hazard curves at the site of the case study office building (the soil type is determined to be B/C at the site). Even though the presented component-based curve represents lower-bound estimates of MDR as described above, its predicted MDR values exceed those from the Cat models for all spectral acceleration levels. It is important to note that the demonstrated Cat model values do not necessarily represent the *pure* vulnerability curves developed by their respective model developers. The Cat model values in Fig. 4 demonstrate the average of model-produced content loss values against spectral acceleration which is influenced by integration of other sources of uncertainties. Moreover, this comparison is limited by the fact that the type of content components cannot be defined in commercial Cat models the same way they are defined in Table 1 to develop the component-based vulnerability function.



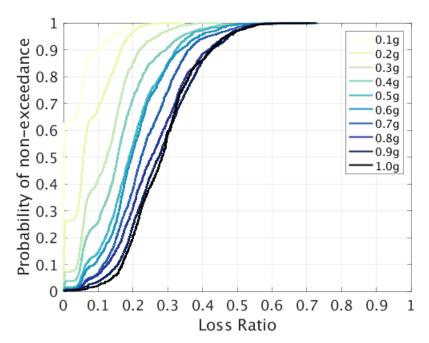


Fig. 3. Component-based driven probability distributions of Content Loss Ratios for a few levels of Spectral Accelerations at building's natural period of vibration (0.94sec). The total content replacement value for this four-story RC space moment resisting frame in Los Angeles, CA, is assumed to be 800,000 US Dollars.

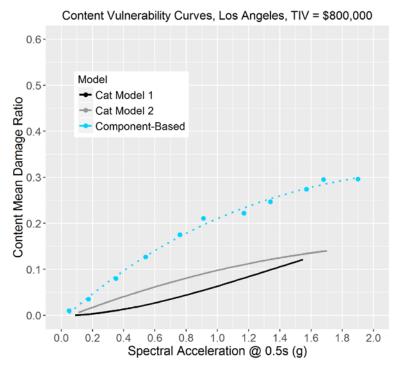


Fig. 4. Comparing the mean content damage ratios against Spectral Accelerations at 0.5 seconds among two commercial Cat models and the component-based approach. The Spectral accelerations associated with the component-based curve have been adjusted from 0.94 seconds to 0.5 seconds using the USGS hazard curves at the location of this building for B/C soil conditions. TIV stands for Total Insured Value and is assumed to be equal to the total replacement value in the development of these curves.



Although these limitations make a direct comparison difficult, they highlight the opportunity that a component-based framework presents to the PC insurers. Using this framework, the insurers can perform site-customized risk assessments in a way which is not feasible through commercial Cat models which use generic vulnerability functions. For example, given the same amount of total replacement value, a different combination of components in Table 1 brings about new vulnerability functions which are customized to the specific situation in hand. While the component-based framework can take advantage of the type and value of content components in a building for site-specific estimations, the users are unable change the output of Cat models. For important and custom-designed buildings, this advantage can be essential to better price the risk.

Developing enhanced fragility functions through considering more damage states and failure modes for the content components directly influences the generated vulnerability functions. Studying previous events reveals that the absence of certain fragility and consequence functions for very common damage states limits the scope of component-based analyses. For example, water damage to carpet and furniture comprise a significant portion of total damage to building components [19]. However, PACT and OpenQuake currently lack the ability to consider pipe breakage and leakage damage either directly or indirectly. At larger spectral accelerations, damage to content components is no longer governed by direct floor acceleration or displacement, but rather by falling non-structural components and other content objects.

6. Conclusions

Evaluation of losses to building contents poses a challenge for property insurance industry as there are very limited historical data available for development of vulnerability functions. The component-based framework presented in FEMA P-58 presents an opportunity to enhance heuristic content vulnerability functions thorough specific modeling of content components. This paper presents one such framework to develop content vulnerability functions for stochastic loss assessments by performing incremental dynamic analysis in OpenSEES. We presented a case study in which the vulnerability functions were developed for a four-story, reinforced concrete, office building in Los Angeles, California. While direct comparisons with the vulnerability functions from commercial catastrophe models are difficult for lack of access to the models' true vulnerability functions, the authors compared the component-based function against catastrophe models output. Vulnerability functions created through our proposed framework can be customized if specific information is available on the type and combination of content components in a building. This customization presents an advantage over commercial models which can only use their default, generic vulnerability functions. To improve componentbased content vulnerability functions, new fragility functions are required to consider damage modes such as water leakage and falling objects, particularly at higher hazard intensity levels. In the meantime, the framework can help insurers form their view of risk and enhance site-specific risk assessments through custom-made vulnerability functions.

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